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Dynamical string tension theories with target space scale invariance leading to 4D

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Abstract The string tensions can be dynamical, as we have studied in recent publications, for example in the case when we formulate string theories in the modified measure formalism. Then string and brane tensions appear, but as an additional dynamical degree of freedom. It can be seen, however, that these string or brane tensions may not be universal, but rather each string and each brane generates its own tension, which can have a different value for each string or brane. The case where there are strings that can have different spontaneously generated tensions has been considered in previous publication. To have a real dynamical string tension, we consider new background fields that can couple to the strings, the tension scalar which is capable of changing locally along the world sheet the value of the tension of the extended object. When all string tensions are equal, there is now an unbroken target space scale invariance. By considering possible effective actions of gravity and strings as matter in D dimensions, we determine D by requiring that the effective action be, as the fundamental theory, target space scale invariant, which singles out D=4.

1 Introduction

String and Brane Theories have been studied as candidates for the theory of all matter and interactions including gravity, in particular string theories [1,2]. But string theory has a dimensionful parameter, the tension of the string, in its standard formulation, the same is true for brane theories in their better known formulations. Another troublesome aspect of String theory is the fact that it has to be formulated in space times dimensions higher than four, although we should of course end up with a four dimensional space time for the low energy effective theory.

The appearance of a dimensionful string tension and brane theories from the start appears somewhat unnatural. Previously however, in the framework of a Modified Measure Theory, a formalism originally used for gravity theories, see for example [3–10], the tension was derived as an additional degree of freedom [11–17]. See also the treatment by Townsend and collaborators [18,19].

A floating cosmological is a generic feature of the modified measure theories of gravity [3–10], including the covariant formulation of the unimodular theory [20], which is in fact a particular case of a modified measure theory, as reviewed in [21].

The tension of the string plays a very similar role to the cosmological constant in four dimensional gravity, but the analogous situation and the role of the cosmological constant is quite different to that of the string tension, because while several world sheets of strings can exist in the same universe and in this way many strings can probe at the same time the same region of space time,but the same is not the case for the cosmological constant, where every cosmological constant defines necessarily a different universe.

This paper is organized as follows. After this section, the introduction, in Sect. 2 we review the modified-measure approach in the string context. In Sect. 3 we review the modified-measure theory in the brane context. The modified-measure theories of strings and branes rely on two basic elements, the modified measure and the existence of internal gauge fields in the strings and branes, the equations of motion of these gauge fields lead to an equation of motion whose integration constant is the Tension of the extended object. In Sect. 4 we discuss the fact that this tension generation (the integration constant) could take place independently for each

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world sheet separately, which would mean that the string or brane tension is not a fundamental coupling in nature and it could be different for different strings or branes. In Sect. 5 we discuss possible background fields that can be introduced into the theory for the bosonic case and find that a new field can be introduced that changes the value of the tension of the extended object along its world sheet, we call this the tension scalar . We present first the coupling of gauge fields in the extended objects to currents in the world sheet of the extended object that couple to the gauge fields, as a consequence this coupling induces variations of the tension along the world sheet of the extended object. Then we consider a bulk scalar and how this scalar naturally can induce this world sheet current that couples to the internal gauge fields, the equation of motion of the internal gauge field lead to the remarkably simple equation that the local value of the tension along the string is given by $T = g\phi + T_i$, where g is a coupling constant that defines the coupling of the bulk scalar to the world sheet gauge fields and T_i is an integration constant which can be different for each string in the universe. In Sect. 6 we introduce the target space scale invariance or space time scale symmetry of the theory, which does not exist in the standard string theory , We do this following [22], but in that paper the focus was on the study of strings living in the same region of space time, which lead to interesting phenomena, like braneworlds, avoidance of swampland constraints, etc, but in this paper we focus on the case where all string tensions are equal, leading to strict, unbroken target space scale invariance. Then, in Sect. 7, we study effective theories of gravity and strings as matter. The effective theory is required to inherit the target space scale invariance of the fundamental theory, which then implies that the dimensionality of the effective theory must be four space time dimensions.

2 The modified measure theory string theory

The standard world sheet string sigma-model action using a world sheet metric is [23–25]

$$S_{sigma-model} = -T\int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \tag{1}$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^μ we get the following equations of motion:

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu} = 0, \tag{2}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0, \tag{3}$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection for the external metric.

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. The modified-measure theory is an example of such a theory.

In the framework of this theory two additional worldsheet scalar fields $\varphi^i (i = 1, 2)$ are introduced. A new measure density is

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i \partial_b\varphi^j. \tag{4}$$

Then the modified bosonic string action is (as formulated first in [11] and latter discussed and generalized also in [12])

$$S = -\int d^2\sigma \Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\right), \tag{5}$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a: $F_{ab} = \partial_a A_b - \partial_b A_a$.

It is important to notice that the action (5) is invariant under conformal transformations of the intrinsic measure combined with a diffeomorphism of the measure fields,

$$\gamma_{ab} \to J\gamma_{ab}, \tag{6}$$

$$\varphi^i \to \varphi^{'i} = \varphi^{'i}(\varphi^i) \tag{7}$$

such that

$$\Phi \to \Phi^{'} = J\Phi \tag{8}$$

Here J is the jacobian of the diffeomorphim in the internal measure fields which can be an arbitrary function of the world sheet space time coordinates, so this can called indeed a local conformal symmetry.

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (5).

The variation with respect to φ^i leads to the following equations of motion:

$$\epsilon^{ab}\partial_b\varphi^i \partial_a\left(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right) = 0. \tag{9}$$

It implies

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M = const. \tag{10}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0. \tag{11}$$

We see that these equations are the same as in the sigma-model formulation (2), (3). Namely, taking the trace of (11) we get that $M = 0$. By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}$ from (24) (with $M = 0$) we obtain (2).

A most significant result is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b\left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\right) = 0. \tag{12}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{13}$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc. The variation with respect to X^μ leads to the second sigma-model-type equation (3). The idea of modifying the measure of integration proved itself effective and profitable. This can be generalized to incorporate super symmetry, see for example [12–15]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [18,19]. However the fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before. Now we go and review the Modified Measure Brane Theory

3 The Modified measure brane theory

The standard world sheet string sigma-model action using a world sheet metric is [26]:

$$S_{sigma-model} = -T\int d^d\sigma \frac{1}{2}\sqrt{-\gamma}\left(\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + 2\Lambda\right). \tag{14}$$

Notice that now a cosmological term has been added as well, that was not needed in the usual formulation of the string theory or in the modified measure formulation of the string theory. In the modified measure formulation of the brane theory such term will not be required. As it is well known, and as we will review, in the standard formulation , this cosmological term needs to be fine tuned. Here again γ^{ab} is the intrinsic Riemannian metric on the d-dimensional brane worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a brane tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^μ we get the following equations of motion:

$$T_{ab} = \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) g_{\mu\nu} - \gamma_{ab}\Lambda = 0, \tag{15}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a\left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0, \tag{16}$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection for the external metric.

From (15) we see that without the cosmological constant term, or if this term is not fine tuned, the equations of motion are inconsistent upon considering the trace for example. This feature is absent in the modified measure brame theory, where an integration constant, that takes the role of the cosmological term is determined dynamically by the consistency of the equations of motion.

To show this, again, as in the string case, we consider other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold, as we have seen before while reviewing the modified measure string theory, a metric independent measure build along the lines of what we did in the previous section is an example for that

In the framework of a d dimensional brane d additional worldsheet scalar fields $\varphi^i (i = 1, 2, \ldots, d)$ are introduced. The new measure density is defined now as

$$\Phi(\varphi) = \epsilon_{ijk...m}\epsilon^{abc....d}\partial_a\varphi^i\partial_b\varphi^j......\partial_d\varphi^m \tag{17}$$

Then the modified bosonic brane action is

$$S = -\int d^d\sigma \Phi(\varphi)\left(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{abcd...}}{2\sqrt{-\gamma}}F_{abcd...}(A)\right), \tag{18}$$

where $F_{abcd...}$ is the rank d totally antisymmetric field-strength derived from an auxiliary abelian gauge field $A_{acd...}$ of rank $d-1$: $F_{ab....cd} = \partial_a A_{bcd..} + ...$, the totally antisymmetrized curl of a $d-1$ potential $A_{bcd.....}$

In the brane modified theory there is a scale invariance, although it is only a global one, as opposed to the string case, where it is local,

$$\gamma_{ab} \to J\gamma_{ab}, \tag{19}$$

$$\varphi^i \to \varphi^{'i} = \lambda^{ij}(\varphi^j) \tag{20}$$

such that

$$\Phi \to \Phi^{'} = J\Phi \tag{21}$$

That is if $det(\lambda^{ij}) = J$, where J, λ^{ij} are just a constant number and a constant matrix now, and finally the antisymmetric tensor field $A_{bcd......}$ must transform as

$$A_{bcd......} \to J^{\frac{d-2}{2}} A_{bcd......} \tag{22}$$

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (14). To start, the variation with respect to φ^i leads to the following equations of motion:

$$K^a_b \partial_a \left(\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef...}}{\sqrt{-\gamma}} F_{cdef...} \right) = 0. \tag{23}$$

Where the matrix K^a_b is just like the measure, except that one factor of a derivative of a measure field is missing, so that there are two free indices therefore the determinant of K^a_b is a power of the measure, so if the measure is non vanishing, this implies

$$\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef...}}{\sqrt{-\gamma}} F_{cdef...} = M = const. \tag{24}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2} \gamma_{ab} \frac{\epsilon^{cdef...}}{\sqrt{-\gamma}} F_{cdef...} = 0. \tag{25}$$

We will see now that the final equations equations are the same as in the sigma-model formulation (2), (3). Namely, taking the trace of (25), using also (24), and then reinserting into (25), we get that $M \neq 0$ now. By solving $\frac{\epsilon^{cdef..}}{\sqrt{-\gamma}} F_{cdef...}$ from (24) we obtain a relation between the intrinsic metric and the induced metric if $p \neq 1$, (where p is the number of spacelike world sheet coordinates ($p = d - 1$)) that if we do not deal with the string case ($p = 1$),

$$\gamma_{ab} = \frac{1-p}{M} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} \tag{26}$$

Using the global scale symmetry of the modified measure brane theory we can set $\frac{1-p}{M} = 1$ if M is negative if we wish, for $p \neq 1$. That is, the intrinsic metric in the world sheet becomes equal to the induced metric from the embedding metric, while in the string case there is only a conformal equivalence between the intrinsic metric in the world sheet and the induced metric. varying the action with respect to $A_{abc...}$:

$$\epsilon^{abc..d} \partial_d \left(\frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0. \tag{27}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{28}$$

That is how the brane tension T is derived as a world sheet constant of integration, opposite to the standard equation (1) where the tension is put in from the start. One can see indeed that the string energy excitations will be proportional to T in exactly the same way as it is the case in the standard case. The variation with respect to X^μ leads to the second sigma-model-type equation. The idea of modifying the measure of integration proved itself effective and profitable.

4 Each string and each brane in its own world sheet determines its tension

If we look at a single string, the dynamical string tension theories and the standard string theories appear indeed indistinguishable, there are however more than one string and/or one brane in the universe then, let us now observe indeed that it does not appear that the string tension or the brane tension derived in the sections above correspond to "the" string or brane tensions of the theory. The derivation of the string or brane tensions in the previous sections holds for a given string or brane, there is no obstacle that for another string or brane these could acquire a different string or brane tension. In other words, the string or brane tension is a world sheet constant, but it does not appear to be a universal constant same for all strings and for all branes. Similar situation takes place in the dynamical string generation proposed by Townsend for example [18], in that paper worldsheet fields include an electromagnetic gauge potential. Its equations of motion are those of the Green–Schwarz superstring but with the string tension given by the circulation of the worldsheet electric field around the string. So again, in [18] also a string will determine a given tension, but another string may determine another tension. If the tension is a universal constant valid for all strings, that would require an explanation in the context of these dynamical tension string theories, for example some kind of interactions that tend to equalize string tensions, or that all strings in the universe originated from the splittings of one primordial string or some other mechanism.

In any case, if one believes for example in strings , on the light of the dynamical string tension mechanism being a process that takes place at each string independently, we must ask whether all strings have the same string tension.

5 Equations for the background fields and a new background field

As discussed by Polchinski for example in [27,28], gravity can be introduced in two different ways in string theory. One way is by recognizing the graviton as one of the fundamental excitations of the string, the other is by considering the effective action of the embedding metric, by integrating out the string degrees of freedom and then the embedding metric and other originally external fields acquire dynamics which is enforced by the requirement of a zero beta function. These equations fortunately appear to be string tension independent for the critical dimension $D = 26$ in the bosonic string for example, so they will not be changed by introducing different strings with different string tensions, if these tensions are constant along the the world sheet .

However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [29]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings and Branes that is induced by such external scalar field. This is very much in accordance to the philosophy of Schwinger [30] that proposed long time ago that a field theory must be understood by probing it with external sources.

As we will see however, there will be a fundamental difference between this background field and the more conventional ones (the metric, the dilaton field and the two index anti symmetric tensor field) which are identified with some string excitations as well. Instead, here we will see that a single string does not provide dynamics for this field, but rather when the condition for world sheet conformal invariance is implemented for two strings which sample the same region of space time, so it represents a collective effect instead.

5.1 Introducing world sheet currents that couple to the internal gauge fields in strings and branes

If to the action of the brane (18) we add a coupling to a world-sheet current $j^{a_2 \ldots a_{p+1}}$, $p = d - 1$ (the case $p = 1$ represents a string) i.e. a term

$$S_{\text{current}} = \int d^{p+1}\sigma A_{a_2 \ldots a_{p+1}} j^{a_2 \ldots a_{p+1}}, \tag{29}$$

see [31**?** –34] for different applications of this. Then the variation of the total action with respect to $A_{a_2 \ldots a_{p+1}}$ gives

$$\epsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \left(\frac{\Phi}{\sqrt{-\gamma}} \right) = j^{a_2 \ldots a_{p+1}}. \tag{30}$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

5.2 Coupling to a bulk scalar field, the tension field

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^{a_1 \ldots a_{p+1}} = e \partial_\mu \phi \frac{\partial X^\mu}{\partial \sigma^a} \epsilon^{a a_2 \ldots a_{p+1}} \equiv e \partial_a \phi \epsilon^{a a_2 \ldots a_{p+1}}, \tag{31}$$

where e is some coupling constant.

Then (30) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \tag{32}$$

The constant of integration T_i may vary from one string to the other.

6 Target space scale invariance and its SSB in the dynamical string tension theory

Notice that the string theory, has world sheet conformal invariance at the classical level, and this world sheet conformal invariance is requires to be extended to the quantum level.

At the classical level, the ordinary string theory does not have target space scale invariance, which is very much related to the fact that there is a definite scale in the theory, the string tension.

Indeed, in the ordinary string theory, a scale transformation of the background metric

$$g_{\mu\nu} \to \omega g_{\mu\nu}$$

where ω is a constant, is not a symmetry of the Polyakov action, but in the dynamical tension string theory, this transformation is a symmetry provided the world sheet gauge fiends and the measure transforms as

$$A_a \to \omega A_a$$
$$\Phi(\varphi) \to \omega^{-1} \Phi(\varphi)$$

and the tension field transforms in a similar way,

$$\phi \to \omega^{-1} \phi$$

As we have seen, the integration of the equations of motions leads to the spontaneous generation of the string tension and at the same time, the spontaneous generation of the target space global scale invariance, since for this case, (32) is satisfied. or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \tag{33}$$

Notice that the interaction is metric independent. Notice that in the absence of these constants of integration, i.e. , if $T_i = 0$ there is no breaking of the Target space scale invariance, since the measure and the tension field transform in the same way, but the introduction of non zero constants of integration introduces a spontaneous breaking of the Target space scale invarince. The role of the constants of integrations T_i is analogous to the role of the integrations M_i that we discussed in the context of the gravitational theories.

One may interpret (33) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i. The consequences of an independent quantization of many strings with different T_i covering the same region of space time was study in Ref. [22]. A similar exercise can be considered for the target space scale invariance and its spontaneous breaking for the modified measure dynamical brane tension theory.

Our emphasis in this paper will be on unbroken Target space time scale symmetry which is achieved for the case where all the constants of integration Ts are zero, or if they are equal, since if they are all equal they can be also eliminated by a shift of the tension field.

7 Effective model for strings and gravity with target space scale invariance determines D = 4

We want to consider for the effective theory a modified measure theory, because then the gravitational part of the action, linear in the curvature scalar, can be scale invariant. The appropriate integration measure in the space of D scalar fields φ_a, $(a = 1, 2, ...D)$, that is (we call the coordinates relevant to the effective theory A, B, C,...) to differentiate them from the spacetime of the fundamental theory, for which we use Greek indices, we also use G for the metric in the effective theory instead of g

$$dV = d\varphi_1 \wedge d\varphi_2 \wedge \cdots \wedge d\varphi_D \equiv \frac{\Phi}{D!} d^D x \tag{34}$$

where

$$\Phi \equiv \varepsilon_{a_1 a_2 \cdots a_D} \varepsilon^{A_1 A_2 \cdots A_D} (\partial_{A_1} \varphi_{a_1})(\partial_{A_2} \varphi_{a_2}) \cdots (\partial_{A_D} \varphi_{a_D}). \tag{35}$$

Accordingly, the total action in the D-dimensional space-time should be written in the form

$$S = \int \Phi L d^D x \tag{36}$$

Our choice for the total Lagrangian density is

$$L = -\frac{1}{\kappa} R(\Gamma, G) + L_m \tag{37}$$

and as mentioned before, the R part is multiplied by the measure can be target space scale invariant, we will see if this is also possible for the matter part, which is represented by L_m is the matter Lagrangian density $R(\Gamma, G)$ the scalar curvature is given, in the first order formalism by

$$R(\Gamma, G) = G^{AB} R_{AB}(\Gamma) \tag{38}$$

$$R_{AB}(\Gamma) = R^C_{ABC}(\Gamma) \tag{39}$$

7.1 Connection and local symmetries

We can take the point of view that the appearance of gravity, represented by the scalar curvature in the effective action arises from the integration of the closed strings, while the open strings remain as the matter, that is, source of gravity. First consider here the case where L_m does not depend on Γ^A_{BC}, that is fermions and curvature are not present in L_m. Then, the curvature tensor is invariant under the λ-transformation

$$\Gamma'^A_{BC} = \Gamma^A_{BC} + \delta^A_B \lambda,_C \tag{40}$$

which was discovered by Einstein and Kaufman [35]. Although this symmetry was discussed in Ref. [35] in the very specific unified theory, it turns out that λ-symmetry has a wider range of validity and in particular it is useful in our case.

In fact, for a wide class of matter models, the matter Lagrangian density L_m is invariant under the λ transformation too. This is obvious if L_m does not include the connection Γ^A_{BC} at all Varying the action (36), (37) with respect to Γ^A_{BC}, we get

$$\begin{aligned} &-\Gamma^A_{BC} - \Gamma^D_{EB} G^{EA} G_{DC} + \delta^A_C \Gamma^D_{BD} + \delta^A_B G^{DE} \Gamma^F_{DE} G_{FC} \\ &\quad - G_{DC} \partial_B G^{DA} + \delta^A_B G_{DC} \partial_E G^{DE} - \delta^A_C \frac{\Phi,_B}{\Phi} + \delta^A_B \frac{\Phi,_C}{\Phi} = 0 \end{aligned} \tag{41}$$

We will look for the solution (up to a λ-symmetry transformation) of the form

$$\Gamma^A_{BC} = \{^A_{BC}\} + \Sigma^A_{BC} \tag{42}$$

where $\{^A_{BC}\}$ are the Christoffel's connection coefficients. Then Σ^A_{BC} satisfies equation

$$\begin{aligned} &-\sigma,_C G_{AB} + \sigma,_A G_{BC} - G_{BD} \Sigma^D_{CA} - G_{AD} \Sigma^D_{BC} \\ &\quad + G_{AB} \Sigma^D_{CD} + G_{BC} G_{DA} G^{EF} \Sigma^D_{EF} = 0 \end{aligned} \tag{43}$$

where

$$\sigma \equiv \ln \chi, \quad \chi \equiv \frac{\Phi}{\sqrt{-g}} \tag{44}$$

The general solution of Eq. (43) is

$$\Sigma^A_{BC} = \delta^A_B \lambda,_C + \frac{1}{D-2} (\sigma,_B \delta^A_C - \sigma,_D G_{BC} G^{AD}) \tag{45}$$

where λ is an arbitrary function. It appeared because we allowed the connection to have an antisymmetric part. In a formulation that allows for this antisymmetric part it is a gauge artifact due to the existence of the Einstein–Kaufman λ-symmetry [35]. If we choose the gauge $\lambda = \sigma/(D-2)$, then the antisymmetric part of Σ^A_{BC} disappears and we get finally

$$\Sigma^A_{BC}(\sigma) = \frac{1}{D-2} (\delta^A_B \sigma,_C + \delta^A_C \sigma,_B - \sigma,_D G_{BC} G^{AD}) \tag{46}$$

In the presence of fermions, for the case $D = 4$, in addition to the σ-dependent contribution to the connection(42), there is the usual fermionic contribution $\Sigma^{(f)A}_{BC}$ which does not depend on σ (see for example Ref. [36]).

As we will see, in the vacuum, the σ-contribution to the connection can be eliminated by a conformal transformation of the metric accompanied by a corresponding transformation of the fields defining the measure Φ. Indeed, in the vacuum the action (36), (37) is invariant under local transformations

$$G_{AB}(x) = J^{-1} G'_{AB}(x) \tag{47}$$

$$\Phi(x) = J^{-1}(x)\Phi'(x) \tag{48}$$

For $J = \chi^{2/(D-2)}$ we get $\chi' \equiv 1$, $\Sigma'^{A}_{BC}(\sigma) \equiv 0$ and $\Gamma'^{A}_{BC} = \{^{A}_{BC}\}'$, where $\{^{A}_{BC}\}'$ are the Christoffel's coefficients corresponding to the new metric G'_{AB}.

For the case where the measure Φ is given by Eq. (35), the transformation (48) can be the result of a diffeomorphism $\varphi_a \longrightarrow \varphi'_a = \varphi'_a(\varphi_b)$ in the space of the scalar fields φ_a . Then of course $J = Det(\frac{\partial\varphi'_a}{\partial\varphi_b})$.

7.2 Equations of motion

First we study equations that originate from the variation with respect to the measure fields. If the measure is defined using the antisymmetric tensor field $A_{A_2 \cdots A_D}$ as the dynamical variable, we obtain

$$\epsilon^{A_1 \cdots A_D} \partial_{A_D}\left[-\frac{1}{\kappa}R(\Gamma, G) + L_m\right] = 0 \tag{49}$$

which means that

$$-\frac{1}{\kappa}R(\Gamma, G) + L_m = M = constant \tag{50}$$

If we consider the case where the measure is defined as in Eq. (35), we obtain instead of (49), the equation

$$A^B_b \partial_B[-\frac{1}{\kappa}R(\Gamma, G) + L_m] = 0 \tag{51}$$

where $A^B_b = \varepsilon_{a_1 \ldots a_{D-1} b}\varepsilon^{A_1 \cdots A_{D-1} B}(\partial_{A_1}\varphi_{a_1}) \ldots (\partial_{A_{D-1}}\varphi_{a_{D-1}})$. Since $A^A_b \partial_A \varphi_{b'} = D^{-1}\delta_{bb'}\Phi$ it follows that $Det(A^A_b) = \frac{D^{-D}}{D!}\Phi^{D-1}$, so that if $\Phi \neq 0$, eq.(50) is again obtained.

Let us now study the equations that originate from variation with respect to G^{AB}. For simplicity we present here the calculations for the case where there are no fermions. Performing the variation with respect to G^{AB} we get

$$-\frac{1}{\kappa}R_{AB}(\Gamma) + \frac{\partial L}{\partial G^{AB}} = 0. \tag{52}$$

Contracting Eq. (52) with G^{AB} and making use Eq. (50) we get the constraint

$$G^{AB}\frac{\partial(L_m - M)}{\partial G^{AB}} - (L_m - M) = 0. \tag{53}$$

This constraint has to be satisfied for all components (in the functional space) of the function L_m. In particular, for the constant part denoted $< L_m >$, which is relevant to a maximally symmetric vacuum state, we get

$$< L_m > -M = 0 \tag{54}$$

Inserting (54) in Eq. (50) we see that in the maximally symmetric vacuum the scalar curvature $R(\Gamma)$ is equal to zero. As we have seen in the previous section, the σ-contribution to the connection can be eliminated in the vacuum by the transformations (47), (48) Notice that a constant part of the matter Lagrangian density $< L_m >$ does not alter the result that the action (36) in the vacuum is invariant under the transformations (47), (48). This is because the measure Φ is a total derivative and therefore constant part of the Lagrangian density does not contribute into equations of motion). Then in terms of the new metric G'_{AB}, the scalar curvature $R(\Gamma, G)$ becomes the usual scalar curvature $R(G'_{AB})$ of the Riemannian space-time with the metric G'_{AB}. Therefore we conclude that the Riemannian scalar curvature vanishes in the maximally symmetric vacuum. In the presence of fermions the constraint (53) has to be generalized.

7.3 String matter satisfies the constraint (53) in 4D

String as source od gravity in the first and second order formalism was studied for example in [3] and it was shown the constraint (53) was verified by representing the string action in the D-dimensional form where G_{AB} plays the role of a background metric. For example, bosonic strings, according to our formulation, where the measure of integration in a D dimensional space-time is chosen to be $\Phi d^D x$, will be governed by an action of the form:

$$S_m = \int L_{string}\Phi d^D x, \tag{55}$$

$$L_{string} = -T\int d\sigma d\tau \frac{\delta^D(x - X(\sigma, \tau))}{\sqrt{-G}}\sqrt{Det(G_{AB}X^A_{,a}X^B_{,b})} \tag{56}$$

where $\int L_{string}\sqrt{-G}d^D x$ would be the action of a string embedded in a D-dimensional space-time in the standard theory; a, b label coordinates in the string world sheet and T is the string tension. Notice that under a transformation (47), $L_{string} \to J^{(D-2)/2}L_{string}$, therefore concluding that L_{string} is a homogeneous function of G^{AB} of degree one, that is constraint (53) with $M = 0$ is satisfied only if $D = 4$.

8 Relation between the string tension and the Planck scale

It would appear that we have introduced a dimension full string tension T and a dimension full Planck scale through $\frac{1}{\kappa}$, but this is not so, since in $4D$, through the transformation

$G^{AB} \rightarrow \omega G^{AB}$, $T \rightarrow \omega T$ and $\frac{1}{\kappa} \rightarrow \omega\frac{1}{\kappa}$, so, only the ratio of the string tension to M_P^2 is meaningful. This ratio will be dependent on details of the compactification to four dimensions, etc.

9 Review of results in this paper

The consideration of dynamical tension generation lead us to theories where each string generates dynamically its own string tension. This has profound consequences. We in particular discuss a new background scalar field, the "tension" field that can change dynamically the value of the tension along the world sheet of the extended objects. This new field introduced in the theory of extended objects appears to have very important consequences in various cosmological and other scenarios.

We have studied in particular how from the integration of the world sheet gauge fields, that control the dynamics of the dynamical tension, that the tension in a given string labeled by i is given by $e\phi + T_i$, where T_i is a constant of integration that may differ for different extended objects.

If all tensions of the extended objects are equal, when all the integration constants Ts are equal and they can be absorbed into a shift redefinition of the tension field,

This leaves us with an unbroken target space scale invariant theory and we then look at effective theories with gravity and string matter and find studying the gravity theory which must be a modified measure theory, that there is a consistency condition that needs to be satisfied and we find that this condition is satisfied only for these effective theories when formulated in four space time dimensions, The condition happens to be that the target space scale invariance is satisfied for the string matter. It would appear that we have introduced in the effective theory a dimension full string tension T and a dimension full Planck scale through $\frac{1}{\kappa}$, but this is not so, since in four dimensions, through the transformation $G^{AB} \rightarrow \omega G^{AB}$, $T \rightarrow \omega T$ and $\frac{1}{\kappa} \rightarrow \omega\frac{1}{\kappa}$, Since $\kappa = 8\pi G = 1/M_P^2$, only the ratio of the string tension to M_P^2 is meaningful. This ratio will be dependent on details of the compactification to four dimensions, etc.

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