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MULTIPLICITY DISTRIBUTIONS IN HIGH ENERGY COLLISIONS DERIVED FROM THE STATISTICAL BOOTSTRAP MODEL

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A B S T R A C T

Charged multiplicity distributions calculated in the framework of the Statistical Bootstrap Model (SBM) describe the shape of the experimentally observed distributions rather well. The SBM distribution is fairly close to the negative binomial distribution. It has two parameters, the average number $\bar{N}$ of clusters and the temperature T. Their dependence on $\sqrt{s}$ and on rapidity cuts is intuitively obvious. The behaviour of SBM multiplicity distributions is governed by the existence of a singularity of the model at some critical temperature T_0 where the phase transition from hadrons to a quark-gluon plasma is expected.

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Negative binomial (NB) distributions $P^{NB}(n;\bar{n},k)$ are known to describe well KNO violating (and non-violating) multiplicity distributions, both for full momentum space and for cut (pseudo-) rapidity regions $|y| \leq y_{cut}$, and this for various processes and energies[1)-6)]. The NB distribution is given by

$$P^{NB}(n;\bar{n},k) = \frac{k(k+1)\ldots(k+n-1)}{n!} \cdot \frac{\bar{n}^{n}k^{k}}{(\bar{n}+k)^{n+k}} \tag{1}$$

In the Giovannini-Van Hove "cascade-clan" model[7)-9)], NB results from a convolution of (logarithmic) multiplicity distributions $R(\nu_i,\bar{\nu})$ for the number ν of particles in a "clan" (i.e., a set having a common "ancestor") with a Poisson distribution $Q(N;\bar{N})$ for the number N of different clans:

$$P(n;\bar{n},k) = \sum_{N=0}^{\infty} Q(N;\bar{N}) \sum_{n_i} \delta\left(n - \sum_{i=1}^{N} \nu_i\right) \prod_{i=1}^{N} R(\nu_i;\bar{\nu}) \tag{2}$$

(In fact this type of formula holds for any convolution.) The experimental dependence of $k(\sqrt{s},y_{cut})$ and $\bar{n}(\sqrt{s},y_{cut})$ can then be re-interpreted in terms of the more intuitive physical parameters

$$\begin{aligned} \bar{\nu}(\sqrt{s}, y_{cut}) &= \bar{n}\left[k \ln(1+\bar{n}/k)\right]^{-1} \\ \bar{N}(\sqrt{s}, y_{cut}) &= \bar{n}/\bar{\nu} \end{aligned} \tag{3}$$

Still, neither the choices of the distributions (Poisson and logarithmic) nor the empirical behaviour of the parameters k and $\bar{n}$ appear peremptory.

The SBM philosophy is that there are two stages of interaction; first among partons (soft and hard) and then hadronization into SBM clusters which move along the collision axis, become larger and more numerous with increasing $\sqrt{s}$ and decay in a well-defined way[10)-17)].

Obviously, the main concepts of the Giovannini-Van Hove model and of SBM are similar. We thus use the same general convolution formula (2) to obtain the total SBM multiplicity distribution, with the difference that in our approach we have the parameters $\bar{N}$ and the temperature T and that the multiplicity distribution $R(\nu;\bar{\nu})$ within a clan is replaced by that within a SBM cluster, $R(\nu;T)$.

A SBM cluster is determined by the "Bootstrap Equation" (BE)[12)-17)] for the mass spectrum $\tau(M^2)$ of clusters, which turns out to increase exponentially $\sim M^{-3}\exp(M/T_0)$, where T_0 is a critical temperature, interpreted as the temperature where a hadron gas becomes a quark-gluon plasma[18)]. The not exactly known value of T_0 must a priori lie in the range of 150-200 MeV; we adopt the value T_0 = 190 MeV[17)]. For simplicity we consider only one sort of final hadron which we call pion ($m=m_\pi$) although in SBM it is treated as a Boltzmann scalar. We neglect here baryons, strange and charmed hadrons.

For our analysis in terms of the SBM, we need a few basic formulae of the model.

After "putting the BE in a heat bath at temperature T", i.e., by Laplace-transforming it, it takes a very convenient from[14)-17)]

$$G(\phi) = \phi + \exp[G(\phi)] - G(\phi) - 1 \tag{4}$$

$$\phi(T) := 2\pi H T m K_1(m/T) \tag{5}$$

$G(\phi)$ is essentially the Laplace transform of the cluster mass spectrum $\tau(M^2)$; solving Eq. (4) for G is equivalent to knowing $\tau(M^2)$. Equation (4) can be solved by a power series[14)]

$$G(\phi) = \sum_{\ell=1}^{\infty} g_\ell \phi^\ell \tag{6}$$

with the coefficients g_ℓ obeying a recursion relation

$$\ell g_\ell = -(\ell-1) g_{\ell-1} + \ell \sum g_k g_{\ell-k}$$

$$g_0 = 0 \quad ; \quad g_1 = 1$$

$$g_\ell \sim \phi^{-\ell} \ell^{-3/2} \sqrt{\frac{\phi_0}{4\pi}} \quad \text{for } \ell \to \infty \tag{7}$$

$\phi_0 = \ln 4 - 1$ is the convergence radius of the series (6); $G_0 = G(\phi_0) = \ln 2$ [to see that, draw $\phi(G)$ from Eq. (4)]. The relation $\phi(T_0) = \phi_0$ fixes T_0 if H is given (or vice versa).

The coefficients g_ℓ, incidentally, are related to the total number S_ℓ of possible decay trees ending with exactly ℓ final pions (if $M/m > \ell$): $S_\ell = \ell! g_\ell$. In this purely combinatorial sense, Eq. (4) and the coefficients g_ℓ have been known for more than a century[19].

To calculate the SBM multiplicity distribution we need three ingredients:
(a) The probability distribution $R(\nu;M^2)$ that a cluster of given mass M decays into ν pions[16]:

$$R(\nu;M^2) = \frac{g_\nu \Omega_\nu(M^2)}{H\tau(M^2)} \tag{8}$$

with $\Omega_\nu(M^2)$ being the invariant ν-pion momentum space;
(b) The probability $W(M^2,T)dM^2$ that a cluster, chosen randomly from the hot SBM cluster gas, has a mass in the interval $\{M^2,dM^2\}$[16]:

$$W(M^2,T)dM^2 = \frac{\tau(M^2)\phi(M,T)}{G[\phi(m,T)]}dM^2 \tag{9}$$

essentially: mass spectrum times Boltzmann factor. Here $\phi(M,T)$ is as in Eq. (5) with M replacing m (only here). The probability that a randomly chosen cluster decays into ν pions is obtained by combining (8) and (9):

$$R(\nu;T) = \int dM^2 W(M^2,T)R(\nu;M^2) = \frac{g_\nu \phi^\nu}{G(\phi)} \tag{10}$$

the explicit mass spectrum cancels out but is still hidden in the coefficients g_ν;
(c) The probability distribution $Q(N;\bar{N})$, that there are N clusters, is in SBM a Poisson distribution, since the <u>interacting</u> SBM hadron gas is <u>formally</u> a gas of <u>non-interacting</u> clusters (all interactions being represented by the mass spectrum and a Van der Waals volume correction[17]); freely created and absorbed non-interacting particles are Poisson distributed. In equilibrium the mean value $\bar{N}$ increases from zero (at T = 0) to some maximum and then drops to one at $T = T_0$, where all clusters collapse into one infinite supercluster: they "condense into a quark-gluon liquid"[17]. This is the grand canonical equilibrium situation; in a collision, however, we are in a microcanonical non-equilibrium situation, where infinite clusters are impossible (energy fixed). Moreover, the violent longitudinal forward-backward motion will tear any large cluster into small ones. The two mechanisms: merging of small clusters into large ones when $T \to T_0$ and

destroying of large clusters by the longitudinal motion neutralize each other to some extent, so that the mean number $\bar{N}$ of clusters might only slowly vary with $\sqrt{s}$. This guess is corroborated by other models (multiperipheral with cluster vertices[20] and bremsstrahlung[21]) which suggest $d\bar{N}/dy$ to be independent of $\sqrt{s}$ and y. Since we cannot calculate $\bar{N}$, we take it as a free parameter to be compared after fits with our above expectations. Thus

$$Q(N;\bar{N}) = e^{-\bar{N}}\,\bar{N}^N/N! \quad ; \quad \bar{N} \text{ free parameter} \tag{11}$$

This fixes everything. We insert (10) and (11) into the general convolution formula (2) and obtain the following normalized distribution

$$P^{SBM}(n;\bar{N},T) = e^{-\bar{N}}\, c_n(\bar{N},T)\,\phi(T)^n \tag{12}$$

the coefficients c obey the recursion relation

$$c_n(\bar{N},T) = \frac{\bar{N}}{G[\phi(T)]}\cdot\frac{1}{n}\sum_{k=1}^{n} k g_k\, c_{n-k} \quad ; \quad c_0 = 1 \tag{13}$$

with the g_k given by (7).

The mean value $\bar{n}$ and the dispersion $D^2 = \overline{n^2} - \bar{n}^2$ are found to be

$$\begin{aligned} \bar{n} &= \bar{N}\,\bar{\nu} \\ D^2 &= \bar{N}\,\overline{\nu^2} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \bar{\nu} &= \phi G'/G \\ \overline{\nu^2} &= (\phi G' + \phi^2 G'')/G \\ G' &:= dG/d\phi \quad \text{etc.} \end{aligned} \tag{15}$$

Experimentally not total, but charged multiplicities are available. We therefore must still project out the charged multiplicity distribution (isospin projection[22] is too complicated here): assume a total charge zero, then the probability to find 2k charged among a total of n particles is, for $2k \leq n$

$$W(k;n) = \frac{n!}{(k!)^2(n-2k)!} \Bigg/ \sum_{\ell} \frac{n!}{(\ell!)^2(n-2\ell)!} \tag{16}$$

and zero for $2k > n$. Combining (12) and (16) gives the normalized charged multiplicity distribution (2n charged)

$$P_{ch}^{SBM}(2n;\bar{N},T)=\sum_{m=0}^{\infty}W(n;m)\,P^{SBM}(m;\bar{N},T) \tag{17}$$

This is our final result to be compared to experiments. We fix the parameters $\bar{N}$ and T (or $\bar{N}$ and ϕ) by a least square fit of (17) to corrected data[23]. Figure 1a presents a plot of (17) with these data, while Fig. 1b compares the same data with a NB fit. NB represents the data better than SBM but the difference is limited mainly to charged multiplicities $\gtrsim 100$, where the experimental errors are significant.

Preliminary fits at other energies and also with (pseudo-) rapidity cuts show a similarly satisfactory agreement with the data. The temperature T monotonously tends towards T_0 with increasing $\sqrt{s}$, but is rather insensitive to (pseudo-) rapidity cuts if $y_{cut} \gtrsim 1$. The mean number $\bar{N}$ of clusters grows somewhat less than logarithmically with $\sqrt{s}$ and about linearly with y_{cut}. The mean multiplicity per cluster $\bar{\nu}$ grows roughly like $\ell n\sqrt{s}$ and is rather insensitive to rapidity cuts.

That the SBM distribution is larger than the corresponding NB distribution at large n is due to the different cluster/clan decay structures - essentially to the factor $1/\sqrt{n}$ by which the asymptotic g_n [see Eq. (7)] differ from the $1/n$ of the logarithmic clan decay distribution. The SBM used here is, however, only one of a large class of possible SBM's [Ref. 16), App. B] containing also models with asymptotic $g_n \sim 1/n$ [namely, when $\tau(M^2) \sim M^{-5/2}\exp(M/T_0)$]. The present version is the simplest and physically most appealing one and moreover the only one worked out in detail[10)-17)]; the multiplicity distribution presented here is just a by-product.

A more detailed account of our work and confrontation with data at other energies will be published elsewhere[24)].

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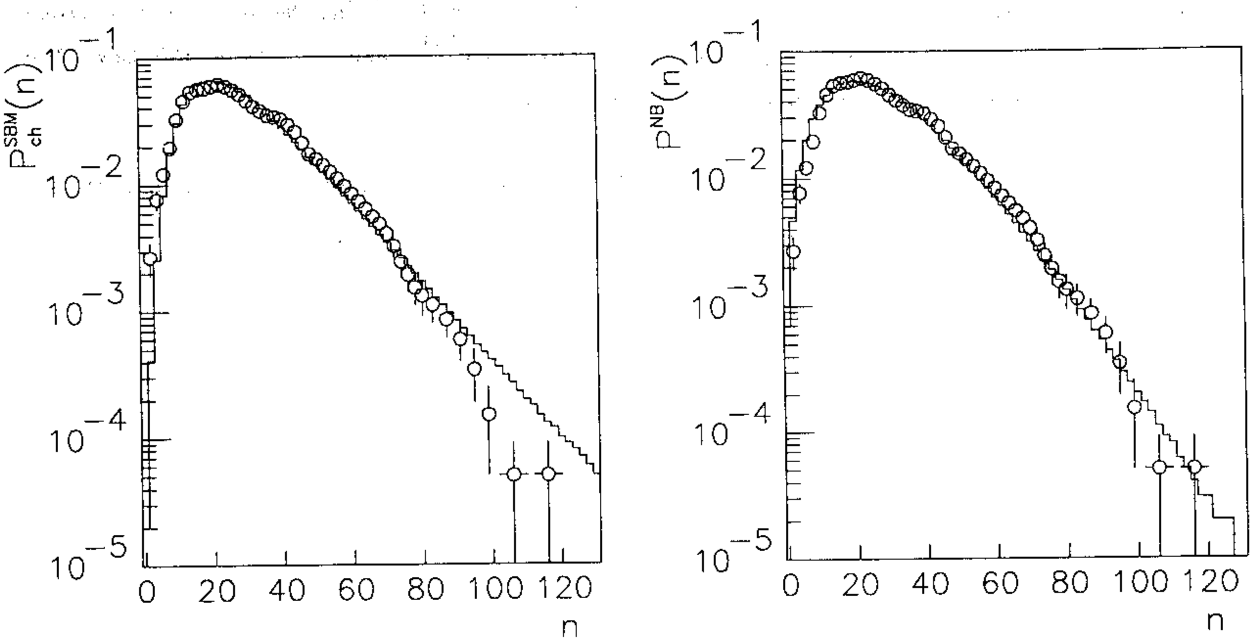
FIGURE CAPTION

Fig. 1 (a) The charged multiplicity distribution P_{ch}^{SBM} (n; $\bar{N}$=16.1, ϕ = 0.3708) compared to corrected experimental data[23)] taken at $\sqrt{s}$ = 540 GeV. The value of ϕ corresponds to T = 187 MeV.

(b) The same data compared to P^{NB} (n; $\bar{n}$=28.3, k=3.69); the values of the parameters $\bar{n}$ and k are also taken from Ref. 23).

Figure 1



$P^{SBM}_{ch}(n)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0 20 40 60 80 100 120
n
$P^{NB}(n)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0 20 40 60 80 100 120
n

(a) (b)