DUALITY AND REGGE CUTS IN INELASTIC 2-BODY PROCESSES

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1. INTRODUCTION

The purpose of this paper is to review some of the recent considerations in the phenomenology of the Regge cuts. This review will neither be complete nor completely unbiased. We shall almost exclusively consider the inelastic 2-body processes and we shall spend most of our time examining the phenomenological consequences of introducing cuts, when the duality between Regge poles and direct channel resonances is postulated[(1)]. In the framework of the pure Regge pole model, this duality leads to a considerably simplified description of the high energy scattering phenomena, providing an elegant justification for the exchange degeneracy constraints[(*)]. It is tempting to keep these constraints even when the Regge cuts are introduced into the game[(**)]. This attitude is partly motivated by aesthetic reasons, but also by the hope (largely justified, as will be seen) that it will be possible to preserve in a more complete theory the progress of understanding of high energy phenomena, which we owe to the duality concept. Finally I should mention that this talk addresses mostly to experimentalists and that often I shall sacrify rigour in favour of simplicity.

2. WHY DOES ONE NEED CUTS ?

Several purely theoretical arguments (study of Feynman diagrams and of the unitarity corrections to the Regge pole exchange amplitudes, the necessity of "shielding" the Gribov-Pomeranchuk fixed singularities etc.[(3)]) indicate that Regge cuts are needed in the theory of strong interactions.

(*) This has been reviewed at the 4th Moriond Meeting by P. Sonderegger.

(**) Notice that our approach will essentially be that of the Argonne group[(2)].

Unfortunately, until now, these considerations have yielded no really reliable predictions for the strength of the discontinuity across the cut. Several experimental facts suggest, however, that cut effects are quite sizable. The strongest experimental evidence for the importance of the cut effects comes from the observed features of certain inelastic 2-body reactions :

(i) Analyticity and factorization constraints relate among them the forward behavior of the differential cross sections for different reactions if a pure Regge pole model is adopted[4]. These predictions are often in disagreement with the experimental data. For example, a conspiring pion is needed to get the sharp forward peak of the np → pn differential cross section. However, a conspiring pion produces a forward dip in $\pi N \to \rho \Delta$; it is a peak that is observed[5].

(ii) Polarization in pion charge-exchange seems to decrease with energy less rapidly than expected if it resulted from the interference between the leading Regge pole (ρ) and a secondary pole.

I mentioned the most drastic and best-known facts. I do not mean that they cannot be explained with Regge poles alone. It appears, however, that such an explanation would be rather artificial and not economical.

The elastic scattering also presents certain features, like the cross-over effect(*), whose description requires unpleasant ad hoc assumptions in the framework of pure Regge polology and which become relatively easily understandable once the cuts are introduced[6,7].

Of course, since a fundamental theory of the Regge cuts is lacking, we are bounded to use phenomenological models which also rest upon rather ad hoc assumptions. The advantage of these models is to provide a unified description of apparently distinct phenomena.

(*) The differential cross sections for AB → AB and $\bar{A}B \to \bar{A}B$ cross each other at a small value of $|t|$.

3. SOME GENERAL FEATURES OF THE LEADING CUTS IN THE PHENOMENOLOGICAL MODELS

Consider for definiteness(*) the eikonal model and for the sake of simplicity let us neglect spins. We write the scattering amplitude for the transition i → j in the form :

$$f_{ji}(s, \cos\theta) = i\frac{\sqrt{s}}{k}\sum_{\ell}(\ell + \tfrac{1}{2})\, P_{\ell}(\cos\theta)(I - e^{i\chi})_{ji} \tag{3.1}$$

where $\chi(\ell)$ is the "eikonal matrix"(8). Following Arnold(2) we postulate

$$\chi_{ji}(\ell) = \frac{k}{\sqrt{s}}\int_{-1}^{1} d\cos\theta\, P_{\ell}(\cos\theta)\, f^{B}_{ji}(s, \cos\theta) \tag{3.2}$$

where f^{B}_{ji} is the "Born" amplitude of the model, in occurrence an amplitude for the transition i → j parametrized as in a simple Regge pole exchange model. Eq. (3.2) constitutes the essential dynamical postulate and replaces the integral expression, well known in non-relativistic quantum mechanics, relating the phase shift to the potential in the eikonal approximation.

It is easy to see how the Regge cuts emerge. Let $\chi^{R}_{ji}(\ell)$ be the contribution to $\chi_{ji}(\ell)$ from the Regge pole R. Assuming linear trajectories and smooth residues one has

$$R_1 \otimes R_2 \otimes \ldots R_n \cong -i\frac{\sqrt{s}}{k}\sum_{\ell}(\ell + \tfrac{1}{2})\, P_{\ell}(\cos\theta)\, [\prod_{k=1}^{n}(i\,\chi^{R_n})]_{ji} \underset{s\to\infty}{\sim} \frac{s^{\alpha_c(t)}}{(\log s)^{n-1}} \tag{3.3}$$

with

$$\alpha_c(t) = \sum_{k=1}^{n}\alpha_k(0) - n + 1 + t/\,[\sum_{k=1}^{n} 1/\alpha'_k\,] \tag{3.4}$$

($\alpha_n(0)$ and α'_n are the intercept and the slope of the n^{th} Regge trajectory).

The asymptotic estimation in Eq. (3.3) is most easily obtained if the replacements

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(*) It is worthwhile to remark that if one parametrizes the Regge input à la Veneziano, the Baker-Blankenbecler model(9) is more suitable(10).

$$\sum_{\ell} (\ell + \tfrac{1}{2})\, P_{\ell}(\cos\theta)\ldots \to k^2 \int b\, db\, J_o\,(b\sqrt{-t})\ldots$$

$$\int_{-1}^{1} d\cos\theta\, P_{\ell}(\cos\theta)\ \ldots \to \frac{1}{k^2}\int \sqrt{-t}\, d\sqrt{-t}\, J_o\,(b\sqrt{-t})\ \ldots$$

valid for $\theta \ll 1$ and $s \to \infty$, are made in (3.2) and (3.3). In particular, it is a simple exercise to derive the following formula, which will be useful later

$$e^{At} \otimes e^{Bt} \sim \frac{i}{s(A+B)}\, e^{t/[\frac{1}{A} + \frac{1}{B}]} \tag{3.5}$$

When $A = \alpha_1'(\log \frac{s}{s_o} - \frac{i\pi}{2})$ and $B = \alpha_2'(\log \frac{s}{s_o} - \frac{i\pi}{2})$ Eq. (3.5) implies Eq. (3.4)(with $n = 2$).

The asymptotic behavior in (3.3) is that of a Regge cut contribution. It is important that the study of Feynman diagrams also leads to Eq. (3.4).

A leading cut associated with a Regge pole R will be defined by the requirement

$$\alpha_c(0) = \alpha_R(0) \tag{3.6}$$

One obtains a leading cut by compounding R with pomerons $(\alpha_p(0) = 1)$.

It is evident from (3.2) and (3.3) that the leading cut contribution to the amplitude is a linear functional of the Regge pole contribution R. We denote this functional by $F(R)$. Hence

$$F\,(\sum_k x_k R_k) = \sum_k x_k F(R_k) \tag{3.7}$$

when x_n are independent of θ.

In the approximation of a flat Pomeranchon, $\alpha_p' = 0$, the Pomeranchon contribution to (3.2) is purely imaginary and one gets from (3.3)

$$F^*(R) = F(R^*) \qquad (\alpha_p' = 0) \tag{3.8}$$

i.e. $F(R)$ is a real functional.

A Regge cut can be considered as a continuum of Regge poles

$$F(R^{\pm}) = \int^{\alpha_c(t)} g_{\pm}(\alpha, t) \, (1 \pm e^{-i\pi\alpha})(s/s_o)^{\alpha} \, d\alpha \qquad (3.9)$$

where the superscript $\pm$ refers to the signature of the Regge pole. We used the fact, which can be obtained from (3.3), that in this model the signature of the cut is the same as that of the pole(*). When the discontinuity function $g_{\pm}(\alpha, t)$ is real, the reality of F implies

$$g_{+}(\alpha, t) = g_{-}(\alpha, t) \qquad (3.10)$$

and one gets the following asymptotic expression

$$F(R^{\pm}) \underset{s \to \infty}{\sim} (\text{real function independent of } \pm) \times [1 \pm e^{-i\pi\alpha_R(0)}] \qquad (3.11)$$

4. AN EXAMPLE : THE CROSS-OVER EFFECT

As an application of the preceding considerations let us consider the cross-over problem. Take two reactions related by the $s \to u$ crossing : $AB \to AB$ and $\bar{A}B \to \bar{A}B$. Let $AB \to AB$ be an exotic channel (like $NN \to NN$ or $K^+N \to K^+N$). The common experimental situation is that $\sigma_{tot}(\bar{A}B \to \bar{A}B) >$ $\sigma_{tot}(AB \to AB)$. Assume for simplicity that $\alpha'_p = 0$. Duality implies

$$f^B(AB \to AB) = ip - r$$
$$f^B(\bar{A}B \to \bar{A}B) = ip - r \, e^{-i\pi\alpha(t)} \qquad (4.1)$$

where r and p are real functions ($r > 0$). In the cut model one has for

(*) As for the other quantum numbers of the cut, they are those of the set of Regge poles which are "compounded". E.g. each leading cut $R \otimes P$... will have the g-parity of the R (since the g-parity of P is +1) but no definite parity (because the relative angular momentum of R and P is left undetermined).

$s \to \infty$

$$\begin{aligned} f(AB \to AB) &= i\,\tilde{p} - r - c \\ f(\bar{A}B \to \bar{A}B) &= i\,\tilde{p} - r\,e^{-i\pi\alpha(t)} - c\,e^{-i\pi\alpha(0)} \end{aligned} \qquad (4.2)$$

where

$$\begin{aligned} \tilde{p} &= p + F(p) \\ c &= F(r) \end{aligned}$$

One gets

$$\begin{aligned} \frac{d\sigma}{dt}(AB \to AB) &= \tilde{p}^2 + O(r^2) \\ \frac{d\sigma}{dt}(\bar{A}B \to \bar{A}B) &= \tilde{p}^2 + 2\tilde{p}\,[r \sin \pi\alpha(t) + c \sin \pi\alpha(0)] + O(r^2) \end{aligned} \qquad (4.3)$$

Without cuts the cross-over appears at $\alpha(t) = 0$. With cuts it is shifted toward the point $t = 0$ provided $0 < -c < r$.

When $\alpha(t) = \frac{1}{2} + t$ the cross-over is at $t = t_o$ where

$$t_o = -\tfrac{1}{2} - \frac{1}{\pi} \arcsin(c/r)_{t=t_o}$$

One gets $t_o \approx -.2$ when $(c/r)_{t=t_o} \approx -80\ \%$.(*) This gives an idea about the magnitude of the cut contribution to the elastic scattering amplitudes since for P_{lab} of the order of 10 GeV/c the cross-over appears precisely near

(*)This corresponds roughly to $(c/r)_{t=0} \approx -50\ \%$ when one uses (3.5) (the slope of the Pomeron exchange amplitude is roughly 4 GeV^{-2}. In eikonal model fits the slope of the "genuine" Regge pole amplitudes is usually also roughly 3-4 GeV^{-2} for P_{lab} of the order of 10 GeV/c). The eikonal model calculations typically yield $(c/r)_{t=0} \approx -30\ \%$. This is the reason why usually the cross-over obtained in these calculations appears at slightly too large a value of $|t|$.

$t = -.2$. Since $r \sim s^{\alpha(t)}$ shrinks and $c \sim s^{\alpha(0)}/\log s$ one expects that $t_o \to 0$ logarithmically as $s \to \infty$.

5. THE SOPKOVICH FORMULA

Let us assume that the off-diagonal elements of the eikonal matrix in (3.1) are small. Treating the inelastic transitions as a perturbation one easily gets

$$(1 - e^{i\chi})_{jj} = 1 - e^{i\chi_{jj}} \tag{5.1}$$

and(*)

$$(1 - e^{i\chi})_{ji} = e^{\frac{i}{2}\chi_{jj}} \chi_{ji} e^{\frac{i}{2}\chi_{ii}} + O[(\chi_{jj} - \chi_{ii})^2] \quad i \neq j \tag{5.2}$$

The quantity on the LHS of (5.2) is the partial wave amplitude $T^{(\ell)}_{ji}$ for the inelastic reaction $i \to j$. According to the Arnold ansatz we consider χ_{ji} as the partial wave "Born" amplitude $B^{(\ell)}_{ji}$ for this reaction. Denoting by $S^{(\ell)}_{jj}$ the S-matrix element for the elastic transition $j \to j$ and using (5.1) we can rewrite (5.2) in the more familiar form

$$T^{(\ell)}_{ji} = [S^{(\ell)}_{jj}]^{\frac{1}{2}} B^{(\ell)}_{ji} [S^{(\ell)}_{ii}]^{\frac{1}{2}} \tag{5.3}$$

known as the Sopkovich formula[11].

The sum of the contribution from a Regge pole R and from all the associated leading cuts to the elastic scattering amplitude is obtained from (5.1)

$$[R + F(R)]^{(\ell)} \cong [i\, \chi^R_{jj}]\, e^{i\chi^P_{jj}} \equiv i\, \chi^R_{jj} - i\, \chi^R_{jj}[1 - e^{i\chi^P_{jj}}] \tag{5.4}$$

It is evident that the cut contribution to the amplitude subtracts from that of the Regge pole. We have seen in the preceding section that this is indeed a requirement needed to get the cross-over effect. Furthermore the fits to

(*) Here, we also assume that the elastic scattering in states i and j is not too different.

the elastic scattering indicate that the cut contribution as given by (5.1) is of correct order of magnitude(*).

Similarly, in the Sopkovich formula, the cuts reduce the input "Born" amplitude especially for low partial waves. This leads to a simple intuitive picture : the situation looks as if the inelastic transition occurred in a semi-transparent medium, where both the incident and final waves are absorbed.

It is important to stress that both (5.1) and the Sopkovich formula rest heavily upon the perturbative treatment of the inelastic transitions. When one attempts to construct models where intermediate inelastic transitions (in particular the diffraction dissociation processes) are at least partly taken into account, then it becomes evident that the perturbative arguments which lead to Eq. (5.1) and to the Sopkovich formula respectively, have a different physical meaning. Hence the validity of (5.1) does not imply that of (5.2) and vice versa. Indeed, the cross-over phenomenon is an argument in favour of (5.1), or at least of the "absorptive" sign of the cut. However, in fitting inelastic reaction data one often needs cuts stronger than predicted by the Sopkovich ansatz. Even the "absorptive" sign of the cut, which predicts correctly the polarization in pion-nucleon charge-exchange reactions and the sharp peak in nucleon charge exchange, seems doubtful in certain cases (like the hypercharge-exchange processes or photoproduction).

6. DUALITY-PRESERVING REGGE CUTS[12-15]

Among the consequences of duality (supplemented by the usual assumption about the absence of exotic resonances) the most clear-cut are probably those which state that linear combinations of certain scattering amplitudes are purely real. For example

$$\mathrm{Im}\, f(K^+ n \to K^0 p) = 0 \tag{6.1}$$

or equivalently

(*) See, however, the footnote on

$$\mathrm{Im}\ [f(K^+ n \to K^+ n) - f(K^+ p \to K^+ p)] = 0 \qquad (6.2)$$

When Regge cuts are taken into account these predictions, in general, break down. However, as discussed in Sec. 3, in the approximation $\alpha'_p = 0$, the leading cuts satisfy the reality condition (3.8). If the contribution of the Regge poles, ΣR_i, is real then combining (3.7) and (3.8) one has

$$\mathrm{Im}\ [\Sigma\, F(R_i)] = \mathrm{Im}\ F(\Sigma\, R_i) = 0 \qquad (6.3)$$

For obvious reasons we say then that the leading cuts are duality preserving : Eqs. (6.1) and (6.2) remain true in the approximation when pole ⊗ pole cuts (which are, of course, duality non-preserving) are neglected. Thus one predicts a vanishing polarization

$$p(K^+ n \to K^o p) = 0 \qquad (6.4)$$

and(using (6.2) at $t = 0$)

$$\sigma_T(K^+ n) = \sigma_T(K^+ p) \qquad (6.5)$$

Similarly, since both $np \to np$ and $pp \to pp$ channels are exotic :

$$\sigma_T(n\, p) = \sigma_T(p\, p) \qquad (6.6)$$

It is worthwhile to note that Eqs. (6.5) and (6.6) are very well satisfied experimentally

7. POLARIZATION AND DIPS (A GENERAL DISCUSSION)

Everybody knows that a vanishing polarization is predicted in the pure Regge pole model for reactions where only one Regge pole is exchanged. This prediction extends to processes dominated by the exchange of several Regge poles if the hypothesis of strong exchange degeneracy is adopted. A non-vanishing polarization is, however, observed in charge and hypercharge exchange reactions. These polarizations can be explained by the interference of the Regge pole(s) with the associated Regge cut(s) and a slow (logarithmic) decrease with energy of the polarizations is then expected in agreement with the data. This is a well-known fact which is often put forward to emphasize the importance of the cut effects.

Let us consider for definiteness the pion-nucleon charge exchange scattering, assuming that only the ρ pole and the associated leading cuts contribute significantly to the amplitude. Provided the leading cuts are duality-preserving, one can write the scattering amplitude at very high energy as

$$f_\lambda \cong r_\lambda(1 - e^{-i\pi\alpha(t)}) + c_\lambda(1 - e^{-i\pi\alpha(0)}) \qquad (7.1)$$

where $\lambda = 0$ or 1 for the helicity nonflip and flip respectively and r_λ, c_λ are real functions of s and t.

Apart from an SU_3 Clebsch-Gordan coefficient, the same r_λ appears in $K^+ n \rightarrow K^o p$. The Gell-Mann ghost-killing mechanism for A_2 and the exchange degeneracy, necessary to ensure (6.1), imply that r_λ is finite.

One obtains from (7.1) the following expression for the polarization

$$\frac{1}{2}\, p \frac{d\sigma}{dt} \underset{s \to \infty}{\sim} [(r_1 c_o - r_o c_1) \sin\frac{\pi}{2}\alpha(t) \sin\frac{\pi}{2}\alpha(0)] \sin\frac{\pi}{2}[\alpha(t) - \alpha(0)] \qquad (7.2)$$

The necessary zeros of p are evident from the above formula. These zeros have, however, two distinct origins :

(i) Exchange degeneracy yields the factor $\sin\frac{\pi}{2}\alpha(t)$ which vanishes e.g. at $\alpha(t) = 0$ (for $\pi^- p \rightarrow \eta n$ the corresponding factor is $\cos\frac{\pi}{2}\alpha(t)$ and the first zero of p appears at $\alpha(t) = -1$).

(ii) The factor $\sin\frac{\pi}{2}[\alpha(t) - \alpha(0)]$ comes from the difference of phase between the pole and the cut contributions and has to be considered more carefully.

The cut has a phase $-\pi\,\alpha(0) + O(1/\log s)$. In writing (7.1) we neglected the correction term $O(1/\log s)$, which is a bad approximation since the principal term in the phase of the cut cancels with the phase of the pole at $t = 0$. The polarization has a kinematical zero at $t = 0$ (angular momentum conservation) and the dynamical zero (cf. (7.2)) which coincides with the kinematical one at infinite energy, but which appears at $t \neq 0$ for any finite energy.

It is easy to show that

$$c_1/c_o = O(1/\log s) \tag{7.3}$$

if $\alpha'_p = 0$. Indeed, using the impact parameter representation for convenience, one has[7]

$$F(R) = k\sqrt{s} \int b\, db\, J_\lambda\, (b\sqrt{-t})\, \chi_\lambda(b, s)\, (e^{i\chi} - 1) \tag{7.4}$$

with

$$\chi_o = \frac{1}{k\sqrt{s}} \int \sqrt{-t}\, d\sqrt{-t}\, J_o(b\sqrt{-t})\, R_o(s, t) \tag{7.5a}$$

$$\chi_1 = \frac{1}{k\sqrt{s}} \int \sqrt{-t}\, d\sqrt{-t}\, J_1(b\sqrt{-t})\, R_1(s, t)$$

$$= -\frac{1}{k\sqrt{s}}\, \frac{d}{db} \int \sqrt{-t}\, d\sqrt{-t}\, J_o(b\sqrt{-t})\, \left[\frac{R_1(s, t)}{\sqrt{-t}}\right] \tag{7.5b}$$

As $s \to \infty$, the behavior of χ_λ is determined by the Regge factor $(s/s_R)^{\alpha(t)}$ common to R_o and $R_1/\sqrt{-t}$. Roughly

$$\chi_o \underset{s \to \infty}{\simeq} \frac{1}{k\sqrt{s}} R_o(s, 0) \exp\left[-b^2/4 \log(s/s_R)\right]$$

$$\chi_1 \underset{s \to \infty}{\sim} \frac{b}{4k\sqrt{s}} \left[\frac{R_1(s, t)}{\sqrt{-t}}\right]_{t=0} \exp\left[-b^2/4 \log(s/s_R)\right] / \log(s/s_R) \tag{7.6}$$

Eq. (7.3) is readily obtained substituting (7.6) into (7.4), provided the absorptive cut-off becomes energy independent at large s, which is precisely the case when $\alpha'_p = 0$.

Using (3.5) with $A = \alpha'(\log s/s_R - \frac{i\pi}{2})$ we see that the corrective term in the phase of c_o is positive at small $|t|$, where the first leading cut is the most important. Neglecting c_1 (cf. (7.3)) the following replacement should be made in (7.2)

$$\sin \frac{\pi}{2} [\alpha(t) - \alpha(0)] \to \sin \frac{\pi}{2} [\alpha(t) - \alpha(0) + a(t)/\log s/s_o] \ , \ a(0) > 0 \tag{7.7}$$

and we find that p vanishes at

$$t \approx - a(0)/ \alpha' \log s/s_o \tag{7.8}$$

This zero tends logarithmically toward the point $t = 0$ as $s \to \infty$. Such a dynamical moving zero of the polarization is typical for cut models[16].

As discussed in Sec. 4 the Sopkovich formula yields the following predictions for the sign of the cut

$$\mathrm{sgn}[c_\lambda] = - \mathrm{sgn}[r_\lambda] \tag{7.9}$$

Phenomenological fits (with or without cuts) give[7, 17]

$$\sqrt{-t}\ r_o/r_1 \approx - 20 \% \tag{7.10}$$

Using (7.3) we find

$$(r_1 c_o - r_o c_1) \approx r_1 c_o > 0 \tag{7.11}$$

and $p > 0$ for small $|t|$, in agreement with the experimental data on $\pi^- p \to \pi^0 n$.

To close this section let us mention the problem of dips. Actually, there are two kinds of dips in cut models :

(a) Dips (or breaks) that occur at relatively large values of $|t|$, and which are interpreted as resulting from multiple scattering of the projectile on the target "stuff"[18-21]. As an example one can quote the break in $\frac{d\sigma}{dt}$ (pp → pp) for $|t| \approx 1 - 1.2\ GeV^2$. These dips are expected to become more and more pronounced as energy increases.

(b) Dips at small values of $|t|$ which reflect the structure of the Regge pole amplitude (usually at $\alpha_{Regge}(t) = 0$). In general, we can expect that, as energy increases and for fixed t, the cut progressively dominates the pole (since $\alpha'_{cut} < \alpha'_{pole}$) and the dip becomes less and less marked. This is indeed observed for several reactions, for example in the π^0 photoproduction [22]. However, two points should be kept in mind. First, the cut amplitude is an integral, whose integrand involves as a factor the Regge pole amplitude. It can be shown that if the Regge amplitude changes sign, then results a cancellation which reduces the magnitude of the cut amplitude at t which roughly corresponds to the zero of the Regge amplitude [6]. Secondly, when the helicity-flip Regge amplitude is particularly large, the cut is less important (cf. (7.3)) and consequently the dip is less affected by the existence of the cut. This seems to be the case in $\pi^- p \to \pi^0 n$: the dip (due to the vanishing of Regge amplitudes at $\alpha(t) = 0$, cf. (7.1)) survives at highest energies where experiments were done, although the peak that follows it is progressively smeared out [23].

8. DUALITY NON-PRESERVING REGGE CUTS [14]

In the preceding discussion we often made use of the approximation $\alpha'_p = 0$, which for leading cuts implies the important reality condition $F(R^*) = F^*(R)$. However, the elastic scattering results obtained with the Serpukhov accelerator point toward a non-zero slope of the Pomeron. If $\alpha'_p \neq 0$, then even the leading cuts are duality non-preserving : $F(R^*) \neq F^*(R)$. Nevertheless, some of the duality predictions are not very sensitive to the approximation $\alpha'_p = 0$ and can be maintained if α'_p is not too large (say $\alpha'_p \leqslant 4\ GeV^2$).

In the following we shall, for definiteness, limit ourselves to the $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ scattering. Let us write the Regge pole amplitudes

$$R^{\pm}_{\lambda} = \beta_{\lambda}(\sqrt{-t})^{\lambda}\ (s/s_R)^{\alpha_R(t)}\ [1 \pm e^{-i\pi\alpha_R(t)}] \qquad (8.1)$$

(notice that R^+ and R^- are strongly degenerate) and the Pomeron exchange amplitude(*)

$$P = i\gamma(s/s_P)^{1+\alpha'_p t}\, e^{-\frac{i}{2}\pi\alpha'_p t} \qquad (8.2)$$

At very high energy, and with a proper choice of the scale parameters s_R and s_P, it is reasonable to neglect the t-dependence of the residues in (8.1) and (8.2). Consider only the first leading cut (i.e. $R \otimes P$; it dominates over the other cuts at small $|t|$). One easily finds from (7.4) and (7.5) [χ is given by a formula identical to (7.5a), with R replaced by P] that

$$|F(R^+ + R^-)| = |F(R^+ - R^-)| \qquad (8.3)$$

up to terms of the relative order $0(1/\log^2 s)$, and that (7.3) is replaced by

$$\frac{(R_o^+ \pm R_o^-)\, F(R_1^+ \pm R_1^-)}{(R_1^+ \pm R_1^-)\, F(R_o^+ \pm R_o^-)} = \alpha'_c/\alpha'_R + 0(1/\log s) \qquad (8.4)$$

where α'_c is the slope of the moving cut :

$$\begin{aligned}\alpha_c(t) &= \alpha_c(0) + \alpha'_c t \\ &= \alpha_R(0) + \frac{\alpha'_R\,\alpha'_p}{\alpha'_R + \alpha'_p}\, t \end{aligned} \qquad (8.5)$$

Set

$$\begin{aligned}\text{Arg } F(R_\lambda^+ + R_\lambda^-) &= a_\lambda - b_\lambda \\ \text{Arg } F(R_\lambda^+ - R_\lambda^-) &= -\pi\,\alpha_c(t) + a_\lambda + b_\lambda\end{aligned} \qquad (8.6)$$

Then

$$\begin{aligned}F(R_\lambda^+) &= c_\lambda \cos\left[\tfrac{\pi}{2}\,\alpha_c(t) - b_\lambda\right] e^{-i\pi\alpha_c(t)/2 + ia_\lambda} \\ F(R_\lambda^-) &= i\, c_\lambda \sin\left[\tfrac{\pi}{2}\,\alpha_c(t) - b_\lambda\right] e^{-i\pi\alpha_c(t)/2 + ia_\lambda}\end{aligned} \qquad (8.7)$$

(*)Notice that with this convention, P becomes t-independent (a nonsense) when one sets $\alpha'_p = 0$. Thus one should not take the limit $\alpha'_p \to 0$ in the formulae derived from (8.2).

When α'_p is small the cut flip contribution to the polarization is not very important. We neglect it for simplicity. The position of the "moving zero" of p is determined by the equation

$$\pi\,\alpha_R(t) - \pi\,\alpha_c(t) + 2a_o = 0 \qquad (8.8)$$

where the Regge pole amplitude is $R^{\pm}$ (e.g. $\pi^- p \to \pi^o n$ or $\pi^- p \to \eta\, n$ with ρ and A_2 exchange respectively) and by

$$\pi\,\alpha_R(t) - \pi\,\alpha_c(t) + a_o + b_o = 0 \qquad (8.9)$$

when the Regge pole amplitude is $R^+ - R^-$ (e.g. $K^- p \to \bar{K}^o n$ with ρ and A_2 exchange). Only when the cut is duality-preserving, the position of the moving zero is common to the three reactions, since $a_\lambda = b_\lambda$.

Of course, when $F^*(R) \neq F(R^*)$ the polarization $p(K^+ n \to K^o p) \neq 0$. The sign of this polarization depends on the relative magnitude of a_λ and b_λ . It is impossible to make a truly model-independent prediction. However, asymptotic expressions for a_λ , b_λ can be derived in the same manner as Eqs. (8.3) and (8.4) :

$$\begin{aligned} a_\lambda &= \frac{\pi}{2\log s/s_o} + O(1/\log^2 s) \\ b_\lambda &= \frac{\pi(1+\lambda)\alpha'_c}{2\alpha'_p \log s/s_o} + \frac{\pi(\alpha'_c)^2}{2\alpha'_p}\, t + O(1/\log^2 s) \end{aligned} \qquad (8.10)$$

Eqs. (8.10) with different scale parameter s_o are asymptotically equivalent. One may hope that Eqs. (8.10) can be used for rough estimations as non-asymptotic energies provided s_o is properly chosen. Hopefully, this parameter is determined by the position of the moving zero of p . We postpone the discussion of polarizations to the following sections . Anticipating slightly, we can say that the qualitative features of polarizations that are allowed to be non-zero by the unbroken duality remain valid (the signs of polarizations, the existence of the moving zero, the zeros due to the exchange degeneracy).

Assuming that roughly $|c_o/r_o|_{t=0} \approx 30\,\%$ and using (8.10) one finds

$$|\sigma_T(K^+n) - \sigma_T(K^+p)| \approx |\sigma_T(K^-n) - \sigma_T(K^-p)| \times \left|\frac{c_o \sin(a_o - b_o)}{r_o \sin \pi \alpha_R(0)}\right| \approx .06 \text{ mb}$$

$$\text{at 6 GeV/c} \qquad (8.11)$$

(we set $\alpha'_p = .4$ and assumed that $p(\pi^- p \to \pi^0 n) = 0$ at $t = -.35$ GeV^2 for 11.2 GeV/c). It is seen that the violation of Eq. (6.5) is indeed well within experimental errors ($\gtrsim .1$ mb).

9. PROCESSES RELATED BY LINE REVERSAL

It has been pointed out by Gilman[24] (see also Mathews[25]) that the comparison of the behavior of reactions related by the $s \to u$ crossing provides a very sensitive test of the exchange degeneracy hypothesis.

We shall consider the pseudoscalar meson-baryon scattering. The contribution of the Regge poles with positive signature to the amplitudes of reactions related by $s \to u$ crossing is identical while the corresponding contribution of the Regge poles with negative signature changes sign. For example if

$$f(AB \to CD) = \beta^+(t)\, s^{\alpha_+(t)} [1 + e^{-i\pi\alpha_+(t)}] + \beta^-(t)\, s^{\alpha_-(t)} [1 - e^{-i\pi\alpha_-(t)}] \qquad (9.1)$$

then

$$f(\bar{C}B \to \bar{A}D) = \beta^+(t)\, s^{\alpha_+(t)} [1 + e^{-i\pi\alpha_+(t)}] - \beta^-(t)\, s^{\alpha_-(t)} [1 - e^{-i\pi\alpha_-(t)}] . \qquad (9.2)$$

With weak exchange degeneracy, which postulates $\alpha^+ = \alpha^-$, one predicts (we assume that the energy is high enough to neglect the kinematical factor due to the difference of masses) :

$$\frac{d\sigma}{dt}(AB \to CD) = \frac{d\sigma}{dt}(\bar{C}B \to \bar{A}D) \qquad (9.3)$$

$$p(AB \to CD) = - p(\bar{C}B \to \bar{A}D) \qquad (9.4)$$

Strong exchange degeneracy, which postulates $\alpha^+ = \alpha^-$ <u>and</u> $\beta^+ = -\beta^-$, predicts a vanishing polarization in both $s \to u$ crossed reactions

$$p(AB \to CD) = p(\bar{C}B \to \bar{A}D) = 0 \qquad (9.5)$$

The experimental situation has recently been summarized by Sonderegger(26) and Kwan Wu Lai and Louie(27). It is in a drastic disagreement with the above predictions, except perhaps for the kaon charge exchange where polarizations have not yet been measured and $\frac{d\sigma}{dt}(K^-p \to \bar{K}^o n)$ becomes equal to $\frac{d\sigma}{dt}(K^+n \to K^o p)$ near 5 GeV/c (however, there is no data above 5.5 GeV/c).

(i) The polarization in hypercharge exchange reactions, $K^-p \to \pi^-\Sigma^o$ (near 3 GeV/c) and $K^-p \to \pi^o\Lambda^o$ (up to 4.5 GeV/c), is large and at small $|t|$ it is of the same sign as the polarization in respective line crossed reactions(*) (one might object, however, that the energies where the experiments were performed are not high enough).

(ii) The differential cross sections change substantially under $s \to u$ crossing. An amusing empirical regularity is observed for hypercharge exchange reactions : out of the two processes related by $s \to u$ crossing, the one where "hypercharge annihilation"(**) (using the terminology of Van Hove and collaborators(28)) is possible in final and initial states, has larger $d\sigma/dt$. This rule is apparently not true for charge exchange, since $\frac{d\sigma}{dt}(K^-n \to \bar{K}^o\Delta^-) < \frac{d\sigma}{dt}(K^+p \to K^o\Delta^{++})$. However, the data on $K^-n \to \bar{K}^o\Delta^-$ do not extend beyond 5 GeV/c. Also $\frac{d\sigma}{dt}(K^-p \to \bar{K}^o n) < \frac{d\sigma}{dt}(K^+n \to K^o p)$ at low energy, but the two cross sections become equal at 5 GeV/c. On the other hand, among the two nucleon charge-exchange reactions, $pn \to np$ and $p\bar{p} \to n\bar{n}$, the one where annihilation (now I mean the "usual", baryon number annihilation) is possible in final and initial states has $d\sigma/dt$ larger by a factor of four !

At high energy and small $|t|$, the Sopkovich formula predicts

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(*) The strong exchange degeneracy of K* and K** in hypercharge exchange reactions follows from the postulated absence of exotic resonances in meson-meson and baryon-baryon systems, duality and factorization.

(**) Two particles with hypercharge +1 and −1 respectively produce n particles with hypercharge 0 : e.g. $K^-p \to (n-1)\pi + \Sigma^+$.

$$\frac{3}{2}\pi > |\arg(\text{cut}) - \arg(\text{pole})| \equiv \Delta\phi > \frac{\pi}{2} \tag{9.6a}$$

$$|\text{cut}| < |\text{pole}| \times |\cos \Delta\phi| \tag{9.6b}$$

Since

$$d\sigma \equiv |f_{in}|^2 = |\text{pole}|^2 + |\text{cut}|^2 + 2|\text{pole}||\text{cut}| \cos \Delta\phi \tag{9.7}$$

it is evident(*) that

$$\left.\begin{array}{l}\text{(a) } d\sigma \text{ increases when } |\pi - \Delta\phi| \text{ increases} \\ \\ \text{(b) } d\sigma \text{ decreases when } |\text{cut}| \text{ increases}\end{array}\right. \tag{9.8}$$

First, let us take into account the leading cut only, assuming for simplicity that $\alpha'_p = 0$. At high energy and small $|t|$ $|(r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}) \otimes P|$ weakly depends on η_1 (**), and (cf. (3.5))

$$\Delta\phi' \equiv \arg[(r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}) \otimes P] - \arg[r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}] = \pi + \pi\alpha'_1 t\eta_1 + \phi_o\eta_1 \tag{9.9}$$

with $\phi_o \sim O(1/\log s)$. Hence

$$|\Delta\phi'(\eta_1 = 1) - \pi| > |\Delta\phi'(\eta_1 = 0) - \pi| \tag{9.10}$$

and one predicts

$$\frac{d\sigma}{dt}(K^+n \to K^o p) < \frac{d\sigma}{dt}(K^-p \to \bar{K}^o n) \tag{9.11}$$

and also, provided the P-baryon couplings do not violate the SU_3 symmetry too badly,

---

(*) Take $\partial[d\sigma]/\partial|\text{cut}|$.

(**) $\eta_1 = 1$ (0) means that the Regge amplitude has the phase factor $e^{-i\pi\alpha}$ (1)

$$\frac{d\sigma}{dt}(K^- p \to \pi^- \Sigma^+) < \frac{d\sigma}{dt}(\pi^+ p \to K^+ \Sigma^+)$$
$$\frac{d\sigma}{dt}(K^- n \to \pi^- \Lambda^o) < \frac{d\sigma}{dt}(\pi^- p \to K^o \Lambda^o) \tag{9.12}$$

in complete disagreement with experiment (the processes on the LHS correspond to $\eta_1 = 0$, those on the RHS of the above inequalities to $\eta_1 = 1$).

We know that in elastic scattering the secondary Regge trajectories (with intercept $\approx 1/2$) are responsible for large differences between particle and antiparticle total cross sections. Through absorption these secondary trajectories can lead to non-negligible cut effects in inelastic reactions. Let us therefore consider the pole $\otimes$ pole cuts also. Again we neglect the dependence of $|(r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}) \otimes (r_2(t) e^{-i\pi\alpha_2(t)\eta_2})|$ on η_1 and η_2, and we write ($r_2(t) < 0$, cf. (4.1)) :

$$\Delta\phi'' = \arg[(r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}) \otimes (r_2(t)\, e^{-i\pi\alpha_2(t)\eta_2})]$$
$$- \arg[r_1(t)\, e^{-i\pi\alpha_1(t)\eta_1}] = \frac{3}{2}\pi + \pi\,\alpha_1'\, t\eta_1 - \pi\,\alpha_2(0)\,\eta_2 + \phi_1\eta_1 + \phi_2\eta_2 \tag{9.13}$$

where $\phi_{1,2} \sim 0(1/\log s)$. We can safely put $\alpha_2(0) = 1/2$:

$$\Delta\phi'' = \begin{cases} 3/2\,\pi , & \eta_1 = 0 \quad \eta_2 = 0 \\ 3/2\,\pi + \pi\alpha_1' t + \phi_1 , & \eta_1 = 1 \quad \eta_2 = 0 \\ \pi + \phi_2 , & \eta_1 = 0 \quad \eta_2 = 1 \\ \pi + \pi\alpha_1' t + \phi_1 + \phi_2 , & \eta_1 = 1 \quad \eta_2 = 1 \end{cases} \tag{9.14}$$

For K charge-exchange, since

$$|\Delta\phi''(\eta_1 = 1,\ \eta_2 = 1) - \pi| < |\Delta\phi''(\eta_1 = 0,\ \eta_2 = 0) - \pi| \tag{9.15}$$

the effects of the leading and secondary cuts are going in the opposite directions and the prediction (9.11) is abandoned.

Using the conventional duality arguments (see Rosner(34)) one finds that the secondary Regge pole contribution to the $K^+\Sigma^+ \to K^+\Sigma^+$ amplitude is real ($\eta_2 = 0$). Since

$$|\Delta\phi''(\eta_1 = 0, \eta_2 = 1) - \pi| < |\Delta\phi''(\eta_1 = 1, \eta_2 = 0) - \pi| \qquad (9.16)$$

the effects of the leading cut and of the secondary cut, associated with absorption in the final state, are going in the same direction as far as the pair of reactions, $\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$ is concerned ; the prediction (9.12) is confirmed. The same is true for other hypercharge-exchange process s. (Michael(15)).

Strictly speaking the above arguments are valid at a sufficiently high energy. When one goes down with energy the situation becomes more complex, but the conclusions remain essentially the same. There are several ways out. The first is to abandon simply the exchange degeneracy arguments. Secondly, one can assume that SU_3 is strongly broken in the Pomeron couplings so that not only $\sigma_T^\infty(\pi^+ p) > \sigma_T^\infty(K^- p)$ (which is true) but also $\sigma_T^\infty(K^+ \Sigma^+) > \sigma_T^\infty(\pi^- \Sigma^+)$, in such a manner that the effect of the secondary trajectories at finite energies is compensated(*). The third possibility, which I prefer personally, is to call into question the universal applicability of the Sopkovich formula. We mentioned in Sec. 5 that it rests upon the perturbative treatment of inelastic transitions and that the relative strength of the cuts may be modified by the effect of the intermediate inelastic states. Also the signs of the cuts become then uncertain. The fact that the inequality (9.6a) has been verified in elastic scattering and charge exchange reactions does not mean that it is true in all possible cases.

Detailed numerical calculations have been done in Orsay(29), in order to see whether it is possible to fit simultaneously the reactions

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(*)Notice that if SU_3 is broken in the Pomeron pseudoscalar meson couplings only, then $\sigma_T^\infty(\pi^+ p) + \sigma_T^\infty(K^+ \Sigma^+) \approx \sigma_T^\infty(K^- p) + \sigma_T^\infty(\pi^- \Sigma^+)$.

$\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$ in the energy range 5-16 GeV/c, taking the exchange degeneracy à la lettre. The rule (3.3) for calculating cut contributions was kept but the relative importance of different cuts was allowed to be arbitrary. The conclusion is that exact exchange degeneracy can be maintained provided the global contribution of cuts is of the opposite sign to that predicted by the absorption model. It is amusing to note that if the Regge pole flip and non-flip helicity amplitudes for $\pi^+ p \to K^+ \Sigma^+$ are of opposite sign (as in $K^- p \to \bar{K}^o n$, where mesons belonging to the same vector and tensor octets are exchanged(*)), the polarization is then expected to be negative at small $|t|$ and positive at large $|t|$ (it is what is observed)(**).

A very important difference is observed between the slope of $\frac{d\sigma}{dt}(K^- n \to \pi^- \Lambda)$ ($\approx 4.0 \pm 0.6$ GeV^{-2}) and that of $\frac{d\sigma}{dt}(\pi^- p \to K^o \Lambda)$ ($\approx 7.3 \pm 0.8$ GeV^{-2}). This can perhaps be interpreted as an evidence for a cut dominating the pole in $K^- n \to \pi^- \Lambda$ (cf. (3.5)). However the data on $K^- n \to \pi^- \Lambda$ do not extend beyond 6 GeV/c and a serious conclusion can hardly be formulated.

On the other hand the much larger $d\sigma/dt$ for $p\bar{p} \to n\bar{n}$ than for $pn \to np$ can be understood with an evasive pion. For small $|t|$, the cuts dominate over the pole. The sharp forward peak is obtained, when a very rapidly varying pion exchange amplitude $\sim t/(\mu^2 - t)$ is subtracted from much less varying cut contributions (here we definitely need the absorptive sign of the cut). But the absorption is much stronger (and the cut amplitude larger) in $p\bar{p} \to n\bar{n}$, which simply reflects the presence of annihilation channels in nucleon-anti-nucleon scattering.

Let us turn now to the polarization problem[14]. When the cuts are duality non-preserving, the reaction which corresponds to an improper duality diagram (e.g. $K^- p \to \pi^- \Sigma^+$) presents a non-vanishing polarization. This polar-

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(*) This can be checked using the fits to the $\bar{K}N$ scattering by Dass, Michael and Phillips[30].

(**) However, this can also be explained by properly choosing the F/D ratio for helicity flip and non-flip amplitudes respectively.

ization comes entirely from the phase of the cut (since the Regge pole amplitude is real). This phase is expected to vary slowly with t and consequently the polarization will be of constant sign in a large interval of the momentum transfers. On the contrary, the reaction obtained by crossing the mesonic line corresponds to a proper duality diagram. The polarization has therefore the usual moving zero at small $|t|$. When the equations (8.10) are used one moreover predicts that the polarizations in two reactions related by $s \to u$ crossing are of the same sign at small $|t|$.

The above description of the behavior of polarizations is based on asymptotic arguments. Background effects of all types could alter these predictions. It is interesting, however, that this description seems to correspond to reality even at relatively small energies. Thus Eqs. (8.10) predict very correctly $p\frac{d\sigma}{dt}(K^- p \to \pi^- \Sigma^+)$ and $p\frac{d\sigma}{dt}(K^- n \to \pi^0 \Lambda)$ when the data on $\pi^+ p \to K^+ \Sigma^+$ and $\pi^- p \to K^0 \Lambda$ (at $\approx$ 3 GeV/c) are used as input data(*).

10. CHARGE-EXCHANGE SCATTERING OF PSEUDOSCALAR MESONS AND SU_3 (14,31)

Interesting equalities can be derived when it is assumed that Regge pole couplings obey the SU_3 symmetry, the Pomeron being a pure SU_3 singlet(**). Then the functional F does not charge when one passes from one reaction to another, provided the colliding particles are replaced by their "partners" in an SU_3 multiplet. Because of the linearity of F, the couplings of a Regge pole exchanged in different reactions are in the same ratio as the couplings of the associated leading cuts.

(*) In this calculation, α'_p appearing in (8.10) should probably be interpreted as an effective slope in the (Pomeron + secondary poles) amplitudes describing the elastic scattering (the pole $\otimes$ pole cuts are important if they are at the origin of the difference between differential cross sections in s and u channels).

(**) In fact the ratio of the $(P\pi\pi)$ to the $(PK\bar{K})$ coupling deviates from unity by about 15-20 % , whenever one uses pure Regge poles or cut models to fit the data. Nevertheless, it is encouraging that the relations of the type derived in this section are often pretty well satisfied by experiment.

Consider the following set of reactions

(a) $\pi^- p \to \pi^0 n$

(b) $\pi^- p \to \eta_8 n$

(c) $K^- p \to \bar{K}^0 n$

(d) $K^+ n \to K^0 p.$

The Regge poles which dominate the above reactions are ρ and A_2. Let f_λ^A and f_λ^ρ denote respectively the helicity amplitudes associated with $(A_2 + A_2 \otimes P + \ldots)$ and $(\rho + \rho \otimes P + \ldots)$. Neglecting non-leading cuts we can write the helicity amplitudes for reactions (a)-(d) as follows

$$\begin{aligned} f_\lambda^a &= \sqrt{2}\, f_\lambda^\rho \\ f_\lambda^b &= -\sqrt{2/3}\, f_\lambda^A \\ f_\lambda^c &= -f_\lambda^\rho - f_\lambda^A \\ f_\lambda^d &= f_\lambda^\rho - f_\lambda^A \end{aligned} \tag{10.1}$$

Hence

$$\begin{aligned} f_\lambda^c + f_\lambda^d &= -\sqrt{6}\, f_\lambda^b \\ f_\lambda^c - f_\lambda^d &= -\sqrt{2}\, f_\lambda^a \end{aligned} \tag{10.2}$$

and

$$f_\lambda^c f_{\lambda'}^{c*} + f_\lambda^d f_{\lambda'}^{d*} = f_\lambda^a f_{\lambda'}^{a*} + 3\, f_\lambda^b f_{\lambda'}^{b*} \tag{10.3}$$

Taking the trace of (10.3) one gets

$$\frac{d\sigma^c}{dt} + \frac{d\sigma^d}{dt} = \frac{d\sigma^a}{dt} + 3\,\frac{d\sigma^b}{dt} \tag{10.4}$$

Equating the imaginary parts of both sides of Eq. (10.3) one finds

$$\left(p\,\frac{d\sigma}{dt}\right)^c + \left(p\,\frac{d\sigma}{dt}\right)^d = \left(p\,\frac{d\sigma}{dt}\right)^a + 3\left(p\,\frac{d\sigma}{dt}\right)^b \tag{10.5}$$

Eq. (10.4) has been derived several years ago by Barger and Cline[32] and is well verified experimentally.

Linear relations between products of helicity amplitudes, analogous to (10.3), can similarly be derived for sets of reactions involving resonance production. Some of these relations can be found dispersed in the literature ; they reflect merely the fact that exchanged "objects" have a well defined SU_3 assignment. As will be seen, the exchange degeneracy leads to further relations between pairs of $\mathrm{Im}[f^k_\lambda f^{k*}_{\lambda'}]$. We limited our discussion to the charge-exchange of pseudoscalar mesons since in this case one has experimental data on $\mathrm{Im}[f^k_\lambda f^{k*}_{\lambda'}] \sim [p\,\frac{d\sigma}{dt}]^{(k)}$. In the case of resonance production the exchange degeneracy constraints concern the imaginary parts of density matrix elements and can hardly be tested with existing data.

Let us rewrite Eqs. (10.1) explicitly separating the pole and cut contributions to the amplitudes and using (8.9), (8.6) and (8.7)

$$
\begin{aligned}
f^a_\lambda &= i\sqrt{2}\, r_\lambda \sin\frac{\pi}{2}\alpha(t)\, e^{-i\pi\alpha(t)/2} + i\sqrt{2}\, c_\lambda \sin[\frac{\pi}{2}\alpha_c(t) - b_\lambda] e^{-\frac{i\pi}{2}\alpha_c(t) + ia_\lambda} \\
f^b_\lambda &= -\sqrt{2/3}\, r_\lambda \cos\frac{\pi}{2}\alpha(t) e^{-i\pi\alpha(t)/2} - \sqrt{2/3}\, c_\lambda \cos[\frac{\pi}{2}\alpha_c(t) - b_\lambda] e^{-i\frac{\pi}{2}\alpha_c(t) + ia_\lambda} \\
f^c_\lambda &= -r_\lambda\, e^{-i\pi\alpha(t)} - c_\lambda\, e^{-i\pi\alpha_c(t) + i(a_\lambda + b_\lambda)} \\
f^d_\lambda &= -r_\lambda - c_\lambda\, e^{i(a_\lambda - b_\lambda)}
\end{aligned}
\tag{10.6}
$$

If according to (8.4) and (8.10) respectively, we put $(r_o c_1/r_1 c_o) = \alpha'_c/\alpha'$ and $a_1 = a_o$, the following three equations are easily obtained :

$$
\begin{aligned}
&(p\,\frac{d\sigma}{dt})^a \cos\frac{\pi}{2}\alpha(t)\{\cos[\frac{\pi}{2}\alpha_c(t) - b_o] - \frac{\alpha'_c}{\alpha'}\cos[\frac{\pi}{2}\alpha_c(t) - b_1]\} \\
&\qquad = 3(p\frac{d\sigma}{dt})^b \sin\frac{\pi}{2}\alpha(t)\{\sin[\frac{\pi}{2}\alpha_c(t) - b_o] - \frac{\alpha'_c}{\alpha'}\sin[\frac{\pi}{2}\alpha_c(t) - b_1]\}
\end{aligned}
\tag{10.7a}
$$

$$
\begin{aligned}
&(p\frac{d\sigma}{dt})^a\{\sin[\pi\,\alpha(t) - \pi\,\alpha_c(t) + a_o + b_o] - \frac{\alpha'_c}{\alpha'}\sin[\pi\alpha(t) - \pi\,\alpha_c(t) + a_o + b_1]\} \\
&= 2(p\frac{d\sigma}{dt})^c \sin\frac{\pi}{2}\alpha(t)\,\sin[\frac{\pi}{2}\alpha(t) - \frac{\pi}{2}\alpha_c(t) + a_o]\{\sin[\frac{\pi}{2}\alpha_c(t) - b_o] - \frac{\alpha'_c}{\alpha'}\sin[\frac{\pi}{2}\alpha_c(t) - b_1]\}
\end{aligned}
\tag{10.7b}
$$

$$\left(p\frac{d\sigma}{dt}\right)^{a}\{\sin(a_o - b_o) - \frac{\alpha'_c}{\alpha'}\sin(a_o - b_1)\} = 2\left(p\frac{d\sigma}{dt}\right)^{d}\sin\frac{\pi}{2}\alpha(t)\sin\left[\frac{\pi}{2}\alpha(t) - \frac{\pi}{2}\alpha_c(t) + a_o\right] \times$$

$$\times\{\sin[\frac{\pi}{2}\alpha_c(t) - b_o] - \frac{\alpha'_c}{\alpha'}\sin[\frac{\pi}{2}\alpha_c(t) - b_1]\} \qquad (10.7c)$$

The qualitative features of the polarizations read from the above equations are precisely those discussed in Sec. 7 : the zeros of $p(\pi^- p \to \pi^o n)$ and $p(\pi^- p \to \eta_8 n)$ due to the factors $\sin \pi\alpha/2$ and $\cos \pi\alpha/2$ respectively, reflect the exchange degeneracy. The factor $\sin[\pi/2\ \alpha(t) - \pi/2\ \alpha_c(t) + a_o]$ yields a moving zero, in $p(\pi^- p \to \pi^o n)$ and $p(\pi^- p \to \eta_8 n)$. The moving zero of $p(K^- p \to \bar{K}^o n)$ is slightly displaced.

The polarization in $K^+ n \to K^o p$ (which is $\neq 0$ since we take duality non preserving cuts) is of the same sign as in $K^- p \to \bar{K}^o n$ for small $|t|$, as it should be according to the preceding section for two processes related by $s \to u$ crossing.

We give in Table I the polarization for $\pi^- p \to \eta_8 n$, $K^- p \to \bar{K}^o n$ and $K^+ n \to K^o p$ predicted from Eqs. (10.7a-c) using $p(\pi^- p \to \pi^o n)$ and data on differential cross sections as input data. Neglecting the η-X mixing our prediction for $\pi^- p \to \eta n$ can be compared with the results of Ref.(33) . The agreement is good (but, of course, the experimental errors are very important ; an ideal situation for theoreticians !). One should notice that a relatively large polarization is predicted for $K^- p \to \bar{K}^o n$ scattering (about 20 % at $-.3 < t < 0$ and 11.2 GeV/c).

Table I

Charge-exchange (polarization) × (differential cross section) at 11.2 GeV/c) (in $\mu b/GeV^2$). We assume that the moving zero of $p(\pi^- p \to \pi^0 n)$ is at t = -.35.

-t	.03	.06	.09	.12	.165	.225	.275
$p\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)$ experimental(23)	18. ± 9.	17. ± 10.	11. ± 7.8	16. ± 7.5	17. ± 5.	9. ± 5.	5.4 ± 3.
$p\frac{d\sigma}{dt}(\pi^- p \to \eta^0 n)$ experimental(33)	3.5 ± 4. for .024 < -t < .135				7.5 ± 3.9 for .135 < -t < .285		
$p\frac{d\sigma}{dt}(\pi^- p \to \eta_8 n)$ calculated	9.4 ± 4.7	9.9 ± 5.8	7.2 ± 5.	12. ± 5.4	14. ± 4.3	9.8 ± 5.5	7.5 ± 4.2
$p\frac{d\sigma}{dt}(K^- p \to \bar{K}^0 n)$ calculated	31. ± 16.	29. ± 17.	19. ± 13.	26. ± 12.	21. ± 6.1	- 1.7 ± 1.	- 24. ± 13.
$p\frac{d\sigma}{dt}(K^+ n \to K^0 p)$ calculated	15. ± 7.6	17. ± 10.	14. ± 9.9	26. ± 12.	40. ± 12.	40. ± 22.	52. ± 29.

Table II

$(p\frac{d\sigma}{dt})^{(K)}/p\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)$ calculated at 5.9 GeV/c using the same value of s_o as in the preceding table.

$-t$	.1	.2	.3	.6	.9
$p\frac{d\sigma}{dt}(\pi^- p \to \eta_8 n)$	.7	1.1	1.7	- 3.8	-.9
$p\frac{d\sigma}{dt}(K^- p \to \bar{K}^o n)$	1.8	1.4	- 2.4	- 25.	- 3.7
$p\frac{d\sigma}{dt}(K^+ n \to K^o p)$	1.3	2.8	8.5	14.	1.9

11. CONCLUSIONS

We have discussed the influence of cuts on the duality predictions for charge and hypercharge exchange scattering (however, the backward scattering has not been considered). We claim that all the successful predictions of duality remain valid in a very good approximation. Moreover the cuts permit to save the exchange degeneracy in several cases, when otherwise it would have to be badly broken (polarizations, dips). The problem of the behavior under $s \to u$ crossing of the differential cross sections in hypercharge exchange processes is, of course, delicate. We think, however, that it is not the exchange degeneracy but the universality of the Sopkovich formula (more precisely, of the absorptive sign of the cut) that is doubtful.

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