Towards constraining realistic Lorentzian wormholes through observations

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Ever since their revival three decades ago, in the seminal work of Morris and Thorne, Lorentzian wormholes in General Relativity have led an uncomfortable existence because they require matter which violates the well-known energy conditions. However, in scalar-tensor and other theories of gravity, realistic wormholes can indeed exist with 'normal matter'. We illustrate this with some known examples and also by explicitly constructing a zero Ricci scalar wormhole in a four dimensional scalar-tensor, on-brane gravity theory arising from the two-brane Randall-Sundrum model with one extra dimension. If such a wormhole could arise as the end-state of some astrophysical process, its ringdown may be studied using gravitational waves. With this aim, we obtain the scalar quasinormal modes in this class of wormholes and choose to identify them as for the 'breathing mode' associated with gravitational waves in scalar-tensor theories. Finally, if a breathing mode is indeed observed in LIGO-like detectors with design sensitivity, and has a maximum amplitude equal to that of the tensor mode that was observed of GW150914, then for a range of values of the wormhole parameters we will be able to discern it from a black hole. If in future observations we are able to confirm the existence of such wormholes, we would, at one go, have some indirect evidence of a modified theory of gravity as well as extra spatial dimensions.

1. Lorentzian wormholes: a quick introduction

The earliest example of a spacetime resembling the Lorentzian wormhole of today is the well-known Einstein-Rosen bridge[1], which, though a two-sheeted geometry, is however non-traversable and has a degenerate metric (the determinant of g_{ij} vanishes at the throat). Subsequent to Einstein-Rosen, wormholes appear in the Misner-Wheeler study of *geons*[2]. Later, in the early 1970s, Ellis and Bronnikov[3] constructed a wormhole solution of the Einstein-scalar equations (the 'phantom' scalar had a wrong-sign kinetic energy). The modern-day Lorentzian wormhole was defined largely in the 1988 paper of Morris and Thorne[4] who also constructed a wormhole time machine[5]. Various other methods, such as *Schwarzschild surgery* were developed by Visser in the 1990s (see[6] for further details and an overview of early work).

Lorentzian wormholes have an existence problem in General Relativity(GR), largely due to the fact that they require matter to violate the *Weak or Null Energy Condition.* The cause behind this is the nature of the geometry which acts as a *defocusing lens* for a congruence of null or timelike geodesics[4]. Such defocusing entails a violation of the *convergence conditions, null or timelike.* Since the convergence conditions are directly related to the energy conditions via Einstein's field equations of GR, we end up violating the energy conditions obn matter as well[4]. Various ways have been suggested in order to resolve this issue[7] within GR, but success, at best, is very limited.

A way out of this impasse is to consider modified theories of gravity. Here we will focus on one class of such theories which are born out of the effective on-brane scalar-tensor gravity resulting from the two-brane Randall-Sundrum model with one orbifolded extra dimension[8]. In such an effective theory one can avoid energy-condition violation, though violation of the convergence condition (as expected for wormholes) remains upheld. The above possibility emerges because we have a *scalar field* contribution on the R. H. S. of the field equations, apart from the energy-momentum of matter. The scalar field piece is not quite 'matter', because its origin lies in the *distance between the branes embedded in the bulk*. We can therefore have *realistic Lorentzian wormholes* in this theory, a fact we demostrate below, in brief.

Given such a possible wormhole with 'good' matter, one is therefore encouraged to see if there are chances of detecting it. If we imagine some astrophysical collision process creating such a wormhole, then, its quasinormal (QNM) ringing will indeed be a signature. It is also important to note that among the allowed gravitational wave polarisations in a scalar-tensor theory there is a *scalar breathing mode*, apart from the standard + and × polarisations which exist in GR. Hence, a study of the scalar wave equation and a determination of the scalar quasinormal modes could be a way of detecting the wormhole. After deriving the frequencies and time-constants of the QNMs as functions of the metric parameters, we use them to estimate the percentage errors in the parameters via the Fisher matrix formalism. We follow this line of investigation in our work. Our findings suggest that observations with multiple detectors can be used to put interesting limits on scalar-mode emissions and, thereby, constrain parameters of such wormholes.

2. A WEC satisfying realistic wormhole in a two-brane RS model

Let us first write down the wormhole spacetime which is a solution in a effective on-brane scalar-tensor theory and is threaded by matter satisfying the WEC. The Einstein field equations for such a scalar-tensor theory are given as[8]:

$$\Phi G_{\mu\nu} = \bar{\kappa} T^b_{\mu\nu} + (\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^\alpha \nabla_\alpha \Phi) \\ -\frac{3}{2(1+\Phi)} \left(\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \Phi \nabla_\alpha \Phi \right) \tag{1}$$

The scalar field equation turns out to be:

$$\Box \Phi = \frac{\bar{\kappa}}{l} \frac{T^b}{2\omega + 3} - \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \nabla^\alpha \Phi \nabla_\alpha \Phi \tag{2}$$

Note that the R. H. S of the Einstein equations has a 'matter stress energy' (the $T^b_{\mu\nu}$) and a contribution from the scalar field ϕ. If one uses the $G_{\mu\nu}$ in the R. H. S. of, say, the null Raychaudhuri equation, one notices that *the convergence condition can be violated, without violating the WEC for matter.* In other words, the negativity required for the violation of the convergence condition can be supplied via the ϕ dependent pieces on the R. H. S. in the Einstein equation, keeping $T^b_{\mu\nu}k^\mu k^\nu$ always positive. The $R = 0$ line element which achieves this, with an everywhere finite and non-zero scalar field is[9–11]:

$$ds^2 = -\left(\kappa + \lambda\sqrt{1 - \frac{2M}{r}}\right)^2 dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2 \tag{3}$$

where $\kappa, \lambda > 0$, $r \geq 2M$. The wormhole throat is at $r = 2M$. It has a spatial slice identical to that for the Schwarzschild black hole.

The WEC inequalities for the required matter ($\rho \geq 0, \rho + \tau \geq 0, \rho + p \geq 0$, where ρ, τ, p are the non-zero, diagonal components of the energy momentum tensor), have been checked and are satisfied[11] as shown in Figs. 1,2,3. In these plots $x = \frac{M}{2r'}$ where r' is the isotropic coordinate.

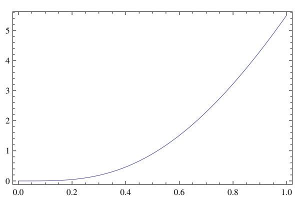

Fig. 1. ρ vs. x

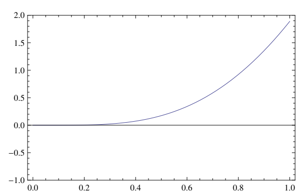

Fig. 2. $\rho + \tau$ vs. x

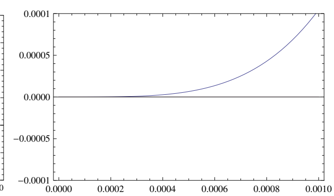

Fig. 3. $10^6(\rho + p)$ vs. x

3. How to detect: observational possibities using GWs

As mentioned earlier, we imagine the end-state of a collision of astrophysical objects (black holes, neutron stars, wormholes?) as a two-parameter (M and $\frac{\kappa}{\lambda}$) wormhole of the type discussed in the previous section. Being a solution of a scalar-tensor theory, where the *breathing mode* polarisation exists apart from the standard GR + and × polarisations[12], it is plausible that scalar perturbations produce it. It can be shown that the scalar wave equation which we wrote earlier, when perturbed (i.e. $\phi \to \phi + \delta\phi$) yields, with a suitable gauge choice, an equation of the form $\Box\delta\phi = 0$. Therefore, by studying the scalar QNMs one can obtain information about the frequencies and damping rates of the ring-down process. Of course, gravitational perturbations must also be present, but here we focus only on the scalar breathing mode. The effective potential as a function of the tortoise coordinate r_* is shown in Fig. 4.[13] Note the double barrier structure of the potential. The time-domain profiles for the ring-down are shown in Figs. 5, 6.[13]

3.1. *Scalar quasinormal modes–breathing mode*

We find the quasinormal modes (fundamentals) by using Prony Fit as well as by using direct integration[13]. The values found using the two methods do agree with each other. In Figs.7,8 the real and imaginary parts of the QNMs are shown as functions of λ, where we have chosen $\kappa + \lambda = 1$. In order to use the QNMs for evaluating the percentage errors we fit the QNM profiles using *NonlinearLeastSquareFit* in *Mathematica 10.0*. The fitted curves are shown as the continuous lines in Figs. 7, 8.

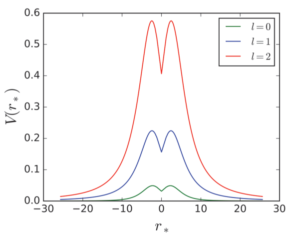

Fig. 4. The effective potential $V_{eff}(r_*)$.

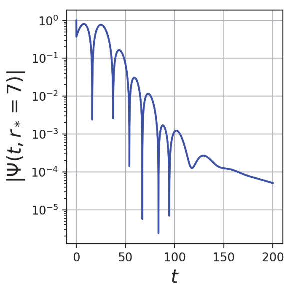

Fig. 5. Time domain profile

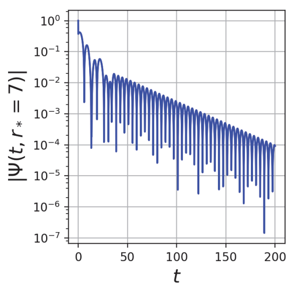

Fig. 6. Time domain profile

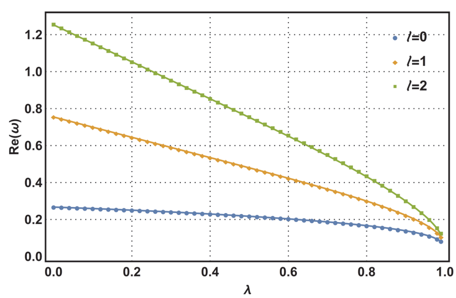

Fig. 7. Real part of QNM vs. λ.

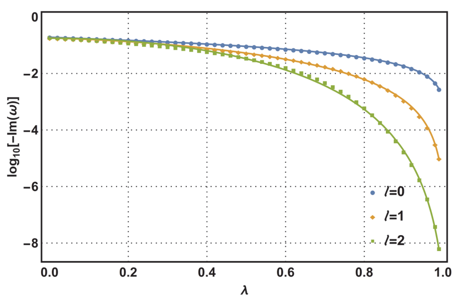

Fig. 8. Imaginary part of QNM vs λ.

An example of the fitted profile function is shown below. For $l = 0$, we have,

$$\nu = \frac{8628.13}{M}\left(1 - 0.29\frac{\lambda}{\lambda+\kappa}\right)\left(1 - \left(\frac{\lambda}{\lambda+\kappa}\right)^{2.36}\right)^{0.24} \tag{4}$$

$$\tau = \frac{M}{38908.58}\left(1 - 0.99\left(\frac{\lambda}{\lambda+\kappa}\right)^{0.94}\right)^{-1.02} \tag{5}$$

For $M = 68M_{sun}$ and $\frac{\kappa}{\lambda} = 0.1$ we obtain $\nu = 64$ Hz.

The breathing mode signal may be written as: $h(t) = A\sin(2\pi\nu t)e^{-t/\tau}$ where the strain amplitude A contains the breathing-mode antenna pattern. Thus, knowing ν and τ from the extracted signal, one can find the $\frac{\kappa}{\lambda}$ and M, in principle. However, one must extract the signal from detector noise for which it is crucial to estimate errors.

3.2. *Error estimates*

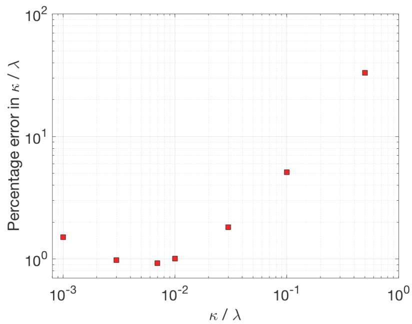

Fig. 9. Percentage error vs. $\frac{\kappa}{\lambda}$

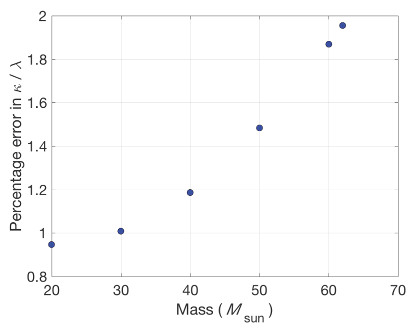

Fig. 10. Percentage error vs. M.

The percentage errors in $\frac{\kappa}{\lambda}$ can be calculated via the Fisher information matrix formalism. We estimate how accurately the wormhole parameters will be measurable using future interferometric detectors like aLIGO. To estimate the error in κ/λ, we compute that matrix for the damped-sinusoid signal in a single aLIGO detector at design sensitivity, for that parameter alone. The matrix is determined by the derivative of the signal h with respect to κ/λ, which influences both the frequency and the damping time-constant of the signal.

We take M to be known. For reference, the maximum QNM strain amplitude is assumed as 10^{-21}, which is approximately the maximum amplitude (quadrupolar) of the GW150914 signal. Finally, we invert the information matrix to derive the estimated variance in the measured values of κ/λ. Its square-root gives the lower bound on the statistical error in κ/λ.

Fig. 9 shows the percentage errors in $\frac{\kappa}{\lambda}$ for $M = 30M_{sun}$. The error intially reduces as $\frac{\kappa}{\lambda}$ increases. The mode frequency gradually shifts to the more sensitive parts of the detector band. For higher values of $\frac{\kappa}{\lambda}$ (less than 1) the error increases due to decreasing time constant (heavy damping). Errors for $\frac{\kappa}{\lambda} > 1$ upto 10 have also been found using Einstein Telescope design sensitivities[13]. In Fig . 10, we have fixed $\frac{\kappa}{\lambda} = 0.01$. The error increases with increasing M because the mode frequency decreases, placing the signal in the less sensitive part of the detector band. More details and other plots are available in[13].

3.3. *Summary and conclusions*

In the above, we have outlined a way of knowing, through GW observations, whether wormholes could exist. Earlier attempts towards detecting wormholes were largely using gravitational lensing. With new GW data, our proposal seems a better prospect. However, one really needs an astrophysical merger model. It is also necessary to study gravitational perturbations in order to obtain a more complete picture.

If we ever see the specific Lorentzian wormhole we proposed here, it will also be indirect evidence for extra dimensions and modified gravity. We hope that observations in the near future would be able to put better constraints and help decide the viability of Lorentzian wormholes in nature.

Acknowledgments

One of the authors (SK) thanks Francisco Lobo for inviting him to give this talk. He (SK) also thanks the organisers of MG15 for their warm hospitality during the conference.

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