PROCEEDINGS OF SCIENCE

Measurement of the anomalous spin precession frequency ω_a in the Muon $g-2$ experiment at Fermilab

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The Muon $g-2$ Experiment at Fermilab aims to measure the muon magnetic moment anomaly, $a_\mu = (g-2)/2$, with a final accuracy of 0.14 parts per million (ppm). A 3.1 GeV muon beam is injected into a storage ring of 14 m diameter, in the presence of a 1.45 T magnetic field. The anomaly a_μ can be extracted by accurately measuring the anomalous muon spin precession frequency ω_a, based on the arrival time distribution of decay positrons observed by 24 calorimeters, and the magnetic field. In 2023, the experiment published its second result based on the two datasets Run-2 and Run-3, reaching the unprecedented sensitivity of 0.21 ppm — a factor ~ 2.2 improvement since its first result published in 2021, based on the first dataset (Run-1). In this paper, we will focus on the measurement of the ω_a frequency, describing the techniques and major sources of systematic uncertainty in the 2023 publication, and outlining the improvements since the 2021 result. We will also cover the status of the ongoing ω_a analysis for the last three datasets, collected from 2020 to 2023, along with the projected uncertainties on the final Muon $g-2$ measurement at Fermilab.

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1. The magnetic moment of the muon

The muon gyromagnetic ratio g_μ is the factor of proportionality between the muon magnetic moment $\vec{\mu}$ and its spin $\vec{S}$: $\vec{\mu} = g_\mu(e/2m_\mu)\vec{S}$. From Dirac's equation, g_μ would be equal to 2, but, in the Standard Model (SM), contributions from QED, electroweak interactions and QCD correct g_μ to a slightly higher value than 2. The muon magnetic anomaly is defined as the fractional difference of g_μ from 2: $a_\mu = (g_\mu - 2)/2$. Figure 1 presents the experimental values of a_μ as measured by BNL E821 [1] and FNAL E989 in its first result (based on the Run-1 dataset, published in 2021) [2] and its second result (based on datasets Run-2/3, published in 2023) [3, 4]. Combining these three measurement brings the uncertainty on the a_μ world average to 0.19 parts per million (ppm).

On the theoretical side, recent developments in the quantum chromodynamics (QCD) sector, which carries the largest uncertainty on the calculation, have prevented a solid comparison with the experimental value. The major contribution comes from hadronic vacuum polarization (HVP), where the energy scale is of the order of the muon mass, well below the region where QCD can be studied perturbatively: a dispersion relation approach can be used to evaluate the contribution, using the experimental hadronic cross section of e^+e^- as an input; lattice QCD is another approach to determine the HVP contribution to a_μ using an ab-initio calculation. In 2020, the Theory Initiative recommended a value for the theoretical prediction of a_μ in a White Paper (WP20) [5], based on the dispersive approach. This value is discrepant from the latest experimental average: $a_\mu^{exp} - a_\mu^{WP20} = (249 \pm 48) \cdot 10^{-11}$, with a significance of 5.1 σ. In 2021, the BMW collaboration presented a prediction of a_μ^{HVP} with lattice QCD with an uncertainty of 0.8% [6], which was in tension with the dispersive approach. Many collaborations which use a lattice approach are working to improve the uncertainty on a_μ^{HVP} and verify the BMW prediction [7]. Tensions in the dispersive approach have also arisen: in 2023, the measurement of the $e^+e^- \to \pi^+\pi^-$ cross section with the CMD-3 detector [8] resulted in a hadronic contribution to a_μ that was significantly larger than the value obtained from previous measurements.

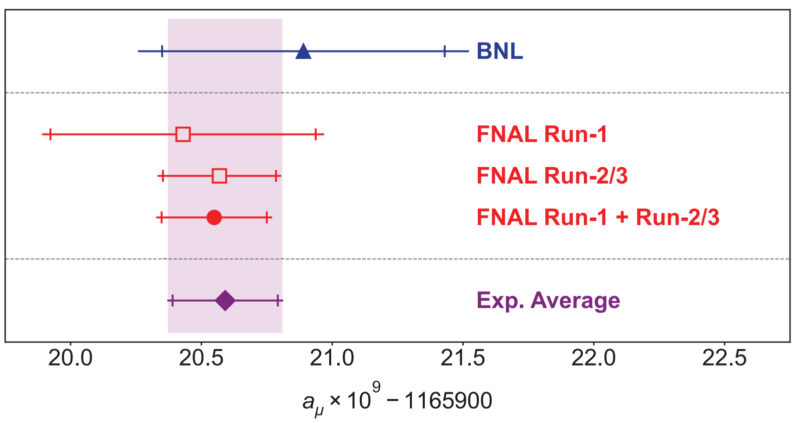

Figure 1: Measured values of a_μ from BNL and FNAL, and new experimental average. The inner tick marks indicate the statistical contribution to the total uncertainties [3].

2. Measurement principle of the Muon $g-2$ (E989) experiment at Fermilab

In the Muon $g-2$ experiment, a spin-polarized beam of 3.1 GeV positively charged muons is injected into a ~ 7 m radius superconducting storage ring, that produces a vertical 1.45 T magnetic field, uniform at the ppm level. Electrostatic quadrupole (ESQ) plates provide weak focusing for vertical confinement. In the storage ring, muons precess with cyclotron frequency ω_C, and their spin also precesses around the direction of the magnetic field, with frequency ω_S. Given e and m the charge and mass of muons, respectively, the anomalous precession frequency ω_a is defined as:

$$\vec{\omega}_a \equiv \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]. \tag{1}$$

$\vec{E}$ is the electric field from ESQ, $\vec{B}$ the magnetic dipole, $\vec{\beta}$ the muons' speed and γ their Lorentz factor. In the Muon $g-2$ experiment, only the first term in square brackets is relevant in the first approximation, because muons travel perpendicularly to the B-field and $\gamma \approx 29.3$ is such that the last parenthesis vanishes. When only the first term is considered, the equation for $\vec{\omega}_a$ becomes:

$$\omega_a = a_\mu \frac{eB}{m} \simeq 1.43\,\mathrm{rad}/\mu\mathrm{s}, \tag{2}$$

with a direct proportionality between a_μ and ω_a/B. The magnetic field is expressed by means of the Larmor precession frequency of free protons ω_p, measured with Nuclear Magnetic Resonance techniques, via $\hbar\omega_p = 2\mu_p|\vec{B}|$, where μ_p is the proton magnetic moment [4, 9]. We account for deviations from the ideal case of Eq. (2) by applying corrections to our measurements, due to beam dynamics or to transient magnetic fields [4, 10].

3. Positron reconstruction and ω_a analysis

This paper will focus on the measurement of ω_a, which is based on the arrival time distribution of decay positrons in the high-energy tail of the spectrum. More details on the ω_a analysis are provided in Ref. [11] for the Run-1 result, and in Ref. [4] for the Run-2/3 result.
Due to parity violation in the muon weak decay, high energy positrons from μ^+ decays are emitted preferentially in the muon's spin direction in the center-of-mass frame. 24 electromagnetic calorimeters are placed along the inner radius of the Muon $g-2$ storage ring, and each is composed of an array of 6×9 lead-fluoride crystals. Positrons generate Cherenkov light in the crystals, which is detected by SiPMs, converted into a voltage signal and recorded for analysis. From template fits on crystal pulses, positrons' energies and times of arrival are reconstructed. When two or more positrons hit the same calorimeter within ~ 10 ns, the reconstruction is not always able to separate the events and the incident particles are reconstructed as a single hit. These pileup events can be identified by studying the distribution of clusters and subtracted from the counts. The resulting energy spectrum of positrons detected in the lab frame changes with time, depending on the angle between muon spin and muon momentum at the time of decay, *i.e.* the anomalous precession phase. The number of all positrons above a fixed energy threshold changes over time, with a distribution that is modulated by the ω_a frequency, described — in the ideal case — by Eq. (3):

$$N(t) = N_0 e^{-t/\gamma\tau}\left[1 + A_0\cos\left(\omega_a t + \phi_0\right)\right], \tag{3}$$

where N_0 is a normalization parameter, A_0 the amplitude of the oscillation, ϕ_0 the initial phase, and $\gamma\tau$ is the muon lifetime in the lab frame. The so-called "Asymmetry-weighted" method has been used since the first FNAL result to yield the lowest statistical uncertainty on ω_a: firstly, we fit the A_0 parameter in Eq. (3) over $\sim$ 10 MeV energy slices, to obtain the energy-dependent asymmetry function; secondly, we count all detected positrons above 1 GeV, weighting them with the asymmetry corresponding to their energies.
The ω_a analysis is performed by several groups, each one employing their own, independent methods for positron reconstruction such as clustering algorithms and pileup subtraction. In addition, the complete ω_a fit function includes terms which account for the muon losses and beam dynamics frequencies, so more floating parameters are implemented with respect to Eq. (3). Figure 2 shows the fit to Run-3 data and the fast fourier transform (FFT) of the residuals: the red dashed curve shows peaks at all the frequencies that are not accounted for in the 5-parameter fit, with the dominant one being the aliased frequency due to the Coherent Betatron Oscillation (CBO) of the muon beam; the solid black curve is the FFT of residuals when fitting the wiggle plot with the full function, which removes all residual frequencies.

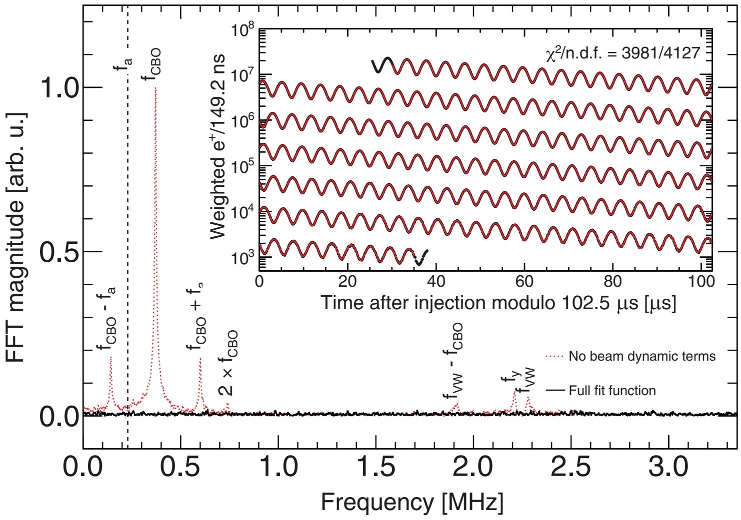

Figure 2: FFT of residuals from the ω_a fit (inset plot) in the case of 5-parameter function (dashed red) or complete fit function in Run-3 (solid black) [3].

The papers in Ref. [4, 12] describe the method to verify the consistency of ω_a results across different analysis groups, and to combine them into one value taking the correlations into account.

4. Improvements from Run-1 to Run-2/3 results

There were several improvements after the Run-1 (2021) result, in terms of running conditions, analysis techniques and systematic studies.
First of all, during Run-1 there were two damaged resistors in the ESQ plates, fixed before Run-2, which strongly affected the stability of beam oscillations and enhanced related systematic effects. Towards the end of Run-3, the non-ferric fast kicker magnet, which is necessary to store the muon

beam at the time of injection, was upgraded in order to achieve the optimal kick, consequently lowering corrections to a_μ related to the ESQ electric field. In Run-2/3, the temperature of the magnet was stabilized with a thermal insulating blanket, to mitigate day-night field oscillations; after Run-2, also the experimental hall's air conditioning system was upgraded to further stabilize the temperature of both the magnet yokes and the detector electronics to better than ±0.5 °C.
In Run-2/3 we collected 4.7 times the number of Run-1 decay positrons, which reduced the statistical uncertainty on ω_a by a factor ~ 2.2. In turn, this increase in statistics allowed to perform more detailed studies on the systematic uncertainties that dominated the Run-1 ω_a result, namely pileup and CBO. Improved empirical modelling of CBO brought down the related uncertainty from 38 ppb to 21 ppb, and new algorithms to better resolve pileup brought down the uncertainty from 35 ppb to 7 ppb. Moreover, a new "Asymmetry-weighted Ratio" method was developed, which consisted in the following steps: first, positron events were subdivided evenly into four wiggle plots; when doing so, each positron event was weighted by the asymmetry A_0 value corresponding to its energy, similarly to the "Asymmetry-weighted" method described in Section 3; in addition, as part of the "Ratio" procedure, one of the four histograms was shifted in time by $+T_a/2$ (half of the anomalous precession period), and another one by $-T_a/2$, whereas the remaining two were not time-shifted; once the four wiggle plots were built, the two time-shifted histograms were summed together in a single wiggle plot labelled $U(t)$, while the remaining two were summed into $V(t)$; finally, the ratio between $V(t) - U(t)$ and $V(t) + U(t)$ was taken, in order to build a wiggle plot where the muon exponential decay was cancelled out. By construction, this method preserved the statistical power in the ω_a fit, whilst reducing sensitivity to many systematics.
With all these improvements, in Run-2 and Run-3 the statistical uncertainties on ω_a were reduced with respect to Run-1, from 434 ppb to 201 ppb, and the systematic uncertainties were improved from 56 ppb to 25 ppb. The overall uncertainty on a_μ was brought down from 0.46 ppm to 0.21 ppm.

5. Conclusion

The goal of the Muon $g-2$ experiment at Fermilab is to measure the muon magnetic anomaly a_μ at the 0.14 ppm level of precision, a fourfold improvement with respect to the previous experiment at BNL [13]. Combining the experiment's results of 2021 and 2023 and the previous BNL result, the new experimental measurement of a_μ has reached the unprecedented precision of 0.19 ppm; in the 2023 result, the systematic uncertainty reached 70 ppb, surpassing the goal of 100 ppb, and with the ongoing analysis of the last three datasets, Run-4/5/6, we expect to reach the goal of 100 ppb in statistical uncertainty. On the ω_a side, further improvements of the systematic are expected in Run-4/5/6: analyzers' task forces were established in order to address the dominant Run-2/3 systematics (like CBO), perform dedicated studies on our modelling of such effects, and ultimately reduce the related systematic uncertainties.

Acknowledgments

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