Numerical Simulations of Nuclear Reaction Scattering Cross-Sections Using Phase Function Method

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1. Introduction

Scattering experiments in nuclear physics mostly measure reaction cross-sections. It's a major challenge for theoretical physicists to build models for determining scattering phase shifts (SPS) and resonances that validate observations. Study of low energy nuclear reactions consisting of nucleon-nucleon, nucleon-nucleus and nucleus-nucleus requires modeling the effective interaction potentials in a reliable and consistent way. In this thesis, the main objective is to focus on phase function method (PFM) for determining SPS, using numerical approach. The advantage of PFM is its dependence directly on the interaction potential, while all other methods rely on the wave functions. This makes PFM suitable for numerical implementation of inversion methods to obtain the effective interaction of low energy nuclear reactions.

Our preliminary investigation with various interaction potentials involving double Hulthen, Malfliet-Tjon, double Gaussisn, Manning-Rosen suggested in literature as compared to Morse function proposed by us, has shown best performance for the later [1]. Further, Morse function has analytical solutions for both bound & scattering states of time independent Schrodinger equation (TISE) for S-wave ($\ell = 0$) and also found to be an ideal choice as reference potential in inversion theory [2]. Hence, we utilized Morse function as interaction potential for studying neutron-proton(np), proton-proton(pp), $\alpha - \alpha$ & $\alpha - ^3He$ systems [1–4] in this research work.

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2. Methodology

The Morse potential is given by

$$V(r) = V_0 \left(e^{-2(\frac{(r-r_m)}{a_m}} - 2e^{-\frac{(r-r_m)}{a_m}} \right) \quad (1)$$

where V_0, r_m and a_m are model parameters. The first order phase equation obtained from TISE [5], for various ℓ values is:

$$\frac{d\delta_\ell(k,r)}{dr} = -\frac{U(r)}{k} \Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr) \Big]^2 \quad (2)$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are Ricatti-Bessel and

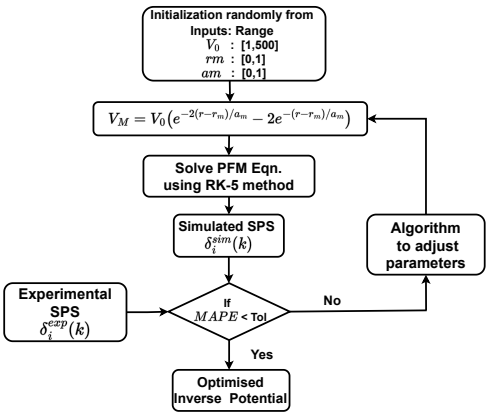

FIG. 1: Flowchart for implementation

Ricatti-Neumann functions. The first major contribution, of this thesis, is design and implementation of a new innovative technique where in Variational Monte-Carlo(VMC) approach is utilised as an optimisation procedure in combination with PFM to obtain best model parameters that fit the experimental SPS by minimising either mean squared error (MSE) or mean absolute percentage error (MAPE).

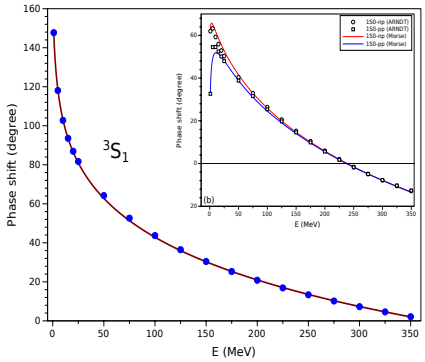

FIG. 2: SPS for $^3S_1$ np-interaction. Inset shows $^1S_0$ SPS for both np and pp.

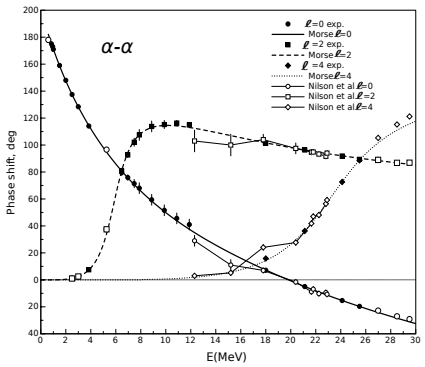

FIG. 3: SPS for $\alpha - \alpha$ interaction.

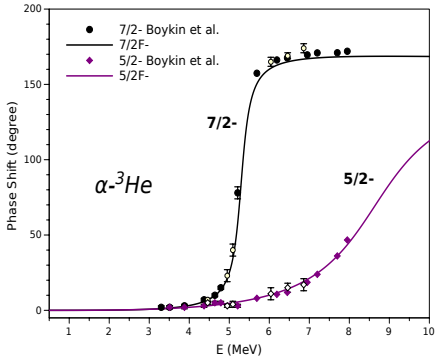

FIG. 4: SPS for $\alpha -^3 He$ interaction.

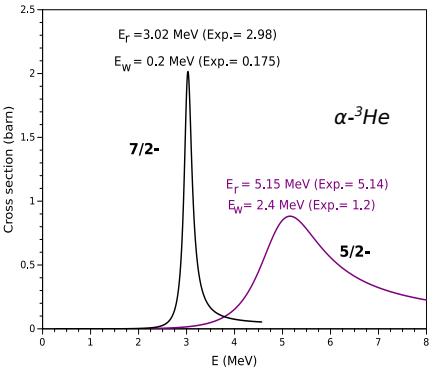

FIG. 5: Scattering cross-sections and resonance.

3. Results and Discussion

$n - p$ and $p - p$ scattering: We obtained SPS for *np* and *pp* scattering, as shown in Fig.2, with MAPE less than 3% for lab energies below 350 MeV and corresponding inverse potentials [2]. Utilising these model parameters, Deuteron properties and form factors were obtained[6] in close validation of experimental values.

$\alpha - \alpha$ scattering: Replacing Nilson's erroneous data (12.3-22.9 MeV) in Afzal's compilation with data of Chien and Brown (18-29.5 MeV), our MAPE analysis [3] resulted in MAPE values of 1.17, 0.69, and 1.77 for ℓ = 0, 2, and 4 respectively. This new compilation can act as benchmark for future SPS calculations by researchers.

$\alpha -^3 He$ scattering: This is an important astrophysical reaction in Sun. SPS were obtained for J = $5/2, 7/2$ states upto 8 MeV, which is proton separation threshold [4]. Resonance energy for two resonant states are found to be in good match with experimental values, as shown in Fig.5. Finally, the study has been applied successfully to n-α and $p - \alpha$ systems[7].

References

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