

COSMOLOGICAL HISTORY OF THE HIGGS VACUUM INSTABILITY

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Abstract

A known property of the Higgs effective potential within the Standard Model is that it develops an instability for very large field values. During inflation, quantum fluctuations can overcome the potential barrier and make the Higgs fall into its true minimum at Planckian scales, forming anti-de Sitter patches that are lethal for the subsequent evolution of our Universe. By analysing the dynamics of the Higgs during and after inflation, we derive a bound on the inflationary Hubble rate that depends on the reheating temperature and on the coupling of the Higgs to the scalar curvature or the inflaton.

1 Introduction

Current measurements of the Higgs boson and top quark masses imply an extremely intriguing result: in the context of the Standard Model with no

additional physics, our universe lies at the edge between stability and instability of the Electro-Weak vacuum ¹⁾. Following the SM Renormalization Group equations, the quartic Higgs coupling becomes negative at a scale around $10^{10} \div 10^{11}$ GeV, with a strong dependence on the precise value of m_H and m_t . What is even more puzzling is the fact that, for the present best fit values of m_H and m_t , we live in the peculiar situation in which the EW vacuum is unstable, but the tunnelling probability is so suppressed that its lifetime is larger than the age of the universe. This fact is usually referred to as “metastability”.

The issue of vacuum instability becomes of particular interest in the early universe. There are three main effects that can modify the situation, and which can be used, in turn, to put bounds on early-time parameters:

1. During inflation, quantum fluctuations of the Higgs field are governed by the size of the Hubble parameter H . If this is large enough, the Higgs can overcome the potential barrier and fall into its deep minimum with negative energy, leading to the creation of regions of anti-de Sitter space.
2. A non minimal coupling of the Higgs to gravity can generate an effective mass term which stabilises the potential. The same could happen as a consequence of a coupling of the Higgs to the inflaton.
3. Thermal effects during the early phases of radiation dominance are twofold: fluctuations can trigger the “jump” of the barrier, while corrections to the effective potential create an additional effective barrier.

In order to study the evolution of the Higgs field h during the inflatorary and (pre-)heating phases, one first has to determine under which conditions h can fall into its true minimum, and second determine the evolution of the regions in which this have happened, under the assumption that the large negative potential energy forces the metric to be anti-de Sitter inside the bubble.

The aim of this talk is to summarize the discussion of ²⁾, in which the full process is reconsidered and new conclusion are drawn on the value of the Hubble parameter during inflation and other relevant physical quantities.

2 Higgs fluctuation during inflation

During inflation, quantum fluctuations of long wavelength modes of the Higgs field are governed, in the absence of a large mass term, by a Langevin stochastic

equation. Starting from $h = 0$ at $t = 0$ and generating a large set of random evolution of h , the resulting distribution is well approximated by a gaussian distribution

$$P(h, N) = \frac{1}{\sqrt{2\pi\langle h^2 \rangle}} \exp\left(-\frac{h^2}{2\langle h^2 \rangle}\right), \quad \sqrt{\langle h^2 \rangle} = \frac{H}{2\pi}\sqrt{N}. \quad (1)$$

The \sqrt{N} behaviour signals the fact that the potential $V(h)$ can be neglected and the evolution is dominated by quantum fluctuations. It's only in the very tail that the distribution becomes non-gaussian: this is due to the fact that the potential term becomes dominant, and the Higgs starts rolling classically down towards its minimum.

2.1 Addition of an effective mass term

Higgs fluctuations during inflation can get damped if the Higgs doublet Φ_H acquires a mass term during inflation. This could happen because of a Higgs-inflaton coupling, because of a non-vanishing temperature generated during inflation by inflaton decays, or thanks to a non minimal coupling of the Higgs to gravity. We will consider here only this last possibility, by adding to the effective lagrangian a term

$$-\xi_H |\Phi_H|^2 R \quad (2)$$

which for constant R produces a large mass $m^2 = \xi_H R = -12\xi_H H^2$. Notice that the presence of this term is unavoidable, since it is generated by RG equations for ξ_H , which have as the only fixed point the conformal value $\xi_H = -1/6$. Assuming $\xi_H < 0$, the potential is stabilized by the effective mass term: if $\xi_H < -3/16$ then fluctuations are exponentially damped, otherwise if $-3/16 < \xi_H < 0$ the distribution at the end of inflation is again quasi gaussian, with

$$\sqrt{\langle h^2 \rangle} = \frac{H}{4\pi\sqrt{-2\xi_H}}. \quad (3)$$

In this case, the presence of the large mass term invalidates the use of the Langevin equation, and the evolution of the probability must be studied by means of a Fokker-Planck equation, taking $P(h = \pm\infty) = 0$ as boundary conditions. Results are summarized in fig.1. Three regions can be distinguished, depending on the value of h at the end of inflation:

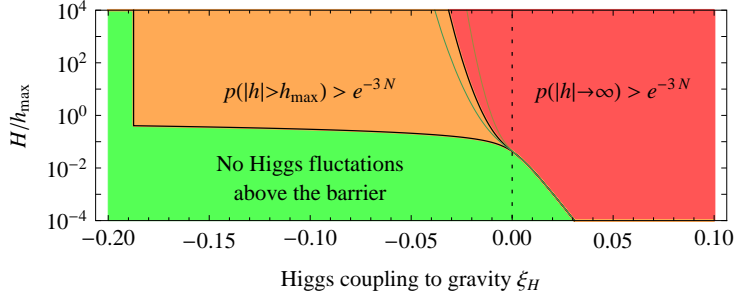


Figure 1: As a function of ξ_H and the Hubble constant in units of the instability scale h_{\max} (and for $N = 60$ e -folds of inflation), we show the three regions where: the probability for the Higgs field to end up in the negative-energy true minimum is larger than e^{-3N} (red); the probability for the Higgs field to fluctuate beyond the potential barrier is larger than e^{-3N} (orange); the latter probability is smaller than e^{-3N} (green). Higgs fluctuations are damped for $\xi_H < -3/16$. The uncertainty on the orange/red boundary corresponds to a fudge factor $1/3 < k < 3$.

1. Regions in which h is smaller than the scale $h_{\max} \approx 5 \times 10^{10}$ GeV at which the potential $V(h)$ has its maximum. After inflation ends, h just rolls down its potential, until it reaches the EW vacuum.
2. Regions in which $h > h_{\max}$ but quantum fluctuations still dominate over classical rolling, so that at the end of inflation they have not fallen into the true minimum yet.
3. Regions in which h falls into its deep minimum and an AdS bubble forms during inflation.

3 Fate of the AdS bubbles

Understanding the fate of the regions in which the Higgs falls into its true minimum is a very complicated task. An involved GR calculation is presented in ²⁾, under the assumptions of spherical bubbles with a thin wall to separate

them from the external background metric. Results can be summarized as follows:

- During inflation (de Sitter background), bubbles can expand or shrink depending on parameters such as the size of the bubble, its internal energy, initial wall velocity and surface tension. Even for expanding bubbles, cosmic expansion is fast enough to hide them behind a de Sitter horizon, so that they don't eat up the whole universe.
- After inflation ends (quasi-Minkowski background), expanding bubbles continue their growth faster than the expansion rate of the universe, and eventually "eat" all space.

As a general conclusion, there is no GR effect that can prevent bubbles from filling the universe. We must then impose that bubbles do not form during inflation: the red region in fig.1 is therefore excluded. As we will discuss in the next section, the orange region can instead be saved by thermal effects during reheating.

4 Thermal effects during radiation dominance

Even if one may naïvely think that the effect of a thermal bath would be that of further destabilize the situation by adding thermal fluctuations, their main consequence is actually the opposite ³⁾: thermal corrections to the effective potential generate a temperature dependent mass term $m^2 \propto T^2$ which stabilizes the potential. During the reheating phase, temperature (and therefore the thermal mass term) rises up to the value T_{\max} , then decreases as $a^{-3/8}$ until it reaches T_{RH} at the end of the reheating phase, and finally starts following the a^{-1} behaviour typical of radiation dominance. If T_{RH} is high, the thermal mass is large enough to change the slope of the effective potential $V(h)$ for values $h > h_{\max}$ (which correspond to the orange region of fig.1). The Higgs field starts rolling towards zero, and if it crosses the critical value h_{\max} before temperature drops then no bubble form. Fig.2 shows the minimal reheating temperature needed in order to avoid bubble formation after inflation, demonstrating that the orange region can be saved by thermal effects.

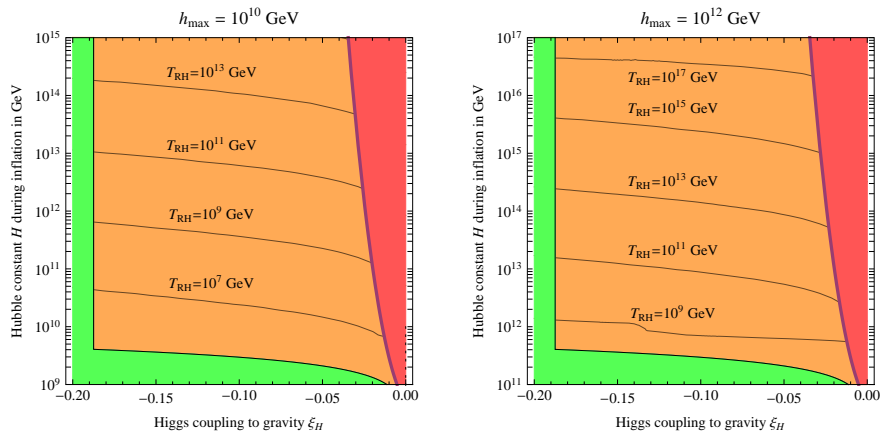


Figure 2: *Minimal reheating temperature T_{RH} needed to prevent the fall of the Higgs down into its deep true vacuum, assuming two different values for the instability scale h_{max} of the Higgs potential.*

5 Conclusion and possible new directions

We studied the evolution of the Higgs field and its instability, during inflation and during the early phases of radiation dominance. Whenever the Higgs falls in its deep minimum, a bubble of AdS forms, (possibly) expands and eventually eats all the visible universe. Bounds can be put on inflationary parameters by requiring that no bubble forms during inflation. Thermal effects after inflation and induced Higgs mass terms (e.g. non minimal coupling to gravity) play a key role in saving the EW vacuum.

References

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