SELECTION OF THE OPTIMAL PARAMETERS FOR A HIGH-ENERGY LINEAR ACCELERATOR*

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Modern linear high-energy accelerators are complex and expensive devices. The problem of how to reduce their cost for given characteristics of the accelerated beam is of paramount importance. In the present paper starting with the criterion of minimum outlays for the construction and operation of the accelerator we determine its optimum parameters taking into account the beam load. We assume here that the cost of construction and operation of accelerators may be written down in the form [1, 2]

$$S = A_1 L + A_2 N \quad (A_i = a_i + b_i t_p), \tag{1}$$

where L is the length of the accelerating system; N is the number of sections; a_i and b_i are constant coefficients determined by the economic analysis; t_p is the total operation time of the accelerator (from the start of its operation to the end of its use). Equation (1) neglects the fixed costs which cannot influence the position of the minimum S and may, consequently, be omitted.

Let us formulate our basic problem. Let us assume that we are given the coefficients A_i, energy W, and average current I_φ of the beam of accelerated electrons, and the characteristics of the hf supply; we also know the power of the source during the pulse P, the frequency of the accelerating field ω, the pulse length τ_p, and the frequency of the pulse repetition, n. We want to determine the values of the basic accelerator parameters, corresponding to the minimum cost, S, of the accelerator: the strength of the accelerating field E_φ (averaged over the sectional length), the length of a single section l, the accelerator efficiency η, the geometrical dimensions of the cells, etc.

The solution of this problem will be given for two accelerating systems:

1) with a field strength which is constant for the entire length $E = E_{av} = \text{const}$;

*This report was not presented.

2) with constant geometry of wave guide units (over the length).

1. THE CONSTANT FIELD ACCELERATOR

It is easy to show that the constant field accelerator satisfies the relations

$$u = \frac{P(l)}{P(0)} = 1 - \frac{2(1+m)E^2 l}{P_r}; \tag{2}$$

$$v = \frac{\tau_f}{\tau_p} = -\frac{1}{\gamma(1+m)} \ln u, \tag{3}$$

where P(0) and P(l) are the values of the hf power at the start and the end of the section, respectively; τ_f is the time for charging the section with electromagnetic energy; m = Ir/E; I is the electron beam current during the pulse; $\gamma = \omega\tau_p/Q$.

During the derivation of equations (2) and (3) we assumed that the Q factor of the accelerating section and the shunt impedance per unit length, r, do not depend on the size of the coupling opening, a/λ. Using equations (2) and (3) and the well-known relationship

$$I = \frac{I_{av}}{n\tau_p(1-v)}, \tag{4}$$

$$L = \frac{W}{E}, \tag{5}$$

$$N = \frac{W}{El} \tag{6}$$

the expression (1) for S may be expressed in the form

$$S = \frac{WA_2}{P} \cdot \frac{I_{av}}{n\tau_p} \sigma = \frac{WA_2}{P} \cdot \frac{I_{av}}{n\tau_p} \Big\{ Cm(1-v) + \frac{1+m}{m(1-v)[1-e^{-\gamma v(1+m)}]} \Big\}, \tag{7}$$

where

$$C = \frac{A_1}{A_2} \cdot \frac{P}{I_{av}^2 r} (n\tau_p)^2 \tag{8}$$

is a constant coefficient which is assumed to be given in the formulation of the problem. The optimum values for the two independent variables, m and v, are determined by the system of equations:

$$\frac{\partial\sigma}{\partial m} = 0; \tag{9}$$

$$\frac{\partial\sigma}{\partial v} = 0. \tag{10}$$

The simultaneous validity of equations (9) and (10) leads to a quadratic equation in m for fixed values of the quantity, u (see equation (2)):

$$m^2 + m\left(2 - \frac{1-u}{\gamma u}\right) + 1 + \frac{\ln u}{\gamma} = 0. \qquad (11)$$

By specifying u one can find m and then the value of v is found from equation (3). The value of C, corresponding to the m and v obtained, can be found, e.g., from equation (8) which in explicit form can be written as

$$C = \frac{1 + [\gamma v(1+m)m - 1]\,e^{-\gamma v(1+m)}}{m^2(1-v)^2[1 - e^{-\gamma v(1+m)}]^2}. \qquad (12)$$

One can easily verify that equations (3), (11), and (12) establish a mutually unique correspondence between the pairs of values (m, v) and (C, γ). It is most convenient to use graphical solutions since because of the transcendental character of the equations the m and v cannot be represented in explicit form as functions of C and γ. The solid lines on Figures 1--3 show the

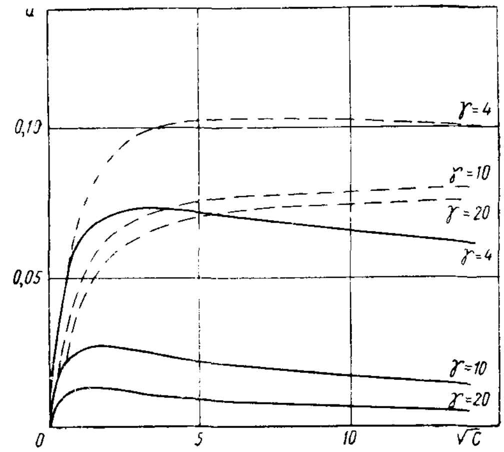

Figure 1

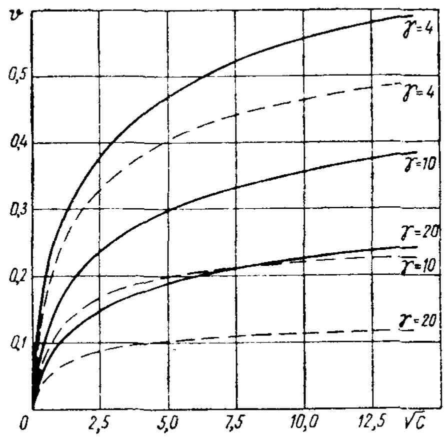

Figure 2

dependence of the optimum values of u, v, and σ on $\sqrt{C}$ for various parameter values of γ. Other important characteristics of the accelerating system such as E, l, and η can be obtained from the equations

$$E\frac{n\tau_p}{I_{av} r} = \frac{1}{m(1-v)}, \qquad (13)$$

$$l\frac{I_{av}^2 r}{P(n\tau_p)^2} = \frac{m^2(1-v)^2}{1+m}[1 - e^{-\gamma v(1+m)}], \qquad (14)$$

$$\eta = \frac{IW_1}{P} = \frac{m}{m+1}[1 - e^{-\gamma v(1+m)}], \qquad (15)$$

where $W_1 = El$ is the electron energy increment per section of the accelerator. Curves illustrating the dependence of the indicated characteristics on $\sqrt{C}$ and γ are shown on Figures 4--6 (solid lines).

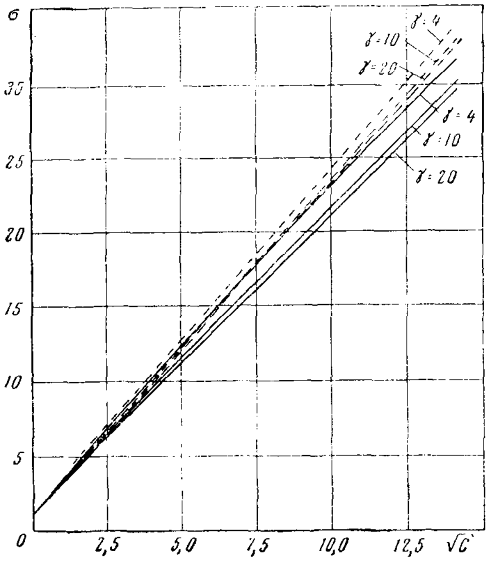

Figure 3

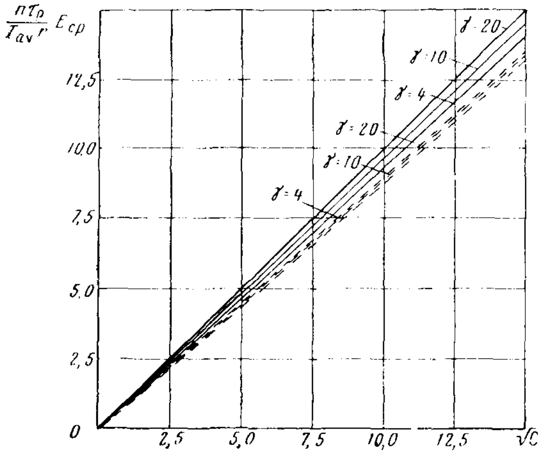

Figure 4

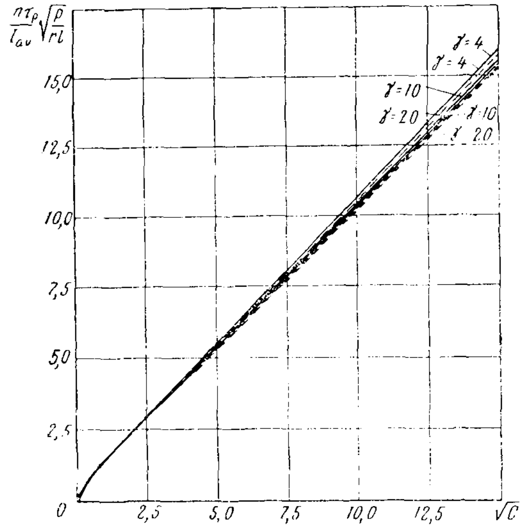

Figure 5

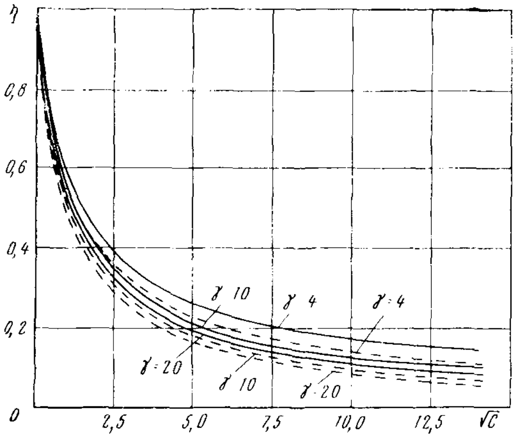

Figure 6

2. THE CONSTANT GEOMETRY ACCELERATOR

This case can be studied basically in the same way as was the case with the constant field accelerator. Therefore, we present only certain basic formulas:

$$m = \frac{Ir}{E_{av}} = \frac{(1-v)\left(1-e^{-\frac{\gamma v}{2}}-\gamma v e^{-\frac{\gamma v}{2}}\right)}{\gamma^2 (1-v)\, e^{-\frac{\gamma v}{2}} - (1+v)\left(1-e^{-\frac{\gamma v}{2}}\right) - (1-3v)\dfrac{\left(1-e^{-\frac{\gamma v}{2}}\right)^2}{\frac{\gamma v}{2}}}; \tag{16}$$

$$\frac{E_{10}}{E_{av}} = \xi = (1+m)\frac{\frac{\gamma v}{2}}{1-e^{-\frac{\gamma v}{2}}} - m; \tag{17}$$

$$C = \frac{\xi}{m^2 (1-v)^2}\left(\frac{1}{1-e^{-\frac{\gamma v}{2}}} - \frac{\xi}{\gamma v}\right); \tag{18}$$

$$u = \frac{P(l)}{P(0)} = \left[\left(1+\frac{m}{\xi}\right)e^{-\frac{\gamma v}{2}} - \frac{m}{\xi}\right]^2; \tag{19}$$

$$\sigma = Cm(1-v) + \frac{\xi^2}{\gamma v m (1-v)}; \tag{20}$$

$$E_{av}\frac{n\tau_\rho}{I_{av} r} = \frac{1}{m(1-v)}; \tag{21}$$

$$l\frac{I_{av}^2 r}{P(n\tau_\rho)^2} = m^2(1-v)^2\frac{\gamma v}{\xi^2}; \tag{22}$$

$$\eta = m\frac{\gamma v}{\xi^2}. \tag{23}$$

Curves corresponding to a constant geometry accelerator are shown on Figures 1--6 by dashed lines. Figure 7 presents the relationship $\xi(\sqrt{C}, \gamma)$, which permits one to check that the maximum value of the field strength at the beginning of each section of the optimum accelerator does not exceed the breakdown field strength.

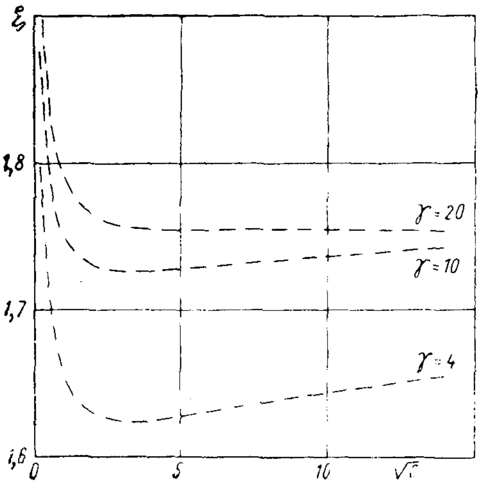

Figure 7

3. THE EVALUATION OF RESULTS

Let us study Figures 3--5. The curves shown can be approximated with sufficient accuracy by a linear function for almost all values of $\sqrt{C}$. The only exception are the regions near zero corresponding to a large loading of the accelerator by the beam (see Figure 6). Such a linear approximation and equation (8) allow us to write the following expressions for the optimum values of S, E_{av}, and l:

$$S = S_0 + S_l = W\left(h\sqrt{\frac{A_1 A_2}{Pr}} + \frac{A_2}{P}\cdot\frac{I_{av}}{n\tau_\rho}\right); \tag{24}$$

$$E_{av} = \frac{2}{h}\sqrt{\frac{A_1}{A_2}Pr}\ ; \tag{25}$$

$$l = \frac{\dfrac{A_2}{A_1}}{1 + \dfrac{2}{h}\sqrt{\dfrac{A_1}{A_2}\cdot\dfrac{r}{P}\cdot\dfrac{I_{av}}{n\tau_\rho}}}. \tag{26}$$

where h is a constant coefficient whose values are given in the table.

Values for the Coefficient h

γ	Constant field	Constant geometry
4	2.20	2.33
10	2.08	2.25
20	2.03	2.23

In this way, the cost, S, of the accelerator (omitting the fixed costs) is equal to the sum of two terms, the first of which, S_0, determines the cost of the nonloaded accelerator, while the second, S_1, takes into account the load of the beam. In extremely high-energy accelerators the role of the second term is usually not very large; for instance, in the Stanford accelerator for 22--45 GeV, S_1 represents approximately 15 percent of S_0. On the contrary, in a high current accelerator, S_1 may be of the same order of magnitude as S_0 or be even larger.

It is easy to show by studying the behavior of the function σ (see equation (7)) for $m \longrightarrow 0$ that optimum accelerators with negligibly small currents I do satisfy the known condition [3]: the cost of the "lengths" of the accelerator must follow the cost of the "sections." This follows also from equation (26). From equation (25) it follows that the optimum field strength and, consequently, the cost of the "length" of the accelerator does not depend on the beam load. At the same time, the cost of the "sections" increases linearly with the increase in I_{av}, as can be seen from equation (24).

Let us elucidate the influence of the power of the hf power supply on the cost of the accelerator. Let us assume that

$$A_2 = d\sqrt{P}, \tag{27}$$

where d is a certain constant. Then the expression for S takes the form

$$S = W\left(h\sqrt{\frac{A_1 d}{r}}\,P^{-\frac{1}{4}} + d\,\frac{I_{av}}{n\tau_p}\,P^{-\frac{1}{2}}\right). \tag{28}$$

from which can be seen that the desired decrease in S can occur only during a sharp increase in P. For instance, the substitution of the twenty megawatt klystron by a hundred megawatt device reduces S_0 by approximately one and a half times. At the same time, in high current accelerators the increase in power of the power supply becomes a very important factor. If we assume that $S_0 = S_1$ then an increase in P by a factor of five reduces S by approximately 1.8 times.

Let us investigate the connection between S and the useful operating time of the accelerator, $n\tau_p$. It is clear from equation (24) and the table of h values that S_0 does not depend on n and is only slightly dependent on τ_p so that an increase in γ above the value $\gamma = 20$ does not change the value of S_0 appreciably. At the same time, S_1 decreases inversely proportionally to $n\tau_p$, i.e., sufficiently rapidly. This yields the solution of the following problem.

Suppose there exist two power supply sources of equal average power but with different powers per pulse and we wish to choose the one which is more suitable. Depending on the relationship between S_0 and S_1 we come to different conclusions:

a) if $S_0 \gg S_1$, then it is better to use the source with the larger pulse power P;

b) if $S_0 \ll S_1$, it is better to use the source with the smaller value of P but with a larger useful time $n\tau_p$;

c) if $S_0 \approx S_1$, both sources are approximately equally good.

We conclude by showing which of the accelerating systems studied should be utilized within a linear accelerator. We can actually distinguish two cases for which the fixed field system shows definite advantages as compared with the fixed geometry systems.

1. If the optimum value E_{av} is within the limits $E_{br}/\xi < E_{av} < E_{br}$, where E_{br} is the breakdown field, then the field strength at the start of the section with a constant geometry $E_0 = \xi E_{av}$ would exceed E_{br}. It is obvious that in this case the only possible solution for the optimum system would be one with the constant field.

2. If the accelerator carries a large current, then the system with a constant field prevents the reduction in the electron momentum in a very efficient manner [4].

An additional argument in favor of the constant field systems originates from the fact that their cost for moderate values of C is 5--6 percent smaller than the cost of the fixed geometry systems.

However, for small values of P (of the order of a few milliwatts) the large damping of the hf power in the optimum section (see Figure 1) leads to a sudden narrowing of the coupling openings within the last units of the constant field section. This may lead to difficulties in channeling of the beam through the accelerator, narrow the band characteristic, and increase the dispersion of the system which is, of course, undesirable. Under these conditions the use of constant geometry systems may turn out to be expedient.

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We formulated and solved the problem concerning the minimum cost of a linear accelerator taking into account the load of the beam. The results are presented in the form of graphs. We gave also approximate formulas for S, E_{av}, and l, which simplify the analysis of the properties of an optimum accelerator during moderate beam loads.

We showed that the optimum accelerator exhibits a large damping of hf power. The power at the exit of the accelerating section does not exceed 7--10 percent of the power at the input.

The cost of the construction and operation of an accelerator increases with the increase of the power of the hf power supply. The increase in P (for a given $n\tau_p$) is particularly convenient in high current accelerators.

The optimum value of E_{av} depends very little on the current I for moderate beam loads (up to $\eta \approx 0.6$--0.7).

A comparison of the accelerating structures having constant field and constant geometry shows that for large values of P and I it is expedient to utilize a constant field accelerator system. On the other hand, the system with constant geometry may prove more convenient in the case of small hf powers (of the order of a few milliwatts).

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