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The Correction of Quantum Tunneling Rate and Entropy of Non-Stationary Spherically Symmetric Black Hole by Lorentz Breaking

Yu-Zhen Liu [1,*], Xia Tan [1], Jie Zhang [1], Ran Li [2] and Shu-Zheng Yang [2]

[1] College of Physics and Electronic Engineering, Qilu Normal University, Jinan 250200, China; 20152329@qlnu.edu.cn (X.T.); zhangjie@qlnu.edu.cn (J.Z.)

[2] College of Physics and Astronomy, China West Normal University, Nanchong 637002, China; ranli007975@gmail.com (R.L.); ysz@cwnu.edu.cn (S.-Z.Y.)

* Correspondence: 20158170@qlnu.edu.cn

Abstract: With the introduction of CFJ correction term, Chiral correction term, and aether-like correction term, based on Lorentz breaking, WKB approximate, and quantum tunneling radiation theory of black holes, the modified fermion dynamics equation is studied in the general non-stationary spherically symmetric black hole space-time, and the new modified expressions of the fermion tunneling rate, the Hawking temperature, and the Bekenstein–Hawking entropy of the black hole are obtained. This black hole has both thermal and non-thermal radiation. In this article, the influence of Lorentz breaking on the energy levels of Dirac particles was also studied, and the distribution characteristics of Dirac energy levels in the space-time and the maximum value of the crossing of positive and negative energy levels were obtained. The necessary discussion and the explanation of the corresponding results are made.

Keywords: general non-stationary spherically symmetric black hole; tunneling radiation; Lorentz breaking; black hole temperature; black hole entropy

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1. Introduction

In classical black hole physics, black holes are considered unable to radiate any particles. However, Hawking found that black holes can actually radiate particles through the quantum effect after considering the quantum effect near the event horizon of black holes, which is known as Hawking radiation. This discovery points to a profound connection between quantum theory, gravitational theory, thermodynamics, and statistical physics. Therefore, the research on Hawking radiation of black holes soon becomes a hot frontier topic in curved space-time quantum field theory. For the explanation of Hawking radiation, we can use the tunneling theory in quantum physics to explain it. It is believed that there are a large number of virtual particles in the event horizon of the black hole. These particles pass through the event horizon of the black hole by the quantum tunneling effect and become real particles, which is called Hawking radiation. Using quantum tunneling theory, Kraus et al. proposed the tunneling radiation theory to calculate the Hawking temperature and black hole entropy of black holes [1]. After the tunneling radiation theory was applied to calculate the Hawking radiation and entropy of black holes, the semi-classical Hamilton–Jacobi method was proposed, which again promoted the research progress of black hole tunneling theory. Subsequently, Kerner et al. started using the semi-classical methods to study the quantum tunneling radiation of Dirac particles [2]. They obtained the tunneling rate and corresponding black hole temperature of Dirac particle tunneling radiation of black holes. Then, people used the tunneling theory to study the Hawking radiation radiation of various types of black holes. In 2009, Yang and Lin proposed a new method to study the tunneling radiation of Dirac particles in black holes [3]. Based on the semi-classical approximation theory, the Dirac equation in curved space-time was reduced to a simpler matrix equation, and the Hamilton–Jacobi equation in curved space-time was obtained by using the commutation relationship of the gamma matrix. The Hawking radiation of the Dirac particle of the black hole is derived by using this semi-classical equation. This result shows that Hamilton–Jacobi equation can be successfully applied to the study of fermion tunneling theory. The development of this theory effectively simplifies the study of fermion tunneling theory, and achieves the unification of quantum tunneling radiation theory. On the other hand, Lorentz dispersion relation has always been considered as the basic relation of modern physics. Today, general relativity and quantum field theory seem to be based on this basic relation.

Lorentz dispersion relation is one of the basic relations in physics. Both general relativity and quantum field theory are based on it. During the research of quantum gravity theory and string theory, people found that gravity cannot be quantized, which is also a big problem in the research of gravitational renormalization and Grand Unity Theory. In recent years, people's research on quantum gravity theory shows that Lorentz relation needs to be modified in high-energy cases. In the case of high energy, Lorentz invariance is broken. Therefore, it is necessary for us to study the physics theory in flat space-time and curved space-time according to Lorentz breaking [4–10]. Now, people are mainly concerned about some corrections caused by Lorentz breaking effect and their physical significance. In the process of studying the forms of various possible Lorentz breaking terms, people put forward a Lorentz breaking term called the aether-like term. The gravitational model of the scalar field, vector field, spinor field, and electromagnetic field containing this term has been widely studied, and a series of meaningful research results have been obtained [11–34]. For different curved space-time, we need to consider different space-time characteristics and field characteristics, so as to make corrections to the quantum tunneling radiation and obtain the correct corrected Hawking temperature and Bekenstein–Hawking entropy. After the aether-like vector is introduced in the case of Lorentz breaking in the literature [30], the scalar particle dynamics equation is modified, and the correction of Hawking temperature and Bekenstein–Hawking entropy of a class of stationary space-time is studied. The literature [35] provides a method to study the dynamics equation of fermions in curved space-time, provides a modified dispersion relationship proposed in the research process of string theory and quantum gravity theory, and studies fermions. After considering Lorentz invariance violation and introducing the gamma matrix, the modified dispersion relation is obtained. At the same time, the literature [35] also introduces the higher-order quantum effect of $\hbar$, and further studies the fermion dynamics equation. The spatiotemporal metric of general dynamic spherically symmetric black holes has its particularity, and the results obtained from studying the radiation of this black hole include the relevant results of the Vaidya–Bonner black hole and Vaidya black hole. Therefore, considering the Lorentz breaking theory, studying the radiation characteristics of general dynamic spherically symmetric black holes is a meaningful research work. In the second section below, the CFJ correction term is introduced. The aether-like term studies the fermion dynamics equation more accurately in curved space-time. In the third section, the fermion dynamics equation is solved to obtain the expressions corresponding to tunneling radiation. In the final section, we analyze and discuss the results obtained.

2. Modified Fermion Dynamics Equation in Curved Space-Time

The Lorentz breaking model of field theory is still a hot research topic, especially the possible expansion of the standard model. The first known Lorentz breaking term is the Carroll–Field–Jackiw (CFJ) term, which has been partially studied [36,37]. In recent years, people have modified the quantum tunneling radiation of a class of black holes by applying Lorentz breaking theory to curved space-time and obtained a series of meaningful results [30,35]. Without considering Lorentz breaking theory, the action of spinor in curved space-time is:

$$S_F = \int d^4x\sqrt{-g}\bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi = \int d^4x\sqrt{-g}\mathcal{L}_F. \tag{1}$$

where:

$$
\begin{aligned}
D_\mu &= \partial_\mu + \frac{i}{2}\Gamma_\mu^{\alpha\beta}\Pi_{\alpha\beta}, \\
\Pi_{\alpha\beta} &= \frac{i}{4}\left[\gamma^\alpha, \gamma^\beta\right],
\end{aligned} \tag{2}
$$

and:

$$
\mathcal{L}_F = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \tag{3}
$$

$\mathcal{L}_F$ is the uncorrected Lagrange density. ψ is the wave function corresponding to the spinor field $\mathcal{L}_F$, and $\bar{\psi}$ is the conjugate of ψ. $\Gamma_\mu^{\alpha\beta}$ is spin connection. It can be obtained from the variational principle and Equations (1) and (3):

$$
\delta S_F = \int d^4x\sqrt{-g}\delta\mathcal{L}_F = 0. \tag{4}
$$

It can be seen from Equations (3) and (4), so there is:

$$
\left(i\gamma^\mu D_\mu - m\right)\psi = 0, \tag{5}
$$

and:

$$
\bar{\psi}\left(i\gamma^\mu D_\mu + m\right) = 0. \tag{6}
$$

Equation (6) is the conjugate form of Equation (5); therefore, we only need to study Equation (5). Equation (5) is a fermions dynamics equation. In curved space-time, taking into account the Lorentz breaking theory and the characteristics of fermion spin, the corrected action of the spinor field includes three parts: 1. CFJ term; 2. aether-like field vector term; 3. Chiral term. After considering the three correction terms, we can modify the action of spinor field in curved space-time to:

$$
\begin{aligned}
S_F{}' &= \int d^4x\sqrt{-g}\bar{\psi}\left[i\gamma^\mu D_\mu\left(1 - \frac{a\hbar^2}{m^2}\gamma^\mu D_\mu\gamma^\nu D_\nu\right) + \frac{b\hbar}{m}u^\mu u^\nu D_\mu D_\nu + \frac{c}{\hbar}\gamma^5 - \frac{m}{\hbar}\right]\psi \\
&= \int d^4x\sqrt{-g}\mathcal{L}_F{}'.
\end{aligned} \tag{7}
$$

where:

$$
\mathcal{L}_F{}' = \bar{\psi}\left[i\gamma^\mu D_\mu\left(1 - \frac{a\hbar^2}{m^2}\gamma^\mu D_\mu\gamma^\nu D_\nu\right) + \frac{b\hbar}{m}\gamma^\mu D_\mu\gamma^\nu D_\nu + \frac{c}{\hbar}\gamma^5 - \frac{m}{\hbar}\right]\psi \tag{8}
$$

The terms related to a, b, and c in the Lagrange density $\mathcal{L}_F$ expression represent the CFJ correction term, the aether-like field correction term, and the Chiral correction term, respectively. According to the variational principle and Equations (7) and (8), we can get:

$$
\delta S_F{}' = \int d^4x\sqrt{-g}\delta\mathcal{L}_F{}' = 0 \tag{9}
$$

that is:

$$
\delta\mathcal{L}_F{}' = \delta\{\bar{\psi}\left[i\gamma^\mu D_\mu\left(1 - \frac{a\hbar^2}{m^2}\gamma^\mu D_\mu\gamma^\nu D_\nu\right) + \frac{b\hbar}{m}u^\mu u^\nu D_\mu D_\nu + \frac{c}{\hbar}\gamma^5 - \frac{m}{\hbar}\right]\psi\} = 0. \tag{10}
$$

As such, the corrected matrix equation satisfied of fermion wave function ψ is:

$$
\left[i\gamma^\mu D_\mu\left(1 - \frac{a\hbar^2}{m^2}\gamma^\mu D_\mu\gamma^\nu D_\nu\right) + \frac{b\hbar}{m}u^\mu u^\nu D_\mu D_\nu + \frac{c}{\hbar}\gamma^5 - \frac{m}{\hbar}\right]\psi = 0. \tag{11}
$$

Its conjugate form is:

$$\bar{\psi}\left[i\gamma^{\mu}D_{\mu}\left[1-\frac{a\hbar^{2}}{m^{2}}\gamma^{\mu}D_{\mu}\gamma^{\nu}D_{\nu}\right]+\frac{b\hbar}{m}u^{\mu}u^{\nu}D_{\mu}D_{\nu}+\frac{c}{\hbar}\gamma^{5}+\frac{m}{\hbar}\right]=0. \tag{12}$$

Equation (12) is the conjugate form of Equation (11). Therefore, we only need to study the field Equation (11). When we study the fermion tunneling rate, we need to start from Equation (11), where the gamma matrix needs to be determined according to the specific black hole space-time metric $g^{\mu\nu}$; it satisfies the following relationship [38]:

$$\begin{aligned}\gamma^{\mu}\gamma^{\nu}+\gamma^{\nu}\gamma^{\mu}&=2g^{\mu\nu}I,\\ \gamma^{\mu}\gamma^{5}+\gamma^{5}\gamma^{\mu}&=0.\end{aligned} \tag{13}$$

The gamma matrix satisfying the above conditions is determined and reflects the basic characteristics of curved space-time. In Equation (11), u^{μ} is an aether-like vector. In curved space-time, u^{μ} needs to meet the following conditions:

$$u^{\mu}u_{\mu}=const. \tag{14}$$

The wave function in Equation (11) is closely related to fermions with different spins. For the fermion with spin $-\frac{1}{2}$, Equation (11) becomes its dynamic equation. For fermions with spin $-\frac{3}{2}$, $\frac{5}{2}$, ..., the dynamic equation should be determined by the Rarita–Schwinger equation. Equations (11), (13), and (14) are the theoretical basis for solving the equation. For fermions with mass m and charge q, D_{μ} in Equation (11) is:

$$\begin{aligned}D_{\mu}&=\partial_{\mu}-\frac{i}{\hbar}\mathrm{eA}_{\mu}+\frac{i}{2}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta},\\ \Pi_{\alpha\beta}&=\frac{i}{4}\left[\gamma^{\alpha},\gamma^{\beta}\right].\end{aligned} \tag{15}$$

γ^{α} and γ^{β} are the gamma matrix corresponding to curved space-time. According to WKB theory, for Dirac particles with spin $-\frac{1}{2}$, its wave function can be expressed as:

$$\psi=\begin{pmatrix}\mathrm{A}\\ \mathrm{B}\end{pmatrix}\mathrm{e}^{\frac{i}{\hbar}\mathrm{s}}. \tag{16}$$

The S in the Equation (16) is the spin$-\frac{1}{2}$ fermion action. Substituting Equation (16) into Equation (11), we have:

$$\left[i\gamma^{\mu}D_{\mu}\left(1-\frac{a\hbar^{2}}{m^{2}}\gamma^{\mu}D_{\mu}\gamma^{\nu}D_{\nu}\right)-\frac{b}{m}u^{\mu}u^{\nu}D_{\mu}D_{\nu}+\frac{c}{m}\gamma^{5}-\frac{m}{\hbar}\right]\begin{pmatrix}A\\ B\end{pmatrix}\mathrm{e}^{\frac{i}{\hbar}\mathrm{s}}=0, \tag{17}$$

and:

$$D_{\mu}\psi=\left(\partial_{\mu}-\frac{i}{\hbar}\mathrm{eA}_{\mu}+\frac{i}{2}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}\right)\begin{pmatrix}A\\ B\end{pmatrix}\mathrm{e}^{\frac{i}{\hbar}\mathrm{s}}=\frac{i}{\hbar}\widetilde{D_{\mu}}\begin{pmatrix}A\\ B\end{pmatrix}\mathrm{e}^{\frac{i}{\hbar}\mathrm{s}}, \tag{18}$$

where:

$$\widetilde{D_{\mu}}=\partial_{\mu}s-\mathrm{eA}_{\mu}+\frac{\hbar}{2}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}. \tag{19}$$

Multiplying $\hbar$ on both sides of the Equation (17), and ignoring the $\mathrm{e}^{\frac{i}{\hbar}\mathrm{s}}$, we have the metric equation as follows:

$$\begin{aligned}&\left\{\gamma^{\mu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left[1+\frac{a}{m^{2}}\gamma^{\mu}\gamma^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)\right]\right.\\ &\left.+bu^{\mu}u^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)-c\gamma_{0}^{5}+m+\frac{1}{2}\hbar\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}\right\}\begin{pmatrix}A\\ B\end{pmatrix}=0.\end{aligned} \tag{20}$$

The condition for this matrix equation to have a solution is that its metrix corresponding to the determinant coefficient value is zero, so:

$$\gamma^{\mu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)=-\left[1-\frac{a}{m^{2}}\gamma^{\mu}\gamma^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)\right]\\ \times\left[\frac{b}{m}u^{\mu}u^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)-\frac{c}{m}\gamma_{0}^{5}+m\right]. \tag{21}$$

where γ_0^5 is the real part of γ^5. We multiply both sides of Equation (22) with $\gamma^{\nu}(\partial_{\nu}S-eA_{\nu})$, and then we get:

$$\begin{aligned}&\gamma^{\mu}\gamma^{\nu}\left(\partial_{\mu}S-eA_{\mu}\right)\left(\partial_{\nu}S-eA_{\nu}\right)\\&=\left[1-\frac{a}{m^{2}}\gamma^{\mu}\gamma^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)\right]^{2}\\&\left[\frac{b}{m}u^{\mu}u^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)-\frac{c}{m}\gamma_{0}^{5}+m+\hbar\gamma^{\mu}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}\right]^{2}\\&=\left[2bu^{\mu}u^{\nu}\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)-2c\gamma_{0}^{5}+m^{2}+\hbar\gamma^{\mu}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}\right]\\&-2a\gamma^{\mu}\gamma^{\nu}\gamma^{\nu}\left(\partial_{\nu}S-eA_{\mu}\right)\left(\partial_{\nu}S-eA_{\nu}\right)\end{aligned} \tag{22}$$

Using the gamma matrix and the curvature space-time metric tensor, Equation (22) can be reduced to:

$$\left(g^{\mu\nu}+2ag^{\mu\nu}-2bu^{\mu}u^{\nu}\right)\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)+2c\gamma_{0}^{5}-m^{2}+\hbar m\gamma^{\mu}\Gamma_{\mu}^{\alpha\beta}\Pi_{\alpha\beta}=0. \tag{23}$$

Equations (20) and (23) are both fermion dynamics equations. In the semi-classical case, ignoring $\hbar$ term, the modified form is:

$$\left(g^{\mu\nu}+2ag^{\mu\nu}-2bu^{\mu}u^{\nu}\right)\left(\partial_{\mu}s-\mathrm{eA}_{\mu}\right)\left(\partial_{\nu}s-\mathrm{eA}_{\nu}\right)+2c\gamma_{0}^{5}-m^{2}=0. \tag{24}$$

After ignoring the $\hbar$ item, Equation (24) is actually a modified field equation. In Equation (24), γ_0^5 is closely related to γ^5. For different curved space-time, it is necessary to construct different γ^5, and the form of γ^5 corresponds to the specific curved space-time. By constructing γ^5 correctly, we can know the specific form of γ_0^5. Only by knowing $g^{\mu\nu}$, u^{μ}, and γ_0^5 can we solve Equation (24). Equation (24) is simplified to a relatively simple matrix equation, which is the Hamilton–Jacobi equation of curved space-time obtained by utilizing the commutative relationship of gamma matrices. The derivation of this equation can effectively simplify the tunneling theory of fermion particle with spin $-\frac{1}{2}$. Next, we will use this half of the classical equation to derive and study the Hawking radiation of the Dirac particle of the relevant black hole.

3. The Corrected Tunnelling Radiation of General Non-Stationary Spherically Symmetric Black Hole

Equation (24) is the fermion dynamics equation after CFJ correction, Chical correction, and aether-like correction. The results obtained from this equation correspond to the semi-classical theory. Equation (24) studies the correction and related problems of the quantum tunneling radiation of fermions with half spin in general non-stationary spherically symmetric complete space-time. The general spherically symmetric non-stationary black hole line element is [39]:

$$\mathrm{d}s^{2}=-e^{2\Psi}\left(1-\frac{2M}{r}+\frac{Q^{2}}{r^{2}}\right)dv^{2}+2e^{\Psi}dvdr+r^{2}\left(d\theta^{2}+\sin^{2}\theta d\phi^{2}\right). \tag{25}$$

The metric determinant and the non-zero components of the inverse metric tensor are, respectively:

$$g=-e^{2\Psi}r^{4}\sin^{2}\theta. \tag{26}$$

$$\begin{aligned} g^{01} &= g^{10} = e^{-\Psi}. \\ g^{11} &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \\ g^{22} &= \frac{1}{r^2}, \\ g^{33} &= \frac{1}{r^2 \sin^2\theta}, \end{aligned} \tag{27}$$

where $\Psi = \Psi(r,v),\ M = M(r,v), Q^2 = Q^2(v)$, v is the advanced Eddington coordinate, and electromagnetic potential $\mathrm{A}_\mu = \mathrm{A}_0 = \frac{-Q}{r}$. It should be noted that $g^{00} = 0$ is calculated from the components of (25) and (26), as well as the metric tensor, which is a unique feature of this space-time. This black hole can be divided into thermal radiation and non-thermal radiation. We first study the quantum tunneling radiation of this black hole, namely Hawking radiation, which is a kind of thermal radiation. To achieve this, it is necessary to build $g^{\mu\nu}, u^\mu, \gamma_0^5$. According to Equations (13), (25), and (26), we can construct the gamma matrix as follows:

$$\begin{aligned} \gamma^v &= \frac{g^{01}}{\sqrt{g^{11}}}\left[i \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} + \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \right], \\ \gamma^r &= \sqrt{g^{11}} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \\ \gamma^\theta &= \sqrt{g^{22}} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \\ \gamma^\phi &= \sqrt{g^{33}} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}. \end{aligned} \tag{28}$$

Construct γ^5 based on (13), (25), and (27), as follows:

$$\begin{aligned} \gamma^5 &= \left[\gamma^v - \frac{g^{01}}{\sqrt{g^{11}}} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \right] \gamma^r \gamma^\theta \gamma^\phi \\ &= \frac{\sin^2\theta}{\sqrt{g}} \frac{g^{01}}{\sqrt{g^{11}}} \sqrt{g^{11} g^{22} g^{33}} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} \\ &= \frac{e^{-2\Psi}}{r^4} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} \\ &= \gamma_0^5 \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \end{aligned} \tag{29}$$

where ${\gamma_0}^5 = \frac{\mathrm{e}^{-2\Psi}}{r^4}$. The conditions are that Equations (28) and (29) must meet Equation (13). I in Equation (28) is the unit matrix and the expression of the Pauli matrix is as follows:

$$\sigma^1 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{30}$$

It can be concluded from Equations (13), (28), and (30) that:

$$\begin{aligned} \gamma^v \gamma^5 + \gamma^5 \gamma^v &= 0 \\ \gamma^r \gamma^5 + \gamma^5 \gamma^r &= 0 \\ \gamma^\theta \gamma^5 + \gamma^5 \gamma^\theta &= 0 \\ \gamma^\phi \gamma^5 + \gamma^5 \gamma^\phi &= 0. \end{aligned} \tag{31}$$

It can be known from Equation (28):

$$\begin{aligned}
&\gamma^v\gamma^r+\gamma^r\gamma^v\\
&=g^{01}\left[\begin{pmatrix} I & 0\\ 0 & -I\end{pmatrix}+\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\right]\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\\
&+g^{01}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\left[\begin{pmatrix} I & 0\\ 0 & -I\end{pmatrix}+\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\right]\\
&=2g^{01}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\\
&=2g^{01}\begin{pmatrix} I & 0\\ 0 & I\end{pmatrix},\\
&\gamma^r\gamma^r+\gamma^r\gamma^r=2g^{11}I,\\
&\gamma^\theta\gamma^\theta+\gamma^\theta\gamma^\theta=2g^{22}I\\
&\gamma^\phi\gamma^\phi+\gamma^\phi\gamma^\phi=2g^{33}I.
\end{aligned} \tag{32}$$

Obviously, the gamma matrix we selected fully satisfies the conditions of Equation (13). In order to solve Equation (24), we construct the aether-like field vector according to Equations (25) and (27):

$$\begin{aligned}
u^v&=\frac{c_v}{\sqrt{g_{vv}}};u^v u_v=u^v u^v g_{vv}=\frac{{c_v}^2}{g_{vv}}g_{vv}={c_v}^2\\
u^r&=\frac{c_r\sqrt{g_{vv}}}{g_{vr}};u^r u_r=u^r u^v g_{vr}=c_v c_r\\
u^\theta&=\frac{c_\theta}{\sqrt{g_{\theta\theta}}};u^\theta u_\theta=u^\theta u^\theta g_{\theta\theta}=\frac{{c_\theta}^2}{g_{\theta\theta}}g_{\theta\theta}={c_\theta}^2\\
u^\phi&=\frac{c_\phi}{\sqrt{g_{\phi\phi}}};u^\phi u_\phi=u^\phi u^\phi g_{\phi\phi}=\frac{{c_\phi}^2}{g_{\phi\phi}}g_{\phi\phi}={c_\phi}^2,
\end{aligned} \tag{33}$$

where c_v, c_r, c_θ, and c_ψ are real constants. Now, by substituting Equations (27) and (29) into Equation (24), we can get the simplified form of the fermion dynamics equation with mass m, charge e, and spin $\frac{1}{2}$:

$$\begin{aligned}
&\left(1+2a-2b{c_r}^2\right)g^{11}\left(\frac{\partial S}{\partial r}\right)^2+2e^{-\Psi}(1+2a-2bc_v c_r)\left(\frac{\partial S}{\partial v}-eA_0\right)\left(\frac{\partial S}{\partial r}\right)\\
&+2be^{-2\Psi}{c_v}^2\left(g^{11}\right)^{-1}\left(\frac{\partial S}{\partial v}-eA_0\right)^2+2c\gamma_0^5-m^2+D(\theta,\phi)=0.
\end{aligned} \tag{34}$$

where $D(\theta,\phi)$ is the item containing θ and ϕ. The dynamic equation in non-stationary curved space-time must be transformed by tortoise coordinates. Therefore, in order to solve Equation (34), by spherical symmetry characteristics, we make the following tortoise coordinate transformation:

$$\begin{aligned}
r_*&=r+\frac{1}{2\kappa}\ln\left[\frac{r-r_H(v)}{r_H(v_0)}\right]\\
v_*&=v-v_0
\end{aligned} \tag{35}$$

It can be concluded that:

$$\begin{aligned}
\frac{\partial}{\partial r}&=\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\\
\frac{\partial}{\partial v}&=\frac{\partial}{\partial v_*}-\frac{\dot{r}_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*}.
\end{aligned} \tag{36}$$

In the special case of spherical symmetry, the particle action S in Equation (34) can be written as:

$$S = R(v_*, r_*) + Y(\theta, \phi). \tag{37}$$

The event horizon of a black hole can be determined according to the following null hypersurface equation:

$$g^{\mu\nu} \frac{\partial F}{\partial x^\mu} \frac{\partial F}{\partial x^\nu} = 0. \tag{38}$$

where the normal vector of hypersurface $F(x^\mu) = 0$ is $\frac{\partial F(x^\mu)}{\partial x^\mu}$. Equation (38) is the definition of null hypersurface. Substituting Equation (27) into Equation (38), we can get the equation that the event horizon r_H of this black hole satisfies as follows:

$$2\mathrm{e}^{-\Psi} \dot{r}_H + 1 - \frac{2M}{r_H} + \frac{Q^2}{{r_H}^2} = 0, \tag{39}$$

where $\dot{r}_H = \frac{\partial r}{\partial v}|_{r=r_H}$. In fact, because of the macroscopic quantum tunneling radiation of the black hole, when a particle with mass of m and charge e is emitted from the black hole, r_H of the black hole will have a very weak contraction, namely $\mathrm{M} \to \mathrm{M} - \mathrm{m},\ \mathrm{Q} \to \mathrm{Q} - \mathrm{e}$; as such, Equation (40) should add a very small real constant ε term, that is:

$$2\mathrm{e}^{-\Psi} \dot{r}_H + 1 - \frac{2M}{r_H} + \frac{Q^2}{{r_H}^2} + \varepsilon = 0. \tag{40}$$

Considering Equations (36)–(38), Equation (36) is reduced to:

$$\begin{aligned}
&\left(1 + 2a - 2b{c_r}^2\right) g^{11} \left[\frac{2\kappa(r - r_H) + 1}{2\kappa(r - r_H)} \frac{\partial R}{\partial r_*}\right]^2 \\
&+ 2e^{-\Psi}(1 + 2a - 2bc_v c_r) \left[\frac{2\kappa(r - r_H) + 1}{2\kappa(r - r_H)} \frac{\partial R}{\partial r_*}\right] \left[\frac{\partial R}{\partial v_*} - \frac{\dot{r}_H}{2\kappa(r - r_H)} \frac{\partial R}{\partial r_*} - eA_0\right] \\
&+ 2be^{-2\Psi}{c_v}^2 \left(g^{11}\right)^{-1} \left[\frac{\partial R}{\partial v_*} - \frac{\dot{r}_H}{2\kappa(r - r_H)} \frac{\partial R}{\partial r_*} - eA_0\right]^2 + 2c\gamma_0^5 - m^2 + D_0 = 0,
\end{aligned} \tag{41}$$

where D_0 is the constant generated by separating variables and γ_0^5 is a real constant. Equation (41) can consider the case of $r \to r_H$:

$$\begin{aligned}
&[2\kappa(r - r_H)]^{-1} \left\{ \left[\left(1 + 2a - 2b{c_r}^2\right) g^{11}(r_H)\right] + 2b{c_v}^2 \dot{r}_H^2 e^{2\Psi} \left(g^{11}(r_H)\right)^{-1} + \right. \\
&\left. 2e^{-\Psi} \dot{r}_H (1 + 2a - 2bc_r c_v) \right\} \left(\frac{\partial R}{\partial r_*}\right)^2 |_{r \to r_H} \\
&+ \left[2e^{-\Psi'}(1 + 2a - 2bc_r c_v) - 4be^{2\Psi}{c_v}^2 \left(g^{11}(r_H)\right)^{-1} \dot{r}_H\right] \left(\frac{\partial R}{\partial r_*}\right) \left(\frac{\partial R}{\partial v_*}\right) |_{r \to r_H} \\
&+ \left[4\dot{r}_H b{c_v}^2 e^{2\Psi} \left(g^{11}(r_H)\right)^{-1} - 2e^{-\Psi} \dot{r}_H (1 + 2a - 2bc_r c_v) eA_0\right] \left(\frac{\partial R}{\partial r_*}\right) |_{r \to r_H} = 0.
\end{aligned} \tag{42}$$

The radiation particle energy is expressed by ω, and at the event horizon of the black hole, the Kodama vector condition [40]:

$$\frac{\partial R}{\partial v_*} = -\omega. \tag{43}$$

Let:

$$\begin{aligned}
&\mathrm{A} = g^{11}(r_H)\left(1+2a-2bc_r{}^2\right) + 2bc_v{}^2\dot{r}_H^2 e^{2\Psi'} g^{11}(r_H) + 2\dot{r}_H e^{-\Psi}(1+2a-2bc_v c_r) \\
&\mathrm{B} = e^{-\Psi}(1+2a-2bc_v c_r) - 2be^{2\Psi} c_v{}^2 \left(g^{11}(r_H)\right)^{-1}\dot{r}_H \\
&\mathrm{C} = 2be^{2\Psi} c_v{}^2 \left(g^{11}(r_H)\right)^{-1}\dot{r}_H - e^{-\Psi}(1+2a-2bc_v c_r) eA_0(r_H) \\
\omega_0 &= \frac{C}{B}.
\end{aligned} \tag{44}$$

Equation (42) is further reduced to:

$$\frac{A}{\kappa(r-r_H)B}|_{r\to r_H}\left(\frac{\partial R}{\partial r_*}\right)^2|_{r\to r_H} - 2(\omega-\omega_0)\frac{\partial R}{\partial r_*}|_{r\to r_H} = 0. \tag{45}$$

Let:

$$\lim_{r\to r_H}\frac{A}{\kappa(r-r_H)B} = 1, \tag{46}$$

where $g^{11}(r_H) = 1 - \frac{2m}{r_H{}^2} + \frac{Q^2}{r_H}$. Equation (45) is reduced to:

$$\left(\frac{\partial R}{\partial r_*}\right)^2|_{r\to r_H} - 2(\omega-\omega_0)\frac{\partial R}{\partial r_*}|_{r\to r_H} = 0. \tag{47}$$

According to Equation (46) and Lopida's law:

$$\frac{\left(1+2a-2bc_r{}^2\right)\left(\frac{2M}{r_H{}^2} - \frac{2Q^2}{r_H{}^3}\right) - 2\dot{r}_H(1+2a-2bc_v c_r)\Psi' e^{-\Psi}}{\kappa B} = 1, \tag{48}$$

where $\Psi' = \frac{\partial\Psi}{\partial r}$. As such:

$$\kappa = \frac{\left(1+2a-2bc_r{}^2\right)\left(\frac{2M}{r_H{}^2} - \frac{2Q^2}{r_H{}^3}\right) - 2\dot{r}_H(1+2a-2bc_v c_r)\Psi' e^{-\Psi}}{e^{-\Psi} g^{11}(r_H)(1+2a-2bc_v c_r) - 2b\left[g^{11}(r_H)\right]^{-1} c_v{}^2 \mathrm{e}^{2\Psi}\dot{r}_H}. \tag{49}$$

Equation (47) can be obtained:

$$\frac{\partial R_\pm}{\partial r} = [(\omega-\omega_0) \pm (\omega-\omega_0)], \tag{50}$$

and:

$$\frac{\partial R_\pm}{\partial r} = \frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}|_{r\to r_H}\frac{\partial R_\pm}{\partial r_*} = \frac{1}{2\kappa(r-r_H)}|_{r\to r_H}[(\omega-\omega_0) \pm (\omega-\omega_0)]. \tag{51}$$

According to residue theorem:

$$R_\pm = \pm\frac{i\pi}{\kappa}(\omega-\omega_0). \tag{52}$$

According to the WKB theory and the tunneling radiation theory of black holes, we obtain that the tunneling rate is:

$$\Gamma \sim \exp(-2ImR_\pm) = \exp\left[-\frac{4\pi}{\kappa}(\omega-\omega_0)\right] = \exp\left(-\frac{\omega-\omega_0}{T_H}\right). \tag{53}$$

where:

$$T_H = \frac{\kappa}{4\pi}. \tag{54}$$

κ is shown in Equation (49). κ is the surface gravity of the event horizon of this black hole, and T_H is the Hawking temperature at the event horizon of this black hole. Obviously, the specific expression of κ in Equation (35) is shown in Equation (53), and Equation (49) is a new form of the event horizon surface gravity of the black hole. Equation (54) is a new expression for the Hawking temperature at the event horizon of the black hole, due to Lorentz breaking and the CFJ correction term, the aether-like vector correction term, and the Chiral correction term. There is no Chiral correction term in Equations (49), (53) and (54). The above has made more accurate corrections to the Dirac particle quantum tunneling rate and Hawking temperature in the black hole space-time, which will inevitably lead to the correction of the black hole entropy.

The study of black hole thermodynamics also reveals the correlation between gravity theory and thermodynamics. In this field and the related fields derived from it, researchers have carried out decades of research and debate. Until now, the issues related to the evolution of black hole thermodynamics are still hot. Among them, Beckenstein believes that the entropy of black holes is directly proportional to their area, and formally introduces entropy into the thermodynamic study of black holes. At the same time, taking into account the quantum effect, Beckenstein recalculated the temperature of the black hole, and found that the temperature of the black hole is not absolute zero. Nowadays, the thermodynamic laws discovered by Beckenstein have become the fundamental laws of black holes. According to the first law of black hole thermodynamics, the entropy S_{BH} of black hole satisfies the following equation:

$$dS_{BH} = \frac{dM - VdQ}{T_H}, \tag{55}$$

$$\begin{aligned} S_{BH} = \int dS_{BH} &= \int \frac{dM - VdQ}{T_h}\Big[1 - \Big(2a - 2bc_v c_r - b{c_r}^2\Big) + \cdots\Big] \\ &= S_{bh}\Big[1 - \Big(2a - 2bc_v c_r - b{c_r}^2\Big) + \cdots\Big]. \end{aligned} \tag{56}$$

According to Equations (49) and (54), the corrected Hawking temperature is:

$$T_H = \frac{1}{4\pi}\frac{\left(1 + 2a - 2b{c_r}^2\right)\left(\frac{2M}{{r_H}^2} - \frac{2Q^2}{{r_H}^3}\right) - 2\dot{r}_H(1 + 2a - 2bc_v c_r)\Psi' e^{-\Psi}}{e^{-\Psi}(1 + 2a - 2bc_v c_r) + 2b\left(g^{11}(r_H)\right)^{-1}{c_v}^2\dot{r}_H e^{2\Psi}}. \tag{57}$$

T_h is the Hawking temperature of the general non-stationary spherically symmetric black hole before correction. S_{bh} is the original Bekenstein–Hawking entropy. S_{BH} is a new expression of the entropy of a general non-stationary black hole after taking into account the Lorentz breaking theory. If the entropy change is expressed by ΔS_{BH}, the tunneling rate can be re-expressed by:

$$\Gamma \sim \exp(\Delta S_{BH}). \tag{58}$$

According to Equations (53), (54), (56), and (57), these revised new expressions are all independent of the coefficient c of the Chiral correction term. In order to reflect the influence of the Chiral correction term, we need to study the non-thermal radiation of this black hole. In relativity and quantum field theory, all positive energy states in vacuum are empty, and all negative energy states are filled with a fermions particle state. For the non-thermal radiation of this black hole, the first step is to find the distribution characteristics of the positive and negative energy levels of particles.

In order to study the distribution characteristics of the Dirac particles level of the general non-stationary spherically symmetric black hole, we separate the variables (θ, ϕ) and (v_*, r_*); then, we can get the $R(r)$ from Equation (42):

$$\begin{aligned}&\Big\{[2\kappa(r-r_H)+1]^2\Big(1+2a-2bc_r{}^2\Big)g^{11}\\&-2e^{-\Psi}(1+2a-2bc_vc_r)\dot{r}_H[2\kappa(r-r_H)+1]-2b\Big(g^{11}\Big)^{-1}c_v{}^2e^{-2\Psi}\dot{r}_H^2\Big\}\left(\frac{\partial R}{\partial r_*}\right)^2\\&+2\kappa(r-r_H)[2\kappa(r-r_H)+1]\Big[2\mathrm{e}^{-\Psi}(1+2a-2bc_vc_r)+4beQr_H{}^{-1}\dot{r}_He^{-2\Psi}\Big]\left(\frac{\partial R}{\partial v_*}\right)\left(\frac{\partial R}{\partial r_*}\right)\\&+2\kappa(r-r_H)\Big[2e^{-\Psi}(1+2a-2bc_vc_r)eQr_H{}^{-1}+4beQr_H{}^{-1}\dot{r}_He^{-2\Psi}c_v{}^2\Big(g^{11}\Big)^{-1}\Big]\left(\frac{\partial R}{\partial r_*}\right)\\&+[2\kappa(r-r_H)]^2\left[2c\gamma_0^5-m^2+\left(\frac{\partial R}{\partial v_*}-eA_0\right)^2\right]=0.\end{aligned}\tag{59}$$

Simplifying Equation (42), we obtain:

$$A'\left(\frac{\partial R}{\partial r_*}\right)^2-2\kappa(r-r_H)B'\left(\omega+\frac{C'}{B'}\right)\frac{\partial R}{\partial r_*}+[2\kappa(r-r_H)]^2\left[2c\gamma_0^5-m^2+\left(\frac{\partial R}{\partial v_*}-\frac{eQ}{r}\right)^2\right]=0,\tag{60}$$

where:

$$\begin{aligned}\lim_{r\to r_H}A'&=A\\\lim_{r\to r_H}B'&=B\\\lim_{r\to r_H}C'&=C.\end{aligned}\tag{61}$$

From Equation (62), it can be concluded that:

$$\frac{\partial R}{\partial r_*}=\frac{1}{2A'}2(r-r_H)(\omega B'+C')\pm\frac{2\kappa(r-r_H)B'(\omega B'+C')}{2A'}\left[1-4A'\frac{2c\gamma_0^5-m^2+\left(\frac{\partial R}{\partial v_*}-\frac{eQ}{r}\right)^2}{(\omega B'+C')^2}\right]^{1/2},\tag{62}$$

$\frac{\partial R}{\partial r_*}$ is the particle generalized momentum of the radial direction and is a real number in order to conform to its physical meaning. It can be obtained from Equation (66):

$$1-4A'\frac{2c\gamma_0^5-m^2+\left(\frac{\partial R}{\partial v_*}-\frac{eQ}{r}\right)^2}{(\omega B'+C)^2}\geq 0,\tag{63}$$

thereby:

$$(\omega B'+C')^2-4A'\left[2c\gamma_0^5-m^2+\left(\frac{\partial R}{\partial v_*}-\frac{eQ}{r}\right)^2\right]\geq 0.\tag{64}$$

By simplifying Equation (64), it can be obtained that:

$$\begin{aligned}(\omega B'+C')^2-4A'\left[2c\gamma_0^5-m^2+\left(\omega+\frac{eQ}{r}\right)^2\right]\geq 0\\\omega^2\Big[(B')^2-4A'\Big]+\omega\left(2B'C'-8A'\frac{eQ}{r}\right)+C'^2-4A'\left(2c\gamma_0^5-m^2+\frac{e^2Q^2}{r^2}\right)\geq 0.\end{aligned}\tag{65}$$

Taking the equal sign to obtain:

$$\begin{aligned}\omega^{\pm}=&-\frac{1}{2\Big[(B')^2-4A'\Big]}\left(2B'C'-8A'\frac{eQ}{r}\right)\\&\pm\frac{1}{2\Big[(B')^2-4A'\Big]}\left\{\left(2B'C'-8A'\frac{eQ}{r}\right)^2-4\Big[B'^2-4A'\Big]\left[C'^2-4A'\left(2c\gamma_0^5-m^2+\frac{e^2Q^2}{r^2}\right)\right]\right\}^{1/2}.\end{aligned}\tag{66}$$

As such, the Dirac energy level distribution of the tunneling particle is:

$$\begin{aligned} \omega &> \omega^{+} \\ \omega &< \omega^{-}. \end{aligned} \tag{67}$$

For $r \to r_H$, due to $r \to r_H$, $\lim\limits_{r \to r_H} \mathrm{A}' = 0$, there is:

$$\omega^{\pm}|_{r \to r_H} = \frac{C}{B} = \omega_0. \tag{68}$$

$\omega^{\pm}|_{r \to r_H} = \omega_0$ indicates that due to the charge and dynamic characteristics of this black hole, the positive and negative energy levels of particles are crossing, which leads to the formation of positive energy particles through quantum tunneling radiation effect barriers. ω_0 is the crossing maximum value of the particle positive and negative energy level. For $\omega^{\pm}|_{r \to \infty} \to \pm\sqrt{m^2\left(1 - \frac{2c}{m^2}\gamma_0^5\right)}|_{r \to \infty} = \pm m\sqrt{1 - \frac{2c}{m^2}\gamma_0^5}|_{r \to \infty} \to \pm m$, the energy distribution of the particles is:

$$m < \omega \leq \omega_0. \tag{69}$$

Obviously, the correction coefficient c has a certain impact on the energy level of Dirac particles. Equations (68) and (69) show that there is a cross between positive and negative energy levels at the event horizon of the black hole, which will inevitably lead to the occurrence of non-thermal radiation; that is to say, at the event horizon of this black hole, when $\omega_0 > m$, non-thermal radiation occurs, which is the Starobinsky–Unruh process, that is, the spontaneous radiation process of this black hole. This quantum effect is non-thermal, that is, independent of the temperature of the black hole. The maximum energy of non-thermal particles is ω_0, and its expression is as follows:

$$\omega_0 = \frac{C}{B} = \frac{2be^{2\Psi}c_v^2\dot{r}_H\left[g^{11}(r_H)\right]^{-1} - e^{-\Psi}(1 + 2a - 2bc_vc_r)eA_0(r_H)}{e^{-\Psi}(1 + 2a - 2bc_vc_r) - 2be^{2\Psi}c_v^2\dot{r}_H\left[g^{11}(r_H)\right]^{-1}}, \tag{70}$$

where $\Psi = \Psi(r_H,\ v),\ A_0 = -\frac{Q}{r_H}$. ω_0 is a new expression, and ω_0 is related to $\dot{r}_H$, a, b, c_v, c_r. $\omega^{\pm}$ is not only related to $\dot{r}_H$, a, b, c_v, c_r, but $\omega^{\pm}$ is related to the Chiral correction term.

4. Discussion

Equation (24) is a modified form of the fermion dynamics equation describing the spin $-\frac{1}{2}$, which includes the CFJ term, aether-like term, and Chiral term. This is an applicable field equation that has been corrected by Lorentz breaking. We applied this equation to study the modified quantum tunneling radiation characteristics in general dynamic spherically symmetric black hole space-time. The results show that the quantum tunneling radiance, Hawking temperature, and Beckenstein–Hawking entropy of the black hole are related to the CFJ term and aether-like term. These results are not related to $m^2 - 2c\gamma_0^5$. Mathematically speaking, $A|_{r \to r_H} = 0$, $A(m^2 - 2c\gamma_0^5)|_{r \to r_H} = 0$, m and c will not appear in expressions due to T_H. In the physical sense, both the black hole temperature and the surface gravity of the black hole event horizon are related to the space-time characteristics. Therefore, $m^2 - 2c\gamma_0^5$ will not appear in the expression. However, when we study the distribution of Dirac energy levels, the term $m^2 - 2c\gamma_0^5$ will definitely appear in the Dirac energy level $\omega^{\pm}$. It is worth explaining further that when $r \to \infty$, $\gamma_0^5 \to 0$, the Dirac energy level $\omega^{\pm}|_{r \to \infty} \to \pm m$. This is consistent with known conclusions.

For the above results, it is necessary to explain $\Psi = 0, m = m(v),\ Q^2 = Q^2(v)$, which can be obtained from Equation (57):

$$T_H = \frac{1}{4\pi}\frac{1 + 2a - 2bc_r{}^2M/r_H{}^2}{(1 + 2a - 2bc_v\mathrm{c}_r) + 2b\left(g^{11}(r_H)\right)^{-1}c_v{}^2\dot{r}_H}, \tag{71}$$

$$m < \omega \leq \omega_0 = \frac{(1 + 2a - 2bc_v c_r) - 2b{c_v}^2 \left(g^{11}(r_H)\right)^{-1} \dot{r}_H}{2b{c_v}^2 \left(g^{11}(r_H)\right)^{-1} \dot{r}_H - (1 + 2a - 2bc_v c_r) eA_0}. \tag{72}$$

Here, Equations (71) and (72) are the Hawking temperatures and the energy level distribution of the Vaidya–Bonner black hole after correction. Respectively, when $\Phi = 0, Q^2 = 0$, T_H correspond to the Hawking temperature of the Vaidya black hole. The above research results show that due to the existence of Lorentz breaking, the CFJ term, aether-like field term, and Chiral term are introduced into the study of the fermion dynamics equation in curved space-time. The correction of the quantum tunneling rate of different black holes and the correction of entropy of different black holes are different, and specific conclusions need to be analyzed. In general, the Chiral term has no effect on the entropy, temperature, and tunneling radiance of the black hole, but the energy level distribution of the Dirac particle, the Chiral term, the aether-like term, and the CFJ term all have an effect on it.

Two specific points need to be explained here. Firstly, for different black holes, γ^μ, γ^5, and u^μ must be selected based on the specific black hole space-time. For boson, we cannot use the above method to correct it. We need to use the scalar field theory to correct the dynamics equation of boson. The second point to note is that for static black holes, there is only thermal radiation and no non-thermal radiation, because there is no crossing of positive and negative energy levels in the space-time of static black holes.

In astrophysics, the study of black hole physics has always been at the forefront and hot. In the theory of gravity, the study of black holes can be divided into astronomical observations and black hole physics. The new progress in astronomical observation today is that LIGO detected the gravitational wave generated by the merger of two black holes in 2016, and obtained the picture of the M87 black hole through the event horizon telescope in 2021. These observations not only prove the existence of black holes, but also promote people's research on black holes. In terms of theoretical physics, in the current situation that gravity cannot be renormalization and Grand Unified Theory, the relevant theory of Lorenz breaking has been proposed, and it is an interesting hotspot to combine it with black hole physics. Under the current energy scale, although there is no conclusive evidence to prove the existence of Lorentz breaking in the experiments conducted by researchers, it is often believed that the current equipment can provide a limited energy scale, so there is currently no exact signal to determine the existence of Lorentz breaking. However, the results in these experiments provide a reference for the coefficient lower limit of Lorentz's breaking theory. However, in astrophysics, its energy scale may satisfy the signal prerequisite for the occurrence of the Lorentz breaking phenomenon. Through correct scientific measurement and observation, it is possible to prove the existence of the Lorentz breaking phenomenon. From the above two research directions, black holes related to the Lorentz breaking theory are being used for popular research, such as photon orbitals, accretion disks, and black hole shadows. The coupling term that may cause Lorentz breaking in this type of popular research can, to some extent, reflect how it affects the relevant physical quantities of black holes. In the detection of gravitational waves, combined with the Lorentz breaking theory, we can screen out the potentially correct Lorentz breaking theory through the detection of gravitational waves. For example, the data obtained from the gravitational wave detection experiment can be compared with alternative theories related to Lorentz breaking, which may be able to select a suitable theory. In summary, the study of the Lorentz breaking theory will be an extremely important research direction.

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