The Profiled Feldman-Cousins method for confidence interval estimation in the presence of nuisance parameters

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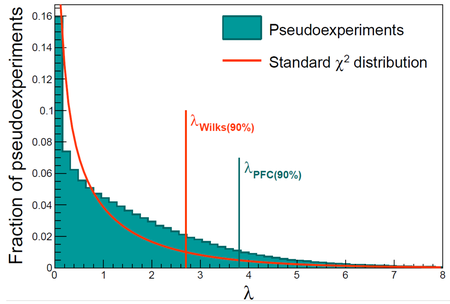
Motivation

The **Unified Approach** [1] or **Feldman-Cousins** approach is commonly used in particle physics to construct **confidence intervals** when Wilks' theorem [2] conditions are not satisfied.

Pseudoexperiments are generated and fitted to build empirical test statistic distributions which may deviate from standard χ^2 distributions.



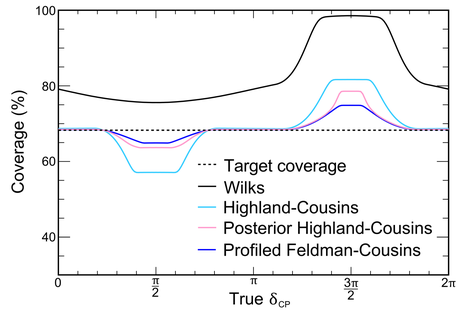
How to handle the nuisance parameters when generating Feldman-Cousins pseudoexperiments?

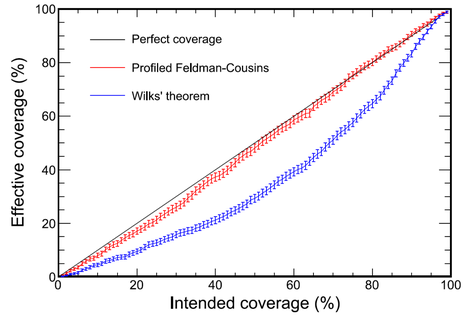
The Profiled Feldman-Cousins method

Several plausible approaches to generate the FC pseudoexperiments have been investigated. For each set of parameters of interest:

- set the nuisance parameters to a chosen **prior estimate**.
- chose the nuisance parameters that yield the most **conservative** critical values.
- chose a range of reasonable values for the nuisance parameters and estimate a p-value from pseudoexperiments generated at those points. **Berger-Boos** [3] propose to keep the largest p-value.
- generate the nuisance parameters from their prior probability distributions, following the **Highland-Cousins** [4] recommendation.
- generate the nuisance parameters from their posterior probability distributions given the observed data, called here **Posterior Highland-Cousins**.
- **profile the nuisance parameters given the data and set the nuisance parameters to their best fit values.**

Using a toy model, the most accurate coverage was achieved with our **Profiled Feldman-Cousins approach.**





Implementation in the NOvA analysis

NOvA [5] is a long-baseline neutrino oscillation experiment aiming to constrain **the neutrino oscillation parameters**.

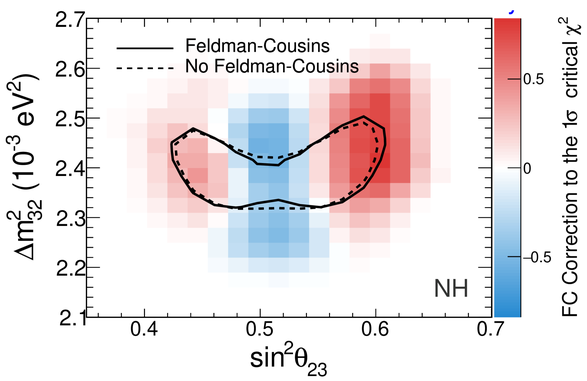
Wilks' theorem violation in NOvA:

1- Sample size: neutrino interactions are rare.

2- Effective boundaries: oscillation probabilities are bounded, δ_{CP} is cyclical.

3- Nested hypothesis: mass ordering and θ_{23} octant are binary questions.

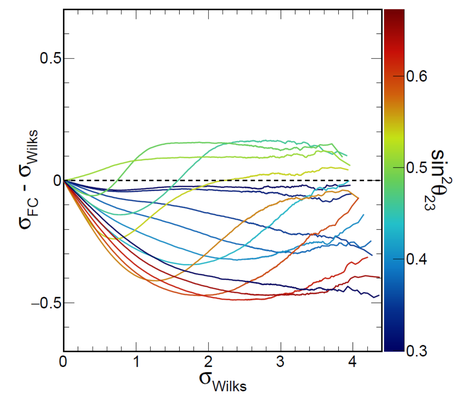
Millions of **FC pseudoexperiments** are generated and fitted to correct NOvA's constraints on Δm^2_{32}, θ_{23}, and δ_{CP}.



Impact of PFC corrections on the NOvA Δm^2_{32} vs. $\sin^2\theta_{23}$ 1σ constraint. [6]

Features and limitations

- No way to **predict** the amplitude of corrections.
- Cannot **extract significance** of a set of parameters of interest from 1D/2D significance plots.
- Profiled nuisance parameters can be very different for similar parameters of interest → **significance discontinuities** in 1D/2D plots.
- **Computationally expensive** → supercomputers [7].
- **Combining** likelihoods from **separate experiments** requires complete knowledge of the nuisance parameters and the generation of new FC pseudoexperiments.



Change of significance from the PFC procedure for different significance levels and at different $\sin^2\theta_{23}$ values.

References: [1] G. Feldman, R. Cousins. doi:10.1103/PhysRevD.57.3873 [2] S. Wilks, doi:10.1214/aoms/1177732360 [3] R. Berger, D. Boos doi:10.2307/2290928 [4] R. Cousins, V, Highland. doi:10.1016/0168-9002(92)90794-5
[5] Ayres, D. et al. (2007) doi:10.2172/935497 [6] M.A. Acero et al. arXiv:2108.08219 [7] A. Sousa et al. 10.1051/epjconf/201921405012

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