

## HIGH PRECISION QCD PHYSICS AT FCC-EE

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### Abstract

The Future Circular Collider (FCC) is a post-LHC project aiming at direct and indirect searches for physics beyond the SM in a new 100 km tunnel at CERN. In addition, the FCC-ee offers unique possibilities for high-precision studies of the strong interaction in the clean environment provided by  $e^+e^-$  collisions, thanks to its broad span of center-of-mass energies ranging from the  $Z$  pole to the top-pair threshold, and its huge integrated luminosities yielding  $10^{12}$  and  $10^8$  jets from  $Z$  and  $W^\pm$  bosons decays, respectively, as well as  $10^5$  pure gluon jets from Higgs boson decays. In this contribution, we will summarize studies on the impact the FCC-ee will have on our knowledge of the strong force including: (i) Quantum Chromodynamics (QCD) coupling extractions with per-mille uncertainties, (ii) parton radiation and parton-to-hadron fragmentation functions, (iii) jet properties (light-quark-gluon discrimination,  $e^+e^-$  event shapes and multijet rates, jet substructure, etc.), (iv) heavy-quark jets (dead cone effect, charm-bottom separation, gluon  $\rightarrow c\bar{c}$ ,  $b\bar{b}$  splitting, etc.); and (v) non-perturbative QCD phenomena (color reconnection, baryon and strangeness production, Bose-Einstein and Fermi-Dirac final-state correlations, etc.).

### 1 Introduction

A crucial aspect for many physics measurements is a precise understanding of QCD. An accurate determination of the strong coupling constant  $\alpha_S$  is mandatory to improve the precision of the production cross sections and decays calculation. The computation of higher-order corrections up to next-to-next-to-leading order ( $N^3\text{LO}$ ) and next-to-next-to-leading logarithm ( $N^2\text{LL}$ ) is also central because it can increase the precision in observables predictivity. Another pivotal ingredient is a precise picture of jet substructure, parton showering, hadronization and colour reconnection, whose understanding benefits any hadronic final state.

The FCC-*ee* program <sup>1)</sup>, with its large integrated luminosities and clean environment, offers a rich QCD program. QCD studies with an unprecedented precision can be performed due to the large expected number of events at the FCC-*ee* of roughly  $\sim 10^{11}$   $Z$  at  $\sqrt{s} = 91$  GeV,  $\sim 10^7$   $W^+W^-$  at  $\sqrt{s} = 160$  GeV and  $\sim 10^6$   $ZH$  at  $\sqrt{s} = 240$  GeV.

## 2 The strong coupling constant

The least precisely known of all interaction coupling constant is  $\alpha_S$ , with an overall uncertainty at per-mille level,  $\delta\alpha_S \sim 10^{-3}$ . Currently,  $\alpha_S$  is determined by comparing 7 experimental observables to perturbative QCD (pQCD) predictions, plus a global average at the  $Z$  pole scale. The relevant observable for  $e^+e^-$  collisions are  $e^+e^-$  jet shapes and hadronic  $\tau$  leptons and  $W/Z$  bosons decays.

### 2.1 $\alpha_S$ from $e^+e^-$ event shapes and jet rates

As already done at LEP <sup>2)</sup>, the thrust ( $\tau$ ) and the  $C$ -parameter defined in Eq. 1 can be used to extract  $\alpha_S$ :

$$\tau = 1 - T = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|} \quad C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{i,j}}{(\sum_i |\vec{p}_i|)^2}, \quad (1)$$

with  $\theta_{i,j}$  the angle between particle  $i$  and  $j$  and  $\vec{p}_{i,j}$  the momentum respectively. Other quantities which are sensitive to  $\alpha_S$  are the  $n$ -jet rates,  $R_n = \frac{\sigma_{n\text{-jet}}}{\sigma_{\text{tot}}}$ , and therefore were used to extract the strong coupling constant. The comparison between the experimental measurements and N<sup>3</sup>LO+N<sup>2</sup>LL predictions yields  $\alpha_S(m_Z) = 0.1171 \pm 0.0027$  ( $\pm 2.6\%$ ).

At lower  $\sqrt{s}$ , the  $n$ -jet rates up to 7 jets could be studied <sup>3)</sup>, while runs at higher  $\sqrt{s}$  could be used to study jet rates in regimes where the probability of hard gluon emission increases. Moreover, a better understanding of hadronization mechanism and improvements in logarithmic resummation to N<sup>3</sup>LL for jet rates would allow the extraction of  $\alpha_S$  at  $\delta\alpha_S/\alpha_S < 1\%$  at the FCC-*ee*.

### 2.2 $\alpha_S$ from hadronic $\tau$ decays

The very precise LEP and B-factories  $e^+e^- \rightarrow \tau^+\tau^-$  data, together with higher-order pQCD corrections to the hadronic  $\tau$  width, allow a remarkably accurate  $\alpha_S$  extraction from hadronic  $\tau$  decays. The quantity of interest is the ratio of the hadronic  $\tau$  width and the electron  $\tau$  width, defined as follows:

$$R_\tau = \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C \left( 1 + \sum_{n=1}^4 c_n \left( \frac{\alpha_S}{\pi} \right)^n + \mathcal{O}(\alpha_S^5) + \delta_{\text{np}} \right), \quad (2)$$

where  $S_{\text{EW}}$  represents the pure electroweak (EW) contribution to the ratio,  $N_C$  the number of colours,  $c_n$  the coefficients of the perturbative expansion, and  $\delta_{\text{np}}$  power-suppressed non-perturbative (NP) corrections. Experimentally, this ratio has determined with a  $\pm 0.23\%$  precision, and this leads to a determination of  $\alpha_S(m_Z) = 0.1187 \pm 0.0018$  ( $\pm 1.5\%$ ).

The dominant source of theoretical uncertainty in the extraction of  $\alpha_S$  comes from the discrepancy between the Fixed Order Perturbation Theory (FOPT) and the Contour-Improved Perturbation Theory (CIPT), two different approaches for evaluating the perturbative expansion. Currently, this uncertainty is at the level of  $\pm 1.5\%$ . NP correction are also relevant in the determination of  $\alpha_S$  from hadronic  $\tau$  decays. These can be sizeable for  $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_\tau^2)$  and they can be controlled by new high-precision measurements of the hadronic  $\tau$  spectral function.

Statistical uncertainty will be negligible at the FCC-*ee*, considering the  $\sim 10^{11}$   $\tau$  produced at the  $Z$ -pole, and parametric and systematic uncertainties will dominate. To fully exploit this huge statistics,

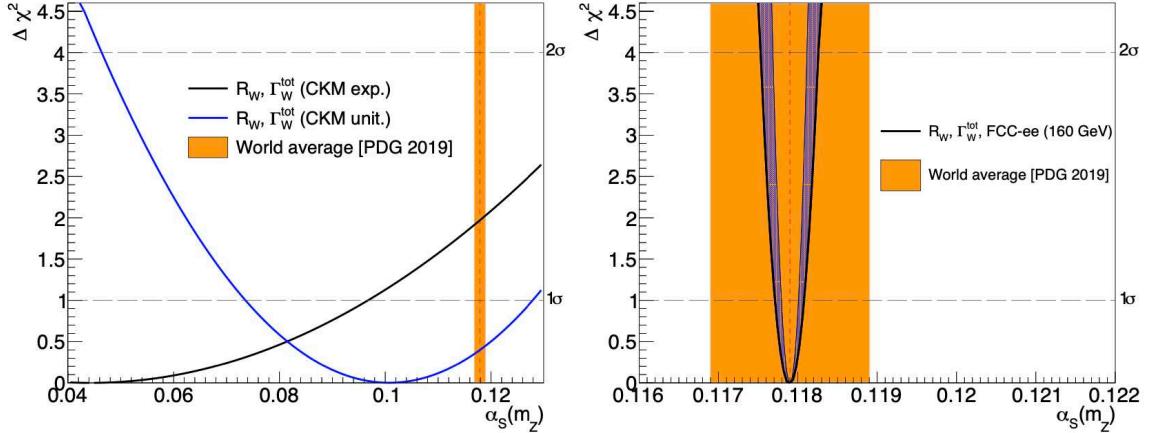


Figure 1:  $\Delta\chi^2$  fit profiles of the  $\alpha_s(m_Z)$  extracted from the combined N<sup>3</sup>LO analysis of the total  $W$  width ( $\Gamma_W^{\text{tot.}}$ ) and hadronic-to-leptonic  $W^\pm$  decay ratio ( $R_W$ ), compared to the current  $\alpha_s(m_Z)$  world average (vertical orange band). Left: Extraction with the present  $W^\pm$  data assuming (blue curve) or not (black curve) CKM unitarity. Right: Extraction expected at the FCC-ee, with the total (experimental, parametric, and theoretical in quadrature) uncertainties (outer parabola) and with the experimental uncertainties alone (inner parabola). These plots are taken from Ref. <sup>4)</sup>.

a reduction in the spread of theoretical determinations of  $R_\tau$  is mandatory. This necessarily implies a better understanding of the discrepancies arising from the CIPT and FOPT comparison. Furthermore, a better determination of the spectral functions entering the  $R_\tau$  calculation is compulsory, and this can be achieved exploiting new data coming from Belle II or the FCC-ee itself. In this way, the uncertainty on  $\alpha_S$  can be reduced well below the current  $\delta\alpha_S/\alpha_S \sim 1\%$  level.

### 2.3 $\alpha_S$ from hadronic $W^\pm$ boson decays

Analogously to the case of the hadronic  $\tau$  decays, the extraction of  $\alpha_S$  from hadronic  $W^\pm$  boson decays can be performed considering the ratio of the hadronic width to the lepton with, as described in Eq. 3

$$R_W(Q) = \frac{\Gamma_W^{\text{had.}}(Q)}{\Gamma_W^{\text{lep.}}(Q)} = R_W^{\text{EW}} \left( 1 + \sum_{i=1}^4 a_i(Q) \left( \frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right) \quad (3)$$

with  $R_W^{\text{EW}}$  representing the pure EW contribution to the ratio,  $a_i(Q)$  the coefficients of the perturbative expansion,  $\delta_{\text{mix}}$  the mixed QCD+EW corrections, and  $\delta_{\text{np}}$  the power-suppressed NP corrections.  $\alpha_S$  is then extracted at N<sup>3</sup>LO from a simultaneous fit of 2  $W$  boson pseudo-observables <sup>4)</sup>:  $R_W$  and  $\Gamma_W^{\text{tot.}}$ . With the assumption of CKM unitarity, a value of  $\alpha_S(m_Z) = 0.101 \pm 0.027$  is obtained (with negligible theoretical and parametric uncertainties), as depicted in Fig. 1 (left). The large uncertainty is mostly due to the poor experimental knowledge of  $R_W$  and  $\Gamma_W^{\text{tot.}}$ , which have been measured in  $e^+e^- \rightarrow W^+W^-$  LEP events. If CKM unitarity is not assumed, the resulting value of the strong coupling constant is basically unconstrained, as shown in Fig. 1 (left).

At the FCC-ee, the uncertainties on  $R_W$  and  $\Gamma_W^{\text{tot.}}$  will be largely reduced, thanks to the high statistics at the  $WW$  threshold. With a factor of 10 reduction of the theoretical uncertainties due to missing  $\alpha_S^5, \alpha^3, \alpha\alpha_S^2$  and  $\alpha^2\alpha_S$  corrections, a final QCD coupling extraction of  $\alpha_S(m_Z) = 0.11790 \pm 0.00023$  with 2 per-mille total error is possible, as illustrated in Fig. 1 (right).

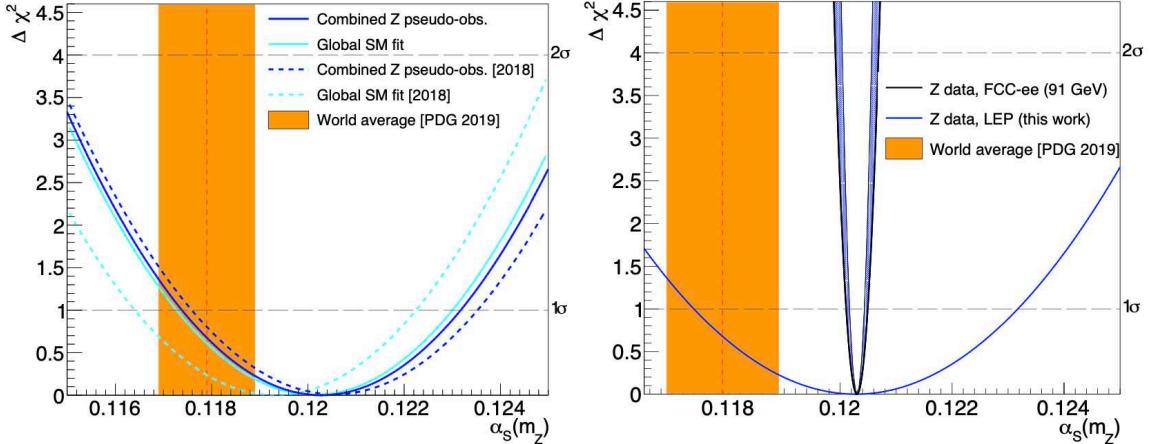


Figure 2:  $\Delta\chi^2$  fit profiles of  $\alpha_s(m_Z)$  extracted from the combined  $Z$  pseudo-observables analysis and/or the global SM fit compared to the current world average (orange band). Left: Current results (solid lines) compared to the previous 2018 fit (dashed lines). Right: Extraction expected at the FCC-ee - with central value (arbitrarily) set to  $\alpha_s(m_Z) = 0.12030$  and total (experimental, parametric, and theoretical in quadrature) uncertainties (outer parabola) and experimental uncertainties alone (inner parabola) - compared to the present one from the combined  $Z$  data (blue line). These plots are taken from Ref. 4).

#### 2.4 $\alpha_s$ from hadronic $Z$ boson decays

Following the same procedure described in Sec. 2.3,  $\alpha_s$  can be extracted at  $N^3\text{LO}$  from a simultaneous fit of 3  $Z$  boson pseudo-observables 4):  $R_Z$ ,  $\Gamma_Z^{\text{tot.}}$  and  $\sigma_Z^{\text{had.}}$ , yielding  $\alpha_s = 0.1203 \pm 0.0029$  ( $\pm 2.3\%$ ), as depicted in Fig. 2 (left).

Having  $10^5$  times more  $Z$  bosons than at LEP, together with an exquisite systematic and parametric precision would allow a remarkable improvement in the theoretical predictions of the  $Z$  boson pseudo observables, and therefore a reduction in the strong coupling uncertainty by almost 2 orders of magnitude. This experimental precision has to be matched by a reduction in the theoretical uncertainties by almost a factor of 5 by computing missing  $\alpha_s^5$ ,  $\alpha^3$ ,  $\alpha\alpha_s^2$  and  $\alpha^2\alpha_s$  corrections. In this way,  $\alpha_s$  can be extracted with a 2 per-mille accuracy, namely  $\alpha_s(m_Z) = 0.11790 \pm 0.00023$ , as reported in Fig. 2 (right).

### 3 Jet substructure

Jet substructure studies play a crucial role in improving our knowledge of parton shower (PS) and hadronization mechanism 5, 6, 7). In particular, jet angularities 8), defined as  $\lambda_\beta^\kappa = \sum_{i \in \text{jet}} z_i^\kappa \theta_i^\beta$  (with  $z_i$  and  $\theta_i$  representing the energy fraction and angular distance to jet axis of constituent  $i$ ), constitute an intriguing starting point. The parameters  $\kappa \geq 0$  and  $\beta \geq 0$  regulate the energy and angular weighting respectively. Multiplicity ( $\kappa = 0$ ,  $\beta = 0$ ), width ( $\kappa = 1$ ,  $\beta = 1$ ), mass ( $\kappa = 1$ ,  $\beta = 2$ ),  $p_T^D$  ( $\kappa = 0$ ,  $\beta = 2$ ) and Les Houches Angularity ( $\kappa = 1$ ,  $\beta = 0.5$ ) are the most common examples. Specifically, this last quantity offers an incredible opportunity to study different PS algorithms between generators.

The FCC-ee would be crucial in addressing such differences in PS and hadronization modelling. For example, the gluon radiation patterns could be studied exploiting the expected  $10^6 e^+e^- \rightarrow ZH(\rightarrow gg)$  events, together with the  $e^+e^- \rightarrow Z \rightarrow b\bar{b}g$  events (assuming that  $b$ -jets are tagged with high efficiency). Therefore, these studies conducted at the FCC-ee would lead directly to improved MC tuning, together

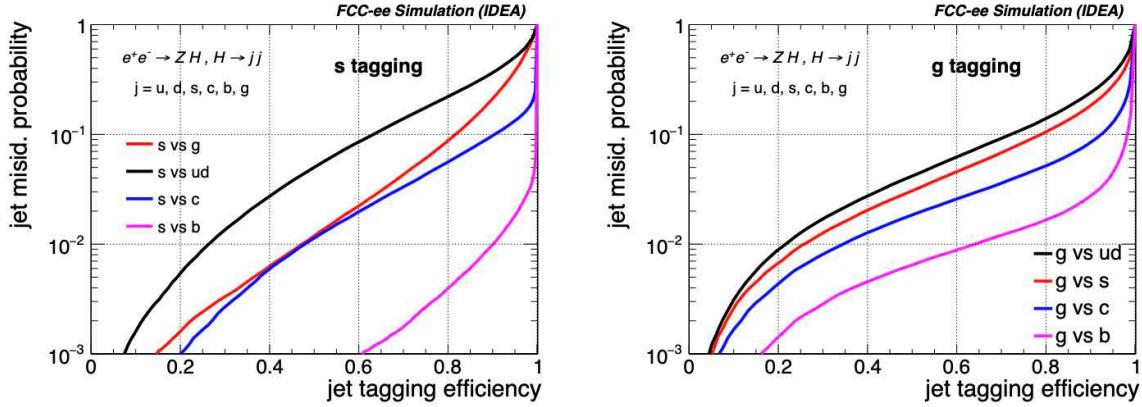


Figure 3: Evaluation of **ParticleNetIdea** performance in terms of a receiver operating characteristic (ROC) curve for the identification of different jet flavours i.e.,  $s$  (left), and  $g$  (right). The different jet flavours considered background are indicated on the labels. The IDEA detector configuration is used. These plots are taken from Ref. <sup>14)</sup>.

with a better understanding of NP QCD.

#### 4 Quark-gluon tagging

One of the most exciting (but challenging) prospects in  $pp$  collisions is light-quark gluon discrimination. Being able to efficiently identify the flavour of the parton which initiates the jet is critical for the success of the physics program of future EW factories <sup>9)</sup>. An accurate light quark-gluon discrimination would allow precise Beyond the Standard Model (BSM) searches for signals without leptons,  $b$ - or top-quarks, as well as would produce an enhancement of light quark-rich signals i.e.  $t\bar{t}H$  or pure EW  $W/Z + \text{jets}$ . Recently, a new generation of advanced machine learning based jet tagging algorithms has been developed <sup>10, 11, 12, 13)</sup>, bringing almost 2 orders of magnitude improvement in background rejection when comparing to the traditional approaches in Heavy Flavour and gluon tagging. In particular, within the context of the FCC-ee, the **ParticleNetIdea** <sup>14)</sup> has been developed, and Figure 3 shows its high performances in discriminating light quark jets from  $s$ -quark (left) and gluons (right).

#### 5 Conclusion

To fully exploit present and future collider programs, a precise understanding of both perturbative and NP QCD is highly needed. At the FCC-ee, a plethora of unique QCD studies would be possible. Among them, the most relevant are the extraction of the strong coupling constant  $\alpha_S$  from jet event shapes and hadronic  $\tau/W^\pm/Z$  decays with a per mille level accuracy and jet substructure studies, which could greatly improve our current knowledge of parton shower and hadronization. Thanks to the large pure quark/gluon samples in the extremely clean environment of a lepton collider, precise quark-gluon discrimination studies would be carried out with a much better discriminating power than the one in  $p\bar{p}/pp$  collisions. Finally, due to the large number of expected  $e^+e^- \rightarrow W^+W^-$ , the huge statistics ( $\times 10^4$  LEP) could be exploited to measure the  $W$  boson mass,  $m_W$ , both (semi-)leptonically and hadronically to constrain colour reconnection at the 1% level or below.

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