Electric Dipole Moment Measurements at Storage Rings

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Abstract. Electric Dipole Moments (EDM) of elementary particles, including hadrons, are considered as one of the most powerful tools to discover CP-violation beyond the already known sources of the Standard Model. Such CP-violating mechanisms are required to explain the dominance of matter over anti-matter in our universe.

Up to now experiments concentrated on neutral systems. Storage rings offer the possibility to measure EDMs of charged particles by observing the influence of the EDM on the spin motion. Different options and a strategy towards storage ring EDM measurements will be discussed.

1. Introduction & Motivation

The existence of electric dipole moments (EDMs) of (sub-)atomic particles (e.g. leptons, atoms, certain molecules, hadrons) is only possible if parity (P) and time reversal (T) symmetry are violated. Assuming that the CPT theorem holds, T-violation is equivalent to CP-violation. Note that in this context we talk only about *permanent* EDMs. The well known EDMs of certain molecules (e.g. H_20, NH_3) are not of this nature and don't require violation of P and T symmetries. These molecules appear to have a permanent EDM because of two almost degenerated energy levels of opposite parity. This implies that the energy levels grow linearly with an applied electric field – a sign of a permanent EDM. However, in very small electric fields E, the energy levels grow quadratically with the electric field strength (quadratic Stark effect). This is the case if the interaction energy eE is smaller than the energy difference of the two almost degenerated energy levels. A more detailed discussion can be found in Ref. [1].

The measurement of EDMs has a long history. Starting 60 years ago with the measurement of the neutron EDM by Smith, Purcell and Ramsey [2]. Fig. 1 shows an overview of experimental results for various particles. Up to now all measurements show results consistent with zero. The resulting upper limits for various particles together with predictions from super-symmetric models (SUSY) and the Standard Model are shown. Most of the measurements were performed on neutral systems. The proton limit was deduced from an EDM measurement of the mercury atom for example. The limit shown in Fig. 1 for the muon was obtained at a storage ring experiment where the main purpose was to measure the anomalous magnetic moment of the

[1] http://collaborations.fz-juelich.de/ikp/jedi/
[2] http://pbc.web.cern.ch/edm/edm-default.htm

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muon [3]. Based on this idea, experiments are proposed to measure EDMs of charged particles in storage rings [4, 5]. The principle will be discussed in the next section.

2. Principle of Storage Ring EDM Experiments

For an elementary particle, the spin is the only vector defining a direction. A possible EDM has to be aligned along this axis. If an EDM exists, the spin vector will experience a torque in addition to the one caused by the magnetic moment. For a particle kept on a horizontal orbit by a radial magnetic and/or vertical electric field the EDM causes a rotation of the spin vector around the radial axis as indicated in Fig. 2. The magnetic moment causes a rotation around the vertical axis. For a particle ensemble with a spin polarisation initially aligned along the momentum vector, this torque causes a build-up of a polarisation component in the vertical direction. The polarisation direction can be determined by scattering the beam off a carbon target and analyzing the azimuthal distribution of the scattered particles. A vertical polarisation results in an left-right asymmetry in the detector.

Quantitatively the spin motion relative to the momentum vector in electric and magnetic fields is governed by the Thomas-BMT equation [6–8]:

$$\frac{d\vec{S}}{dt} = (\vec{\Omega}_{\mathrm{MDM}} + \vec{\Omega}_{\mathrm{EDM}}) \times \vec{S}, \tag{1}$$

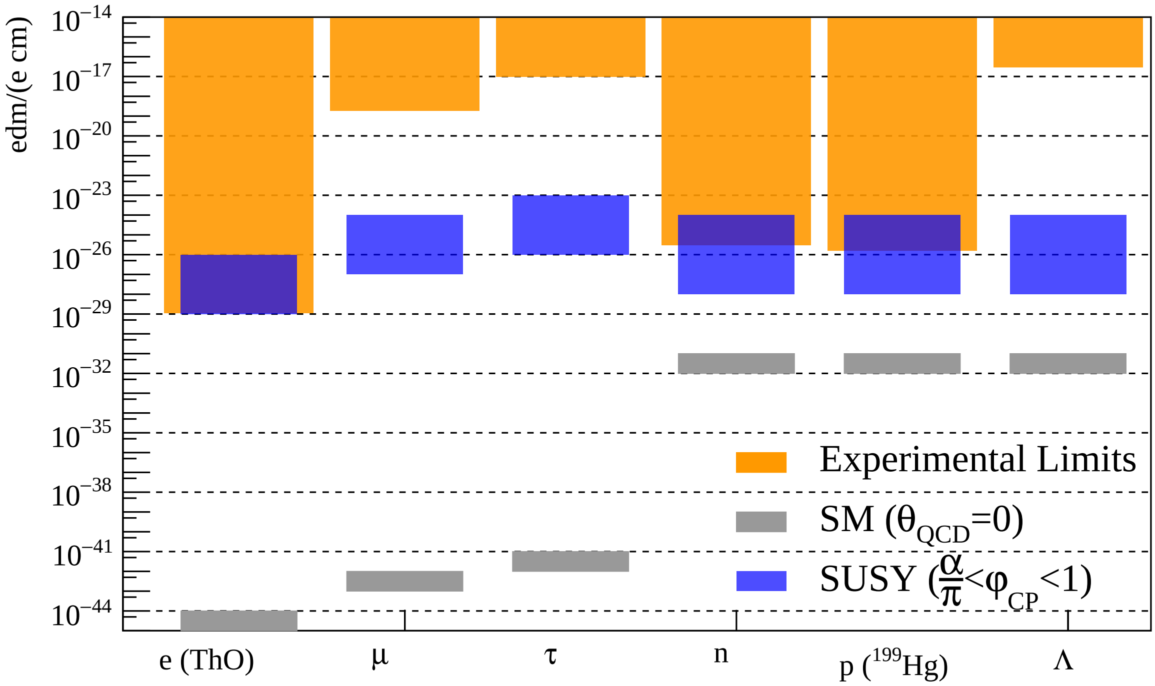

Figure 1. Experimental limits of EDMs together with predictions from super symmetric models (SUSY) and the Standard Model (SM).

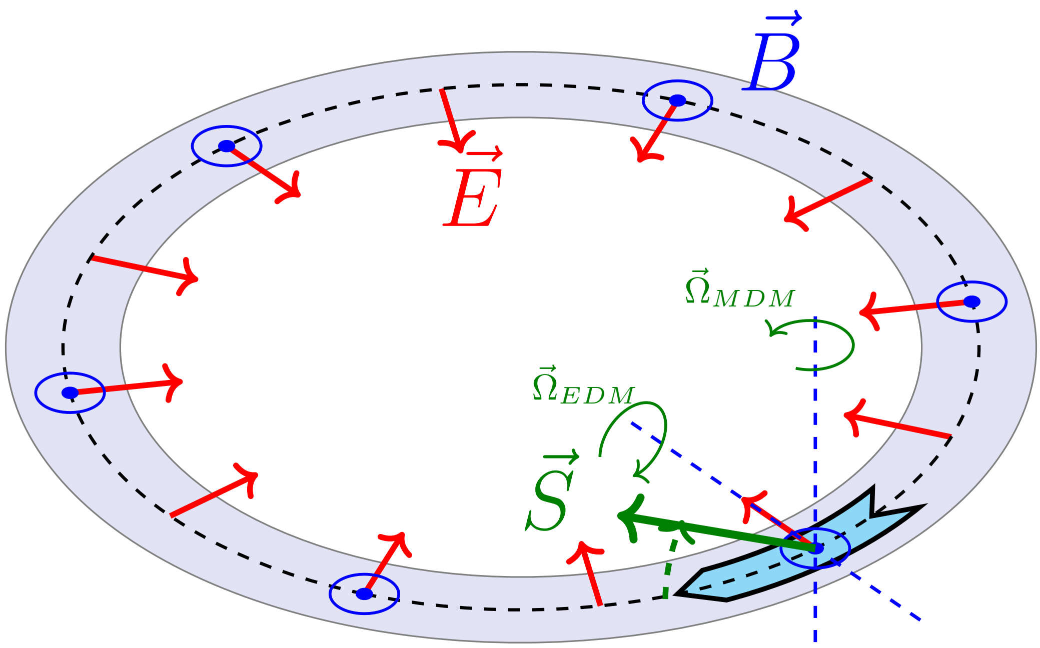

Figure 2. Principle of a storage ring EDM measurement.

$$\vec{\Omega}_{\text{MDM}} = -\frac{q}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right], \tag{2}$$

$$\vec{\Omega}_{\text{EDM}} = -\frac{\eta q}{2mc}\left[\vec{E} + c\vec{\beta}\times\vec{B}\right]. \tag{3}$$

$\vec{S}$ in this equation denotes the spin vector in the particle rest frame, t the time in the laboratory system, β and γ the relativistic Lorentz factors, and $\vec{B}$ and $\vec{E}$ the magnetic and electric fields in the laboratory system, respectively. To simplify the notation terms including $\vec{\beta}\cdot\vec{B}$ and $\vec{\beta}\cdot\vec{E}$ were omitted.

The magnetic dipole moment $\vec{\mu}$ and electric dipole moment $\vec{d}$ both pointing in the direction of the particle's spin $\vec{S}$ are related to the dimensionless quantities G (magnetic anomaly) and η in eq. 1:

$$\vec{\mu} = g\frac{q\hbar}{2m}\vec{S} = (1+G)\frac{q\hbar}{m}\vec{S} \quad \text{and} \quad \vec{d} = \eta\frac{q\hbar}{2mc}\vec{S}\,. \tag{4}$$

The difficulty of the experiment is that in general the magnetic moment (terms proportional to G in eq. 1) causes a precession of the spins orders of magnitude higher compared to the expected effect of the EDM. The G factors of proton and deuteron are 1.79 and -0.14 respectively. The factor η amounts to $2\cdot 10^{-15}$ for an EDM of $d = 10^{-29}e\,\text{cm}$.

Analyzing eq. 1, one has several possibilities. The most favorable option is to run in a so called frozen-spin condition where the precession in the horizontal plane is suppressed by a suitable choice of an electric and magnetic field combination, i.e. $\Omega_{\text{MDM}} = 0$, see e.g. [9]. In order to achieve this

$$G\vec{B} - \left(G - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c} = 0 \tag{5}$$

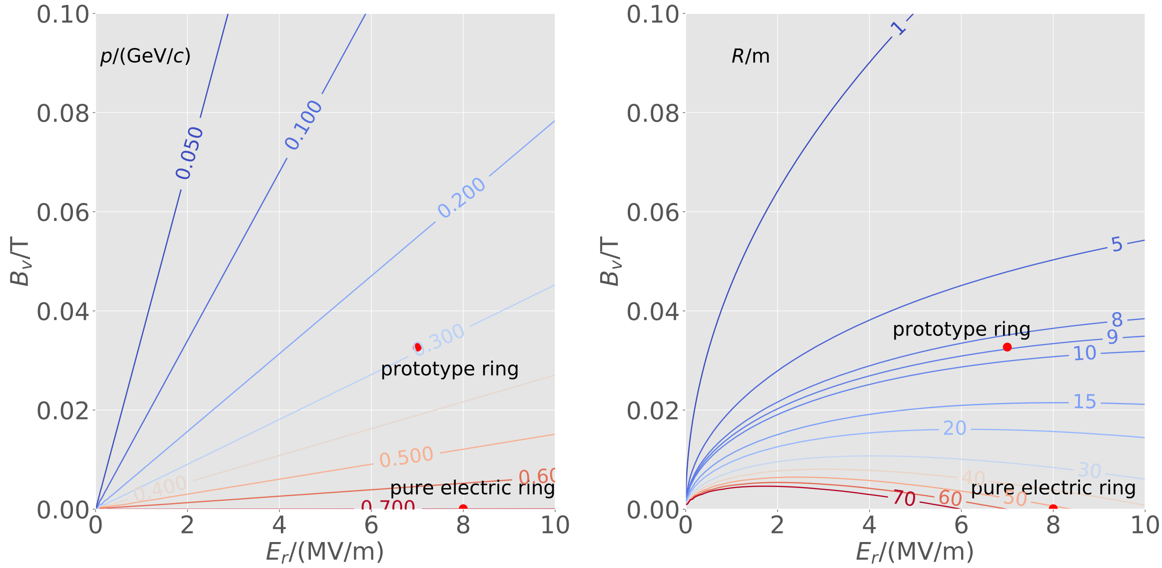

Figure 3. Proton momentum (left) and storage ring radius (right) for different values of the magnetic and electric field. For a pure electric ring (i.e. $B = 0$) the momentum is fixed to $p = 0.7007\,\mathrm{GeV}/c$.

has to be fulfilled. Assuming radial electric fields (E_r) and vertical magnetic fields (B_v) this can be written as

$$\left(1 - G\beta^2\gamma^2\right) E_r = G B_v c \beta \gamma^2 \,. \tag{6}$$

Fig. 3 and 4 show the particle momentum and bending radius of a storage ring for possible combinations of magnetic and electric fields for protons and deuterons where eq. 6 is fulfilled. For particles with $G > 0$ this can be achieved with a pure electric storage ring (i.e. $B = 0$). For protons ($G \approx 1.79$) this means that the momentum has to be $p = 0.7007\,\mathrm{GeV}/c$ in order to have $G - \frac{1}{\gamma^2-1} = 0$. For particles with $G < 0$ (e.g. deuterons with $G \approx -0.14$) one needs both electric and magnetic fields. Also note that for particles with $G < 0$, fulfilling eq. 6 means that the bending of electric and magnetic field act in opposite directions.

If the polarisation vector points initially in the direction of the momentum vector, the EDM causes a rotation out of the horizontal plane. In practice, since the effect of the EDM is expected to be very small, the observable is just a build-up of a tiny vertical polarisation.

In a pure magnetic storage ring, it is impossible to null Ω_{MDM}. This prevents a continuous build-up of a vertical polarisation due to the EDM, because the spin precesses in the horizontal plane such that the build-up due to Ω_{EDM} is half of the time in the positive and half of the time in the negative direction. The net effect is an oscillation of the vertical polarisation with an amplitude $\beta\eta/(2G)$. This signature is used in the muon $g-2$ experiment to measure the muon EDM [3]. For hadrons this method is less sensitive because $G_{\mathrm{hadron}} \gg G_\mu \approx 10^{-3}$. For $p = 970\,\mathrm{MeV}/c$ deuterons the amplitude is only $3 \cdot 10^{-15}$ for an EDM of $d = 10^{-29} e\,\mathrm{cm}$.

To allow for a build-up of the vertical polarization proportional to the EDM, a radio-frequency (RF) Wien-filter has to be operated [10, 11]. Running at a resonance frequency of the spin precession, it leads to the desired build-up of the vertical polarisation proportional to the EDM. Qualitatively this can be understood as follows. Running at resonance with the spin precession frequency the Wien filter advances/slows down the spin motion in the horizontal plane in such a way that on average it points more parallel than anti-parallel to the momentum vector. This leads finally to a build-up proportional to the EDM due to the term $c\vec{\beta} \times \vec{B}$ in eq. 1, because

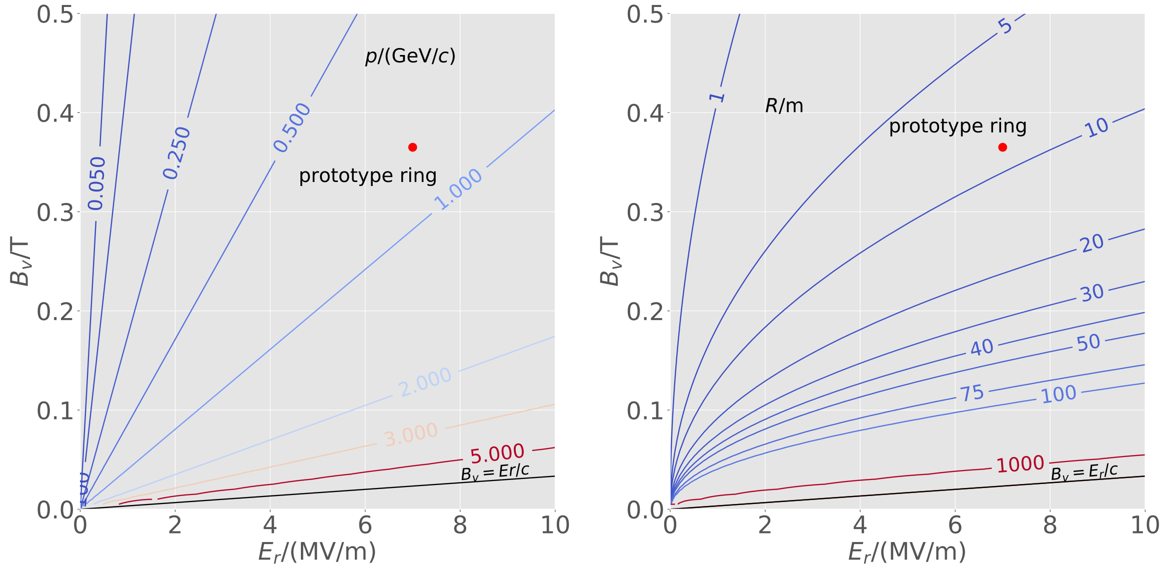

Figure 4. Deuteron momentum (left) and storage ring radius (right) for different values of the magnetic and electric field. Note that for the deuteron ($G < 0$) a magnetic field $B > E/c$ is always required to maintain the frozen spin condition.

Table 1. Different options for storage ring EDM experiments.

Options	pros	cons
1.) pure electric ring	no $\vec{B}$ field needed, CW/CCW beams simultaneously	works only for particles with $G > 0$ (e.g. p)
2.) combined ring	works for p, d, ^{3}He, ...	both $\vec{E}$ and $\vec{B}$ required
3.) pure magnetic ring	existing (upgraded) COSY ring can be used, shorter time scale	lower sensitivity, precession due to G, i.e. no frozen spin

now the average over the ring $< \vec{S} \times \vec{\beta} \times \vec{B} > \neq 0$.

Tab. 1 lists the options discussed. The main advantages of option 1 is the possibility to run counter circulating beams at the same time. As will be discussed in sec. 3 this is a big advantage for the cancellation of systematic effects. The disadvantage is that it works only for particles with $G > 0$. Option 2 works for all G, but to run a beam in opposite direction the magnetic field has to be reversed. Option 3 finally has the advantage that magnetic rings, like the Cooler Synchrotron COSY at Forschungszentrum Jülich already exist. The reachable statistical sensitivity is worse as explained in eq. 3.

3. Statistical and Systematic Sensitivities

3.1. Statistical Sensitivities

Assuming $\Omega_{\rm MDM} = 0$, the EDM can be determined from the measurement of $\Omega \equiv \Omega_{\rm EDM}$:

$$\Omega_{\rm EDM} = \frac{dE}{s\hbar} \,. \tag{7}$$

Here we assume a pure electric ring. For the combined ring E has to be replaced by $E \pm \beta c B$.

Assuming a polarisation vector initially along the momentum vector and $\Omega_{\rm EDM}\tau \ll 1$, we get

$$\dot{P}_v = \Omega_{\rm EDM} P = \frac{dE}{\hbar s}\, P\,, \tag{8}$$

where P is the polarisation of the beam and $P_v \ll P$ the vertical component.

This results in

$$d = \frac{s\hbar \dot{P}_v}{EP} = \frac{s\hbar}{EP}\frac{(P_v(\tau) - P_v(0))}{\tau}\,, \tag{9}$$

where τ is the duration of the measurement. This describes the most simple scenario where half of the beam is extracted at $t = 0$ to measure $P_v(0)$ and half of the beam at $t = \tau$ to measure $P(\tau)$.

The statistical error on the polarization measurement from the azimuthal distribution of events is given by

$$\sigma^2(P_v) = \frac{2}{A^2(fN/2)}\,, \tag{10}$$

where A is the analyzing power of the scattering process. N is the number of particles in the beam, f the fraction of particles elastically scattered and detected in the polarimeter. For the polarization difference one finds

$$\sigma^2(P_v(\tau) - P_v(0)) = \frac{4}{A^2(fN/2)}\,. \tag{11}$$

We finally obtain

$$\sigma_{\rm EDM} = \sqrt{8}\,\frac{s\hbar}{\sqrt{Nf}APrE\tau}\,. \tag{12}$$

In a more realistic scenario where the polarisation is continuously monitored instead of just taking two measurements at $t = 0$ and $t = \tau$, the rhs has to be multiplied by a factor $\sqrt{3}$. The factor r accounts for the fraction of the ring where the electric and magnetic fields are present.

For option 3 using the Wien filter, eq. 12 has to be modified as follows: the electric field E has to be replaced by the electric field in the Wien filter and r by the fraction of the ring equipped with the Wien filter. A factor $(2G\gamma^2)/(G + 1)$ has to be included as well. In Eq. 12 it is assumed that the beam polarisation is constant during the measurement duration τ. Tab. 2 gives the statistical error for the three different options discussed.

3.2. Systematic Sensitivities

The major sources of systematic errors and ways to mitigate them will be discussed. The discussion is restricted to the frozen spin method with electric fields only.

The observable is $\Omega_{\rm EDM}$. For an EDM of $10^{-29} e\,$cm and an electric field of $E = 8\,$MV/m for protons ($s = 1/2$) it is

$$\Omega_{\rm EDM} = \frac{dE}{s\hbar} = 2.4 \cdot 10^{-9}\,{\rm s}^{-1}\,. \tag{13}$$

We will now discuss several systematic effects and compare their contribution to $\vec{\Omega}$ to the one in eq. 13.

(i) Radial magnetic field: A remanent radial magnetic field of $B_r = 10^{-17}\,$T leads to an

$$\Omega_{B_r} = \frac{eGB_r}{m} = 1.7 \cdot 10^{-9}\,{\rm s}^{-1} \tag{14}$$

which is similar to $\Omega_{\rm EDM}$.

	pure magnetic ring with Wien filter	combined ring	pure electric ring
P		0.8	
N	10^9	$2 \cdot 10^9$	$4 \cdot 10^{10}$
f		0.005	
A	0.6	0.2	0.6
τ		1000 s	
E, B	3 kV/m, 22 μT	7.3 MV/m, 0.03 T	8 MV/m, −
r	1/184	0.55	0.65
$\sigma_{\rm EDM}$(1fill)/e cm	$8.6 \cdot 10^{-21}$	$5.5 \cdot 10^{-26}$	$4.6 \cdot 10^{-27}$
$\sigma_{\rm EDM}$(1year)/e cm 10000 fills	$8.6 \cdot 10^{-23}$	$5.5 \cdot 10^{-28}$	$4.6 \cdot 10^{-29}$

Table 2. The statistical uncertainty for the three different options proposed assuming a continuous extraction of the beam. The beam polarisation P is assumed to be constant during the measurement time τ.

(ii) Geometric Phases: Imagine the following sequence of rotations:
A rotation around the vertical y-axis followed by a rotation around the longitudinal z-axis by an angle ϑ compensated by rotations by $-\vartheta$ around the y-axis and finally the longitudinal z-axis. This results, like the EDM, in a net rotation around the radial x-axis of approximately ϑ^2. Such rotations can for example be caused by misalignment of ring elements. Assume for example alternating B =1 nT longitudinal and vertical magnetic fields in four segments of 90° in the ring [12]. In each segment the rotation angle is

$$\vartheta = \frac{eGB}{m} \frac{1}{4 f_{\rm rev}} = 8.6 \cdot 10^{-8} \tag{15}$$

where $f_{\rm rev} \approx 0.5\,$MHz is the revolution frequency. This leads to

$$\Omega_{\rm GP} = \vartheta^2 f_{\rm rev} \approx 3.7 \cdot 10^{-9}\,{\rm s}^{-1}, \tag{16}$$

which is again of the same order as $\Omega_{\rm EDM}$.

Ways to fight these systematic effects are described in Refs. [4, 13] and are still under investigation. Here we just describe the main ideas: The key point is to use clockwise (CW) and counter clockwise circulating (CCW) beams. One measures thus two frequencies or rather polarisation build-ups, $\Omega_{\rm CW}$ and $\Omega_{\rm CCW}$. Schematically one can write

$$\Omega_{\rm CW} = \Omega_{\rm EDM} + \Omega_{\rm GP} + \Omega_{B_r}\,, \tag{17}$$

$$\Omega_{\rm CCW} = \Omega_{\rm EDM} - \Omega_{\rm GP} + \Omega_{B_r}\,. \tag{18}$$

Taking the sum $\Omega_{\rm CW} + \Omega_{\rm CCW}$ the contribution from geometrical phases, $\Omega_{\rm GP}$ will cancel. The effect of a remanent radial magnetic field Ω_{B_r} does not but it will cause a vertical displacement of the two beams which can be used to disentangle a build-up due to an EDM or a radial B-field.

At this level of precision even effects of gravity have to be taken into account. There are two contributions, a direct one [14–16]

$$\Omega_{\rm grav} = \frac{2\gamma + 1}{\gamma + 1} \frac{\beta g}{c} = 3 \cdot 10^{-8}\,{\rm s}^{-1} \tag{19}$$

on the spin rotation, with $g = 9.81\,\mathrm{ms}^{-2}$, and an indirect one. To prevent the particles from falling, a restoring force either in form of a vertical electric or radial magnetic field has to be present. If gravity is compensated by a radial magnetic field its magnitude is

$$B_r = \frac{(2\gamma^2 - 1)mg}{\gamma e \beta c} = 1.2 \cdot 10^{-15}\,\mathrm{T} \tag{20}$$

which will cause in turn a fake EDM effect as described above [17]. Note that for a correct treatment of the gravity and other systematic effects eq. 1 is no more applicable because is uses the simplified assumption that the particle is kept on a circular orbit simply by a radial electric and/or vertical magnetic field. Nevertheless, to estimate the order of magnitude of various contributions, it can be used.

From the examples discussed above, it is evident that the only possibility to disentangle systematic effects from the EDM build-up is to operate a storage ring with counter rotating beams. The main advantage of the pure electric ring is that two beams can circulate at the same time on the same orbit, except for small shifts due to B_r. This assures that Ω_{GP} occurring in Ω_{CW} and Ω_{CCW} is really identical. Note that in other EDM experiments one either takes data shifted in time (e.g. by a few minutes in the neutron EDM experiment [18]) or in space (two cells with opposite E-fields separated by a few cm in the Hg-EDM experiment [19]). This is also the case for a combined ring. One has to reverse the magnetic field to circulate a beam in the opposite direction and take two consecutive runs apart by the measurement duration of $\tau \approx 1000\,\mathrm{s}$. In this case conditions may have changed such that the cancellation in $\Omega_{\mathrm{CW}} + \Omega_{\mathrm{CCW}}$ is not perfect.

The examples discussed above is not an exhaustive list. Other sources of systematic errors are still under consideration. These include differences of beam intensities and phase space of the two counter circulating beams and off axis passage of the beams in the RF cavity.

4. Status & Strategy

In December 2018 a document [20] has been submitted to the European Strategy for Particle Physics (ESPP). It outlines a strategy for a storage ring EDM experiment. It starts with a continuation of the so called precursor experiment at the magnetic ring COSY aiming at a first measurement of the deuteron EDM. Several milestones have already been reached:

(i) a spin coherence times of over 1000 s needed to achieve the statistical sensitivity [21],

(ii) a precise determination of of the horizontal spin precession frequency [22] and

(iii) a polarisation feedback system [23] was developed needed in a dedicated ring to maintain the frozen spin condition.

A vertical polarisation build-up could also be observed in a first test run using the Wien filter method. At this stage the build-up is dominated by systematic effects which are still under investigation.

As can be judged from Fig. 3, a pure electric storage ring will have a radius of around 50 m in order to keep the electric field at a achievable level of $\approx 8\,\mathrm{MV/m}$. Before starting the construction of such a storage ring it is proposed to construct a prototype ring of smaller size with a ring radius of about 9 m [24]. Many components could be tested in such a ring. Operating the ring only with electric fields, one could test simultaneously circulating counter-rotating beams, albeit not in the frozen spin mode. To achieve this, an additional magnetic field of about 0.03 T (see Fig. 4) is necessary. This would allow to measure the proton EDM. With a magnetic field of 0.38 T the deuteron EDM could also be measured.

5. Acknowledgments
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References

[1] Wirzba A, Bsaisou J and Nogga A 2017 *Int. J. Mod. Phys.* **E26** 1740031 (*Preprint* 1610.00794)
[2] Smith J H, Purcell E M and Ramsey N F 1957 *Phys. Rev.* **108** 120–122
[3] Bennett G W *et al.* (Muon (g-2)) 2009 *Phys. Rev.* **D80** 052008 (*Preprint* 0811.1207)
[4] Anastassopoulos V *et al.* 2016 *Rev. Sci. Instrum.* **87** 115116 (*Preprint* 1502.04317) URL https://doi.org/10.1063/1.4967465
[5] JEDI Collaboration, proposals available from http://collaborations.fz-juelich.de/ikp/jedi/
[6] Bargmann V, Michel L and Telegdi V L 1959 *Phys. Rev. Lett.* **2** 435–436 URL https://doi.org/10.1103/PhysRevLett.2.435
[7] Nelson D F, Schupp A A, Pidd R W and Crane H R 1959 *Phys. Rev. Lett.* **2** 492–495
[8] Fukuyama T and Silenko A J 2013 *Int. J. Mod. Phys.* **A28** 1350147 URL https://doi.org/10.1142/S0217751X13501479
[9] Farley F J M, Jungmann K, Miller J P, Morse W M, Orlov Y F, Roberts B L, Semertzidis Y K, Silenko A and Stephenson E J 2004 *Phys. Rev. Lett.* **93** 052001 (*Preprint* hep-ex/0307006)
[10] Rathmann F, Saleev A and Nikolaev N N (JEDI, srEDM) 2013 *J. Phys. Conf. Ser.* **447** 012011
[11] Morse W M, Orlov Y F and Semertzidis Y K 2013 *Phys. Rev. ST Accel. Beams* **16**(11) 114001 URL http://link.aps.org/doi/10.1103/PhysRevSTAB.16.114001
[12] Hacmerolu S, Orlov Y F and Semertzidis Y K 2019 *Nucl. Instrum. Meth.* **A927** 262–266 (*Preprint* 1812.02381)
[13] Abusaif F *et al.* 2019 (*Preprint* 1912.07881) URL https://arxiv.org/abs/1912.07881
[14] Y Semertzidis Y Orlov E F 2012 *Phys. Lett.* **A376** 2822–2829
[15] Khriplovich I B and Pomeransky A A 1998 *J. Exp. Theor. Phys.* **86** 839–849 [Zh. Eksp. Teor. Fiz.113,1537(1998)] (*Preprint* gr-qc/9710098)
[16] Kobach A 2016 *Nucl. Phys.* **B911** 206–211 (*Preprint* 1603.00127)
[17] Silenko A J and Teryaev O V 2007 *Phys. Rev.* **D76** 061101 (*Preprint* gr-qc/0612103)
[18] Baker C A, Doyle D D, Geltenbort P, Green K, van der Grinten M G D, Harris P G, Iaydjiev P, Ivanov S N, May D J R, Pendlebury J M, Richardson J D, Shiers D and Smith K F 2006 *Phys. Rev. Lett.* **97**(13) 131801 URL http://link.aps.org/doi/10.1103/PhysRevLett.97.131801
[19] Graner B, Chen Y, Lindahl E G and Heckel B R 2016 *Phys. Rev. Lett.* **116** 161601 (*Preprint* 1601.04339)
[20] Abusaif F *et al.* 2018 (*Preprint* 1812.08535) URL https://arxiv.org/abs/1812.08535
[21] Guidoboni G *et al.* (JEDI) 2016 *Phys. Rev. Lett.* **117** 054801 URL https://doi.org/10.1103/PhysRevLett.117.054801
[22] Eversmann D *et al.* (JEDI) 2015 *Phys. Rev. Lett.* **115** 094801 (*Preprint* 1504.00635) URL https://doi.org/10.1103/PhysRevLett.115.094801
[23] Hempelmann N *et al.* (JEDI) 2017 *Phys. Rev. Lett.* **119** 014801 (*Preprint* 1703.07561) URL https://doi.org/10.1103/PhysRevLett.119.014801
[24] RM Talman A Lehrach, S Martin 2018 *PoS(SPIN2018)* **144**