# DIRECT MEASUREMENTS OF RHIC BPM DATA AT THE IP USING LINEAR REGRESSION\*

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## Abstract

Many mature methods to measure the betatron function of a lattice rely on beam position monitor (BPM) data and the model of the whole machine. In this study, specific sections of the Relativistic Heavy Ion Collider (RHIC) were analyzed, taking advantage of BPMs separated by drift spaces near interaction points (IPs) and B3/B4 magnet sections of RHIC. This (local) approach would provide a alternative measure of the linear optics at specific regions which can be compared to previous (global) methods. This process utilizes the phase transfer matrix built from existing BPM data from RHIC using Linear Regression (LR) techniques. Non-AC dipole BPM data as well as AC dipole data was used to measure the linear optics. It was found that the local method yields comparable beta beat to global methods; however, it differs significantly around IP6. This study demonstrates that using LR analysis has advantages and disadvantages, and that further studies are needed to improve the method.

#### INTRODUCTION

RHIC is particle collider with six-fold symmetry for six IPs. Currently, two of these IPs are used for collision experiments to better understand spin and quark-gluon plasma. In order to obtain the desired statistics from these experiments, a large luminosity must be achieved. Luminosity can be determined by many factors. For the optics optimization, the minimum of the beta functions ( $\beta^*$ ) should be where the two bunches collide. Thus, an accurate measurement of the beta function is imperative.

There are countless methods to find the beta function of an accelerator. This includes the three-bpm method and the harmonic analysis to retrieve the twiss parameters [1]. The three-bpm method can also be extended to N bpms, thereby eliminating statistical anomalies from the signals. By utilizing independent component analysis or principal component analysis, one can find the betatron function and phase with more precision [2]. Another popular method is LOCO (Linear Optics from Closed Orbit), used for linear optics correction as well as orbit optimization to ensure beam stability. The method calculates the magnet strengths by taking advantage of the orbit response matrix. This is done by fitting the gradients of the computer model until the model is fitted to the measured response matrix[3].

One method at RHIC utilizes the an equation modeled after describing coherent driven oscillations[4]:

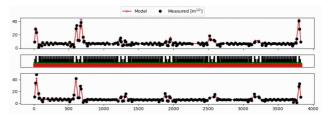


Figure 1: The amplitude function fitted around RHIC lattice using equation 1.

$$x_{co} = A \exp(-2\frac{(\pi n)^2}{B^2})\cos(2\pi\nu n + \phi)$$
 (1)

An exponential term is added to represent the nonlinear beam effects, and the model equation fitted to obtain the amplitude A, tune  $\nu$ , phase offset  $\phi$ , and nonlinear term B that damps the amplitude of oscillation. The amplitude function can be retrieved from the fitting equation 1 to the turn by turn (TBT) data shown in figure 1.

These methods listed above utilize a model beta function such as one from MADx (code to model the accelerator) to find the resulting beta function. These methods usually involve finding the amplitude function first, then using the model to fit the amplitude function to the model to find the optics functions at the BPM locations of the whole ring. These approaches do not take the discrepancy of the model and machine into account, and therefore may introduce bias from the model, possibly leading to an inaccurate beta function.

This paper attempts to specialize around the IPs by deriving the phase space coordinates and utilizing LR analysis in section 2. This will be followed by results and discussion in section 3 from regular and AC dipole TBT data.

## **METHODS**

## Linear Regression

A standalone BPM can only provide the position offset. However, the angle can be retrieved by a pair of BPMs separated by a drift space. The angle coordinates of the particles can be calculated between the start and end points  $x_1$  and  $x_2$  of the region[4]:

$$x_{12}' = \frac{x_2 - x_1}{L} \tag{2}$$

where L is the length of the drift space, and x represents either the horizontal and vertical axis.

We can also model the TBT or phase transfer matrix using a linear system:

$$\widetilde{X} = MX + B. \tag{3}$$

In this case  $\widetilde{X}$  and X are 2x1 matrices, each component representing the initial and final coordinates respectively.

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But when doing a linear regression, these matrices can be represented as Nx2 matrices, where N is the number of turns used. Since M is the 2x2 transfer matrix that takes the initial coordinates to the final coordinates, the matrices can be represented as:

$$X = X_j^i; \widetilde{X} = X_j^{i+1} \tag{4}$$

for the  $i^{th}$  turn and the  $j^{th}$  bpm.

It is important to note that it is possible to make  $\widetilde{X}$  and X 4x1 matrices to get a 4D phase transfer matrix. This was done initially, but it was found that the coupling was minimal

For this study, LR analysis was done using standard scipy packages. Data was selected (explained in the next section) to be fit by the regression method, and the resulting M and Bwere retrieved. The correlation between the prediction from X and  $\widetilde{X}$  was also calculated and visualized. This compares the final coordinates predicted from the LR model vs the actual final coordinates.

## Linear Optics Measurements

After linear regression (LR) is performed, the twiss parameters at the bpm along with det(M) were calculated using the resulting phase transfer matrix:

$$\mathbf{M} = \begin{bmatrix} \cos \phi + \alpha \sin \phi & \beta \sin \phi \\ -\gamma \sin \phi & \cos \phi - \alpha \sin \phi \end{bmatrix}$$
 (5)

The beta beat was also calculated using the model given by the output file of MADx's twiss module.

The number of turns and the starting turn was also varied in this study. 100 turns was chosen for each region when measuring the beta function at a BPM due to the build up of nonlinear effects. The starting turn for the horizontal and vertical axis was usually 35 and 550 respectively since this is where the kick from a corrector is performed to create centroid oscillations.

## RESULTS

## Non-AC Dipole Data

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The BPM data is taken from an sdds file from an APEX run at RHIC. The BPM data was parsed and analyzed using linear regression at 12 sections, four IPs (6, 8, 10, 12) and 2 magnet sections upstream and downstream from each IP. The decoherence effects on LR for the horizontal results for IP8 at the g7 and g8 bpm are shown in figure 2.

For each bpm and each axis, the TBT data was visualized in the first row, and the downstream BPM (in this case g7\_bx) and upstream BPM (g8\_bx) is shown in the left and right column. The second row shows the non-linear effects for a given number of turns in the LR analysis. For example, 100 turns (starting from turn 35) represents the TBT data used in the LR analysis (shown by the dashed lines in the corresponding TBT graph above), and its corresponding y value represents the resulting "scaled" beta beat (beta function difference between the model and the result from LR) as

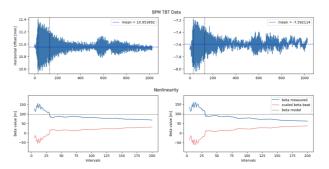


Figure 2: Horizontal TBT data and beta beat measurement due to nonlinear effects at IP8

well as the beta value from LR. The dashed line represents the model beta function.

It was expected that as the number of turns increased, the more apparent the effects of the nonlinearity produced by the ring would be. This is apparent in figure 2; the more turns used, the more the value of beta decreases. It was also a concern that as the number of turns increased, coupling due to the kick from the corrector from the opposing transverse plane along with the background noise would also become more apparent, limiting the accuracy of the LR. However, there are no distinct patterns between the first and second row when considering every region; the decrease in beta happens more so with bpms around IP 6 and 8, and less so around IP 10 and 12, where the red and blue curves were more or less constant. TBT data having the standard shape doesn't imply a smooth or constant nonlinearity curve. This demonstrates a weakened relationship between the LR analysis with nonlinear effects along the ring.

For each bpm, the beta beat was plotted against the correlation between predicted and model coordinates, shown in figure 3.

The top row is missing the bi5 value since the beta value couldn't be measured during the LR analysis (resulted in NaN value) at the 3/4 magnets. The only purpose of the black line is to demonstrate the expected downward trend

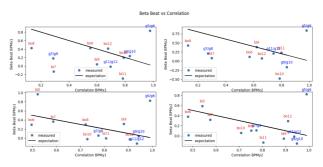


Figure 3: Beta beat vs correlation at each bpm for 100 turns. The top and bottom row show the left and right bpm results and the left and right column show the horizontal and vertical axis respectively. The blue represents the points at IP, the red represents points at 3/4 magnets.

between beta beat and correlation. This is confirmed by the trend of the data points.

A very notable outlier seems to be IP6 (g5/g6 point), as it has a very high correlation and high beta beat. Some possible factors that might lead to this result could be that the LR analysis was done incorrectly, the algorithm could not properly analyze the experimental TBT data and that certain preprocessing steps should be taken first, or that the MADx model beta values at that section are incorrect. Since the trend follows the data well except for IP6, the LR analysis shows some consistency. Another explanation is that the data was taken while an experiment was being run at IP6. One way to check this is to use another dataset and see if there any changes.

Figure 4 shows the average beta beat as well as for IP6/8 and IP10/12 sections for the local and global method respectively. The IP6 and 8 regions were grouped together since they represent experiments from STAR and sPHENIX, respectively.



Figure 4: Average beta beat for each axis using local (blue and red) and global methods (yellow and green).

It is apparent that the global method results from RHIC overall has a lower average beta beat than the local method. Since the NaN value from the bi5 section is not accounted for here, the values for Avg 6/8 and Average beat section for local (shown in blue) may be higher than what is shown. However, the average beat within the 10/12 regions produce similar values between methods. Also, when looking at each section for the horizontal beat, in actuality it is very unclear to quantitatively say which method yields a lower beta beat, as the local method has the lower beat in 12 out of 24 sections, not including the bi5 section. Furthermore, there is no pattern to where it does better; the beat is overall lower in IP8, IP12, bi11, bi7, and a few other sections. This suggests that many methods to find the beta beta should be used and compared, since different methods will yield similar but different results. In this case, LR analysis yields better results in the case of IP8 and IP12 while this global method has overall lower vertical beta beat as well as a better result around the IP6 sections.

# AC Dipole Data

The AC dipole data used was from when the two horizontal and vertical AC dipoles were still installed in the ring before to check the linear optics [5]. The same procedure was performed and most of the results match the non AC dipole data with some minor improvements. However, since the vertical axis data was not provided, the vertical beat couldn't be measured.

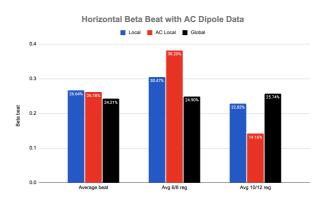


Figure 5: Average beta beat overall and between 6/8 and 10/12 regions for the horizontal axis.

Figure 5 demonstrates the average horizontal beta beat at each section and compares the previous beta beat to the AC Dipole TBT data. Since the bo4/3 magnets downstream of IP5 could be measured this time, the result was a larger beat around the 6/8 region. Closer inspection within those sections show that the beat was either similar or higher than the non AC dipole data. However, the average 10/12 region horizontal beat is lower than the global method's, showing that there is some benefit to using AC dipoles to measure the beta function. But since they were taken out of the ring, this study doesn't show enough improvement to the beta beat to justify bringing them back.

#### **CONCLUSION AND FUTURE STUDIES**

Beta beat is not a complete measure of the accuracy of the beta function, but only a guide since it is always possible that the model is incorrect. However, a high beta beat such as in IP6 in multiple datasets indicates either an incorrect model at that region or that LR analysis could not properly analyze the experimental TBT data at that region due to something unknown at this time. This study demonstrates similar but also some different beta beat results that could be obtained using both local and global methods.

Further studies with other datasets, either AC dipole or non-AC dipole, can be done to reinforce LR techniques and the studies above. Data preprocessing such as ICA and PCA could be done to clean the data as well as potentially uncover patterns that could explain the IP6 discrepancy. Machine learning techniques can also be used to figure out if there are any other factors that contribute to high beta beating.

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#### REFERENCES

- [1] P. Castro-Garcia, "Luminosity and Beta Function Measurement at the Electron-positron Collider Ring LEP", Ph.D. Thesis, CERN, Geneva, Switzerland, Nov. 1996.
- [2] X. Huang, S.Y. Lee, Eric Prebys, and Ray Tomlin, "Application of independent component analysis to Fermilab Booster", Phys. Rev. Spec. Top. Accel. Beams, vol. 8, p. 064001, June. 2005. doi:10.1103/PhysRevSTAB.8.064001
- [3] J. Safranek, "Experimental determination of storage ring optics using orbit response measurements", Nucl. Instrum. Methods

- Phys. Res., Sect. A, vol. 388, p. 27, 1997. doi:10.1016/S0168-9002(97)00309-4
- [4] H. Wiedemann, Particle Accelerator Physics 4th Edition. Berlin, Germany: Springer, 2015, pp.213-231.
- [5] M. Bai, S. Peggs, T. Roser, T. Satogata, and D. Trbojevic, "Measuring Beta Function and Phase Advance in RHIC with an AC Dipole", in Proc. PAC'03, Portland, OR, USA, May 2003, pp. 2204-2206.

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