A strategy for detecting non-gaussianity of stochastic gravitational waves

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We study how to probe non-gaussianity of stochastic gravitational waves with pulsar timing arrays. The non-gaussianity is a key to probe the origin of stochastic gravitational waves. In particular, the shape of the bispectrum which characterize the non-gaussianity carries valuable information of inflation models. We show that an appropriate filter function for three point correlations enables us to extract a specific configuration of momentum triangles in bispectra.

1. Introduction

Stochastic gravitational waves (GWs), the GW analog for the cosmic microwave background, are going through us from all directions. They contain primordial GWs[1,2] produced during inflation[3–6] in addition to GWs of cosmological/astrophysical origin[7]. As the name suggests, stochastic GWs are characterized by statistics such as the power spectrum, bispectrum and higher order correlation functions.

The detection of the power spectrum of stochastic GWs is a clue to probe the early universe. Moreover, the bispectrum of stochastic GWs, which represents the non-gaussianity, is a powerful tool to discriminate astrophysical and primordial origin since the former has a gaussian distribution as long as event rates are high enough to create continuous GWs[7,8].[a] Therefore, the bispectrum of stochastic GWs enables us to probe the early universe. Indeed, the bispectrum of primordial GWs contains the detail of inflation models like nonlinear interactions of the graviton. The shape of bispectra, which depends on inflation models[15–18], allows us to discriminate inflation models.[b] Furthermore, the imprint of new particles with the mass comparable to the Hubble scale during inflation can potentially appear in the squeezed limit of momentum triangles[20–26]. Therefore, the bispectrum is a powerful probe of the early universe and beyond the standard model.

Now, GW detectors are in operation to probe stochastic GWs, although no signal of stochastic GWs has been detected yet. The sensitive frequency band of interferometers like LIGO[27] and Virgo[28] is around 10^2 Hz, while pulsar timing

[a] Alternatively, if the event rate is too low to produce continuous GWs, the distribution is not gaussian. Detectability of such feature is explored in Refs. 9–14.

[b] Recently, it was shown that the detection of power spectrum of primordial GWs is not enough to exclude bouncing universe models[19]. Therefore, the bispectrum is also important to distinguish inflation and bouncing universe models.

arrays such as EPTA [29] and NANOGrav [30] are searching for stochastic GWs with a frequency range 10^{-9}-10^{-7} Hz. The constraints on the energy density of stochastic GWs are $\Omega_{GW} < 1.2 \times 10^{-9}$ (EPTA), $\Omega_{GW} < 3.4 \times 10^{-10}$ (NANOGrav), and $\Omega_{GW} < 1.1 \times 10^{-11}$ (PPTA), respectively. In future, the space interferometers, LISA [31] and DECIGO [32], will be launched in a few decades. The pulsar timing array project SKA [33] will start in 2020 and significantly improve the current sensitivity. Its possible upper limit is $\Omega_{GW} < 1.0 \times 10^{-13}$. [34] Therefore, it is worth exploring a new theoretical research area for forthcoming observations.

In this paper, we investigate a method for detecting the bispectrum of stochastic GWs with pulsar timing arrays [35]. In particular, we explain how to utilize a filter function not only to maximize the signal to noise ratio (SNR), but also to extract a specific configuration of momentum triangles in the bispectrum.

2. GW signal in pulsar timing arrays

In the Minkowski spacetime, GWs as tensor perturbations of the metric can be expanded with plane waves:

$$h_{ij}(t, \vec{x}) = \sum_A \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\hat{\Omega}\, e^{2\pi \mathrm{i}\left(ft - |f|\hat{\Omega}\cdot\vec{x}\right)}\, \tilde{h}_A(f, \hat{\Omega}) e^A_{ij}(\hat{\Omega})\,, \tag{1}$$

where $\hat{\Omega}$ is the direction of propagation of GWs. Polarization tensors, which satisfy $e^A_{ij}(\hat{\Omega}) e^{A'}_{ij}(\hat{\Omega}) = 2\delta^{AA'}$, can be defined by

$$e^{+}_{ij}(\hat{\Omega}) = \hat{m}_i\hat{m}_j - \hat{n}_i\hat{n}_j \ , \quad e^{\times}_{ij}(\hat{\Omega}) = \hat{m}_i\hat{n}_j + \hat{n}_i\hat{m}_j \ . \tag{2}$$

Here, $\hat{m}$ and $\hat{n}$ are unit vectors perpendicular to $\hat{\Omega}$ and one another.

GW detectors have their specific response to GWs. For instance, pulsars can be utilized as a detector. A pulsar is a neutron star which emits periodic electromagnetic fields very accurately. If gravitational waves $h_A(f, \hat{\Omega})$ exist continuously between the Earth and a pulsar (the direction $\hat{p}$), we observe the redshift of an emitted pulse as [36]

$$\tilde{Z}(f, \hat{\Omega}) = \left(e^{-2\pi \mathrm{i} L\left(f + |f|\hat{\Omega}\cdot\hat{p}\right)} - 1\right) \sum_A \tilde{h}_A(f, \hat{\Omega}) F^A(\hat{\Omega}, \hat{p}) \ , \tag{3}$$

where

$$F^A(\hat{\Omega}, \hat{p}) \equiv e^A_{ij}\, \frac{1}{2}\, \frac{\hat{p}^i\hat{p}^j}{1 + \hat{\Omega}\cdot\hat{p}} \tag{4}$$

is the pattern function. It represents a geometrical factor, namely, dependence of the sensitivity on the configuration of the detector and GWs. Furthermore, we integrate Eq. (3)

$$\tilde{z}(f) = \int \mathrm{d}\hat{\Omega}\, \tilde{Z}(f, \hat{\Omega}) \ , \tag{5}$$

because stochastic GWs propagate toward all directions. The quantity that is actually measured is the residual defined by

$$R(t) \equiv \int_0^t \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}f\, e^{2\pi \mathrm{i} f t'}\, \tilde{z}(f) \tag{6}$$

$$= \int_0^t \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\hat{\Omega}\, e^{2\pi \mathrm{i} f t'} \left(e^{-2\pi \mathrm{i} L\left(f + |f| \hat{\Omega}\cdot\hat{p}\right)} - 1 \right) \sum_A \tilde{h}_A(f, \hat{\Omega}) F^A(\hat{\Omega}, \hat{p}). \tag{7}$$

Since, for pulsar timing measurement, the minimum frequency is about $0.1\,\mathrm{yr}^{-1}$ and the shortest distance between the Earth and a pulsar is $\sim 100\,\mathrm{ly}$, we have $fL \gtrsim 10$. In this range, the exponential term in the parenthesis of Eq. (7) can be approximated to zero because it oscillates rapidly. Hence, Eq. (5) can be approximated as

$$\tilde{z}(f) \simeq -\sum_A \int \mathrm{d}\hat{\Omega} \tilde{h}_A(f, \hat{\Omega}) F^A(\hat{\Omega}, \hat{p})\ . \tag{8}$$

We find that the correlation of residuals is directly related with that of stochastic GWs. Therefore, observing appropriate correlation function of the signals, we can probe the statistic of stochastic GWs.

3. Probing bispectrum with pulsar timing arrays

Let us define the bispectrum as

$$\langle \tilde{h}_A(f_1, \hat{\Omega}_1) \tilde{h}_{A'}(f_2, \hat{\Omega}_2) \tilde{h}_{A''}(f_3, \hat{\Omega}_3) \rangle = B_{AA'A''}(|f_1|, |f_2|, |f_3|) \delta(f_1 + f_2 + f_3) \times \delta^{(3)}(|f_1|\hat{\Omega}_1 + |f_2|\hat{\Omega}_2 + |f_3|\hat{\Omega}_3)\ . \tag{9}$$

The first delta function denotes that correlations of stochastic GWs are time-independent. The second delta function shows that the momenta form a closed triangle due to homogeneity of the Minkowski spacetime. The bispectral shape varies depending on the inflation model, so that its measurement is a key to probe the early universe.

We define a correlation function of signals in three detectors with a filter function as follows:

$$S_{123} = \iiint_{-T/2}^{T/2} \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3\ s_1(t_1) s_2(t_2) s_3(t_3) Q(t_1, t_2, t_3)\,, \tag{10}$$

where the signal $s_i(t_i)$ in each detector contains the GW signal $z_i(t_i)$ and noises $n_i(t_i)$. We introduce a filter function in a form $Q(t_1, t_2, t_3) = Q(at_1 + bt_2 + ct_3)$, where a, b and c are positive constants. It will turn out that the filter function enables us to extract a specific configuration of the momentum triangle determined by these constants. Moving on to Fourier space, we have

$$S_{123} = \iiint_{-T/2}^{T/2} \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}t_3 \iiiint_{-\infty}^{\infty} \mathrm{d}f_1 \mathrm{d}f_2 \mathrm{d}f_3 \mathrm{d}f\ \tilde{s}_1(f_1) \tilde{s}_2(f_2) \tilde{s}_3(f_3) \tilde{Q}(f) \times e^{2\pi \mathrm{i} f_1 t_1}\, e^{2\pi \mathrm{i} f_2 t_2}\, e^{2\pi \mathrm{i} f_3 t_3}\, e^{-2\pi \mathrm{i} f(at_1 + bt_2 + ct_3)}\ , \tag{11}$$

where we have assumed $\tilde{Q}(-f) = \tilde{Q}(f)$. As in the case of the power spectrum, taking $T \to \infty$, one can carry out the integration:

$$\begin{aligned} S_{123} &= \iiiint_{-\infty}^{\infty} \mathrm{d}f_1 \mathrm{d}f_2 \mathrm{d}f_3 \mathrm{d}f\, \tilde{s}_1(f_1)\tilde{s}_2(f_2)\tilde{s}_3(f_3)\tilde{Q}(f)\delta(f_1 - af)\delta(f_2 - bf)\delta(f_3 - cf) \\ &= \int_{-\infty}^{\infty} \mathrm{d}f\, \tilde{s}_1(af)\tilde{s}_2(bf)\tilde{s}_3(cf)\tilde{Q}(f)\,. \end{aligned} \tag{12}$$

The ensemble average of S_{123} becomes

$$\langle S_{123} \rangle = \int_{-\infty}^{\infty} \mathrm{d}f\, \langle \tilde{z}_1(af)\tilde{z}_2(bf)\tilde{z}_3(cf) \rangle \tilde{Q}(f)\ . \tag{13}$$

It should be noted that Eq. (13) is valid even for a single pulsar case as long as the noise is gaussian, namely, $\langle n_i(t)n_i(t)n_i(t)\rangle = 0$. From Eqs. (8), (9) and (13), one can deduce

$$\begin{aligned} \langle S_{123} \rangle = 2T &\sum_{A,A',A''} \int_0^{\infty} \mathrm{d}f \frac{1}{f^3} B_{AA'A''}(af, bf, cf)\tilde{Q}(f) \frac{\sin\left(\pi(a+b+c)fT\right)}{\pi(a+b+c)fT} \\ &\times \frac{(4\pi)^2}{abc} \Gamma^{AA'A''}(a, b, c; \hat{p}_1, \hat{p}_2, \hat{p}_3)\ , \end{aligned} \tag{14}$$

where the ORF for the three point correlation is defined by [c]

$$\begin{aligned} \Gamma^{AA'A''}(a, b, c; \hat{p}_1, \hat{p}_2, \hat{p}_3) = -\frac{abc}{(4\pi)^2} &\iiint \mathrm{d}\hat{\Omega}_1 \mathrm{d}\hat{\Omega}_2 \mathrm{d}\hat{\Omega}_3 \delta^{(3)}(a\hat{\Omega}_1 + b\hat{\Omega}_2 + c\hat{\Omega}_3) \\ &\times F^A(\hat{\Omega}_1, \hat{p}_1) F^{A'}(\hat{\Omega}_2, \hat{p}_2) F^{A''}(\hat{\Omega}_3, \hat{p}_3)\ . \end{aligned} \tag{15}$$

In Eq. (14), we see that the frequency and the angular integrals are separated due to the filter function, although there appears a suppression factor, $\int_{-T/2}^{T/2} e^{2\pi i(a+b+c)ft}\mathrm{d}t = \frac{\sin\left(\pi(a+b+c)fT\right)}{\pi(a+b+c)f}$, in the first line.[d] Eq. (15) shows that a specific configuration of the momentum triangle determined by a, b and c is extracted.[e] Therefore, we can probe the shape of the bispectrum with changing those parameters. For this purpose, we need to carry out the angular integration to evaluate the ORF. Although we do not study it in this paper, a full discussion is found in the paper [35].

4. Conclusion

In this paper, we investigate a method for detecting the bispectrum of stochastic GWs with pulsar timing arrays [35]. We showed that an appropriate filter function in

[c] Note that we have defined the ORF to be scale invariant with respect to a,b and c.

[d] Allowing a, b and c to be negative, one can remove the suppression factor when $a + b + c = 0$, i.e., in the collinear limit. It implies that pulsar timing arrays is more sensitive to collinear limits of the bispectrum. However, we only focus on positive constants case to probe general momentum triangles in this paper.

[e] In an equal-time three point correlation function, all momentum triangles are integrated. Such case is well studied in the context of LISA [37–39].

three point correlations enables us to extract a specific configuration of momentum triangles in the bispectrum of stochastic GWs. Therefore, one can probe the bispectral shape, which carries important information of the early universe, by adjusting the filter function. Although we did not evaluate the ORF in this paper, a full discussion is found in the paper[35].

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