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$\bar{K}N$ Feshbach Resonance in The Skyrme Model

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We study the $\Lambda(1405)$ resonance as a $\bar{K}N$ Feshbach resonance in the Skyrme model. To describe the $\bar{K}N$ Feshbach resonance, we combine the Callan-Klebanov's bound state approach for hyperons and our modified bound state approach for kaon-nucleon systems. Our numerical result shows that the width of the Feshbach resonance seems to be narrow.

KEYWORDS: Skyrmions, Hadron mass models and calculations, Hyperons

1. Introduction

$\Lambda(1405)$ is considered to be a candidate of the exotic hadrons, whose properties can not be easily explained by a three-quark state. To determine the properties of $\Lambda(1405)$ is one of the important tasks in hadron physics. There are long-standing discussions for the $\Lambda(1405)$ resonance. According to Refs [1, 2], it was identified with the $\bar{K}N$ quasi-bound state embedded in the $\pi\Sigma$ continuum. In recent studies of the chiral unitary approach, $\Lambda(1405)$ is considered to be a resonance of the $\bar{K}N$ and $\pi\Sigma$ channels [3]. However, their detailed structures are under debate.

In the previous works, we have discussed the kaon-nucleon systems and their interactions with a modified bound state approach in the Skyrme model [4,5], where the order of projection and variation is different from the Callan-Klebanov's bound state approach [6, 7]. We have shown there exists one bound state in $\bar{K}N\left(J^P = 1/2^-, I = 0\right)$ channel with the binding energy of order ten MeV in the previous work [4]. In this article, we explain how to describe the $\Lambda(1405)$ as a $\bar{K}N$ Feshbach resonance as an extension of our previous study.

2. Method

Let us start with the SU(3) Skyrme Lagrangian [8–10]

$$L = \frac{1}{16}F_\pi^2 \mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{WZ} + L_{SB}, \tag{1}$$

where U is the SU(3)-valued chiral field. In the Lagrangian Eq. (1), the first and second terms are the normal Skyrme Lagrangians, the third one, L_{WZ}, is the contribution of the chiral anomaly called Wess-Zumino term [11–13], and the last, L_{SB}, is the explicit symmetry breaking term due to the finite masses of pseudo-scalar mesons. In this study, we consider the chiral limit for the u and d sectors, $m_u = m_d = 0, m_s \neq 0$. There are three model parameters: the pion decay constant F_π, the Skyrme parameter e, and the mass of the kaon m_K.

In the ordinary SU(3) Skyrme model, the ansatz U is composed of a 3×3 matrix defined by SU(3) Nambu-Golodstone bosons. However, in this study, we use the following two ansatze to investigate

$\Lambda\,(1405)$ as a $\bar{K}N$ Feshbach resonance,

$$U_{CK} = A\,(t)\,\sqrt{U_\pi} U_K \sqrt{U_\pi} A\,(t) \tag{2}$$

and

$$U_{EH} = A\,(t)\,\sqrt{U_\pi} A^\dagger\,(t)\, U_K A\,(t)\,\sqrt{U_\pi} A^\dagger\,(t)\,, \tag{3}$$

where $A\,(t)$ is an SU(2) isospin rotation matrix, U_π is the hedgehog soliton embedded in an SU(3) matrix,

$$U_\pi = U_\pi\,(\boldsymbol{r}) = \begin{pmatrix} \xi^2\,(\boldsymbol{r}) & 0 \\ 0 & 1 \end{pmatrix}, \quad \xi^2\,(\boldsymbol{r}) = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}F\,(r)\right], \tag{4}$$

and

$$U_K = U_K\,(\boldsymbol{r},t) = \exp\left[\frac{2\sqrt{2}i}{F_\pi}\begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}\right], \quad K = K\,(\boldsymbol{r},t) = \begin{pmatrix} K^+\,(\boldsymbol{r},t) \\ K^0\,(\boldsymbol{r},t) \end{pmatrix}. \tag{5}$$

The upper ansatz is proposed by Callan and Klebanov to investigate hyperons [6, 7] (CK approach) and the lower one is introduced in our previous work [4] to describe the kaon-nucleon systems (EH approach). The difference in the CK and EH approaches is discussed in Ref. [4].

Before moving on to the $\bar{K}N$ Feshbach resonance in the Skyrme model, we consider an effective Lagrangian method for the decay of $\Lambda\,(1405)$. The relevant Lagrangian is given by,

$$\mathcal{L} = g_{\Lambda^*\pi\Sigma}\bar{\psi}^a_\Sigma \pi^a \psi_{\Lambda^*} + (\text{h.c.})\,, \tag{6}$$

where $g_{\Lambda^*\pi\Sigma}$ is a dimensionless coupling constant for the $\Lambda\,(1405)$-$\pi\Sigma$ vertex, $a = 1, 2, 3$ are the isospin indices, and (h.c.) stands for the Hermitian conjugate to the first term. Using Eq. (6), the decay width of the $\Lambda\,(1405) \to \pi\Sigma$ process is given by,

$$\Gamma_{\Lambda^*\to\pi\Sigma} = g^2_{\Lambda^*\pi\Sigma}\frac{3}{2\pi}\frac{\left|\vec{p}\right|(E_\Sigma + m_\Sigma)}{2m_{\Lambda^*}}, \tag{7}$$

where masses of particles are denoted by m, the particle energies is denoted by E, and the relative momentum of π and Σ is denoted by $\left|\vec{p}\right|$, which are determined by the energy-momentum conservation.

In this study, we identify the coupling constant with the matrix element of the interaction Lagrangian,

$$g_{\Lambda^*\pi\Sigma} \equiv \langle\pi\Sigma|\,\mathcal{L}_{int}\,|\Lambda\,(1405)\rangle = \frac{2}{F_\pi}\langle\pi|\,\partial_\mu\pi^a\,|0\rangle\,\langle\Sigma|\,J^{5,a}_\mu\,|\Lambda\,(1405)\rangle\,, \tag{8}$$

where we employ a current-current type Lagrangian,

$$\mathcal{L}_{int} = \frac{2}{F_\pi}\partial_\mu\pi^a J^{5,a}_\mu. \tag{9}$$

In Eq. (9), $\partial_\mu\pi^a$ is the one-pion axial current, and $J^{5,a}_\mu$ is the baryon axial current. When we evaluate the matrix element Eq. (8), a nontrivial value is the matrix element of the baryon axial current. To calculate it, we combine the CK and EH approaches; the initial $\Lambda\,(1405)$ is described as a $\bar{K}N$ bound state in the EH approach while the final Σ is generated as an s-quark and di-quark bound state in the CK one.

In the rest of this section, we show how to evaluate the matrix element. First, the axial current is obtained from the SU(3) Skyrme Lagrangian Eq. (1) as the Noether's current associated with the SU(3) axial transformation,

$$J^{5,\mu,a} = \frac{iF_\pi^2}{16}\mathrm{tr}\left[T^a\left(R^\mu - L^\mu\right)\right] + \frac{i}{16e^2}\mathrm{tr}\left[T^a\left\{\left[R^\nu,\left[R_\nu,R^\mu\right]\right] - \left[L^\nu,\left[L_\nu,L^\mu\right]\right]\right\}\right]$$
$$-\frac{N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left[\frac{T^a}{2}\left(L_\nu L_\alpha L_\beta + R_\nu R_\alpha R_\beta\right)\right], \tag{10}$$

where T^a $(a = 1, 2, \cdots 8)$ are the Gell-Mann matrices,

$$R_\mu = U\partial_\mu U^\dagger, \quad L_\mu = U^\dagger\partial_\mu U, \tag{11}$$

and the variable U is given by,

$$U = \sqrt{U_\pi}U_K\sqrt{U_\pi}. \tag{12}$$

In Eq. (10), the first term derived from the second derivative term, the second from the Skyrme term, and the last from the Wess-Zumino term in the SU(3) Skyrme Lagrangian Eq. (1).

Next, expanding U_K with respect to the kaon field K, we obtain,

$$J^{5,\mu,a} = J^{5,\mu,a,(0)} + J^{5,\mu,a,(2)} + O\left(K^3\right), \tag{13}$$

where superscripts (0) and (2) stand for the order of the kaon field. To describe the $\bar{K}N$ Feshbach resonance, we concentrate on $J^{5,\mu,a,(2)}$ which contains two kaon fields, K and $K^\dagger$. We identify K $(K^\dagger)$ with a kaon annihilation (creation) operator to perform a quantization. Here, what is important is that the kaon is quantized as a physical kaon in our approach [4] while it is quantized as an s-quark in the CK approach [6, 7] after collective quantization,

$$\begin{cases} K \to K_{EH} & \text{for the EH approach} \\ K \to A(t)\,K_{CK} & \text{for the CK approach.} \end{cases} \tag{14}$$

The relation between fields and operators is summarized in Tab. I. In the following discussion, we

Table I. Identification of creation and annihilation operators for the kaon and s-quark.

	Kaon	Anti-kaon	s-quark	$\bar{s}$-quark
K	Annihilation	Creation	Creation	Annihilation
$K^\dagger$	Creation	Annihilation	Annihilation	Creation

follow the table. On the other hand, the Hedgehog soliton is collective-quantized by the isospin rotation,

$$\xi(\boldsymbol{r}) \to A(t)\,\xi A^\dagger(t). \tag{15}$$

Third, we consider the initial and final states. The former is constructed by combining isospin 1/2 anti-kaon and nucleon to form isospin 0 by using the Clebsch-Gordan coefficients,

$$|\Lambda(1405)\rangle = |\bar{K}N\rangle = \sqrt{\frac{1}{2}}\,|pK^-\rangle + \sqrt{\frac{1}{2}}\,|n\bar{K}^0\rangle. \tag{16}$$

For the Σ state, it is given by

$$|\Sigma\,(J_3 = 1/2)\rangle = |d\,(J = 1)\,s\,(J = 1/2)\rangle = \sqrt{\frac{2}{3}}\,|d\,(J_3 = 1)\,s_{\downarrow}\rangle - \sqrt{\frac{1}{3}}\,|d\,(J_3 = 0)\,s_{\uparrow}\rangle\,, \tag{17}$$

where J is the spin, J_3 is the third component of J, and $s_{\downarrow}$ ($s_{\uparrow}$) stands for the s-quark with spin down (up).

Finally, we show the wave functions for the particles. The anti-kaon and s-quark wave functions are obtained by solving the equations of motion for the bound states and the equations of motion are shown in Refs. [4, 6], respectively. On the other hand, the nucleon and di-quark wave functions defined in the SU(2) isospin space and they are anti-symmetric and symmetric, respectively, in the isospin space [14]. For example, the nucleon wave functions are given by [15],

$$|p\uparrow\rangle = \frac{1}{\pi}\,(a_1 + ia_2)\,, \quad |p\downarrow\rangle = -\frac{i}{\pi}\,(a_0 - ia_3) \tag{18}$$

$$|n\uparrow\rangle = \frac{i}{\pi}\,(a_0 + ia_3)\,, \quad |n\downarrow\rangle = -\frac{1}{\pi}\,(a_1 - ia_2)\,, \tag{19}$$

where $\uparrow$ ($\downarrow$) stands for the spin up (down).

3. Result

We have numerically calculated the matrix element Eq. (8) and derived the width from Eq. (7). For numerical calculations, we keep mass of the kaon at 495 MeV and consider several parameter sets for F_π and e. Our preliminary numerical calculation shows that the width of the $\bar{K}N$ Feshbach resonance is found to be narrow around ten MeV depending on the choice of the parameters. Details will be discussed elsewhere. The result indicates that the Skyrme model can naturally accommodate $\Lambda\,(1405)$ as a Feshbach resonance of $\bar{K}N$, supporting its molecular like structure.

4. Conclusion

In this article, we show an outline to describe the $\Lambda\,(1405)$ as a $\bar{K}N$ Feshbach resonance in the Skyrme model. To do that, what is essential idea is to combine the CK and EH approach. The former is used to describe the final Σ as a bound state of an s-quark and di-quark. On the other hand, the latter is for describing $\Lambda\,(1405)$ as the $\bar{K}N$ bound state. As a result, the $\Lambda\,(1405)$ resonance is realized as a narrow resonance. In the future, we will show more detailed discussions for the $\bar{K}N$ Feshbach resonance in the Skyrme model.

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