

Pion sub-leading twist generalized transverse momentum dependent parton distributions E_2 in light-front quark model

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Introduction

Understanding the internal structure of the hadrons in terms of quarks and gluons degree of freedom is the most challenging task in quantum chromodynamics (QCD) from both theoretical and experimental point of view [1]. Study of hadron structure covers both non-perturbative and perturbative areas of QCD where formerly valence quarks play a vital role and in the latter gluons contribution is more dominating.

The three-dimensional internal structure of the hadrons can be revealed through the parton distribution functions (PDFs) and their generalized forms known as generalized parton distributions (GPDs) and transverse momentum dependent parton distributions (TMDs). PDFs provide only one-dimensional information about the longitudinal momentum fraction (x) carried by a quark within its parent hadron. PDFs can be accessed by the deep-inelastic scattering process.

GPDs can be accessed through the deep virtual Compton scattering (DVCS) process and provide spatial information about the distribution of partons in position space. TMDs can be accessed through the semi-inclusive deep inelastic scattering (SIDIS) or Drell-Yan (DY) process, providing information on the

distribution of partons in momentum space. In this work, we focus on the sub-leading twist T-even TMD E_2 for the pion. In literature, various studies are present for the nucleons, but rarely available for the pion. Generalized transverse momentum dependent parton distributions (GTMDs) also called as mother distributions which contains all the information about the internal structure of the hadrons. We consider the light-front quark model (LFQM) to calculate the sub-leading twist GTMDs for the pion. We consider the minimal Fock state, in which a meson can be expressed as $|M\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}}$.

Light-Front Quark Model

LFQM provides a useful framework to understand and model the hadron structure using quarks and gluons degrees of freedom. Using the light-front Fock state expansion, a meson state can be expressed as

$$|M\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}} + \sum |q\bar{q}g\rangle\psi_{q\bar{q}g} + \dots \quad (1)$$

In this model, we express minimal state description in the form of a quark-antiquark pair as

$$|M(P, S_z = 0)\rangle = \sum_{\lambda_i, \lambda_j} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \Psi(x, \mathbf{k}_\perp, \lambda_i, \lambda_j) |x, x\mathbf{P}_\perp + \mathbf{k}_\perp, \lambda_i, \lambda_j\rangle, \quad (2)$$

where P is the four momentum of meson state, x and $1-x$ is the longitudinal momen-

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tum fraction of quark and anti-quark respectively. The light-front meson wave function $\Psi(x, \mathbf{k}_\perp, \lambda_i, \lambda_j)$ describes the meson state with different spin and helicity projections of the constituent quarks.

GTMDs

For pion, valence quark GTMDs can be expressed in terms of quark-quark correlator as

$$\Phi_q^{[\Gamma]} = \int \frac{dz^- d^2 \mathbf{z}_\perp}{32\pi^3} e^{i\mathbf{k}\cdot\mathbf{z}} \langle M(P^*) | \bar{\psi}(-z/2) \Gamma \mathcal{W}(-z/2, z/2) \psi(z/2) | M(P) \rangle, \quad (3)$$

where Γ represent the gamma matrices, whose specific form depends on the choice of GTMDs sandwiched between the initial and final hadronic states. For the present work, we consider only $\Gamma = 1, i\sigma^{ij}\gamma_5$ which leads to GTMD $E_2(x, \mathbf{k}_\perp, \Delta_\perp, \mathbf{k}_\perp \cdot \Delta_\perp)$ and $H_2(x, \mathbf{k}_\perp, \Delta_\perp, \mathbf{k}_\perp \cdot \Delta_\perp)$. From the Eq.(3), we express it as [2]

$$\Phi_q^{[1]} = \frac{M_\pi}{P_+} E_2(x, \mathbf{k}_\perp, \Delta_\perp, \mathbf{k}_\perp \cdot \Delta_\perp), \quad (4)$$

and their explicit expressions are

$$E_2 = \frac{1}{16\pi^3} \left[\left(m_u^2 + \mathbf{k}_\perp^2 - \frac{(1-x)^2 \Delta_\perp^2}{4} + (1-x) \frac{\Delta_\perp^2}{2} \right) \right] \frac{\psi^*(x, \mathbf{k}'_\perp) \psi^*(x, \mathbf{k}_\perp)}{\omega^+ \omega^-}, \quad (5)$$

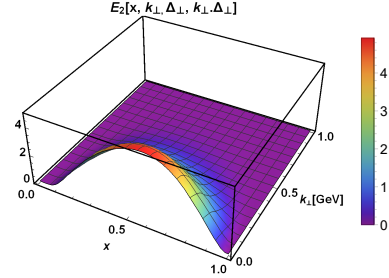
where

$$\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x) \frac{\Delta_\perp}{2}, \quad (6)$$

$$\mathbf{k}_\perp = \mathbf{k}_\perp - (1-x) \frac{\Delta_\perp}{2}, \quad (7)$$

$$\omega^+ = \sqrt{x_1 x_2 \left[\frac{(\mathbf{k}'_\perp{}^2 + m_u^2)}{x_1 x_2} - (m_u^2 - m_d^2)^2 \right]}, \quad (8)$$

$$\omega^- = \sqrt{x_1 x_2 \left[\frac{(\mathbf{k}_\perp{}^2 + m_u^2)}{x_1 x_2} - (m_u^2 - m_d^2)^2 \right]}. \quad (9)$$



Conclusion

In this work, we calculate the sub-leading twist GTMD E_2 for the pion using light-front quark model. GTMD E_2 is calculated from the overlap of light-front wave functions. We observed that E_2 exhibits positive distribution and peaks around $x = 0.5$ for fixed value of $\Delta_\perp = 1\text{GeV}$. E_2 GTMD has direct connection with quark PDF $e(x)$ and offers important information about the physics of widely-unexplored quark-gluon correlations. Its integral over x gives scalar charge of the nucleon and pion-nucleon σ -term. Additionally one can also find out the results for the GPDs and TMDs in the certain limits over transverse momentum. One can extend this work, by considering the higher Fock states to achieve a more comprehensive description of pion internal structure.

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