Published for SISSA by Springer

Received: *November 19, 2022*
Revised: *February 3, 2023*
Accepted: *March 1, 2023*
Published: *March 20, 2023*

Texture-zero patterns of lepton mass matrices from modular symmetry

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Abstract: Texture zeros in fermion mass matrices have been widely considered in tackling the Standard Model flavour puzzle. In this work, we perform a systematic analysis of texture zeros in lepton mass matrices in the framework of $\Gamma'_3 \cong T'$ modular symmetry. Assuming that the lepton fields transform as irreducible representations of T', we obtain all possible texture-zero patterns for both charged-lepton and neutrino mass matrices which can be achieved from T' modular symmetry. We provide representative models for the phenomenologically-viable textures which can accommodate the experimental data. The predictions for lepton mixing angles, CP-violating phases, light neutrino masses and effective neutrino mass relevant for neutrinoless double beta decay, are discussed. We find that the minimal viable lepton model depends on only 7 real free parameters including the modulus τ (the corresponding charged-lepton mass matrix contains 4 vanishing entries, and the neutrino mass matrix has 1 texture zero). Finally, we study in detail three benchmark models, one for each neutrino mass generation mechanism considered (Dirac, Majorana via Weinberg operator and Majorana via minimal type-I seesaw mechanism).

Keywords: CP Violation, Flavour Symmetries, Neutrino Mixing, Theories of Flavour

ArXiv ePrint: 2211.08136

https://doi.org/10.1007/JHEP03(2023)141

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1 Introduction

Understanding the fermion flavour pattern is still one of the most challenging and outstanding problems in particle physics. In the last decades, the quark mass and mixing parameters have been precisely measured, while neutrino oscillation experiments have firmly established the existence of neutrino masses and lepton mixing. Among all aspects related to the *flavour problem*, perhaps the most intriguing one is the huge hierarchy among the quark and lepton masses. Namely, the lightest neutrino mass is (at most) of $\mathcal{O}(1\,\text{eV})$, while the heaviest (top) quark mass is over eleven orders of magnitude heavier. Moreover, the quark and lepton mixing patterns turn out to be drastically different: the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in quark sector is close to the identity with small mixing angles, while the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix features two large (atmospheric and solar) mixing angles, and the small (reactor) one. In the Standard Model (SM), the fermion mass matrices are determined by the Yukawa couplings, which are arbitrary complex numbers, unconstrained by the SM gauge symmetry. The fact that the number of free parameters is much larger than the number of physical observables makes the SM unpredictive in the flavour sector.

With the purpose of finding a solution for the flavour puzzle, several approaches have been developed. One of the ways of reducing the number of free parameters in fermion mass matrices, is to assume some of their entries to be vanishing [1–3], i.e. to assume what are commonly known as texture-zero *ansätze*. A typical example is the Fritzsch-type quark mass matrices which can relate the Cabibbo angle θ_C to the ratio between the down and strange quark masses [4, 5]. The phenomenology of texture zeros in both quark and lepton sectors have been widely studied in literature — see refs. [6, 7] for reviews. Systematical and complete analyses of all possible texture zeros have been carried out for both quark [8] and lepton mass matrices [9]. It is found that the predictivity of pure texture zero models is quite weak and the predictive mass matrices need relations among the non-zero matrix elements. The most straightforward way to impose vanishing Yukawa couplings is by enforcing them with Abelian flavour symmetries [10–12]. In such case, the non-zero entries of the fermion mass matrices are uncorrelated since the three generations of matter fields are not linked by the Abelian symmetry group.

Recently, modular symmetries have been proposed to address the flavour puzzle [13]. In this approach, the Yukawa couplings are level-N modular forms, which are holomorphic functions of a single complex scalar field — the modulus τ — and transform nontrivially under the action of the modular group. The matter fields are usually assumed to be in irreducible representations of the modular group. Models with modular flavour symmetries can be quite predictive, in the sense that fermion masses and mixing parameters depend on few inputs. The phenomenology of modular invariance has been widely studied, and a plethora of modular-invariant models for lepton masses and mixing have been constructed by using the inhomogeneous finite modular group Γ_N for $\Gamma_2 \cong S_3$ [14–17], $\Gamma_3 \cong A_4$ [13–15, 18–41], $\Gamma_4 \cong S_4$ [28, 42–50], $\Gamma_5 \cong A_5$ [47, 51, 52] and $\Gamma_7 \cong \mathrm{PSL}(2, Z_7)$ [53], the homogeneous finite modular group Γ'_N for $\Gamma'_3 \cong T'$ [54–56], $\Gamma'_4 \cong S'_4$ [57–59], $\Gamma'_5 \cong A'_5$ [60, 61] and Γ'_6 [62], and the finite metaplectic group $\widetilde{\Gamma}_N$ [61, 63]. The most general finite modular groups beyond Γ_N and Γ'_N are discussed in [64] from the view of vector-valued modular form.

Texture-zero patterns for fermion mass matrices can naturally arise from modular symmetries. In comparison to the models based on Abelian flavour symmetries, the texture-zero models relying on modular invariance are more predictive since the nonzero entries of the mass matrices are related by the modular symmetry. In ref. [55], texture zeros in quark mass matrices from T' modular group symmetry have been investigated . In the same token, the specific case of texture zeros of quark mass matrices with nearest neighbor interaction has been discussed in [65]. Regarding the lepton sector, two-zero textures of the Majorana neutrino mass matrix with diagonal charged-lepton mass matrix have been studied in the framework of the A_4 modular group [29]. However, all matter fields were assigned to A_4 singlets. As a consequence, the modular forms appearing in the lepton mass matrices can be absorbed into the Yukawa coupling constants. Since the triplet representation of A_4 is not used, the effect of A_4 modular symmetry is equivalent to an Abelian flavour symmetry and, thus, the correlations among nonzero entries of lepton mass matrices are lost. Thus, a more general and systematic study of lepton texture-zero in the context of modular symmetries is needed.

In this work, we will extend the analysis of [55] to the lepton sector, following a systematic approach in seeking the modular-symmetry realisations of lepton texture zeros and studying their phenomenology. We shall use the modular group $\Gamma_3' \cong T'$ as cornerstone. In contrast with ref. [29], we will not impose any specific basis for charged leptons, and all possible ways of assigning the lepton fields to irreducible representations of the T' modular group will be explored. Since the particle nature of neutrinos is still unclear, we will consider both Dirac and Majorana neutrino masses. For the latter, we analyse two kinds of mass generation mechanisms: the effective Weinberg operator and the type-I seesaw mechanism. To the combination of a given pattern for the charged-lepton and neutrino mass matrix realised with a specific representation and modular weight assignment we call a *texture-zero lepton model* (TZLM).

This paper is organised as follows: in section 2, we briefly review some key aspects about modular symmetry and modular forms of level 3. In section 3, we systematically analyse the texture-zero patterns of the charged-lepton mass matrix stemming from T' modular symmetry by considering all possible representation assignments of matter fields. This procedure is repeated for the neutrino mass matrices in section 4. The combination of charged-lepton and neutrino patterns is carried out in section 5, and the phenomenologically viable pairs are identified in section 5.1 by means of a complete numerical analysis in view of latest neutrino experimental data. Among all viable cases, we focus on three benchmark TZLMs which will be studied in section 6. Finally, we draw ourconclusions in section 7. Group-theoretical aspects of the modular group T' are covered in appendix A. In particular, we provide the level 3 modular forms of weight 2, 3, 4, 5 and 6 in appendix B. The details of the representative models for the viable textures of lepton mass matrices and the corresponding predictions for lepton observables are summarized in appendix C.

2 Modular symmetry and modular forms of level 3

The modular group $\Gamma = \mathrm{SL}(2,\mathbb{Z})$ is the special linear group of two-dimensional matrices with integer entries defined as:

$$\Gamma = \mathrm{SL}(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{Z}, ad-bc=1 \right\} . \tag{2.1}$$

The group Γ has infinite elements and it can be generated by two generators S and T, namely

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \tag{2.2}$$

which satisfy the relations

$$S^4 = (ST)^3 = \mathbb{1}_2, \quad S^2T = TS^2, \tag{2.3}$$

where $\mathbb{1}_2$ denotes the 2×2 unit matrix and $S^2 = -\mathbb{1}_2$. Notice that eq. (2.3) implies $(TS)^3 = \mathbb{1}_2$. The modular group Γ acts on the complex modulus τ with fractional linear transformations:

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad \text{with} \ \ \mathrm{Im}\tau > 0, \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma, \tag{2.4}$$

being the action of S and T on τ

$$S : \tau \to -\frac{1}{\tau}, \qquad T : \tau \to \tau + 1. \tag{2.5}$$

We see that the modulus τ transforms in the same way under the action of γ and $S^2\gamma = -\gamma$. Hence, the group of fractional linear transformations is isomorphic to the projective matrix group $\mathrm{PSL}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})/\{\mathbb{1}_2, -\mathbb{1}_2\}$. Moreover, the modular transformation of a set of chiral supermultiplets Φ_i under the action of γ is given by

$$\Phi_i \xmapsto{\gamma} (c\tau + d)^{-k_\Phi} \rho_{ij}(\gamma) \Phi_j, \tag{2.6}$$

where $k_\Phi \in \mathbb{Z}$ is the modular weight of the superfield multiplet Φ_i, and ρ is the unitary representation of Γ with finite image. For a modular flavour symmetry [13], it is assumed that the representation matrix $\rho(\gamma)$ is a unit matrix when γ belongs to the principal congruence subgroup of level N[1]

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}), \ \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ (\mathrm{mod}\ N) \right\}, \tag{2.7}$$

which is an infinite normal subgroup of $\mathrm{SL}(2,\mathbb{Z})$. We see $\Gamma(1) = \mathrm{SL}(2,\mathbb{Z})$ and $T^N \in \Gamma(N)$. The fundamental theorem of homomorphisms implies that ρ is effectively a representation of the quotient group $\Gamma'_N \equiv \mathrm{SL}(2,\mathbb{Z})/\Gamma(N) \cong \mathrm{SL}(2, Z_N)$ which is called homogeneous finite modular group. Γ'_N can be obtained by further imposing the condition $T^N = 1$ besides those in eq. (2.3):

$$\Gamma'_N = \langle S, T | S^4 = (ST)^3 = T^N = 1, \ S^2T = TS^2 \rangle. \tag{2.8}$$

[1]In fact, one can start from any irreducible representation ρ of $\mathrm{SL}(2,\mathbb{Z})$ with finite image [64], and here $\rho(\Gamma(N)) = 1$ is a particular choice.

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Additional relations are necessary in order to render the group Γ'_N finite for $N \geq 6$ [66]. The Γ'_N group can also be expressed in terms of three generators S, T and R as follow

$$\Gamma'_N = \langle S, T, R | S^2 = R, (ST)^3 = T^N = R^2 = 1, \; RT = TR \rangle \, . \tag{2.9}$$

If ρ cannot distinguish between γ and $-\gamma$ with $\rho(S^2) = 1$, ρ would be the representation of the inhomogeneous finite modular group $\Gamma_N \equiv \mathrm{SL}(2, \mathbb{Z})/\pm\Gamma(N) \cong \Gamma'_N / Z_2^{S^2}$, where $Z_2^{S^2}$ is the order-two cyclic group generated by S^2. For $N \leq 5$, the defining relations of Γ_N are

$$\Gamma_N = \langle S, T | S^2 = (ST)^3 = T^N = 1 \rangle \, . \tag{2.10}$$

Note that $\Gamma'_2 = \Gamma_2$ since $S^2 \in \Gamma(2)$, while Γ'_N is the double cover of Γ_N for $N \geq 3$ due to $S^2 \notin \Gamma(N)$. It is notable that the groups Γ_N for $N = 2, 3, 4, 5$ are isomorphic to the permutation groups S_3, A_4, S_4 and A_5 respectively.

Implementation of modular flavour symmetries requires modular forms $Y(\tau)$ of weight k and level N. $Y(\tau)$ is a holomorphic function of τ with a well-defined transformation property under $\Gamma(N)$:

$$Y\left(\gamma\tau\right) = (c\tau + d)^k Y(\tau), \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma(N) \, , \tag{2.11}$$

where k is a generic non-negative integer.[2] The modular forms of weight k and level N span a linear space $\mathcal{M}_k(\Gamma(N))$ of finite dimension, they are invariant under $\Gamma(N)$ but transform under the quotient group Γ'_N. It is always possible to choose a basis in the linear space $\mathcal{M}_k(\Gamma(N))$ such that the modular forms can be arranged into some modular multiplets $Y_{\mathbf{r}}^{(k)} = (Y_1(\tau), Y_2(\tau), \ldots)^T$ which transform as irreducible representation $\mathbf{r}$ of the finite modular group Γ'_N or Γ_N [13, 54]:

$$Y_{\mathbf{r}}^{(k)}(\gamma\tau) = (c\tau + d)^k \rho_{\mathbf{r}}(\gamma) Y_{\mathbf{r}}^{(k)}(\tau) \quad \text{for} \;\; \forall\, \gamma \in \mathrm{SL}(2, \mathbb{Z}) \, , \tag{2.12}$$

where γ is the representative element of the coset $\gamma\Gamma(N)$ in Γ'_N, and $\rho_{\mathbf{r}}(\gamma)$ is the representation matrix of the element γ in the irreducible representation $\mathbf{r}$.

The superpotential $\mathcal{W}(\Phi_I, \tau)$ can be expanded in power series of the supermultiplets Φ_I,

$$\mathcal{W}(\Phi_I, \tau) = \sum_n Y_{I_1 \ldots I_n}(\tau) \Phi_{I_1} \ldots \Phi_{I_n} \, , \tag{2.13}$$

where the sum is taken over all possible combinations of the fields $\{I_1, \ldots, I_n\}$. The supermultiplet Φ_{I_i} is assumed to transform in the representation ρ_{I_i} of Γ'_N, being $-k_{I_i}$ its modular weight. Modular invariance requires $\mathcal{W}(\Phi_I, \tau)$ to be invariant under the finite modular group Γ'_N and, thus, the total weight of each of its terms must vanish. As a consequence, $Y_{I_1 \ldots I_n}(\tau)$ should be a modular multiplet of weight k_Y transforming in the representation ρ_Y of Γ'_N, i.e.

$$Y_{I_1 \ldots I_n}(\tau) \to Y_{I_1 \ldots I_n}(\gamma\tau) = (c\tau + d)^{k_Y} \rho_Y(\gamma) Y_{I_1 \ldots I_n}(\tau) \, , \tag{2.14}$$

[2]The rational weight modular forms have been studied in [61, 63].

with k_Y and ρ_Y satisfying

$$k_Y = k_{I_1} + \ldots + k_{I_n}, \qquad \rho_Y \otimes \rho_{I_1} \otimes \ldots \otimes \rho_{I_n} \ni \mathbf{1}\,, \tag{2.15}$$

where $\mathbf{1}$ denotes the trivial singlet representation of Γ'_N.

2.1 Weight-1 modular forms of level $N = 3$

The linear space $\mathcal{M}_k(\Gamma(3))$ spanned by level-3 and weight-k modular forms has dimension $k+1$, and can be explicitly constructed by using the Dedekind eta function $\eta(\tau)$:

$$\mathcal{M}_k(\Gamma(3)) = \sum_{m+n=k;m,n\geq 0} c_{mn}\frac{\eta^{3m}(3\tau)\eta^{3n}(\tau/3)}{\eta^{m+n}(\tau)} = \sum_{m+n=k;m,n\geq 0} c_{mn}\left[\frac{\eta^3(3\tau)}{\eta(\tau)}\right]^m\left[\frac{\eta^3(\tau/3)}{\eta(\tau)}\right]^n, \tag{2.16}$$

where c_{mn} are general complex coefficients and

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n), \quad q = e^{2\pi i\tau}\,. \tag{2.17}$$

Hence, $\mathcal{M}_k(\Gamma(3))$ can be generated by polynomials of degree k in the two functions $u_1(\tau)$ and $u_2(\tau)$:

$$u_1(\tau) = \frac{\eta^3(3\tau)}{\eta(\tau)}, \quad u_2(\tau) = \frac{\eta^3(\tau/3)}{\eta(\tau)}\,, \tag{2.18}$$

which are the two linearly independent weight-1 modular forms at level 3. The functions $u_1(\tau)$ and $u_2(\tau)$ transform under the modular generators S and T as

$$\begin{aligned}
&u_1(\tau) \xmapsto{T} e^{2\pi i/3}u_1(\tau), && u_2(\tau) \xmapsto{T} 3\sqrt{3}e^{-i\pi/6}u_1 + u_2\,,\\
&u_1(\tau) \xmapsto{S} 3^{-3/2}(-i\tau)u_2(\tau), && u_2(\tau) \xmapsto{S} 3^{3/2}(-i\tau)u_1(\tau)\,.
\end{aligned} \tag{2.19}$$

and can be arranged in a T' doublet $Y^{(1)}_{\mathbf{2}}$ up to the automorphy factor $c\tau + d$ [54]. In the group representation basis given by eq. (A.3), we find that the modular form doublet $Y^{(1)}_{\mathbf{2}}(\tau)$ is defined as

$$Y^{(1)}_{\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau)\\ Y_2(\tau)\end{pmatrix}\,, \tag{2.20}$$

with

$$Y_1(\tau) = \sqrt{2}\,u_1(\tau), \qquad Y_2(\tau) = -u_1(\tau) - \frac{1}{3}u_2(\tau)\,. \tag{2.21}$$

One can check that $Y^{(1)}_{\mathbf{2}}(\tau)$ transforms under the modular generator S and T as expected, namely

$$Y^{(1)}_{\mathbf{2}}(-1/\tau) = -\tau\rho_{\mathbf{2}}(S)Y^{(1)}_{\mathbf{2}}(\tau), \qquad Y^{(1)}_{\mathbf{2}}(\tau+1) = \rho_{\mathbf{2}}(T)Y^{(1)}_{\mathbf{2}}(\tau)\,, \tag{2.22}$$

where the representation matrices $\rho_{\mathbf{2}}(S)$ and $\rho_{\mathbf{2}}(T)$ are given in appendix A. The q-expansion of the weight-1 modular forms $Y_{1,2}(\tau)$ reads

$$\begin{aligned}
Y_1(\tau) &= \sqrt{2}\,q^{1/3}\left(1 + q + 2q^2 + 2q^4 + q^5 + 2q^6 + q^8 + 2q^9 + 2q^{10} + 2q^{12} + \ldots\right),\\
Y_2(\tau) &= -1/3 - 2q - 2q^3 - 2q^4 - 4q^7 - 2q^9 - 2q^{12} - 4q^{13} + \ldots\,.
\end{aligned} \tag{2.23}$$

Notice that $Y_1(\tau)$ and $Y_2(\tau)$ are algebraically independent. The level-3 modular form of weight $k \geq 2$ can be expressed as polynomials of degree k in $Y_1(\tau)$ and $Y_2(\tau)$, as shown explicitly in appendix B.

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3 Texture zeros in the charged-lepton mass matrix

As antecipated in the Introduction, in this work we will carry out a systematic analysis of texture zeros in the lepton mass matrices stemming from a $\Gamma_4 \cong T'$ modular symmetry. The matter fields are assumed to transform as irreducible representations of T' which has three one-dimensional irreducible representations denoted by $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, three two-dimensional irreducible representations denoted by $\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$ and a three-dimensional irreducible representation denoted by $\mathbf{3}$. Thus, the lepton fields can be assigned to any of these representations.

The transformation assignments of the three generations of left-haned (LH) leptons and the right-handed (RH) charged leptons can be classified into the following cases, i.e.,

$$L \equiv \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} \sim \mathbf{3}\,, \quad \text{or} \quad L_D \equiv \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \sim \mathbf{2}^i, \quad L_3 \sim \mathbf{1}^j\,, \quad \text{or} \quad L_a \sim \mathbf{1}^{j_a}, \quad \text{with} \quad a = 1,2,3\,, \tag{3.1}$$

$$E^c \equiv \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix} \sim \mathbf{3}\,, \quad \text{or} \quad E^c_D \equiv \begin{pmatrix} e^c \\ \mu^c \end{pmatrix} \sim \mathbf{2}^k, \quad \tau^c \sim \mathbf{1}^l\,, \quad \text{or} \quad E^c_a \sim \mathbf{1}^{l_\alpha}, \quad \text{with} \quad \alpha = 1,2,3\,. \tag{3.2}$$

where $i, j, k, l, j_{1,2,3}, l_{1,2,3} = 0, 1, 2$ with $\mathbf{1} \equiv \mathbf{1}^0$, $\mathbf{1}^{'} \equiv \mathbf{1}^1$, $\mathbf{1}^{''} \equiv \mathbf{1}^2$ ($\mathbf{2} \equiv \mathbf{2}^0$, $\mathbf{2}^{'} \equiv \mathbf{2}^1$, $\mathbf{2}^{''} \equiv \mathbf{2}^2$) for the singlet (doublet) representations. E^c_a stands for e^c, μ^c, τ^c when $a = 1, 2, 3$, respectively. The above assignment for the three lepton generations is not unique in the sense that permutations among them can the be considered. This amounts to multiplying the charged-lepton mass matrix on the left and/or right by permutation matrices. However, this does not change lepton mixing and, at the end, we can identify nine classes of charged-lepton mass matrices according to how the lepton fields transform under T'.

In table 1, we summarize all possible texture-zero patterns for the charged-lepton mass matrix that can be obtained with T' modular group. In the following, we discuss how those structures can be achieved by properly assigning the representations and weights of the lepton fields under the T' modular group. We should emphasize that the matrix entries marked with $\times$ in the mass matrices considered in this work are not arbitrary as in usual texture-zero frameworks based on Abelian symmetries. As already mentioned in the Introduction, this is due to the fact that extra relations among the non-zero entries appear as a result of the T' modular symmetry. Therefore, compatibility of a given texture with arbitrary non-vanishing entries does not guarantee its viability in the present framework.

We will now discuss the 9 distinct assignments of lepton fields, and require the charged-lepton mass matrix to have rank three in order to accommodate the three nonzero masses of the electron, muon and tau.

$\mathcal{C}_1^{(1)}$:	$\begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$							
$\mathcal{C}_2^{(1)}$:	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$	,	$\mathcal{C}_2^{(2)}$:	$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}$	,	$\mathcal{C}_2^{(3)}$:	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$	
$\mathcal{C}_3^{(1)}$:	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}$	,	$\mathcal{C}_3^{(2)}$:	$\begin{pmatrix} \times & 0 & \times \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	,	$\mathcal{C}_3^{(3)}$:	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	
$\mathcal{C}_4^{(1)}$:	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$	,	$\mathcal{C}_4^{(2)}$:	$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ \times & \times & \times \end{pmatrix}$	,	$\mathcal{C}_4^{(3)}$:	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	
$\mathcal{C}_6^{(1)}$:	$\begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$							

Table 1. Texture-zero classification for the (rank-3) charged-lepton mass matrices which can be realised from T' modular symmetry, up to row and column permutations.

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}$, $\quad E^c \equiv (e^c, \mu^c, \tau^c)^T \sim \mathbf{3}$

  In this case, the most general effective Yukawa terms for the charged leptons in the superpotential are given by

$$\mathcal{W}_E = \sum_{\mathbf{r},a} g^E_{\mathbf{r},a} \left[E^c L Y^{(k_L+k_{E^c})}_{\mathbf{r}a} \right]_{\mathbf{1}} H_d \,, \tag{3.3}$$

  where $g^E_{\mathbf{r},a}$ are coupling constants, and $Y^{(k)}_{\mathbf{r}a}$ stands for the modular form of weight k and representation $\mathbf{r}$, with a possibly labelling linearly independent multiplets of the same type. Notice that all possible contractions into the T' singlet should be considered. As shown in table 13, the allowed representation $\mathbf{r}$ of modular forms in eq. (3.3) depends on the modular weights $k_L + k_{E^c}$. From eq. (3.3), we can read out the expressions of the elements of charged lepton mass matrix,

$$\begin{aligned}(M_E)_{\alpha\beta} = v_d \sum_{a,b,c,d} \Big\{ &\Big[g^E_{\mathbf{3}_S,a}(3\delta_{\alpha\beta}-1) + g^E_{\mathbf{3}_A,a}(1-\delta_{\alpha\beta})(-1)^{(\alpha-\beta)|3} \Big] Y^{(k_L+k_{E^c})}_{\mathbf{3}a,3-(\alpha+\beta)|3} \\ &+ g^E_{\mathbf{1},b}\delta_{2,(\alpha+\beta)|3} Y^{(k_L+k_{E^c})}_{\mathbf{1}b} + g^E_{\mathbf{1}',c}\delta_{1,(\alpha+\beta)|3} Y^{(k_L+k_{E^c})}_{\mathbf{1}'c} + g^E_{\mathbf{1}'',d}\delta_{0,(\alpha+\beta)|3} Y^{(k_L+k_{E^c})}_{\mathbf{1}''d} \Big\} \,,\end{aligned} \tag{3.4}$$

  where $v_d = \langle H_d^0 \rangle$ is the vacuum expectation value (VEV) of the Higgs field H_d and $a|3 \equiv (a \bmod 3)$. From now on, $Y^{(k_L+k_{E^c})}_{\mathbf{r}_a,j}$ denotes the j^{th} component of the modular form $Y^{(k_L+k_{E^c})}_{\mathbf{r}_a}$ in the vector notation of eq. (2.20) and eqs. (B.1)–(B.5). The charged-lepton mass matrix M_E is defined in the right-left basis with $E^c_\alpha (M_E)_{\alpha\beta} L_\beta$. We find

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that only texture $\mathcal{C}_6^{(1)}$ can be reproduced, requiring total weight $k_L + k_{E^c} = 0$. None of the other cases can be obtained for a generic value of the modulus τ when the three lepton doublets and singlets are in triplet representations of T'. It turns out that the modular realisation of $\mathcal{C}_6^{(1)}$ is not viable since, in this case, modular symmetry imposes $(M_E)_{12} = (M_E)_{21} = (M_E)_{33}$, leading to a fully degenerate charged-lepton mass spectrum.

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}$, $\quad E_D^c \equiv (e^c, \mu^c)^T \sim \mathbf{2}^k$, $\quad E_3^c \equiv \tau^c \sim \mathbf{1}^l$

  In this case, the superpotential terms relevant for charged-lepton mass generation are:

$$\mathcal{W}_E = \sum_{\mathbf{r},a,b} g^E_{\mathbf{r},a} \left[E_D^c L Y_{\mathbf{r}_a}^{(k_L + k_{E_D^c})} \right]_{\mathbf{1}} H_d + g^E_{\mathbf{3},b} \left[E_3^c L Y_{\mathbf{3}b}^{(k_L + k_{E_3^c})} \right]_{\mathbf{1}} H_d \,. \tag{3.5}$$

  where $g^E_{\mathbf{r},a}$, $g^E_{\mathbf{3},b}$ are coupling constants. Modular invariance requires the representation $\mathbf{r}$ to be $\mathbf{2}, \mathbf{2}', \mathbf{2}''$, for which the corresponding modular forms of weight $k = k_L + k_{E_D^c}$ are given in table 13. The elements in the first two rows of M_E read

$$\begin{aligned}(M_E)_{\alpha\beta} = v_d \textstyle\sum_{a,b,c} \Big\{ & g^E_{\mathbf{2}^{2-k},a} (1-\delta_{1,\beta-\alpha}) \Big[\delta_{1,(\alpha-\beta)|3} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{2-k}a,1} (\delta_{\alpha,2} - \sqrt{2}\delta_{\alpha,1}) + \delta_{\alpha\beta} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{2-k}a,2} (\delta_{\alpha,1} + \sqrt{2}\delta_{\alpha,2}) \Big] \\ & + g^E_{\mathbf{2}^{(1-k)|3},b} \Big(1 - \delta_{1,(\alpha-\beta)|3}\Big) \Big[\delta_{\alpha\beta} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{(1-k)|3}b,1} (\delta_{\alpha,2} - \sqrt{2}\delta_{\alpha,1}) + \delta_{1,\beta-\alpha} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{(1-k)|3}b,2} (\delta_{\alpha,1} + \sqrt{2}\delta_{\alpha,2}) \Big] \\ & + g^E_{\mathbf{2}^{(-k)|3},c} (1 - \delta_{\alpha\beta}) \Big[\delta_{1,\beta-\alpha} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{(-k)|3}c,1} (\delta_{\alpha,2} - \sqrt{2}\delta_{\alpha,1}) + \delta_{1,(\alpha-\beta)|3} Y^{(k_{E_D^c}+k_L)}_{\mathbf{2}^{(-k)|3}c,2} (\delta_{\alpha,1} + \sqrt{2}\delta_{\alpha,2}) \Big] \Big\} .\end{aligned} \tag{3.6}$$

  with $\alpha = 1, 2$ and $\beta = 1, 2, 3$, while the elements in the third row are given by

$$(M_E)_{3\beta} = \sum_b g^E_{\mathbf{3},b} v_d Y^{(k_L + k_{E_3^c})}_{\mathbf{3}b,3-(l+\beta+1)|3} \,, \; \beta = 1, 2, 3. \tag{3.7}$$

  In this case, only the $\mathcal{C}_2^{(1)}$ texture-zero pattern can be realised by the modular symmetry with $k_{E_D^c} + k_L = 1$, and there are no zero elements in M_E for the remaining weights.

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}$, $\quad E_\alpha^c \sim \mathbf{1}^{l_\alpha}$

  With the LH leptons transforming as a triplet of T', and the RH charged leptons transforming as singlets of T', the Yukawa terms for charged leptons are

$$\mathcal{W}_E = \sum_{\alpha=1}^{3} \sum_a g^E_{\alpha\mathbf{3},a} \left[E_\alpha^c L Y_{\mathbf{3}a}^{(k_L + k_{E_\alpha^c})} \right]_{\mathbf{1}} H_d \,, \tag{3.8}$$

  where $g^E_{\alpha\mathbf{3},a}$ are coupling constants. The matrix elements of M_E are in this case:

$$(M_E)_{\alpha\beta} = \sum_a g^E_{\alpha\mathbf{3},a} v_d Y^{(k_L + k_{E_\alpha^c})}_{\mathbf{3}a,3-(l_\alpha+\beta+1)|3} \,. \tag{3.9}$$

  Demanding that M_E is of rank 3, no texture zeros can be obtained in this case.

- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i$, $\quad L_3 \sim \mathbf{1}^j$, $\quad E^c \equiv (e^c, \mu^c, \tau^c)^T \sim \mathbf{3}$

  This case can be related to the second case described above by exchanging the assignments of LH and RH lepton fields. Hence, one can obtain the corresponding M_E transposing the mass matrix given in eqs. (3.6) and (3.7)). Consequently, only texture $\mathcal{C}_2^{(1)}$ can be realised for $k_{L_D} + k_{E^c} = 1$ in this case.

- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i$, $\quad L_3 \sim \mathbf{1}^j$, $\quad E^c_D \equiv (e^c, \mu^c)^T \sim \mathbf{2}^k$, $\quad E^c_3 \sim \mathbf{1}^l$

  In the case that both the left- and RH charged leptons transform as direct sums of one- and two-dimensional representations of T', the superpotential $\mathcal{W}_E$ is given as

$$\begin{aligned}\mathcal{W}_E = &\sum_{\mathbf{r},a,b,c,d} g^E_{1\mathbf{r},a}\left[E^c_D L_D Y^{(k_{L_D}+k_{E^c_D})}_{\mathbf{r}a}\right]_{\mathbf{1}} H_d + g^E_{2\mathbf{r},b}\left[E^c_D L_3 Y^{(k_{L_3}+k_{E^c_D})}_{\mathbf{r}b}\right]_{\mathbf{1}} H_d \\ &+ g^E_{3\mathbf{r},c}\left[E^c_3 L_D Y^{(k_{L_D}+k_{E^c_3})}_{\mathbf{r}c}\right]_{\mathbf{1}} H_d + g^E_{4\mathbf{r},d}\left[E^c_3 L_3 Y^{(k_{L_3}+k_{E^c_3})}_{\mathbf{r}d}\right]_{\mathbf{1}} H_d\,. \end{aligned} \tag{3.10}$$

  The corresponding charged-lepton mass matrix can be divided into four blocks which correspond to 2×2, 2×1, 1×2 and 1×1 sub-matrices. Using the Kronecker products and the CG coefficients of T' given in appendix A, we find the explicit forms of the four submatrices:

$$\begin{aligned}(M_E)_{\alpha\beta} = v_d \sum_{a,b,c,d} \Bigg\{ & g^E_{\mathbf{13},a}(\sqrt{2})^{\delta_{\alpha\beta}}(-1)^{\delta_{2,\alpha+\beta}} Y^{(k_{E^c_D}+k_{L_D})}_{\mathbf{3}a,3-(k+i-\alpha-\beta)|3} + (1-\delta_{\alpha\beta})(-1)^{(\alpha-\beta)|3}\delta_{3,\alpha+\beta} \\ & \times\Bigg[g^E_{\mathbf{11},b}\delta_{2,(i+k)|3} Y^{(k_{E^c_D}+k_{L_D})}_{\mathbf{1}b} + g^E_{\mathbf{11}',c}\delta_{1,(i+k)|3} Y^{(k_{E^c_D}+k_{L_D})}_{\mathbf{1}'c} \\ & + g^E_{\mathbf{11}'',d}\delta_{0,(i+k)|3} Y^{(k_{E^c_D}+k_{L_D})}_{\mathbf{1}''d}\Bigg]\Bigg\}, \end{aligned} \tag{3.11}$$

$$\begin{aligned}(M_E)_{\alpha3} = v_d \sum_{a,b,c} \Bigg[& g^E_{\mathbf{22},a}\delta_{2,(k+j)|3}(-1)^{\alpha+1} Y^{(k_{E^c_D}+k_{L_3})}_{\mathbf{2}a,3-\alpha} + g^E_{\mathbf{22}',b}\delta_{1,(k+j)|3}(-1)^{\alpha+1} Y^{(k_{E^c_D}+k_{L_3})}_{\mathbf{2}'b,3-\alpha} \\ & + g^E_{\mathbf{22}'',c}\delta_{0,(k+j)|3}(-1)^{\alpha+1} Y^{(k_{E^c_D}+k_{L_3})}_{\mathbf{2}''c,3-\alpha}\Bigg], \end{aligned} \tag{3.12}$$

$$\begin{aligned}(M_E)_{3\beta} = v_d \sum_{a,b,c} \Bigg[& g^E_{\mathbf{32},a}\delta_{2,(i+l)|3}(-1)^{\beta+1} Y^{(k_{E^c_3}+k_{L_D})}_{\mathbf{2}a,3-\beta} + g^E_{\mathbf{32}',b}\delta_{1,(i+l)|3}(-1)^{\beta+1} Y^{(k_{E^c_3}+k_{L_D})}_{\mathbf{2}'b,3-\beta} \\ & + g^E_{\mathbf{32}'',c}\delta_{0,(i+l)|3}(-1)^{\beta+1} Y^{(k_{E^c_3}+k_{L_D})}_{\mathbf{2}''c,3-\beta}\Bigg], \end{aligned} \tag{3.13}$$

$$\begin{aligned}(M_E)_{33} = v_d \sum_{a,b,c} \Bigg[& g^E_{\mathbf{41},a}\delta_{0,(j+l)|3} Y^{(k_{E^c_3}+k_{L_3})}_{\mathbf{1}a} + g^E_{\mathbf{41}',b}\delta_{2,(j+l)|3} Y^{(k_{E^c_3}+k_{L_3})}_{\mathbf{1}'b} \\ & + g^E_{\mathbf{41}'',c}\delta_{1,(j+l)|3} Y^{(k_{E^c_3}+k_{L_3})}_{\mathbf{1}''c}\Bigg], \alpha,\beta=1,2. \end{aligned} \tag{3.14}$$

The possible forms of the sub-matrices for different representation and weight assignments are summarised in table 2. When combined, these submatrix configurations may lead to the following 9 texture-zero patterns for M_E:

$$\mathcal{C}_1^{(1)},\ \mathcal{C}_2^{(1)},\ \mathcal{C}_2^{(2)},\ \mathcal{C}_2^{(3)},\ \mathcal{C}_3^{(3)},\ \mathcal{C}_4^{(1)},\ \mathcal{C}_4^{(2)},\ \mathcal{C}_4^{(3)},\ \mathcal{C}_6^{(1)}\,. \tag{3.15}$$

Note that the texture $\mathcal{C}_6^{(1)}$ can be achieved only if $k_{L_D} + k_{E_D^c} = 0$ and $k + i = 2\,(\mathrm{mod}\ 3)$, with M_E satisfying $(M_E)_{12} = -(M_E)_{21}$ and $(M_E)_{33} \neq 0$, thus featuring two degenerate charged-lepton masses.

- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i$, $\quad L_3 \sim \mathbf{1}^j$, $\qquad E_\alpha^c \sim \mathbf{1}^{l_\alpha} \qquad$ with $\alpha = 1, 2, 3$

  The modular invariant superpotential is now given by

$$\mathcal{W}_E = \sum_{\alpha=1}^{3} \sum_{\mathbf{r},a,b} g_{\alpha 1 \mathbf{r},a}^E \left[E_\alpha^c L_D Y_{\mathbf{r}a}^{(k_{L_D}+k_{E_\alpha^c})} \right]_{\mathbf{1}} H_d + g_{\alpha 2\mathbf{r},b}^E \left[E_\alpha^c L_3 Y_{\mathbf{r}b}^{(k_{L_3}+k_{E_\alpha^c})} \right]_{\mathbf{1}} H_d \,. \tag{3.16}$$

  The corresponding expressions of the elements in the M_E can be obtained as

$$\begin{aligned}(M_E)_{\alpha\beta} \;=\; v_d \sum_{a,b,c} \Big[& g_{\alpha 1\mathbf{2},a}^E \delta_{2,i+l_\alpha} (-1)^{\beta+1} Y_{\mathbf{2}a,3-\beta}^{(k_{E_\alpha^c}+k_{L_D})} + g_{\alpha 1 \mathbf{2}',b}^E \delta_{1,(i+l_\alpha)|3} (-1)^{\beta+1} Y_{\mathbf{2}'b,3-\beta}^{(k_{E_\alpha^c}+k_{L_D})} \\ & + g_{\alpha 1\mathbf{2}'',c}^E \delta_{0,(i+l_\alpha)|3} (-1)^{\beta+1} Y_{\mathbf{2}''c,3-\beta}^{(k_{E_\alpha^c}+k_{L_D})} \Big] ,\end{aligned} \tag{3.17}$$

$$\begin{aligned}(M_E)_{\alpha 3} \;=\; v_d \sum_{a,b,c} \Big[& g_{\alpha 2\mathbf{1},a}^E \delta_{0,(j+l_\alpha)|3} Y_{\mathbf{1}a}^{(k_{E_\alpha^c}+k_{L_3})} + g_{\alpha 2\mathbf{1}',b}^E \delta_{2,(j+l_\alpha)|3} Y_{\mathbf{1}'b}^{(k_{E_\alpha^c}+k_{L_3})} \\ & + g_{\alpha 2\mathbf{1}'',c}^E \delta_{1,(j+l_\alpha)|3} Y_{\mathbf{1}''c}^{(k_{E_\alpha^c}+k_{L_3})} \Big] ,\end{aligned} \tag{3.18}$$

  where $\alpha = 1, 2, 3$ and $\beta = 1, 2$. In this case, we can obtain five possible texture-zero patterns in M_E, namely

$$\mathcal{C}_1^{(1)}\,,\ \mathcal{C}_2^{(2)}\,,\ \mathcal{C}_2^{(3)}\,,\ \mathcal{C}_3^{(1)}\,,\ \mathcal{C}_4^{(1)}\,. \tag{3.19}$$

- $L_\alpha \sim \mathbf{1}^{j_\alpha}$, $\quad E^c \equiv (e^c, \mu^c, \tau^c)^T \sim \mathbf{3}$

  The corresponding charged-lepton mass matrix is the transpose of that in eq. (3.9), and no texture zeros can be achieved.

- $L_\alpha \sim \mathbf{1}^{j_\alpha}$, $\quad E_D^c \equiv (e^c, \mu^c)^T \sim \mathbf{2}^k$, $\quad E_3^c \equiv \tau^c \sim \mathbf{1}^l$

  As in the previous case, the resulting M_E can be obtained by transposing the that of eqs. (3.17) and (3.18) by switching the transformation properties of LH and RH charged leptons. In this case, the allowed texture zeros are

$$\mathcal{C}_1^{(1)}\,,\ \mathcal{C}_2^{(2)}\,,\ \mathcal{C}_2^{(3)}\,,\ \mathcal{C}_3^{(2)}\,,\ \mathcal{C}_4^{(1)}\,. \tag{3.20}$$

- $L_\beta \sim \mathbf{1}^{j_\beta}$, $\quad E_\alpha^c \sim \mathbf{1}^{l_\alpha}$

  All lepton multiplets are assigned to singlets of T' and the Yukawa superpotential for the charged-lepton masses reads

$$\mathcal{W}_E = \sum_{\alpha,\beta=1}^{3} \sum_{\mathbf{r},a} g_{\alpha\beta\mathbf{r},a}^E \left(E_\alpha^c L_\beta Y_{\mathbf{r}a}^{(k_{L_\beta}+k_{E_\alpha^c})} \right)_{\mathbf{1}} H_d \,. \tag{3.21}$$

The elements of M_E are now given by:

$$(M_E)_{\alpha\beta} = v_d \sum_{a,b,c} \left[g^E_{\alpha\beta\mathbf{1},a} \delta_{0,(j_\beta+l_\alpha)|3} Y_{\mathbf{1}a}^{(k_{E^c_\alpha}+k_{L_\beta})} + g^E_{\alpha\beta\mathbf{1}',b} \delta_{2,(j_\beta+l_\alpha)|3} Y_{\mathbf{1}'b}^{(k_{E^c_\alpha}+k_{L_\beta})} \right.$$
$$\left. + g^E_{\alpha\beta\mathbf{1}'',c} \delta_{1,(j_\beta+l_\alpha)|3} Y_{\mathbf{1}''c}^{(k_{E^c_\alpha}+k_{L_\beta})} \right]. \tag{3.22}$$

In this case, the representation and modular weight of each lepton field can be adjusted so that several texture-zero patterns in M_E can be obtained. Since we have $S = 1$, $T = \omega^a$ in the singlet representations $\mathbf{1}^a$, the flavour symmetry is essentially the Z_3 subgroup generated by T rather than T' for this assignment. As a consequence, a larger number of free parameters would have to be introduced, leading to less predictive power. Moreover, the contributions of all modular forms can be absorbed into the coupling constants, as can be seen from eq. (3.22). Thus, the advantage of the modular symmetry is lost in this case. In the present work, we are mainly concerned on achieving texture-zero patterns with small number of free parameters by taking the most of T' modular symmetry. Therefore, from now on we will not consider the case in which all lepton fields are in singlets of T'.

4 Texture zeros in the neutrino mass matrix

To carry out a full analysis of lepton models with texture-zero patterns realised by modular T' flavour symmetry, we now turn our attention to the neutrino sector. Given that the particle nature of neutrinos is still unclear, we will consider both scenarios in which neutrinos are Dirac or Majorana particles. In the latter case, we explore two possibilities depending on wether neutrino masses are generated via: *i*) an effective dimension-five Weinberg operator (without specifying the underlying full theory) or *ii*) a minimal type-I seesaw model.

4.1 Dirac neutrinos

If neutrinos are Dirac particles, three RH (singlet) neutrino fields N^c are necessary, leading to neutrino mass terms similar to the charged-lepton ones. As in section 3, we can obtain the possible texture-zero patterns for the neutrino mass matrix by considering all different representation assignment of the lepton fields L and N^c. Given that neutrino oscillation data requires that at least two neutrinos are massive, the rank of the neutrino mass matrix can be either three or two, in contrast with rank three of the charged-lepton mass matrix. Hence, the set of texture-zero patterns for the Dirac neutrino mass matrix M_D is larger than for M_E, as can be seen in table 3. Besides those textures given in table 1 for M_E, we find the following additional ones which correspond to M_D of rank two:

$$\mathcal{D}_3^{(4)}, \mathcal{D}_3^{(5)}, \mathcal{D}_4^{(4)}, \mathcal{D}_4^{(5)}, \mathcal{D}_4^{(6)}, \mathcal{D}_5^{(1)}, \mathcal{D}_5^{(2)}, \mathcal{D}_5^{(3)}, \mathcal{D}_5^{(4)}, \mathcal{D}_5^{(5)}, \mathcal{D}_5^{(6)}, \mathcal{D}_6^{(1)}, \mathcal{D}_6^{(2)}, \mathcal{D}_7^{(1)}. \tag{4.1}$$

Similarly to the charged-lepton sector, both $\mathcal{D}_6^{(3)}$ and $\mathcal{D}_7^{(1)}$ lead to degeneracy between the two nonvanishing neutrino masses and, consequently, will not be considered from now on.

(ψ^c, ψ)	Expressions of Sub-matrix	Weight and representation assignments
$(\mathbf{2}^i, \mathbf{3})$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	① $k_{\psi^c} + k_\psi \leq 0\,,$ ② $k_{\psi^c} + k_\psi = 2, 4, 6, \ldots$
	$\begin{pmatrix} 0 & -\sqrt{2}g_1 Y^{(1)}_{\mathbf{2},1} & g_1 Y^{(1)}_{\mathbf{2},2} \\ \sqrt{2}g_1 Y^{(1)}_{\mathbf{2},2} & 0 & g_1 Y^{(1)}_{\mathbf{2},1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 1,\ i = 0$
	$\begin{pmatrix} -\sqrt{2}g_1 Y^{(1)}_{\mathbf{2},1} & g_1 Y^{(1)}_{\mathbf{2},2} & 0 \\ 0 & g_1 Y^{(1)}_{\mathbf{2},1} & \sqrt{2}g_1 Y^{(1)}_{\mathbf{2},2} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 1,\ i = 1$
	$\begin{pmatrix} g_1 Y^{(1)}_{\mathbf{2},2} & 0 & -\sqrt{2}g_1 Y^{(1)}_{\mathbf{2},1} \\ g_1 Y^{(1)}_{\mathbf{2},1} & \sqrt{2}g_1 Y^{(1)}_{\mathbf{2},2} & 0 \end{pmatrix}$	$k_{\psi^c} + k_\psi = 1,\ i = 2$
	$\begin{pmatrix} g_1 Y^{(3)}_{\mathbf{2}'',2} & -\sqrt{2}g_2 Y^{(3)}_{\mathbf{2},1} & -\sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',1} + g_2 Y^{(3)}_{\mathbf{2},2} \\ g_1 Y^{(3)}_{\mathbf{2}'',1} + \sqrt{2}g_2 Y^{(3)}_{\mathbf{2},2} & \sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',2} & g_2 Y^{(3)}_{\mathbf{2},1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 3,\ i = 0$
	$\begin{pmatrix} -\sqrt{2}g_2 Y^{(3)}_{\mathbf{2},1} & g_2 Y^{(3)}_{\mathbf{2},2} - \sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',1} & g_1 Y^{(3)}_{\mathbf{2}'',2} \\ \sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',2} & g_2 Y^{(3)}_{\mathbf{2},1} & \sqrt{2}g_2 Y^{(3)}_{\mathbf{2},2} + g_1 Y^{(3)}_{\mathbf{2}'',1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 3,\ i = 1$
	$\begin{pmatrix} g_2 Y^{(3)}_{\mathbf{2},2} - \sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',1} & g_1 Y^{(3)}_{\mathbf{2}'',2} & -\sqrt{2}g_2 Y^{(3)}_{\mathbf{2},1} \\ g_2 Y^{(3)}_{\mathbf{2},1} & \sqrt{2}g_2 Y^{(3)}_{\mathbf{2},2} + g_1 Y^{(3)}_{\mathbf{2}'',1} & \sqrt{2}g_1 Y^{(3)}_{\mathbf{2}'',2} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 3,\ i = 2$
	$\begin{pmatrix} g_1 Y^{(5)}_{\mathbf{2}'',2} - \sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',1} & g_2 Y^{(5)}_{\mathbf{2}',2} - \sqrt{2}g_3 Y^{(5)}_{\mathbf{2},1} & -\sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',1} + g_3 Y^{(5)}_{\mathbf{2},2} \\ g_1 Y^{(5)}_{\mathbf{2}'',1} + \sqrt{2}g_3 Y^{(5)}_{\mathbf{2},2} & \sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',2} + g_2 Y^{(5)}_{\mathbf{2}',1} & \sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',2} + g_3 Y^{(5)}_{\mathbf{2},1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 5,\ i = 0$
	$\begin{pmatrix} g_2 Y^{(5)}_{\mathbf{2}',2} - \sqrt{2}g_3 Y^{(5)}_{\mathbf{2},1} & g_3 Y^{(5)}_{\mathbf{2},2} - \sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',1} & -\sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',1} + g_1 Y^{(5)}_{\mathbf{2}'',2} \\ g_2 Y^{(5)}_{\mathbf{2}',1} + \sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',2} & \sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',2} + g_3 Y^{(5)}_{\mathbf{2},1} & \sqrt{2}g_3 Y^{(5)}_{\mathbf{2},2} + g_1 Y^{(5)}_{\mathbf{2}'',1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 5,\ i = 1$
	$\begin{pmatrix} g_3 Y^{(5)}_{\mathbf{2},2} - \sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',1} & g_1 Y^{(5)}_{\mathbf{2}'',2} - \sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',1} & -\sqrt{2}g_3 Y^{(5)}_{\mathbf{2},1} + g_2 Y^{(5)}_{\mathbf{2}',2} \\ g_3 Y^{(5)}_{\mathbf{2},1} + \sqrt{2}g_2 Y^{(5)}_{\mathbf{2}',2} & \sqrt{2}g_3 Y^{(5)}_{\mathbf{2},2} + g_1 Y^{(5)}_{\mathbf{2}'',1} & \sqrt{2}g_1 Y^{(5)}_{\mathbf{2}'',2} + g_2 Y^{(5)}_{\mathbf{2}',1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 5,\ i = 2$
$(\mathbf{2}^i, \mathbf{2}^k)$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	① $k_{\psi^c} + k_\psi < 0\,,$ ② $k_{\psi^c} + k_\psi = 1, 3, 5, \ldots$
	$\begin{pmatrix} 0 & g_1 \\ -g_1 & 0 \end{pmatrix}$	$k_{\psi^c} + k_\psi = 0,\ k + i = 2\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(2)}_{\mathbf{3},2} & g_1 Y^{(2)}_{\mathbf{3},3} \\ g_1 Y^{(2)}_{\mathbf{3},3} & g_1\sqrt{2}Y^{(2)}_{\mathbf{3},1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 2,\ k + i = 0\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(4)}_{\mathbf{3},2} & g_1 Y^{(4)}_{\mathbf{3},3} \\ g_1 Y^{(4)}_{\mathbf{3},3} & g_1\sqrt{2}Y^{(4)}_{\mathbf{3},1} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 4,\ k + i = 0\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -\sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,2} + g_2 Y^{(6)}_{\mathbf{3}II,2}) & g_1 Y^{(6)}_{\mathbf{3}I,3} + g_2 Y^{(6)}_{\mathbf{3}II,3} \\ g_1 Y^{(6)}_{\mathbf{3}I,3} + g_2 Y^{(6)}_{\mathbf{3}II,3} & \sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,1} + g_2 Y^{(6)}_{\mathbf{3}II,1}) \end{pmatrix}$	$k_{\psi^c} + k_\psi = 6,\ k + i = 0\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(2)}_{\mathbf{3},1} & g_1 Y^{(2)}_{\mathbf{3},2} \\ g_1 Y^{(2)}_{\mathbf{3},2} & g_1\sqrt{2}Y^{(2)}_{\mathbf{3},3} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 2,\ k + i = 1\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(4)}_{\mathbf{3},1} & g_2 Y^{(4)}_{\mathbf{1}'} + g_1 Y^{(4)}_{\mathbf{3},2} \\ -g_2 Y^{(4)}_{\mathbf{1}'} + g_1 Y^{(4)}_{\mathbf{3},2} & g_1\sqrt{2}Y^{(4)}_{\mathbf{3},3} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 4,\ k + i = 1\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -\sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,1} + g_2 Y^{(6)}_{\mathbf{3}II,1}) & (g_1 Y^{(6)}_{\mathbf{3}I,2} + g_2 Y^{(6)}_{\mathbf{3}II,2}) \\ (g_1 Y^{(6)}_{\mathbf{3}I,2} + g_2 Y^{(6)}_{\mathbf{3}II,2}) & \sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,3} + g_2 Y^{(6)}_{\mathbf{3}II,3}) \end{pmatrix}$	$k_{\psi^c} + k_\psi = 6,\ k + i = 1\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(2)}_{\mathbf{3},3} & g_1 Y^{(2)}_{\mathbf{3},1} \\ g_1 Y^{(2)}_{\mathbf{3},1} & g_1\sqrt{2}Y^{(2)}_{\mathbf{3},2} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 2,\ k + i = 2\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -g_1\sqrt{2}Y^{(4)}_{\mathbf{3},3} & g_2 Y^{(4)}_{\mathbf{1}} + g_1 Y^{(4)}_{\mathbf{3},1} \\ -g_2 Y^{(4)}_{\mathbf{1}} + g_1 Y^{(4)}_{\mathbf{3},1} & g_1\sqrt{2}Y^{(4)}_{\mathbf{3},2} \end{pmatrix}$	$k_{\psi^c} + k_\psi = 4,\ k + i = 2\,(\mathrm{mod}\ 3)$
	$\begin{pmatrix} -\sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,3} + g_2 Y^{(6)}_{\mathbf{3}II,3}) & (g_1 Y^{(6)}_{\mathbf{3}I,1} + g_2 Y^{(6)}_{\mathbf{3}II,1}) + g_3 Y^{(6)}_{\mathbf{1}} \\ (g_1 Y^{(6)}_{\mathbf{3}I,1} + g_2 Y^{(6)}_{\mathbf{3}II,1}) - g_3 Y^{(6)}_{\mathbf{1}} & \sqrt{2}(g_1 Y^{(6)}_{\mathbf{3}I,2} + g_2 Y^{(6)}_{\mathbf{3}II,2}) \end{pmatrix}$	$k_{\psi^c} + k_\psi = 6,\ k + i = 2\,(\mathrm{mod}\ 3)$
$(\mathbf{2}^i, \mathbf{1}^j)$	$(0, 0)^T$	① $k_{\psi^c} + k_\psi < 0,$ ② $k_{\psi^c} + k_\psi = 0, 2, 4, 6, \ldots,$ ③ $k_{\psi^c} + k_\psi = 1,\ i + j = 0, 1\,(\mathrm{mod}\ 3)\,,$ ④ $k_{\psi^c} + k_\psi = 3,\ i + j = 1\,(\mathrm{mod}\ 3)\,.$
	$(gY^{(3)}_{\mathbf{2}'',2}, -gY^{(3)}_{\mathbf{2}'',1})^T$	$k_{\psi^c} + k_\psi = 3,\ \ i + j = 0\,(\mathrm{mod}\ 3)$
	$(gY^{(5)}_{\mathbf{2}'',2}, -gY^{(5)}_{\mathbf{2}'',1})^T$	$k_{\psi^c} + k_\psi = 5,\ \ i + j = 0\,(\mathrm{mod}\ 3)$
	$(gY^{(5)}_{\mathbf{2}',2}, -gY^{(5)}_{\mathbf{2}',1})^T$	$k_{\psi^c} + k_\psi = 5,\ \ i + j = 1\,(\mathrm{mod}\ 3)$
	$(gY^{(1)}_{\mathbf{2},2}, -gY^{(1)}_{\mathbf{2},1})^T$	$k_{\psi^c} + k_\psi = 1,\ \ i + j = 2\,(\mathrm{mod}\ 3)$
	$(gY^{(3)}_{\mathbf{2},2}, -gY^{(3)}_{\mathbf{2},1})^T$	$k_{\psi^c} + k_\psi = 3,\ \ i + j = 2\,(\mathrm{mod}\ 3)$
	$(gY^{(5)}_{\mathbf{2},2}, -gY^{(5)}_{\mathbf{2},1})^T$	$k_{\psi^c} + k_\psi = 5,\ \ i + j = 2\,(\mathrm{mod}\ 3)$

Table 2. Possible submatrix forms for the charged-lepton mass matrix up to weight-6 modular forms, for the case where the RH charged lepton field ψ^c transform as $\mathbf{2}^i$ and the LH lepton doublet ψ as $\mathbf{3}$, $\mathbf{2}^k$ or $\mathbf{1}^j$ under T' modular symmetry. Their modular weights are denoted by k_{ψ^c} and k_ψ, respectively. The parameters g_i are coupling constants. If the representation assignments of ψ and ψ^c are exchanged, the corresponding submatrix is the transpose of the original one.

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At this point, it is worth stressing that Majorana mass terms for the RH neutrinos can be forbidden by properly choosing the modular weights of the N^c fields, so that the Diracness of light neutrinos stems automatically from modular symmetry. For instance, let us consider a case with the LH leptons and the RH neutrinos transforming as

$$L = (L_1, L_2, L_3)^T \sim \mathbf{3}\,, \quad N_D^c \equiv (N_1^c, N_2^c)^T \sim \mathbf{2}^k\,, \quad N_3^c \sim \mathbf{1}^l\,. \tag{4.2}$$

Then, the superpotential for Dirac neutrino masses can be written as

$$\mathcal{W}_\nu^D = \sum_{\mathbf{r},a,b} g^\nu_{\mathbf{r},a} \left[N_D^c L Y^{(k_1)}_{\mathbf{r}_a}\right]_{\mathbf{1}} H_u + g^\nu_{\mathbf{r},b} N_3^c L Y^{(k_2)}_{\mathbf{3}b} H_u\,. \tag{4.3}$$

Modular invariance requires the weights of the involved modular forms to satisfy

$$k_1 = k_L + k_{N_D^c}\,, \qquad k_2 = k_L + k_{N_3^c}\,, \tag{4.4}$$

which, for any given values of k_1 and k_2, implies

$$k_L = k_1 - k_{N_D^c}\,, \qquad k_{N_3^c} = k_{N_D^c} + k_2 - k_1\,. \tag{4.5}$$

Notice that the value of $k_{N_D^c}$ is free and, thus, we can always choose a negative value for $k_{N_D^c}$, so that $k_{N_3^c}$ is negative as well. If modular weights of all RH neutrino fields are negative, modular-invariant Majorana mass terms of the type

$$\mathcal{W}_N^M \sim N_i^c N_j^c Y^{(k_{N_i^c} + k_{N_j^c})}\,, \quad i, j = D, 3\,, \tag{4.6}$$

cannot be written since there are no modular forms of negative modular weights at level N. The above arguments hold for any other representation assignment of the lepton fields. In short, the modular symmetry can enforce light neutrinos to be Dirac particles if the modular weights of the RH neutrinos are properly (not uniquely) assigned.

4.2 Majorana neutrinos

For Majorana neutrinos, we consider two distinct mass generation mechanisms: the effective Weinberg operator and the type-I seesaw mechanism. In the same token of the discussions for charged leptons and Dirac neutrinos, in table 4 we present all possible texture-zero for the effective Majorana neutrino mass matrix M_ν which can be obtained from T' modular symmetry. Note that $\mathcal{W}_4^{(1)}$, $\mathcal{W}_4^{(4)}$ and $\mathcal{W}_5^{(1)}$ lead to two degenerate neutrino masses, which is excluded by neutrino oscillation data. In the following, we first discuss the textures originated from the effective Weinberg operator, and then turn to the type-I seesaw case.

4.2.1 Neutrino masses via Weinberg operator

The most general modular-invariant Weinberg operator of neutrino masses can be written as

$$\mathcal{W}_\nu \sim \frac{1}{\Lambda} L_i L_j Y^{(k_{L_i} + k_{L_j})} H_u H_u\,, \tag{4.7}$$

where Λ denotes the new physics scale where lepton number is violated by two units, and the Higgs field H_u is assumed to be invariant singlet of T' with a vanishing modular

$\mathcal{D}_1^{(1)} : \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$
$\mathcal{D}_2^{(1)} : \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathcal{D}_2^{(2)} : \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \mathcal{D}_2^{(3)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$
$\mathcal{D}_3^{(1)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \mathcal{D}_3^{(2)} : \begin{pmatrix} \times & 0 & \times \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathcal{D}_3^{(3)} : \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \mathcal{D}_3^{(4)} : \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}$ $\mathcal{D}_3^{(5)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}$
$\mathcal{D}_4^{(1)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad \mathcal{D}_4^{(2)} : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \quad \mathcal{D}_4^{(3)} : \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \mathcal{D}_4^{(4)} : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$ $\mathcal{D}_4^{(5)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{D}_4^{(6)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$
$\mathcal{D}_5^{(1)} : \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{D}_5^{(2)} : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \mathcal{D}_5^{(3)} : \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{D}_5^{(4)} : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ \times & \times & 0 \end{pmatrix}$ $\mathcal{D}_5^{(5)} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathcal{D}_5^{(6)} : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix}$
$\mathcal{D}_6^{(1)} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \mathcal{D}_6^{(2)} : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \quad \mathcal{D}_6^{(3)} : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$
$\mathcal{D}_7^{(1)} : \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Table 3. Texture-zero patterns for the Dirac neutrino mass matrix M_D which can be realised from T' modular symmetry, up to row and column permutations.

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$\mathcal{W}_1^{(1)}: \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$,	$\mathcal{W}_1^{(2)}: \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$		
$\mathcal{W}_2^{(1)}: \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$,	$\mathcal{W}_2^{(2)}: \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$		
$\mathcal{W}_3^{(1)}: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$,	$\mathcal{W}_3^{(2)}: \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}$		
$\mathcal{W}_4^{(1)}: \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}$,	$\mathcal{W}_4^{(2)}: \begin{pmatrix} 0 & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}$,	$\mathcal{W}_4^{(3)}: \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}$,	$\mathcal{W}_4^{(4)}: \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$
$\mathcal{W}_5^{(1)}: \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$			

Table 4. Texture-zero patterns for the Majorana neutrino mass matrix M_ν which can be realised by T' modular symmetry, up to row and column permutations. Notice that only $\mathcal{W}_1^{(1)}$, $\mathcal{W}_2^{(2)}$, $\mathcal{W}_3^{(1)}$, $\mathcal{W}_3^{(2)}$, $\mathcal{W}_4^{(1)}$ and $\mathcal{W}_4^{(4)}$ can be obtained if neutrino masses are described by the Weinberg operator. On the other hand, all the above textures except $\mathcal{W}_4^{(4)}$ can be achieved if neutrino masses are generated via the minimal type I seesaw mechanism.

weight. In this case, the neutrino mass matrix M_ν only depends on the T' representation assignments and modular weights of the LH lepton fields. In the following, we will discuss all possible choices according to eq. (3.1) and infer about the properties of the resulting M_ν in each case.

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}$

  In the case that the three LH lepton fields are assigned to a triplet of T', the effective dimension-five terms in the superpotential can be written as

$$\mathcal{W}_\nu = \sum_{\mathbf{r},a} \frac{g^\nu_{\mathbf{r},a}}{\Lambda} \left[L L Y^{(2k_L)}_{\mathbf{r}a} \right]_{\mathbf{1}} H_u H_u \,, \tag{4.8}$$

  where k_L is the modular weight of L. The explicit form of the M_ν elements is

$$(M_\nu)_{\alpha\beta} = \frac{v_u^2}{\Lambda} \sum_{a,b,c,d} \Big[g^\nu_{\mathbf{3},a} (3\delta_{\alpha\beta} - 1) Y^{(2k_L)}_{\mathbf{3}a,3-(\alpha+\beta)|3} + g^\nu_{\mathbf{1},b} \delta_{2,(\alpha+\beta)|3} Y^{(2k_L)}_{\mathbf{1}b} \\ + g^\nu_{\mathbf{1}',c} \delta_{1,(\alpha+\beta)|3} Y^{(2k_L)}_{\mathbf{1}'c} + g^\nu_{\mathbf{1}'',d} \delta_{0,(\alpha+\beta)|3} Y^{(2k_L)}_{\mathbf{1}''d} \Big] \,, \tag{4.9}$$

  where $\alpha, \beta = 1, 2, 3$. The above expression reveals that M_ν has no zero elements if $k_L > 0$. In the case of $k_L = 0$, only the pattern $\mathcal{W}_4^{(4)}$ of table 4 can be realised, but it leads to two degenerate masses.

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- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i, \quad L_3 \sim \mathbf{1}^j$

  If the three lepton doublets transform as the direct sum of doublet and singlet of T', the most general form of $\mathcal{W}_\nu$ is

$$\mathcal{W}_\nu = \frac{1}{\Lambda} \sum_{a,b,c} \left\{ g^\nu_{1\mathbf{r},a} \left[L_D L_D Y^{(2k_{L_D})}_{\mathbf{r}a} \right]_{\mathbf{1}} + g^\nu_{2\mathbf{r},b} \left[L_D L_3 Y^{(k_{L_D}+k_{L_3})}_{\mathbf{r}b} \right]_{\mathbf{1}} \right.$$
$$\left. + g^\nu_{3\mathbf{r},c} \left[L_3 L_3 Y^{(2k_{L_3})}_{\mathbf{r}c} \right]_{\mathbf{1}} \right\} H_u H_u \,. \tag{4.10}$$

  from which we can read out the expressions for the M_ν matrix elements

$$(M_\nu)_{\alpha\beta} = \sum_a \frac{v_u^2}{\Lambda} \left[g^\nu_{\mathbf{13},a} (\sqrt{2})^{\delta_{\alpha\beta}} (-1)^{\delta_{2,\alpha+\beta}} Y^{(2k_{L_D})}_{\mathbf{3}a,3-(2i-\alpha-\beta)|3} \right], \tag{4.11}$$
$$(M_\nu)_{\alpha 3} = \sum_{a,b,c} \frac{v_u^2}{\Lambda} \left[g^E_{\mathbf{22},a} \delta_{2,(i+j)|3} (-1)^{\alpha+1} Y^{(k_{L_D}+k_{L_3})}_{\mathbf{2}a,3-\alpha} + g^E_{\mathbf{22'},b} \delta_{1,(i+j)|3} (-1)^{\alpha+1} Y^{(k_{L_D}+k_{L_3})}_{\mathbf{2'}b,3-\alpha} \right.$$
$$\left. + g^E_{\mathbf{22''},c} \delta_{0,(i+j)|3} (-1)^{\alpha+1} Y^{(k_{L_D}+k_{L_3})}_{\mathbf{2''}c,3-\alpha} \right], \tag{4.12}$$
$$(M_\nu)_{33} = \sum_{a,b,c} \frac{v_u^2}{\Lambda} \left[g^E_{\mathbf{31},a} \delta_{0,(2j)|3} Y^{(2k_{L_3})}_{\mathbf{1}a} + g^E_{\mathbf{31'},} \delta_{2,(2j)|3} Y^{(2k_{L_3})}_{\mathbf{1'}b} \right.$$
$$\left. + g^E_{\mathbf{31''},c} \delta_{1,(2j)|3} Y^{(2k_{L_3})}_{\mathbf{1''}c} \right], \tag{4.13}$$

  with $\alpha, \beta = 1, 2$. Going through all possible values of i, j and of the modular weights k_{L_D} and k_{L_3}, we find that the following five texture-zero patterns can be obtained,

$$\mathcal{W}_1^{(1)}\,, \mathcal{W}_2^{(2)}\,, \mathcal{W}_3^{(1)}\,, \mathcal{W}_3^{(2)}\,, \mathcal{W}_4^{(1)}\,, \tag{4.14}$$

  as shown in table 4.

- $L_\alpha \sim \mathbf{1}^{j_\alpha}$

  If all LH leptons transform as one-dimensional representations of T', we have

$$\mathcal{W}_\nu = \sum_{\alpha,\beta=1}^{3} \sum_a \frac{g^\nu_{\alpha\beta\mathbf{r},a}}{\Lambda} \left[L_\alpha L_\beta Y^{(k_{L_\alpha}+k_{L_\beta})}_{\mathbf{r}a} \right]_{\mathbf{1}} H_u H_u \,, \tag{4.15}$$

  and the general expression for the neutrino mass matrix is

$$(M_\nu)_{\alpha\beta} = \frac{(1+\delta_{\alpha\beta}) v_u^2}{2\Lambda} \sum_{a,b,c} \left[g^\nu_{\alpha\beta\mathbf{1},a} \delta_{0,(j_\alpha+j_\beta)|3} Y^{(k_{L_\alpha}+k_{L_\beta})}_{\mathbf{1}a} + g^\nu_{\alpha\beta\mathbf{1'},b} \delta_{2,(j_\alpha+j_\beta)|3} Y^{(k_{L_\alpha}+k_{L_\beta})}_{\mathbf{1'}b} \right.$$
$$\left. + g^\nu_{\alpha\beta\mathbf{1''},c} \delta_{1,(j_\alpha+j_\beta)|3} Y^{(k_{L_\alpha}+k_{L_\beta})}_{\mathbf{1''}c} \right]. \tag{4.16}$$

  Analogously to the charged-lepton sector (see the discussion at the end of section 3), we shall not consider this case here since it is less constrained by modular symmetry and, in general, more free parameters would be required. Nevertheless, some texture zeros can be realised [29].

4.2.2 Neutrino masses via type-I seesaw mechanism

If neutrino masses are generated through the type-I seesaw mechanism, at least two RH neutrino fields are required to accommodate present neutrino oscillation data, namely, three nonzero lepton mixing angles and two mass-squared differences. In this section, we will consider the minimal seesaw model with two RH neutrinos [67, 68] for which the Dirac (RH Majorana) neutrino mass matrix M_D (M_N) is a 2×3 (2×2 symmetric) matrix. The RH neutrinos can transform as either doublet or singlet of T', i.e.,

$$N^c \equiv (N_1^c, N_2^c)^T \sim \mathbf{2}^k\,, \quad \text{or} \quad N_\alpha^c \sim \mathbf{1}^{l_\alpha}\,, \tag{4.17}$$

with $\alpha = 1, 2$.

- $N^c \sim \mathbf{2}^k$

  In this case, the mass term of the RH neutrinos can be written as

$$\mathcal{W}_\nu^N = \sum_{\mathbf{r},a} g_{\mathbf{r},a}^N \Lambda \left[N^c N^c Y_{\mathbf{r}a}^{(2k_{N^c})} \right]_{\mathbf{1}}\,, \tag{4.18}$$

  where $g_{\mathbf{r},a}^N$ are coupling constants and k_{N^c} is the modular weight of N^c. From table 2, we can read out the matrix element of M_N:

$$(M_N)_{\alpha\beta} = \sum_a g_{\mathbf{3},a}^N (\sqrt{2})^{\delta_{\alpha\beta}} (-1)^{\delta_{2,\alpha+\beta}} Y_{\mathbf{3}a,3-(2k-\alpha-\beta)|3}^{(2k_{N^c})}\,, \tag{4.19}$$

  where $\alpha, \beta = 1, 2$.

- $N_\alpha^c \sim \mathbf{1}^{l_\alpha}$

  The RH neutrino fields transform as T' singlets, and their mass terms read

$$\mathcal{W}_\nu^N = \sum_{\alpha,\beta=1}^{2} \sum_{\mathbf{r},a} g_{\alpha\beta\mathbf{r},a}^N \Lambda \left[N_\alpha^c N_\beta^c Y_{\mathbf{r}a}^{(k_{N_\alpha^c}+k_{N_\beta^c})} \right]_{\mathbf{1}}\,. \tag{4.20}$$

  The explicit form of the elements of M_N is

$$\begin{aligned}(M_N)_{\alpha\beta} = \frac{(1+\delta_{\alpha\beta})\Lambda}{2} \sum_{a,b,c} \Bigg[& g_{\alpha\beta\mathbf{1},a}^M \delta_{0,(l_\alpha+l_\beta)|3} Y_{\mathbf{1}a}^{(k_{N_\alpha^c}+k_{N_\beta^c})} + g_{\alpha\beta\mathbf{1}',b}^M \delta_{2,(l_\alpha+l_\beta)|3} Y_{\mathbf{1}'b}^{(k_{N_\alpha^c}+k_{N_\beta^c})} \\ & + g_{\alpha\beta\mathbf{1}'',c}^M \delta_{1,(l_\alpha+l_\beta)|3} Y_{\mathbf{1}''c}^{(k_{N_\alpha^c}+k_{N_\beta^c})} \Bigg]\,. \end{aligned} \tag{4.21}$$

Considering that the RH neutrinos can not be massless, we find that M_N can be of one of the following types,

$$\begin{aligned}
&\mathfrak{N}_0^{(1)} : \begin{pmatrix} \times & \times \\ \times & \times \end{pmatrix}, \\
&\mathfrak{N}_1^{(1)} : \begin{pmatrix} 0 & \times \\ \times & \times \end{pmatrix}, \quad \mathfrak{N}_1^{(2)} : \begin{pmatrix} \times & \times \\ \times & 0 \end{pmatrix}, \quad \mathfrak{N}_1^{(3)} : \begin{pmatrix} \times & 0 \\ 0 & \times \end{pmatrix}, \\
&\mathfrak{N}_2^{(1)} : \begin{pmatrix} 0 & \times \\ \times & 0 \end{pmatrix},
\end{aligned} \tag{4.22}$$

up to row and column permutations. All the above textures can be achieved with the singlet assignments $N^c_\alpha \sim \mathbf{1}^{l_\alpha}$, while the doublet assignment $N^c \sim \mathbf{2}^k$ can only give rise to $\mathfrak{N}_0^{(1)}$. Notice that we have included the pattern $\mathfrak{N}_0^{(1)}$ which does not exhibit any zero elements. This is due to the fact that we focus on the texture zero of the effective neutrino mass matrix M_ν which is given by the seesaw formula $M_\nu = -M_D^T M_N^{-1} M_D$. Thus, even if there is no zero elements in M_N, M_ν can still have texture zeros, as long as the Dirac neutrino mass matrix M_D takes suitable form.

The Dirac neutrino mass term arises from the Yukawa couplings of LH leptons and RH neutrinos. For some representation and weight assignments of these fields, texture zeros in the Dirac neutrino mass matrix M_D can be obtained. With two RH neutrinos, we find that M_D can take the following six zero patterns,

$$\begin{aligned}
&\mathfrak{D}_1^{(1)}:\begin{pmatrix}\times & \times & \times\\ \times & \times & 0\end{pmatrix},\\
&\mathfrak{D}_2^{(1)}:\begin{pmatrix}0 & \times & \times\\ \times & 0 & \times\end{pmatrix},\quad \mathfrak{D}_2^{(2)}:\begin{pmatrix}\times & \times & \times\\ 0 & 0 & \times\end{pmatrix},\quad \mathfrak{D}_2^{(3)}:\begin{pmatrix}\times & \times & 0\\ \times & \times & 0\end{pmatrix},\\
&\mathfrak{D}_3^{(1)}:\begin{pmatrix}\times & \times & 0\\ 0 & 0 & \times\end{pmatrix},\\
&\mathfrak{D}_4^{(1)}:\begin{pmatrix}0 & \times & 0\\ \times & 0 & 0\end{pmatrix},
\end{aligned}\tag{4.23}$$

up to row and column permutations. We have omitted the cases in which one row or two columns of M_D are vanishing, since they would lead to two massless neutrinos which are not compatible with experimental data. If all elements of M_D are non-vanishing, M_ν will not have texture zeros too and, consequently, we do not consider this case in the following. We analyse the M_D patterns stemming from all possible lepton field representation assignments.

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}$, $\quad N^c \sim \mathbf{2}^k$

  If the L and N^c fields transform as the triplet and doublet of T', respectively, the superpotential for the Dirac neutrino Yukawa couplings is
  $$\mathcal{W}_\nu^D = \sum_{\mathbf{r},a} g^D_{\mathbf{r},a}\left[N^c L Y^{(k_L+k_{N^c})}_{\mathbf{r}a}\right]_{\mathbf{1}} H_u\,,\tag{4.24}$$
  where $g^D_{\mathbf{r},a}$ are coupling constants. From table 2, we can read out the elements of M_D from those given for M_E in eq. (3.6) by performing the replacements $g^E \to g^D$, $k_{E^c_D} \to k_{N^c}$ and $v_d \to v_u$. We find that only texture $\mathfrak{D}_2^{(1)}$ can be achieved when $k_L + k_{N^c} = 1$.

- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i$, $\quad L_3 \sim \mathbf{1}^j$, $\quad N^c \sim \mathbf{2}^k$

  In this case,
  $$\mathcal{W}_\nu^D = \sum_{\mathbf{r},a,b} g^D_{1\mathbf{r},a}\left[N^c L_D Y^{(k_{L_D}+k_{N^c})}_{\mathbf{r}a}\right]_{\mathbf{1}} H_u + g^D_{2\mathbf{r},b}\left[N^c L_3 Y^{(k_{L_3}+k_{N^c})}_{\mathbf{r}b}\right]_{\mathbf{1}} H_u\,.\tag{4.25}$$

Once more, using the results for M_E given in eq. (3.10), we can obtain the M_D elements from eqs. (3.11) and (3.12) with the replacement $g^E \to g^D$, $k_{E^c_D} \to k_{N^c}$ and $v_d \to v_u$. Doing so, M_D can take the following three types of texture zeros,

$$\mathfrak{D}_2^{(1)}\,,\quad \mathfrak{D}_2^{(3)}\,,\quad \mathfrak{D}_4^{(1)}\,. \tag{4.26}$$

- $L_\beta \sim \mathbf{1}^{j_\beta}, \quad N^c \sim \mathbf{2}^k$

  In case all the LH leptons transform as singlets of T', we have

$$\mathcal{W}_\nu^D = \sum_{\beta=1}^{3}\sum_{\mathbf{r},a} g^D_{\beta\mathbf{r},a}\left[N^c L_\beta Y_{\mathbf{r}_a}^{(k_{L_\beta}+k_{N^c})}\right]_{\mathbf{1}} H_u\,, \tag{4.27}$$

  and the elements of M_D read

$$\begin{aligned}(M_D)_{\alpha\beta} = v_u \sum_{a,b,c}\Big[& g^D_{\mathbf{2},a}\delta_{2,(k+j)|3}(-1)^{\alpha+1}Y^{(k_{L_\beta}+k_{N^c})}_{\mathbf{2}a,3-\alpha} + g^D_{\mathbf{2}',b}\delta_{1,(k+j)|3}(-1)^{\alpha+1}Y^{(k_{L_\beta}+k_{N^c})}_{\mathbf{2}'b,3-\alpha}\\ &+g^D_{\mathbf{2}'',c}\delta_{0,(k+j)|3}(-1)^{\alpha+1}Y^{(k_{L_\beta}+k_{N^c})}_{\mathbf{2}''c,3-\alpha}\Big]\,,\ \alpha=1,2\,,\beta=1,2,3\,.\end{aligned} \tag{4.28}$$

  We find that there is only one allowed texture-zero pattern for M_D, which is $\mathfrak{D}_2^{(3)}$ (up to row and column permutations).

- $L \equiv (L_1, L_2, L_3)^T \sim \mathbf{3}, \quad N^c_\alpha \sim \mathbf{1}^{l_\alpha}$

  With the LH doublet (RH neutrino) fields transforming as triplet (singlets) of T':

$$\mathcal{W}_\nu^D = \sum_{\alpha=1}^{2}\sum_{\mathbf{r},a} g^D_{\alpha\mathbf{r},a}\left[N^c_\alpha L Y_{\mathbf{3}a}^{(k_L+k_{N^c_\alpha})}\right]_{\mathbf{1}} H_u\,, \tag{4.29}$$

  and the general form of M_D can be extracted from eq. (3.9) with the replacements $g^E \to g^D$, $k_{E^c_\alpha} \to k_{N^c_\alpha}$ and $v_d \to v_u$. One row of M_D may vanish in this case, and at least two light neutrino would be massless.

- $L_D \equiv (L_1, L_2)^T \sim \mathbf{2}^i, \quad L_3 \sim \mathbf{1}^j, \quad N^c_\alpha \sim \mathbf{1}^{l_\alpha}$

  Assigning two lepton doublets to a doublet of T' and the remaining fields to singlets, leads to the superpotential

$$\mathcal{W}_\nu^D = \sum_{\alpha=1}^{2}\sum_{\mathbf{r},a,b} g^D_{\alpha 1\mathbf{r},a}\left[N^c_\alpha L_D Y_{\mathbf{r}a}^{(k_{L_D}+k_{N^c_\alpha})}\right]_{\mathbf{1}} H_u + g^D_{\alpha 2\mathbf{r},b}\left[N^c_\alpha L_3 Y_{\mathbf{r}b}^{(k_{L_3}+k_{N^c_\alpha})}\right]_{\mathbf{1}} H_u\,, \tag{4.30}$$

  which is analogous to $\mathcal{W}_E$ given in eq. (3.16). The corresponding expressions of M_D can be obtained from M_E in eqs. (3.17) and (3.18) by performing the same replacements as in the previous case. Consequently, the texture-zero patterns

$$\mathfrak{D}_1^{(1)}\,,\quad \mathfrak{D}_2^{(2)}\,,\quad \mathfrak{D}_2^{(3)}\,,\quad \mathfrak{D}_3^{(1)}\,, \tag{4.31}$$

  can be reproduced.

M_D	M_N	M_ν
$\mathfrak{D}_1^{(1)}$, $\mathfrak{D}_2^{(1)}$, $\mathfrak{D}_3^{(1)}$	$\mathfrak{N}_1^{(1)}$	$\mathcal{W}_1^{(1)}$
$\mathfrak{D}_1^{(1)}$	$\mathfrak{N}_2^{(1)}$	
$\mathfrak{D}_2^{(1)}$	$\mathfrak{N}_1^{(2)}$	
$\mathfrak{D}_2^{(1)}$	$\mathfrak{N}_1^{(3)}$	$\mathcal{W}_1^{(2)}$
$\mathfrak{D}_2^{(1)}$	$\mathfrak{N}_2^{(1)}$	$\mathcal{W}_2^{(1)}$
$\mathfrak{D}_3^{(1)}$	$\mathfrak{N}_1^{(3)}$	$\mathcal{W}_2^{(2)}$
$\mathfrak{D}_2^{(2)}$	$\mathfrak{N}_1^{(2)}$, $\mathfrak{N}_2^{(1)}$	$\mathcal{W}_3^{(1)}$
$\mathfrak{D}_3^{(1)}$	$\mathfrak{N}_1^{(2)}$	
$\mathfrak{D}_2^{(3)}$	$\mathfrak{N}_0^{(1)}$, $\mathfrak{N}_1^{(1)}$, $\mathfrak{N}_1^{(2)}$, $\mathfrak{N}_1^{(3)}$, $\mathfrak{N}_2^{(1)}$	$\mathcal{W}_3^{(2)}$
$\mathfrak{D}_4^{(1)}$	$\mathfrak{N}_0^{(1)}$	
$\mathfrak{D}_3^{(1)}$	$\mathfrak{N}_2^{(1)}$	$\mathcal{W}_4^{(1)}$
$\mathfrak{D}_4^{(1)}$	$\mathfrak{N}_1^{(1)}$, $\mathfrak{N}_1^{(2)}$	$\mathcal{W}_4^{(2)}$
$\mathfrak{D}_4^{(1)}$	$\mathfrak{N}_1^{(3)}$	$\mathcal{W}_4^{(3)}$
$\mathfrak{D}_4^{(1)}$	$\mathfrak{N}_2^{(1)}$	$\mathcal{W}_5^{(1)}$

Table 5. Textures for the effective neutrino mass matrix M_ν in the type-I seesaw mechanism with two RH neutrino fields. $\mathfrak{D}_1^{(1)} - \mathfrak{D}_4^{(1)}$ are the texture-zero patterns for the Dirac neutrino mass matrix M_D and $\mathfrak{N}_0^{(1)} - \mathfrak{N}_2^{(1)}$ are those for the RH Majorana neutrino mass matrix M_N — see (4.22) and (4.23). The explicit forms of the resulting M_ν textures $\mathcal{W}_1^{(1)} - \mathcal{W}_5^{(1)}$ are given in table 4.

- $L_\beta \sim \mathbf{1}^{j_\beta}$, $\quad N^c_\alpha \sim \mathbf{1}^{l_\alpha}$

  If all lepton multiplets transform as one-dimensional representations of T', then

$$\mathcal{W}_\nu^D = \sum_{\alpha=1}^{2} \sum_{\beta=1}^{3} \sum_{\mathbf{r},a} g^D_{\alpha\beta\mathbf{r},a} \left[N^c_\alpha L_\beta Y_{\mathbf{r}a}^{(k_{L_\beta}+k_{N^c_\alpha})} \right]_{\mathbf{1}} H_u \,. \tag{4.32}$$

  The corresponding M_D can be obtained from eq. (3.22). As explained at the end of section 3, we will not discuss this representation assignments.

So far, we have discussed separately the texture-zero patterns of the Dirac neutrino mass matrix M_D and the RH Majorana neutrino mass matrix M_N for the case of two-RH neutrino fields. The possible M_N and M_D textures were summarised in (4.22) and (4.23), respectively. The effective light neutrino mass matrix M_ν is given by the well-known seesaw formula $M_\nu = -M_D^T M_N^{-1} M_D$ and, combining all possible $\mathfrak{D}$ and $\mathfrak{N}$ structures for M_D and M_N, ten possible texture-zero patterns arise for M_ν. These are summarised in table 5, from which one sees that some M_ν textures can be realised from different (M_D, M_N) pairs. For the sake of completeness, we will consider all possible (M_D, M_N) combinations in the following.

Before proceeding to the phenomenological analysis of lepton models based on the texture-zero patterns discussed in the previous sections, it is worth summarising our find-

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$\mathcal{C}_3^{(1)}:$	$3_1^{(l)}$	$\mathcal{D}_4^{(6)}:$	$4_6^{(\nu_D)},\ 4_{10}^{(\nu_D)},\ 4_{14}^{(\nu_D)}\ 4_{20}^{(\nu_D)},\ 4_{25}^{(\nu_D)},\ 4_{27}^{(\nu_D)}$
$\mathcal{C}_3^{(2)}:$	$3_2^{(l)}$	$\mathcal{D}_5^{(1)}:$	$5_3^{(\nu_D)},\ 5_7^{(\nu_D)},\ 5_9^{(\nu_D)}$
$\mathcal{C}_3^{(3)}:$	$3_3^{(l)}$	$\mathcal{D}_5^{(2)}:$	$5_{12}^{(\nu_D)},\ 5_{20}^{(\nu_D)},\ 5_{25}^{(\nu_D)}$
$\mathcal{C}_4^{(1)}:$	$4_4^{(l)}$	$\mathcal{D}_5^{(3)}:$	$5_4^{(\nu_D)},\ 5_5^{(\nu_D)},\ 5_8^{(\nu_D)}$
$\mathcal{C}_4^{(2)}:$	$4_1^{(l)}$	$\mathcal{D}_5^{(4)}:$	$5_{13}^{(\nu_D)},\ 5_{17}^{(\nu_D)},\ 5_{24}^{(\nu_D)}$
$\mathcal{C}_4^{(3)}:$	$4_2^{(l)}$	$\mathcal{D}_5^{(5)}:$	$5_1^{(\nu_D)},\ 5_2^{(\nu_D)},\ 5_6^{(\nu_D)}$
$\mathcal{C}_6^{(1)}:$	$6_1^{(l)}$	$\mathcal{D}_5^{(6)}:$	$5_{10}^{(\nu_D)},\ 5_{11}^{(\nu_D)},\ 5_{19}^{(\nu_D)}\ 5_{21}^{(\nu_D)},\ 5_{26}^{(\nu_D)},\ 5_{27}^{(\nu_D)}$
$\mathcal{D}_2^{(1)}:$	$2_2^{(\nu_D)}$	$\mathcal{D}_6^{(1)}:$	$6_3^{(\nu_D)},\ 6_5^{(\nu_D)},\ 6_7^{(\nu_D)}$
$\mathcal{D}_2^{(2)}:$	$2_1^{(\nu_D)}$	$\mathcal{D}_6^{(2)}:$	$6_{10}^{(\nu_D)},\ 6_{11}^{(\nu_D)},\ 6_{12}^{(\nu_D)}, 6_{14}^{(\nu_D)},\ 6_{15}^{(\nu_D)},\ 6_{16}^{(\nu_D)}$
$\mathcal{D}_2^{(3)}:$	$2_3^{(\nu_D)}$	$\mathcal{D}_6^{(3)}:$	$6_{13}^{(\nu_D)}$
$\mathcal{D}_3^{(1)}:$	$3_3^{(\nu_D)},\ 3_5^{(\nu_D)},\ 3_{10}^{(\nu_D)}$	$\mathcal{D}_7^{(1)}:$	$7_1^{(\nu_D)},\ 7_2^{(\nu_D)},\ 7_3^{(\nu_D)}$
$\mathcal{D}_3^{(2)}:$	$3_8^{(\nu_D)},\ 3_9^{(\nu_D)},\ 3_{11}^{(\nu_D)},\ 3_{13}^{(\nu_D)},\ 3_{16}^{(\nu_D)},\ 3_{17}^{(\nu_D)}$	$\mathcal{W}_1^{(1)}:$	$1_1^{(\nu_L)},\ 1_2^{(\nu_L)},\ 1_3^{(\nu_L)}$
$\mathcal{D}_3^{(3)}:$	$3_{12}^{(\nu_D)}$	$\mathcal{W}_1^{(2)}:$	$1_4^{(\nu_L)},\ 1_5^{(\nu_L)},\ 1_6^{(\nu_L)}$
$\mathcal{D}_3^{(4)}:$	$3_1^{(\nu_D)}$	$\mathcal{W}_2^{(1)}:$	$2_1^{(\nu_L)},\ 2_2^{(\nu_L)},\ 2_3^{(\nu_L)}$
$\mathcal{D}_3^{(5)}:$	$3_7^{(\nu_D)},\ 3_{15}^{(\nu_D)},\ 3_{18}^{(\nu_D)}$	$\mathcal{W}_2^{(2)}:$	$2_{13}^{(\nu_L)},\ 2_{14}^{(\nu_L)},\ 2_{15}^{(\nu_L)}$
$\mathcal{D}_4^{(1)}:$	$4_{12}^{(\nu_D)},\ 4_{18}^{(\nu_D)},\ 4_{21}^{(\nu_D)}$	$\mathcal{W}_3^{(1)}:$	$3_2^{(\nu_L)},\ 3_5^{(\nu_L)},\ 3_{10}^{(\nu_L)}$
$\mathcal{D}_4^{(2)}:$	$4_5^{(\nu_D)},\ 4_9^{(\nu_D)},\ 4_{17}^{(\nu_D)}$	$\mathcal{W}_3^{(2)}:$	$3_{11}^{(\nu_L)},\ 3_{16}^{(\nu_L)},\ 3_{19}^{(\nu_L)}$
$\mathcal{D}_4^{(3)}:$	$4_7^{(\nu_D)},\ 4_{16}^{(\nu_D)},\ 4_{26}^{(\nu_D)}$	$\mathcal{W}_4^{(2)}:$	$4_1^{(\nu_L)},\ 4_2^{(\nu_L)},\ 4_3^{(\nu_L)},\ 4_4^{(\nu_L)},\ 4_5^{(\nu_L)},\ 4_6^{(\nu_L)}$
$\mathcal{D}_4^{(4)}:$	$4_4^{(\nu_D)},\ 4_{13}^{(\nu_D)},\ 4_{24}^{(\nu_D)}$	$\mathcal{W}_4^{(3)}:$	$4_7^{(\nu_L)},\ 4_8^{(\nu_L)},\ 4_9^{(\nu_L)}$
$\mathcal{D}_4^{(5)}:$	$4_1^{(\nu_D)},\ 4_2^{(\nu_D)},\ 4_3^{(\nu_D)}$		

Table 6. Correspondence between our texture-zero patterns $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{W}$ and those of ref. [9]. Since the assignment of the three generations of lepton fields under T' modular symmetry can be freely exchanged, the lepton mass matrices are determined up to independent row and column permutations. Consequently, one texture zero of ours could correspond to several textures of ref. [9].

ings up to this point. As shown in sections 3 and 4, texture zeros of fermion mass matrices can be naturally implemented in the context of modular flavour symmetries, being the non-vanishing elements of those matrices correlated as shown, for instance, in table 2. Thus, the resulting lepton models are expected to be much more predictive than, e.g., models based in Abelian symmetries. In tables 1, 3 and 4, we have listed all possible texture-zero patterns for the charged-lepton, Dirac neutrino and Majorana neutrino mass matrices, respectively, which are obtainable from the T' modular symmetry. Notice that one could get alternative texture-zero patterns from finite modular groups other than T'. It is remarkable that some texture zeros of lepton mass matrices listed in ref. [9] can be reproduced from the T' modular symmetry, as shown in table 6. However, some others cannot be realised in the framework of T' modular group, as it is the case of texture "$5_1^{(l)}$" of ref. [9]. Notice that one texture of ours may correspond to several ones of [9] since our lepton mass matrices are defined up to row and column permutation due to the freedom of representation assignment of matter fields. Moreover, the patterns with one or two zero elements in M_E and one zero element in M_D were not studied in [9]. Thus, $\mathcal{C}_1^{(1)}$, $\mathcal{C}_2^{(1)}$, $\mathcal{C}_2^{(2)}$, $\mathcal{C}_2^{(3)}$ and $\mathcal{D}_1^{(1)}$ discussed in this work have no correspondence in [9].

5 Testing the full set of TZLMs

After having analysed separately the realisation of texture zeros in the charged-lepton and neutrino mass matrices with T' modular forms up to weight 6, we now combine them with the purpose of studying their phenomenology. In practice, we will test all possible texture-zero pairs (M_E, M_ν), where M_ν can be either Dirac or Majorana neutrino mass matrix, taking into account the results of the previous sections. Since the LH charged-lepton and neutrino fields belong to $\mathrm{SU}(2)_L$ doublets, they should transform in the same way under modular symmetry, i.e., they should share the same modular weight and representation assignments. Each of these pairs correspond to a different *texture-zero lepton model* or TZLM. In this paper we are interested in those TZLMs which are, in some sense, predictive. The set of observables against which the test will be performed contains essentially nine experimentally-measured lepton-mass and mixing parameters, namely

$$m_e,\ m_\mu,\ m_\tau,\ \Delta m^2_{21},\ \Delta m^2_{31},\ \theta_{12},\ \theta_{13},\ \theta_{23},\ \delta_{CP}\,, \tag{5.1}$$

where m_e, m_μ, m_τ are three charged-lepton masses, Δm^2_{21}, Δm^2_{31} are two neutrino mass-squared differences, θ_{12}, θ_{13}, θ_{23} are three lepton mixing angles and δ_{CP} is the Dirac CP-violation phase. Note that the present statistical significance of the δ_{CP} measurement is rather weak, and the preferred value of δ_{CP} from global data analyses should be taken with a grain of salt [69].

Given the above set of observables, we shall focus on TZLMs with nine or less real free input parameters. Notice that, in general, all coupling constants appearing in previous sections can be complex. Still, some of them can be made real by rephasing fields. Thus, for a given TZLM, the number of input parameters is counted after removing all unphysical complex phases. Interestingly, one can further constrain the number of inputs, and thus increase the TZLM predictive power, by imposing the generalized CP symmetry (gCP). In the context of modular symmetries, it has been found that the gCP acts on the complex modulus as $\tau \xrightarrow{\mathcal{CP}} -\tau^*$, up to modular transformations [70–74]. In the basis where both modular generators S and T are represented by symmetric matrices in all irreducible representations, gCP reduces to the canonical CP transformation [74]. Hence, all coupling constants are forced to be real if the CG coefficients are real in the symmetric basis. As shown in appendix A, we are indeed working in the symmetric basis of the T' group with real CG coefficients. Consequently, imposing gCP in our TZLMs amounts to considering all coupling real and, as a result, the number of free parameters is reduced. Given this interesting possibility, we will consider both cases with and without gCP. In all cases we require the total number of real input parameters to be less than or equal to nine. Notice that imposing gCP can reduce the number of free parameters in lepton models and, thus, the TZLMs containing nine or less real free input parameters without gCP can be also achieved with gCP. Moreover, some TZLMs without gCP which are excluded by the constraint on the number of free parameters, could depend on nine or less real free parameters after gCP is imposed. Hence gCP can lead to more TZLMs.

For the three different neutrino-mass generation mechanisms considered in this work we have:

Dirac neutrinos. From tables 1 and 3, we see the T' modular symmetry can realise 11 (25) distinct texture-zero patterns for M_E (M_D). As pointed out in sections 3 and 4, texture $\mathcal{C}_6^{(1)}$ for M_E and $\mathcal{D}_6^{(3)}$ and $\mathcal{D}_7^{(1)}$ for M_D, lead to degenerate mass eigenvalues, which is not compatible with experimental data. Thus, they will be neglected hereafter. It is straightforward to conclude that there are $10 \times 23 = 230$ pairs (M_E, M_D). Considering that the LH neutrinos and charged leptons should transform in the same way under modular symmetry, and sticking to our classification of a predictive TZLM, we find that 136 out of those 230 classes of (M_E, M_D) can be realised from T' modular symmetry. If gCP is imposed, the number increases to 174. The allowed patterns of (M_E, M_D) in T' modular symmetry are summarised in tables 8 and 9 (more details on the contents of these tables will be given in the next section).

Majorana neutrinos. For Majorana neutrinos, we have obtained 11 M_ν texture-zero patterns, as summarised in table 4. Textures $\mathcal{W}_4^{(1)}$, $\mathcal{W}_4^{(4)}$ and $\mathcal{W}_5^{(1)}$ predict two degenerate neutrino masses, thus they will be excluded in the following analysis. If light neutrino masses are described by the Weinberg operator, combining the possible constructions in the neutrino and charged-lepton sectors, we find that 29 pairs (M_E, M_ν) can be achieved with T' modular symmetry without gCP, and two additional ones are allowed if gCP symmetry is imposed. These findings are summarized in table 10. For neutrino masses generated via the minimal type-I seesaw mechanism (see section 4.2.2), the possible M_D and M_N textures, as well as the resulting patterns for M_ν, are presented in table 5. Notice that some M_ν given by seesaw formula can be realised from more than one (M_D, M_N). We find 35 (36) possible pairs if gCP is not (is) included in the T' modular models, which are listed in table 11.

5.1 Numerical analysis and TZLM predictions

In order to quantitatively estimate how well the different TZLMs describe the data, we perform a χ^2 analysis to find out the best fit values of the input parameters for each model, as well as the predictions for lepton mass and mixing parameters. As common practice, we consider both normal ordering (NO) and inverted ordering (IO) for the neutrino mass spectrum, depending on whether $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$, respectively. The χ^2 function is defined as usual:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{P_i(x_1, x_2, \ldots, x_m) - \mu_i}{\sigma_i} \right)^2 , \tag{5.2}$$

where P_i are the predictions of a given TZLM for the physical observables θ_{12}, θ_{13}, θ_{23}, δ_{CP}, m_e/m_μ, m_μ/m_τ and $\Delta m_{21}^2/\Delta m_{31}^2$. These are nontrivial functions of the free input parameters in each TZLM. μ_i and σ_i denote the best-fit values and standard deviations of the corresponding quantities extracted from global analysis of the data — see table 7.

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Observables	Normal Ordering		Inverted Ordering	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\delta_{\rm CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3l}}{10^{-3}\mathrm{eV}^2}$	$2.515^{+0.028}_{-0.028}$	$2.430 \to 2.593$	$-2.490^{+0.026}_{-0.028}$	$2.410 \to 2.574$
	bfv $\pm 1\sigma$			
m_e/m_μ	0.004737 ± 0.000040			
m_μ/m_τ	0.05857 ± 0.00047			
m_τ/GeV	1.30234			

Table 7. Allowed ranges for the neutrino oscillation parameters obtained from global analysis of the data, and values of the charged-lepton mass ratios. Here, we use the NuFIT v5.1 results with Super-Kamiokanda atmospheric data [69]. Note that $\Delta m^2_{3l} \equiv \Delta m^2_{31} > 0$ for NO and $\Delta m^2_{3l} \equiv \Delta m^2_{32} < 0$ for IO. The charged-lepton masses are taken from [76] with $\tan\beta = 10$ and SUSY-breaking scale $M_{\rm SUSY} = 10$ TeV.

We adopt the standard parametrization of the lepton mixing matrix [75],

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) \,, \tag{5.3}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, δ_{CP} is the Dirac CP violation phase, and $\alpha_{21,31}$, are Majorana CP phases. If the lightest neutrino is massless, there is a single Majorana phase ϕ and the diagonal phase matrix in the above equation can be replaced by $\mathrm{diag}(1, e^{i\phi/2}, 1)$. Information on the Majorana phases could be potentially extracted from neutrinoless double beta decay ($0\nu\beta\beta$) experiments. If only light neutrino masses provide contributions to $0\nu\beta\beta$, the decay amplitude depends on the effective Majorana neutrino mass $m_{\beta\beta}$,

$$m_{\beta\beta} = |m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13}e^{i\alpha_{21}} + m_3 \sin^2\theta_{13}e^{i(\alpha_{31}-2\delta_{CP})}| \,, \tag{5.4}$$

which, in case the lightest neutrino is massless, reduces to

$$m_{\beta\beta} = \begin{cases} |m_2 \sin^2\theta_{12}\cos^2\theta_{13}e^{i\phi} + m_3 \sin^2\theta_{13}e^{-i2\delta_{CP}}| \,, & m_1 = 0 \text{ (NO)} \,, \\ |m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13}e^{i\phi}| \,, & m_3 = 0 \text{ (IO)} \,. \end{cases} \tag{5.5}$$

In this work we will use the NuFIT v5.1 results [69] for the allowed ranges of the neutrino oscillation parameters (see also refs. [77] and [78]). The charge-lepton masses will

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enter in the χ^2 function in the form of their ratios, being the best fit values (bfv) and 1σ errors taken from ref. [76]. The overall scale of the charged lepton mass matrix does not affect the mass ratios and mixing parameters, and it is fixed by the central value of $m_\tau = 1.30234\,\text{GeV}$ given in table 7. Similarly, the overall scale of the light neutrino mass matrix is fixed by the solar neutrino mass squared difference $\Delta m^2_{21} = 7.42 \times 10^{-5}\text{eV}^2$. It is known that the effects of renormalization group evolution (RGE) on neutrino masses and mixing parameters are suppressed for NO neutrino mass spectrum and small $\tan\beta$, while the RGE effects could be large in the case of IO spectrum if the different terms do not cancel each other [79–81]. Hence, we expect RGE corrections to be generally small for our TZLMs with NO spectrum. The details of neutrino-parameter running depend on the specific models under consideration for IO. In the appendix C, we have presented a large number of representative models for each phenomenological viable TZLM. The detailed RGE analysis for these models with IO spectrum is left for future.

For χ^2 minimisation and consequent determination of the inputs which best fit the data, we use the CERN package `TMinuit` [82]. The input value of the modulus τ will be a random complex number in the fundamental domain $\mathcal{F}$: $|\text{Re}\tau| \leq \frac{1}{2}$, $\text{Im}\tau > 0$ and $|\tau| \geq 1$. The absolute values and phases of all coupling constants are free to vary in the ranges $[0, 10^6]$ and $[0, 2\pi)$, respectively. As a general criterion, we will consider that a given TZLM is compatible with experimental data if the predicted values of the neutrino mass and mixing parameters at the minimum of the χ^2 are within the 3σ ranges given in table 7. For the charged-lepton masses, the model's best-fit values should not deviate from the experimental central values by more than 3σ. Generally, a pair of texture zeros in $(M_E, M_{D(\nu)})$ can be realised in several lepton models differing in the representation and modular weight assignments of the fields. In the numerical analysis, for an allowed texture of $(M_E, M_{D(\nu)})$, we will perform χ^2 analysis for all corresponding lepton models. A texture of $(M_E, M_{D(\nu)})$ is said to be viable if at least one of these lepton models can explain the experimental data. After setting the general grounds of out numerical analysis, we now discuss its results for all allowed TZLMs identified in section 5.

5.1.1 Dirac neutrinos

For Dirac neutrinos, the allowed combinations of (M_E, M_D) realised by T' modular symmetry with and without gCP are presented in table 8 (table 9) for NO (IO). For all texture pairs the symbol inside (outside) the parenthesis corresponds to the case when gCP is (is not) imposed. Also, "-" means that a particular configuration cannot be achieved by modular symmetry, while "✔" ("✘") identifies those cases which are realised by the modular symmetry and (but) are (are not) phenomenologically viable. It is remarkable that gCP allows to obtain more additional TZLMs which are excluded by our requirement on the number of free parameters if gCP is not imposed. From table 8 we can see that for NO and if gCP is not included, 27 out of 136 allowed texture pairs provide a good fit to the experimental data . In table 14 of appendix C, we show a representative model for each viable (M_E, M_D) pair by listing the representation and modular-weight assignments of lepton fields. The corresponding values of $\sin^2\theta_{ij}$, m_i and δ_{CP} at the best-fit point are given in table 15. From table 14, we can see that the minimal models have 8 free real input

M_D \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{D}_1^{(1)}$	✔ (✔)	- (✔)	✘ (✔)	✔ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✔ (✔)
$\mathcal{D}_2^{(1)}$	✘ (✔)	✘ (✔)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✔)	✘ (✘)
$\mathcal{D}_2^{(2)}$	✘ (✔)	- (✔)	✔ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)
$\mathcal{D}_2^{(3)}$	✘ (✔)	- (✔)	✘ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✘ (✔)
$\mathcal{D}_3^{(1)}$	- (✔)	- (✘)	- (✔)	- (✔)	- (-)	- (-)	- (✘)	✔ (✔)	- (✘)	- (✔)
$\mathcal{D}_3^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{D}_3^{(3)}$	✘ (✘)	- (✔)	✘ (✔)	✘ (✘)	- (✔)	- (-)	✘ (✘)	✘ (✔)	✘ (✘)	- (-)
$\mathcal{D}_3^{(4)}$	✘ (✔)	✘ (✔)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✔)	✘ (✔)	✘ (✔)	✔ (✔)
$\mathcal{D}_3^{(5)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(1)}$	✔ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✔)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(2)}$	✘ (✘)	- (✘)	✘ (✔)	✘ (✘)	- (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)
$\mathcal{D}_4^{(3)}$	✘ (✔)	- (✘)	✘ (✔)	✘ (✔)	- (✔)	- (-)	- (-)	✘ (✘)	- (-)	✘ (✘)
$\mathcal{D}_4^{(4)}$	- (✔)	- (✔)	- (✔)	- (✔)	- (✔)	- (-)	- (✘)	✘ (✔)	- (✘)	- (✘)
$\mathcal{D}_4^{(5)}$	✘ (✔)	- (✔)	✘ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(6)}$	- (-)	- (-)	- (✔)	- (-)	- (-)	- (-)	- (✔)	✘ (✘)	- (-)	- (-)
$\mathcal{D}_5^{(1)}$	✔ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(2)}$	✘ (✔)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)	- (-)	✘ (✔)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(3)}$	✘ (✔)	✘ (✘)	✘ (✔)	✘ (✘)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)
$\mathcal{D}_5^{(4)}$	✘ (✔)	- (✘)	✘ (✔)	✘ (✔)	✘ (✘)	- (-)	- (-)	✘ (✘)	- (-)	✘ (✘)
$\mathcal{D}_5^{(5)}$	✘ (✔)	- (✔)	✔ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(6)}$	- (-)	- (-)	- (✔)	- (-)	- (-)	- (-)	- (✔)	✘ (✘)	- (-)	- (-)
$\mathcal{D}_6^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_6^{(2)}$	- (✔)	- (-)	✘ (✘)	- (✔)	- (-)	- (-)	✘ (✘)	✘ (✘)	- (-)	- (-)

Table 8. Allowed combinations of (M_E, M_D) realised by T' modular symmetry with and without gCP for NO neutrino mass spectrum. For all texture pairs the symbol inside (outside) the parenthesis corresponds to the case when gCP is (is not) imposed. Also, "-" means that a particular configuration cannot be achieved by modular symmetry, while "✔" ("✘") identifies those cases which are realised by the modular symmetry and (but) are (are not) phenomenologically viable.

parameters. In the IO case, there are 34 viable pairs of (M_E, M_D), as shown in table 9, and we provide the field assignments of representative models and the corresponding fitting results of lepton parameters in tables 16 and 17, respectively. All representative models are found to contain either 8 or 9 free real parameters.

When gCP is imposed, there are allowed (M_E, M_D) pairs. By performing the χ^2 analysis of the corresponding TZLMs, we obtain 97 (88) phenomenologically-viable ones for NO (IO), as presented in table 8 (table 9). The details of the representative models and corresponding predictions for each compatible case are displayed in tables 14 and 15 (tables 16 and 17) for NO (IO). We find that the number of input parameters for representative models are at least 8, as in the case of no gCP.

M_D \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{D}_1^{(1)}$	✔ (✔)	- (✔)	✔ (✔)	✔ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✔ (✔)
$\mathcal{D}_2^{(1)}$	✘ (✔)	✘ (✔)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✔)	✘ (✘)
$\mathcal{D}_2^{(2)}$	✘ (✔)	- (✘)	✔ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✔)
$\mathcal{D}_2^{(3)}$	✘ (✔)	- (✔)	✘ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✘ (✔)
$\mathcal{D}_3^{(1)}$	- (✔)	- (✘)	- (✔)	- (✘)	- (-)	- (-)	- (✘)	✘ (✘)	- (✘)	- (✘)
$\mathcal{D}_3^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{D}_3^{(3)}$	✘ (✘)	- (✘)	✘ (✘)	✘ (✘)	- (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)
$\mathcal{D}_3^{(4)}$	✔ (✔)	✘ (✔)	✔ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✔)	✔ (✔)	✘ (✘)	✔ (✔)
$\mathcal{D}_3^{(5)}$	✔ (✔)	✘ (✘)	✔ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(2)}$	✘ (✘)	- (✘)	✘ (✘)	✘ (✘)	- (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)
$\mathcal{D}_4^{(3)}$	✘ (✘)	- (✘)	✘ (✘)	✘ (✘)	- (✘)	- (-)	- (-)	✘ (✘)	- (-)	✘ (✘)
$\mathcal{D}_4^{(4)}$	- (✘)	- (✘)	- (✘)	- (✘)	- (✘)	- (-)	- (✘)	✘ (✘)	- (✘)	- (✘)
$\mathcal{D}_4^{(5)}$	✔ (✔)	- (✔)	✔ (✔)	✘ (✔)	- (✔)	- (-)	✘ (✘)	✔ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_4^{(6)}$	- (-)	- (-)	- (✔)	- (-)	- (-)	- (-)	- (✘)	✘ (✘)	- (-)	- (-)
$\mathcal{D}_5^{(1)}$	✔ (✔)	✘ (✘)	✔ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(2)}$	✘ (✘)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(3)}$	✘ (✔)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)
$\mathcal{D}_5^{(4)}$	✘ (✔)	- (✘)	✘ (✔)	✘ (✘)	✘ (✘)	- (-)	- (-)	✘ (✘)	- (-)	✘ (✘)
$\mathcal{D}_5^{(5)}$	✘ (✘)	- (✘)	✔ (✘)	✘ (✘)	- (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_5^{(6)}$	- (-)	- (-)	- (✔)	- (-)	- (-)	- (-)	- (✘)	✘ (✘)	- (-)	- (-)
$\mathcal{D}_6^{(1)}$	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{D}_6^{(2)}$	- (✘)	- (-)	✘ (✘)	- (✘)	- (-)	- (-)	✘ (✘)	✘ (✘)	- (-)	- (-)

Table 9. The same as in table 8 but for IO neutrino mass spectrum.

At this point, it is worth comparing the parameter counting of TZLMs realised by T' modular symmetry with that of texture-zero scenarios where nonvanishing entries in the mass matrices are not correlated. We have seen that, in the case of Dirac neutrinos, and demanding the number of inputs not to exceed 9, viable TZLMs require either 8 or 9 real parameters. We emphasize that this number is much smaller than what is typically found with uncorrelated nonvanishing matrix elements. This is apparent in tables 14 and 16 where we compare the number of free parameters "#P" for each TZLM realisable with T' modular symmetry (third column) with the same number for the same matrix textures but with uncorrelated nonzero entries in the mass matrices ("#P$_0$" in the second column). For all cases #P $\ll$ #P$_0$, showing that modular symmetry increases drastically the predictive power of TZLMs. As an example, in the case $(M_E, M_D) \sim (C_2^{(3)}, D_5^{(1)})$, we find that #P$_0 = 20$, while the same pair is realised in a specific T' modular model with only 8 free

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parameters — see line 17 of table 14. As will become clear in the following, this is a general feature which is present also for Majorana neutrinos.

5.1.2 Majorana neutrinos

We now repeat the analysis presented in the previous section but for TZLMs with Majorana neutrinos. Both cases of M_ν generated via a Weinberg operator and via the minimal type-I seesaw mechanism will be considered. Each model is now classified according to the textures of its corresponding (M_E, M_ν) pair.

Weinberg operator. For neutrino masses generated by a Weinberg operator, and without imposing gCP, there are 29 texture pairs (M_E, M_ν) which can be realised with T' modular symmetry, as shown in table 10 for both NO and IO neutrino masses. By performing their χ^2 analysis, we find that, out of those 29 models, only 5 are compatible with experimental data for NO neutrino masses spectrum (upper rows of the table). As done for Dirac neutrinos, we present in table 18 the representation and weight assignments of a representative model for each phenomenologically viable case, while the best fitting results of physical observables and the model predictions are provided in table 19. For IO neutrino mass spectrum, we obtain 7 phenomenologically viable (M_E, M_ν) pairs, for which the representative models are listed in table 20, and the numerical results in table 21. Note that the minimal viable NO (IO) model $\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$ $(\mathcal{C}_2^{(2)} - \mathcal{W}_3^{(1)})$ — see table 18 (table 20) — is realisable with only 7 (8) input parameters to explain all 9 measured observables.

In case the TZLMs are constructed imposing gCP, 11 out of the 31 allowed (M_E, M_ν) pairs are consistent with the measured data for NO as shown in table 10. The representative models for these cases are given in table 18 and the corresponding fitting results in table 19. The number of input parameters for the representative models vary from 7 to 9. For IO with gCP, we have 10 (M_E, M_ν) pairs compatible with data (see tables 10, 20 and 21 for the representative models and fitting results).

Seesaw mechanism. When Majorana neutrino masses are generated through the (minimal) type-I seesaw mechanism, we find that 35 (M_E, M_ν) pairs are realised by T' modular symmetry if gCP is not considered — see table 11. For NO (IO) neutrino mass spectrum, only 6 (11) of these 35 cases are phenomenologically viable. As illustrated in table 5, some patterns of M_ν can be realised in more than one way with different (M_D, M_N) combinations. For the sake of completeness, we present representative models for distinct (M_D, M_N) pairs leading to the same textures of (M_E, M_ν) in tables 22 and 24 for NO and IO, respectively. The best-fit values of lepton mass and mixing parameters are instead given in tables 23 and 25. All viable NO models contain 9 free real parameters, while for IO this number can be 8 or 9. With gCP, there are 36 allowed pairs of (M_E, M_ν), being 13 and 15 of them phenomenologically viable for NO and IO, respectively (see table 11). The representative models and fitting results analogous to the previous cases are shown in tables 22–25. Due to the fact that the coupling constants are required to be real by gCP, we can find viable models with 8 parameters for NO neutrino masses, while this number does not changed for IO when compared with the non-gCP case.

NO										
M_ν \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{W}_1^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	-(-)	-(✘)	✔ (✔)	-(✘)	✘ (✘)
$\mathcal{W}_2^{(2)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	-(-)	-(-)	✘ (✘)	-(-)	✘ (✘)
$\mathcal{W}_3^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	-(-)	✘ (✘)	-(-)	✘ (✘)	-(-)	✘ (✘)	-(-)
$\mathcal{W}_3^{(2)}$	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	-(-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
IO										
M_ν \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{W}_1^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	- (✘)	✔ (✔)	- (✘)	✘ (✘)
$\mathcal{W}_2^{(2)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	- (-)	✘ (✘)	- (-)	✘ (✘)
$\mathcal{W}_3^{(1)}$	✘ (✘)	✘ (✘)	✔ (✘)	- (-)	✔ (✘)	- (-)	✘ (✘)	- (-)	✘ (✘)	- (-)
$\mathcal{W}_3^{(2)}$	✘ (✔)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)

Table 10. Same analysis as in tables 8 and 9 but now for a Majorana neutrino mass matrix M_ν generated via the Weinberg operator — see table 4 for the general structure of the $\mathcal{W}$ textures. The results for NO (IO) are shown in the upper (lower) part of the table.

We present a grand summary of our results in table 12 where, for the three considered neutrino mass generation mechanisms, we show the number of texture pairs with no more than 9 real input parameters which can be realised via T' modular symmetry (third column) with and without gCP (second column). The number of phenomenologically viable cases (i.e. those which are able to fit the data at the 3σ level) is shown in the fourth column for both NO and IO. We see that more patterns of texture zero and viable cases can be obtained if gCP is imposed.

6 Benchmark models

The analysis presented in the previous section provided a complete view of how modular symmetries drastically increase the predictive power of TZLMs. It is obviously impossible to go through all the listed cases in detail and to present a complete graphical treatment of each model predictions. Nevertheless, we believe it is worth providing a small set of benchmark cases where the quality of the results can be appreciated. With this purpose, in the following we select one TZLM realisable with T' modular symmetry for each neutrino mass generation mechanism.

6.1 Model for Dirac neutrino masses

For the Dirac neutrino benchmark case, we consider the lepton fields transforming under the T' modular symmetry T' as

$$L_D \sim \mathbf{2}\,, \quad L_3 \sim \mathbf{1}'\,, \quad e^c \sim \mathbf{1}\,, \quad \mu^c \sim \mathbf{1}''\,, \quad \tau^c \sim \mathbf{1}'\,, \quad N_D^c \equiv \{N_1^c, N_2^c\} \sim \mathbf{2}\,, \quad N_3^c \sim \mathbf{1}'\,,$$

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NO										
M_ν \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{W}_1^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✔)	✘ (✘)	✘ (✘)
$\mathcal{W}_1^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_2^{(1)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_2^{(2)}$	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	- (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_3^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✘ (✔)	✘ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_3^{(2)}$	✔ (✔)	✘ (✘)	✔ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_4^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_4^{(3)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
IO										
M_ν \ M_E	$\mathcal{C}_1^{(1)}$	$\mathcal{C}_2^{(1)}$	$\mathcal{C}_2^{(2)}$	$\mathcal{C}_2^{(3)}$	$\mathcal{C}_3^{(1)}$	$\mathcal{C}_3^{(2)}$	$\mathcal{C}_3^{(3)}$	$\mathcal{C}_4^{(1)}$	$\mathcal{C}_4^{(2)}$	$\mathcal{C}_4^{(3)}$
$\mathcal{W}_1^{(1)}$	✔ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✔ (✔)	✘ (✘)	✘ (✔)
$\mathcal{W}_1^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_2^{(1)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_2^{(2)}$	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)	- (✘)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_3^{(1)}$	✘ (✔)	✘ (✘)	✘ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_3^{(2)}$	✔ (✔)	✘ (✘)	✔ (✔)	✔ (✔)	✔ (✔)	- (-)	✘ (✘)	✘ (✘)	✘ (✘)	✘ (✘)
$\mathcal{W}_4^{(2)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)
$\mathcal{W}_4^{(3)}$	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)	- (-)

Table 11. Same as in table 10 for M_ν generated via (minimal) type-I seesaw mechanism from M_D and M_N textures given in (4.22) and (4.21), respectively. The $(M_D, M_N) \to M_\nu$ dictionary is provided in table 5.

with modular-weight assignments:

$$k_{L_D} = 2 - x\,, \quad k_{L_3} = 3 - x\,, \quad k_{e^c} = 1 + x\,, \quad k_{\mu^c} = 1 + x\,, \quad k_{\tau^c} = 1 + x\,, \tag{6.1}$$
$$k_{N^c_D} = x\,, \quad k_{N^c_3} = 1 + x\,, \tag{6.2}$$

where x is an arbitrary integer. The corresponding modular-invariant superpotentials relevant for charged-lepton and neutrino masses are given by

$$\begin{aligned}\mathcal{W}_E &= y_1^e\, e^c \left[L_D Y^{(3)}_{\mathbf{2''}}\right]_{\mathbf{1}} H_d + y_2^e\, \mu^c \left[L_D Y^{(3)}_{\mathbf{2}}\right]_{\mathbf{1'}} H_d + y_3^e\, \mu^c L_3 Y^{(4)}_{\mathbf{1}} H_d + y_4^e\, \tau^c L_3 Y^{(4)}_{\mathbf{1'}} H_d\,,\\ \mathcal{W}_\nu &= y_1^d \left[(L_D N^c)_{\mathbf{3}} Y^{(1)}_{\mathbf{3}}\right]_{\mathbf{1}} H_u + y_2^d L_3 N^c_3\, Y^{(4)}_{\mathbf{1'}} H_u\,,\end{aligned} \tag{6.3}$$

where the $y_k^{e,d}$ couplings are, in principle, complex. Notice, however, that their phases can be eliminated by rephasing the lepton supermultiplets. As a result, we find that there are 8 real free parameters in this model: 6 real coupling constants (y^e_{1-4} and $y^d_{1,2}$), and the

Neutrino nature	gCP	Number of textures	Viable
Dirac	no	136	23 (NO)
			27 (IO)
	yes	174	97 (NO)
			57 (IO)
Majorana (Weinberg operator)	no	29	5 (NO)
			7 (IO)
	yes	31	11 (NO)
			10 (IO)
Majorana (Seesaw mechanism)	no	35	6 (NO)
			10 (IO)
	yes	36	13 (NO)
			14 (IO)

Table 12. Grand summary of the TZLM compatibility analysis. We show the number of texture pairs with no more than 9 real input parameters which can be realised via T' modular symmetry (third column) with and without gCP (second column). The number of phenomenologically viable cases (i.e. those which are able to fit the data at the 3σ level) is given in the fourth column for both NO and IO.

complex modulus τ. The charged-lepton and neutrino mass matrices read

$$M_E = \begin{pmatrix} y_1^e Y^{(3)}_{\mathbf{2}'',2} & -y_1^e Y^{(3)}_{\mathbf{2}'',1} & 0 \\ -y_2^e Y^{(3)}_{\mathbf{2},2} & y_2^e Y^{(3)}_{\mathbf{2},1} & y_3^e Y^{(4)}_{\mathbf{1}} \\ 0 & 0 & y_4^e Y^{(4)}_{\mathbf{1}'} \end{pmatrix} v_d\,, \quad M_D = \begin{pmatrix} -y_1^d Y^{(2)}_{\mathbf{3},2} & \frac{y_1^d Y^{(2)}_{\mathbf{3},3}}{\sqrt{2}} & 0 \\ \frac{y_1^d Y^{(2)}_{\mathbf{3},3}}{\sqrt{2}} & y_1^d Y^{(2)}_{\mathbf{3},1} & 0 \\ 0 & 0 & y_2^d Y^{(4)}_{\mathbf{1}'} \end{pmatrix} v_u\,, \tag{6.4}$$

which correspond to the texture-zero pattern $\mathcal{C}_3^{(1)} - \mathcal{D}_4^{(1)}$ for (M_E, M_D) is — see tables 1 and 3 (this TZLM is also presented in table 14). We perform a global fit to the lepton experimental data and, for normally-ordered neutrino masses, the values of the input parameters at the best-fit point are

$$\begin{aligned} &\langle\tau\rangle = -0.30546 + 1.05008i\,, \quad y_2^e/y_1^e = 10.6173\,, \quad y_3^e/y_1^e = 21.5185\,, \quad y_4^e/y_1^e = 0.011167\,, \\ &y_2^d/y_1^d = 3.02666\,, \quad y_1^e v_d = 1.68246 \text{ GeV}\,, \quad y_1^d v_u = 558.654 \text{ meV}\,, \end{aligned} \tag{6.5}$$

to which correspond

$$\begin{aligned} &\sin^2\theta_{12} = 0.3043\,, \quad \sin^2\theta_{13} = 0.02244\,, \quad \sin^2\theta_{23} = 0.4509\,, \quad \delta_{CP} = 208.4^\circ\,, \\ &m_e/m_\mu = 0.004737\,, \quad m_\mu/m_\tau = 0.05857\,, \quad \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.02956\,, \\ &m_1 = 39.43 \text{ meV}\,, \quad m_2 = 40.36 \text{ meV}\,, \quad m_3 = 63.75 \text{ meV}\,, \end{aligned} \tag{6.6}$$

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with $\chi^2_{\rm min} = 0.75$. Notice that, in this case, all best-fit values lie in the 1σ experimental ranges. Using the numerical package `MultiNest` [83, 84], we scan the parameter space of the model, and require all observables to be in the 3σ regions allowed by data. We find that the three mixing angles can nearly take any values within their experimental 3σ regions, while the Dirac CP phase δ_{CP} is sharply predicted to be in the narrow range $\delta_{CP} \in [201^\circ, 215^\circ]$.

For inverted neutrino masses, the best-fit values of the input parameters are

$$\begin{aligned}
&\langle\tau\rangle = 0.21972 + 1.08073i\,, \quad y_2^e/y_1^e = 9.51186\,, \quad y_3^e/y_1^e = 27.9507\,, \quad y_4^e/y_1^e = 0.018387\,,\\
&y_2^d/y_1^d = 5.23555\,, \quad y_1^e v_d = 1.65508 \text{ GeV}\,, \quad y_1^d v_u = 533.206 \text{ meV}\,.
\end{aligned} \tag{6.7}$$

and the corresponding values of lepton mass and mixing parameters:

$$\begin{aligned}
&\sin^2\theta_{12} = 0.3048\,, \quad \sin^2\theta_{13} = 0.02234\,, \quad \sin^2\theta_{23} = 0.5727\,, \quad \delta_{CP} = 315.0^\circ\,,\\
&m_e/m_\mu = 0.004737\,, \quad m_\mu/m_\tau = 0.05857\,, \quad \frac{\Delta m_{21}^2}{\Delta m_{31}^2} = 0.02980\,,\\
&m_1 = 60.72 \text{ meV}\,, \quad m_2 = 61.32 \text{ meV}\,, \quad m_3 = 35.65 \text{ meV}\,,
\end{aligned} \tag{6.8}$$

with $\chi^2_{\rm min} = 2.87$. Similarly to the NO case, the best fit values of all mass and mixing observables are within the 1σ experimental ranges, while δ_{CP} is predicted to be in the 2σ interval. A scanning of the parameter space shows that the allowed range of δ_{CP} is $[307^\circ, 345^\circ]$ and that the 3σ regions of all remaining parameters can be achieved.

6.2 Model for Majorana neutrino masses: Weinberg operator

As a benchmark TZLM for the case of Majorana neutrino masses generated via the Weinberg operator, we take the T' representation and modular-weight assignments:

$$\begin{aligned}
&L_D \sim \mathbf{2}'\,, \quad L_3 \sim \mathbf{1}' \quad e^c \sim \mathbf{1}'\,, \quad \mu^c \sim \mathbf{1}'\,, \quad \tau^c \sim \mathbf{1}''\,,\\
&k_{L_D} = 1\,, \quad k_{L_3} = 0\,, \quad k_{e^c} = 2\,, \quad k_{\mu^c} = 0\,, \quad k_{\tau^c} = 0\,.
\end{aligned} \tag{6.9}$$

for which the superpotentials $\mathcal{W}_{E,\nu}$ are

$$\begin{aligned}
\mathcal{W}_E &= y_1^e e^c \left[L_D Y_{\mathbf{2}}^{(1)}\right]_{\mathbf{1}''} H_d + y_2^e \mu^c \left[L_D Y_{\mathbf{2}}^{(3)}\right]_{\mathbf{1}''} H_d + y_3^e \tau^c L_3 H_d\,,\\
\mathcal{W}_\nu &= \frac{y_1^\nu}{\Lambda}\left[(L_D L_D)_{\mathbf{3}_S} Y_{\mathbf{3}}^{(2)}\right]_{\mathbf{1}} H_u H_u + \frac{y_2^\nu}{\Lambda} L_3 \left[L_D Y_{\mathbf{2}}^{(1)}\right]_{\mathbf{1}''} H_u H_u\,,
\end{aligned} \tag{6.10}$$

where all coupling constant $y^e_{1,2,3}$ and $y^\nu_{1,2}$ can be made real by using the freedom of lepton field redefinition. In total, there are only 7 real free parameters in this model including the real and imaginary parts of τ. The charged-lepton and (Majorana) neutrino mass matrices are

$$M_E = \begin{pmatrix} -y_1^e Y_{\mathbf{2},2}^{(1)} & y_1^e Y_{\mathbf{2},1}^{(1)} & 0 \\ -y_2^e Y_{\mathbf{2},2}^{(3)} & y_2^e Y_{\mathbf{2},1}^{(3)} & 0 \\ 0 & 0 & y_3^e \end{pmatrix} v_d\,, \quad M_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} -y_1^\nu Y_{\mathbf{3},3}^{(2)} & \frac{y_1^\nu Y_{\mathbf{3},1}^{(2)}}{\sqrt{2}} & -y_2^\nu Y_{\mathbf{2},2}^{(1)} \\ \frac{y_1^\nu Y_{\mathbf{3},1}^{(2)}}{\sqrt{2}} & y_1^\nu Y_{\mathbf{3},2}^{(2)} & y_2^\nu Y_{\mathbf{2},1}^{(1)} \\ -y_2^\nu Y_{\mathbf{2},2}^{(1)} & y_2^\nu Y_{\mathbf{2},1}^{(1)} & 0 \end{pmatrix}\,, \tag{6.11}$$

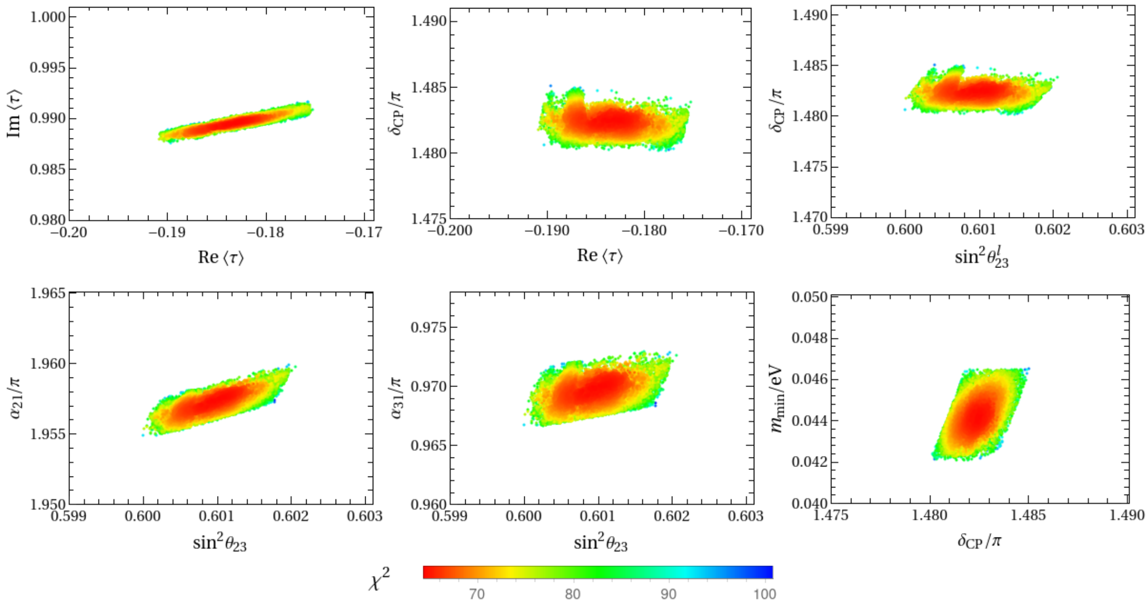

Figure 1. Results of the parameter-space scanning for the benchmark model of neutrino masses generated by the Weinberg operator. The T' representation and modular-weight assignments are given in (6.10). We present the scatter plots in distinct bidimensional planes of the neutrino mixing angles, CP phases and the real/imaginary part of $\langle\tau\rangle$. The colour grading reflects the values of the χ^2 for each point, according to the scale shown at the bottom of the figure.

which fit in the texture-zero pattern $\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$. The best-fit values of the input parameters are, for this case,

$$\begin{aligned} &\langle\tau\rangle = -0.18355 + 0.98944\,i\,, \quad y_2^e/y_1^e = 0.026019\,, \qquad y_3^e/y_1^e = 6.47820\,, \\ &y_2^\nu/y_1^\nu = 0.41163\,, \qquad y_1^e v_d = 201.034 \text{ MeV}\,, \quad \frac{y^\nu v_u^2}{\Lambda} = 348.515 \text{ meV}\,. \end{aligned} \tag{6.12}$$

for which the corresponding values of the lepton observables are

$$\begin{aligned} &\sin^2\theta_{12}=0.3037, \quad \sin^2\theta_{13}=0.02254, \quad \sin^2\theta_{23}=0.6010, \quad \delta_{CP}=266.8^\circ, \\ &\alpha_{21}=352.3^\circ, \quad \alpha_{31}=174.6^\circ, \quad m_e/m_\mu=0.004737, \quad m_\mu/m_\tau=0.05857, \quad \frac{\Delta m_{21}^2}{\Delta m_{31}^2}=0.02957, \\ &m_1=44.19 \text{ meV}, \quad m_2=45.02 \text{ meV}, \quad m_3=66.80 \text{ meV}, \quad m_{\beta\beta}=44.86 \text{ meV}, \end{aligned} \tag{6.13}$$

with $\chi^2_{\min} = 64.21$. Almost all observables lie in the 1σ experimental intervals, except $\sin^2\theta_{23}$ which is close to the 3σ upper limit. The lightest neutrino mass is predicted to be 44.19 meV and $\sum_i m_i$ is about 156 meV. Cosmological data shows that the most stringent bound on the sum of neutrino masses is $\sum_i m_i < 120$ meV at 95% C.L. from the Planck Collaboration results [85]. Thus, the present benchmark would be excluded by this data since the prediction $\sum_i m_i \simeq 156$ meV exceeds that bound. However, as is well known, cosmological bounds on $\sum_i m_i$ significantly depend on the data sets that need to be combined in order to break the degeneracies of the many cosmological parameters [85]. In fact, combining the baryon acoustic oscillation (BAO) data with the cosmic microwave

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background (CMB) lensing reconstruction power spectrum, one has $\sum_i m_i < 600$ meV and, taking this result, the model would be still viable.

The latest bound on the effective Majorana neutrino mass $m_{\beta\beta}$ has been set by the KamLAND-Zen experiment, namely $m_{\beta\beta} < 36 - 156$ meV [86], for which the largest uncertainty arises from the computation of nuclear matrix elements. The model prediction $m_{\beta\beta} = 44.86$ meV respects this bound. With future large-scale $0\nu\beta\beta$-decay experiments aiming at improving the bound on $m_{\beta\beta}$, the present benchmark would be further scrutinised. For instance, the SNO+ Phase II is expected to reach a sensitivity of $19-46$ meV [87], which is nearly the same as the one foreseen by the LEGEND experiment (15-50 meV) by operating 1000 kg of detectors for 10 years [88]. nEXO, the successor of EXO-200, will be able to probe $m_{\beta\beta}$ down to $5.7 - 17.7$ meV after 10 years of data taking [89].

In figure 1, we show the correlations among the input free parameters, neutrino masses and mixing parameters predicted in this model. We can see that the complex modulus τ scatter in a very narrow region. The predictions for the atmospheric mixing angle θ_{23} and the three CP violation phases are very sharp. In particular, the allowed range of δ_{CP} is very close to $3\pi/2$. In conclusion, this model is very predictive, since it is able to describe the 12 masses and mixing parameters for the NO case with only 7 input real parameters. The IO neutrino mass spectrum cannot be accommodated in this case.

6.3 Model for Majorana neutrino masses: seesaw mechanism

In the last benchmark TZLM with neutrino masses are generated via the seesaw mechanism with imposed gCP. The transformation properties and modular weights of the lepton fields are,

$$
\begin{aligned}
&L_D \sim \mathbf{2}\,, \quad L_3 \sim \mathbf{1} \quad e^c \sim \mathbf{1}''\,, \quad \mu^c \sim \mathbf{1}\,, \quad \tau^c \sim \mathbf{1}\,, \quad N^c \equiv \{N_1^c, N_2^c\} \sim \mathbf{2}\,,\\
&k_{L_D} = 1\,, \quad k_{L_3} = -2\,, \quad k_{e^c} = 2\,, \quad k_{\mu^c} = 2\,, \quad k_{\tau^c} = 4\,, \quad k_{N^c} = 3\,,
\end{aligned}
\tag{6.14}
$$

from which the modular-invariant superpotentials

$$
\begin{aligned}
\mathcal{W}_E &= y_1^e e^c \left[L_D Y_{\mathbf{2}}^{(3)}\right]_{\mathbf{1}'} H_d + y_2^e \mu^c \left[L_D Y_{\mathbf{2}''}^{(3)}\right]_{\mathbf{1}} H_d + y_3^e \tau^c \left[L_D Y_{\mathbf{2}''}^{(5)}\right]_{\mathbf{1}} H_d + y_4^e \mu^c L_3 H_d\,,\\
\mathcal{W}_\nu &= y_1^D \left[(L_D N^c)_{\mathbf{3}} Y_{\mathbf{3}}^{(4)}\right]_{\mathbf{1}} H_u + y_1^N \Lambda \left[(N^c N^c)_{\mathbf{3}} Y_{\mathbf{3}I}^{(6)}\right]_{\mathbf{1}} + y_2^N \Lambda \left[(N^c N^c)_{\mathbf{3}} Y_{\mathbf{3}II}^{(6)}\right]_{\mathbf{1}}\,.
\end{aligned}
\tag{6.15}
$$

can be defined, being all coupling constants real due to gCP. The charged-lepton and neutrino mass matrices take the following form,

$$
\begin{aligned}
M_E &= \begin{pmatrix} -y_1^e Y_{\mathbf{2},2}^{(3)} & y_1^e Y_{\mathbf{2},1}^{(3)} & 0 \\ y_2^e Y_{\mathbf{2}'',2}^{(3)} & -\, y_2^e Y_{\mathbf{2}'',1}^{(3)} & y_4^e \\ y_3^e Y_{\mathbf{2}'',2}^{(5)} & -\, y_3^e Y_{\mathbf{2}'',1}^{(5)} & 0 \end{pmatrix} v_d\,, \quad M_D = y_1^D v_u \begin{pmatrix} -Y_{\mathbf{3},2}^{(4)} & \dfrac{Y_{\mathbf{3},3}^{(4)}}{\sqrt{2}} & 0 \\ \dfrac{Y_{\mathbf{3},3}^{(4)}}{\sqrt{2}} & Y_{\mathbf{3},1}^{(4)} & 0 \end{pmatrix}\,,\\
M_N &= \Lambda \begin{pmatrix} -y_2^N Y_{\mathbf{3}II,2}^{(6)} - y_1^N Y_{\mathbf{3}I,2}^{(6)} & \dfrac{y_2^N Y_{\mathbf{3}II,3}^{(6)} + y_1^N Y_{\mathbf{3}I,3}^{(6)}}{\sqrt{2}} \\ \dfrac{y_2^N Y_{\mathbf{3}II,3}^{(6)} + y_1^N Y_{\mathbf{3}I,3}^{(6)}}{\sqrt{2}} & y_2^N Y_{\mathbf{3}II,1}^{(6)} + y_1^N Y_{\mathbf{3}I,1}^{(6)} \end{pmatrix}\,.
\end{aligned}
\tag{6.16}
$$

According to classification given in eqs. (4.22) and (4.23), the above matrices correspond to $\mathfrak{D}_2^3 - \mathfrak{N}_0^1$, while M_ν follows the $\mathcal{W}_3^{(2)}$ pattern, as shown in table 5. Moreover, the texture of M_E is of type $\mathcal{C}_2^{(3)}$. There is a total of 8 effective real parameters in the model. For neutrino masses with NO, the best-fit values of the input parameters are

$$\begin{aligned}&\langle\tau\rangle=0.41547+1.10335i, \quad y_2^e/y_1^e=6.13351\times10^2, \quad y_3^e/y_1^e=3.39303\times10^2, \quad y_4^e/y_1^e=16.5963,\\ &y_2^N/y_1^N=-0.29007, \qquad y_1^e v_d=35.6572\text{ MeV}, \qquad \frac{(y_1^D v_u)^2}{y_1^N\Lambda}=303.537\text{ meV},\end{aligned} \tag{6.17}$$

being the corresponding lepton observables

$$\begin{aligned}&\sin^2\theta_{12}=0.3046, \quad \sin^2\theta_{13}=0.02242, \quad \sin^2\theta_{23}=0.4524, \quad \delta_{CP}=209.4^\circ,\\ &\phi=112.3^\circ, \quad m_e/m_\mu=0.004737, \quad m_\mu/m_\tau=0.05857, \quad \frac{\Delta m_{21}^2}{\Delta m_{31}^2}=0.02957,\\ &m_1=0\text{ meV}, \quad m_2=8.614\text{ meV}, \quad m_3=58.68\text{ meV}, \quad m_{\beta\beta}=1.466\text{ meV},\end{aligned} \tag{6.18}$$

with $\chi^2_{\min} = 0.70$ (remember that for the minimal seesaw considered in this work the lightest neutrino is massless and there is a single Majorana phase ϕ). The above best-fit values are in very good agreement with the data. Correlations between the values of $\sin^2\theta_{23}$ and of the CP-violation phases δ_{CP} and ϕ are shown in the upper panels of figure 2 for NO.

If the neutrino mass spectrum is inverted, the best agreement between model predictions and experimental data is achieved with the following values of the input parameters

$$\begin{aligned}&\langle\tau\rangle=0.000786+1.267779i, \quad y_2^e/y_1^e=9.451055, \quad y_3^e/y_1^e=0.0532938, \quad y_4^e/y_1^e=0.209372,\\ &y_2^N/y_1^N=0.884872, \qquad y_1^e v_d=2.333797\text{ GeV}, \qquad \frac{(y_1^D v_u)^2}{y_1^N\Lambda}=594.172\text{ meV}.\end{aligned} \tag{6.19}$$

At this best-fit point we have:

$$\begin{aligned}&\sin^2\theta_{12}=0.2999, \quad \sin^2\theta_{13}=0.02239, \quad \sin^2\theta_{23}=0.6064, \quad \delta_{CP}=315.6^\circ,\\ &\phi=184.2^\circ, \quad m_e/m_\mu=0.004737, \quad m_\mu/m_\tau=0.05857, \quad \frac{\Delta m_{21}^2}{\Delta m_{31}^2}=0.03068,\\ &m_1=48.418\text{ meV}, \quad m_2=49.179\text{ meV}, \quad m_3=0\text{ meV}, \quad m_{\beta\beta}=18.791\text{ meV},\end{aligned} \tag{6.20}$$

with $\chi^2_{\min} = 9.30$, being all observables in the experimentally allowed 3σ intervals. In the lower panels of figure 2, we show the regions for the real and imaginary parts of τ and the correlation among θ_{23} and two CP-violation phases δ_{CP} and ϕ.

7 Conclusions

The nature of the fermion flavour pattern in the SM is a great puzzle. With the purpose of reducing the number of free parameters in fermion mass matrices, texture-zero patterns have been widely studied in the literature. In the present work, we have performed a systematic analysis of how texture zeros in lepton mass matrices can be realised in the framework of a T' modular symmetry. We show that, by properly assigning representations and modular weights to the matter fields, texture-zero patterns can be naturally

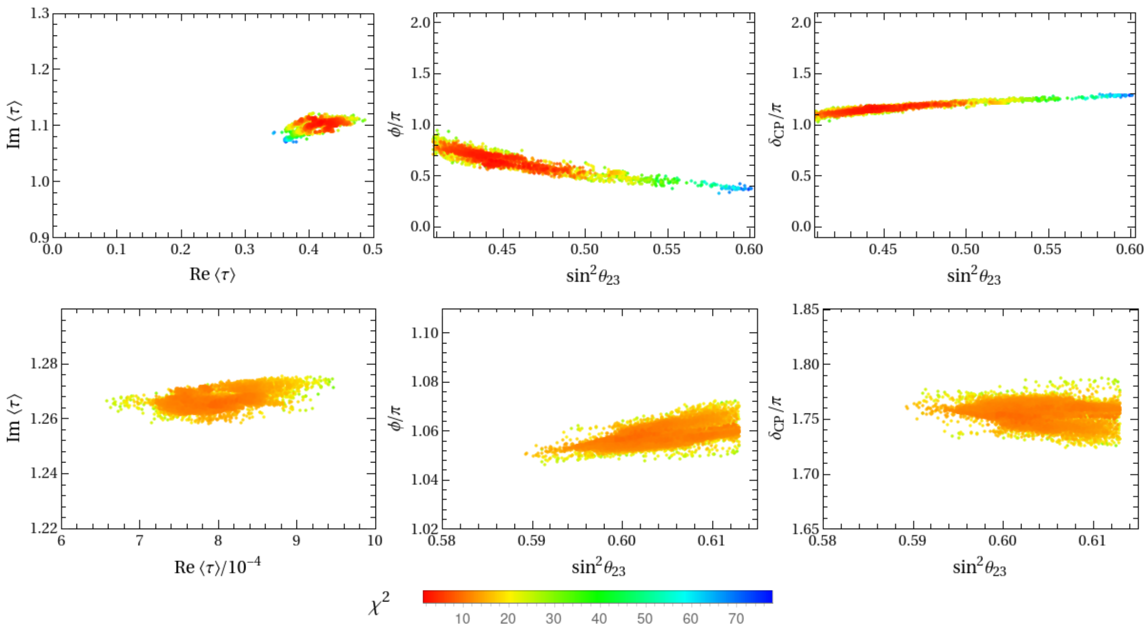

Figure 2. Values of the complex modulus τ compatible with experimental data, and correlations between $\sin^2\theta_{23}$ and the CP-violating phases δ_{CP} and ϕ. The upper (lower) panels correspond to the case in which the neutrino mass specturm is NO (IO).

reproduced, each assignment leading to a specific TZLM. In particular, we have considered all cases in which lepton fields are assumed to transform as singlets ($\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$), doublets ($\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$) or triplet $\mathbf{3}$ of T'. Both cases of Dirac and Majorana neutrino masses (generated by either the Weinberg operator or via the type-I seesaw mechanism) were investigated. The most general form of the lepton mass matrices for all possible representation assignments were found, namely there are 10 (23) [8] texture-zero patterns for the charged-lepton (Dirac neutrino) [effective Majorana neutrino] mass matrix.

Combining the texture-zero patterns for the charged-lepton and neutrino mass matrices, we have obtained 136 allowed (M_E, M_D) pairs which can be realised from T' modular symmetry for Dirac neutrinos. In case of Majorana neutrinos, we have found 29 and 35 pairs of texture zeros in (M_E, M_ν) when neutrinos masses are generated by the Weinberg operator and the type-I seesaw mechanism, respectively. If gCP is introduced, more combinations of texture-zero patterns can be achieved, as shown in table 12. In order to test whether the obtained texture pairs $(M_E, M_D)/(M_E, M_\nu)$ can accommodate experimental data, we have performed a χ^2 analysis for the corresponding TZLMs. It turns out that only part of those models are compatible with the data, as can be seen in tables 8, 9, 10 and 11. For each viable texture pair, we have provided representative models, for which the representation and modular-weight assignments are shown in tables 14, 16, 18, 20, 22 and 24. The corresponding predictions for the lepton observables are summarised in tables 15, 17, 19, 21, 23 and 25, respectively. We found that the minimal model requires 7 real parameters to explain all 9 measured observables. To illustrate our findings, we studied in more detail three benchmark TZLMs for Dirac and Majorana neutrinos in section 6.

In conclusion, we have shown that several texture-zero patterns for lepton mass matrices previously considered in the literature can be realised in the context of modular flavour symmetries. A considerable fraction of those textures are able to accommodate the experimental data, being some of them predictive in the sense that the corresponding TZLMs contain less free parameters than observables. In comparison with typical analyses of texture zeros in the context of Abelian flavour symmetries, the present approach is much more predicitive due to T' modular symmetry which implies correlations among the non-vanishing elements of the mass matrices. Since said correlations depend on the choice of the modular group, it would be surely interesting to explore other possibilities besides T'.

Acknowledgments

G.J.D. and J.N.L. acknowledge the National Natural Science Foundation of China under Grant Nos. 11975224, 11835013 and the Key Research Program of the Chinese Academy of Sciences under grant NO. XDPB15. J.N.L. is supported by the Grants No. NSFC-12147110 and the China Post-doctoral Science Foundation under Grant No. 2021M703106. The work of F.R.J. is supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects CFTP-FCT Unit UIDB/00777/2020 and UIDP/00777/2020, and through the CERN/FCT project CERN/FIS-PAR/0004/2019, which are partially funded through POCTI (FEDER), COMPETE, QREN and EU.

A The T' modular group

The T' group is the double covering of the tetrahedral group A_4. All the elements of T' can be generated by three generators S, T and R which obey the following relations[3] [54]:

$$S^2 = R, \quad (ST)^3 = T^3 = R^2 = 1, \quad RT = TR\,. \tag{A.1}$$

The generator R commutes with all elements of the group, and the center of T' is the Z_2 subgroup generated by R. The 24 elements of T' group belong to 7 conjugacy classes:

$$\begin{aligned}
1C_1 &: \; 1\,,\\
1C_2 &: \; R\,,\\
6C_4 &: \; S,\, T^{-1}ST,\, TST^{-1},\, SR,\, T^{-1}STR,\, TST^{-1}R\,,\\
4C_6 &: \; TR,\, TSR,\, STR,\, T^{-1}ST^{-1}R\,,\\
4C_3 &: \; T^{-1},\, ST^{-1}R,\, T^{-1}SR,\, TSTR\,,\\
4C_3' &: \; T,\, TS,\, ST,\, T^{-1}ST^{-1}\,,\\
4C_6' &: \; ST^{-1},\, T^{-1}S,\, TST,\, T^{-1}R\,.
\end{aligned} \tag{A.2}$$

The T' group has a triplet representation **3** and three singlets representations **1**, **1**$'$ and **1**$''$ in common with A_4. In addition, it has three two-dimensional spinor representations **2**,

[3] Alternatively the T' group can be expressed by only S and T obeying $S^4 = T^3 = (ST)^3 = 1$, $S^2T = TS^2$.

$\mathbf{2}'$ and $\mathbf{2}''$. In our working basis, the generators S, and T are represented by the following symmetric and unitary matrices:

$$
\begin{aligned}
\mathbf{1}: &\quad S = 1, & T &= 1\,, \\
\mathbf{1}': &\quad S = 1, & T &= \omega\,, \\
\mathbf{1}'': &\quad S = 1, & T &= \omega^2\,, \\
\mathbf{2}: &\quad S = -\frac{i}{\sqrt{3}}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}, & T &= \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}, \\
\mathbf{2}': &\quad S = -\frac{i}{\sqrt{3}}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}, & T &= \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \\
\mathbf{2}'': &\quad S = -\frac{i}{\sqrt{3}}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}, & T &= \begin{pmatrix} 1 & 0 \\ 0 & \omega^2 \end{pmatrix}, \\
\mathbf{3}: &\quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, & T &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},
\end{aligned}
\tag{A.3}
$$

with $\omega = e^{i2\pi/3}$. Notice that the two-dimensional representation matrices are related to those of refs. [54, 55] by a similarity transformation, while the remaining ones are the same. The Kronecker products between different irreducible representations of T' are given by

$$
\begin{aligned}
\mathbf{1}^a \otimes \mathbf{r}^b &= \mathbf{r}^b \otimes \mathbf{1}^a = \mathbf{r}^{a+b \ (\mathrm{mod}\ 3)}, \qquad \text{for} \ \ \mathbf{r} = \mathbf{1}, \mathbf{2}\,, \\
\mathbf{1}^a \otimes \mathbf{3} &= \mathbf{3} \otimes \mathbf{1}^a = \mathbf{3}\,, \\
\mathbf{2}^a \otimes \mathbf{2}^b &= \mathbf{3} \oplus \mathbf{1}^{a+b+1 \ (\mathrm{mod}\ 3)}\,, \\
\mathbf{2}^a \otimes \mathbf{3} &= \mathbf{3} \otimes \mathbf{2}^a = \mathbf{2} \oplus \mathbf{2}' \oplus \mathbf{2}''\,, \\
\mathbf{3} \otimes \mathbf{3} &= \mathbf{3}_S \oplus \mathbf{3}_A \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''\,,
\end{aligned}
\tag{A.4}
$$

where $a, b = 0, 1, 2$ and we have denoted $\mathbf{1} \equiv \mathbf{1}^0$, $\mathbf{1}' \equiv \mathbf{1}^1$, $\mathbf{1}'' \equiv \mathbf{1}^2$ for singlet representations and $\mathbf{2} \equiv \mathbf{2}^0$, $\mathbf{2}' \equiv \mathbf{2}^1$, $\mathbf{2}'' \equiv \mathbf{2}^2$ for the doublet representations. The notations $\mathbf{3}_S$ and $\mathbf{3}_A$ stand for the symmetric and antisymmetric triplet combinations respectively. In the following, we report the Clebsch-Gordon (CG) coefficients of the T' group in the chosen basis. We shall use α_i (β_i) to denote the elements of the first (second) representation of the product.

$$
\mathbf{1}^a \otimes \mathbf{1}^b = \mathbf{1}^{a+b \ (\mathrm{mod}\ 3)} \sim \alpha\beta\,, \tag{A.5}
$$

$$
\mathbf{1}^a \otimes \mathbf{2}^b = \mathbf{2}^{a+b \ (\mathrm{mod}\ 3)} \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \end{pmatrix}, \tag{A.6}
$$

$$
\mathbf{1} \otimes \mathbf{3} = \mathbf{3} \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix}, \tag{A.7}
$$

$$\mathbf{1}' \otimes \mathbf{3} = \mathbf{3} \sim \begin{pmatrix} \alpha\beta_3 \\ \alpha\beta_1 \\ \alpha\beta_2 \end{pmatrix}, \tag{A.8}$$

$$\mathbf{1}'' \otimes \mathbf{3} = \mathbf{3} \sim \begin{pmatrix} \alpha\beta_2 \\ \alpha\beta_3 \\ \alpha\beta_1 \end{pmatrix}. \tag{A.9}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{2}' \otimes \mathbf{2}'' = \mathbf{1}' \oplus \mathbf{3} \text{ with } \begin{cases} \mathbf{1}' \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ \mathbf{3} \sim \begin{pmatrix} \alpha_2\beta_2 \\ \frac{1}{\sqrt{2}}(\alpha_1\beta_2 + \alpha_2\beta_1) \\ -\alpha_1\beta_1 \end{pmatrix} \end{cases} \tag{A.10}$$

$$\mathbf{2} \otimes \mathbf{2}' = \mathbf{2}'' \otimes \mathbf{2}'' = \mathbf{1}'' \oplus \mathbf{3} \text{ with } \begin{cases} \mathbf{1}'' \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ \mathbf{3} \sim \begin{pmatrix} -\alpha_1\beta_1 \\ \alpha_2\beta_2 \\ \frac{1}{\sqrt{2}}(\alpha_1\beta_2 + \alpha_2\beta_1) \end{pmatrix} \end{cases} \tag{A.11}$$

$$\mathbf{2} \otimes \mathbf{2}'' = \mathbf{2}' \otimes \mathbf{2}' = \mathbf{1} \oplus \mathbf{3} \text{ with } \begin{cases} \mathbf{1} \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ \mathbf{3} \sim \begin{pmatrix} \frac{1}{\sqrt{2}}(\alpha_1\beta_2 + \alpha_2\beta_1) \\ -\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix} \end{cases} \tag{A.12}$$

$$\mathbf{2} \otimes \mathbf{3} = \mathbf{2} \oplus \mathbf{2}' \oplus \mathbf{2}'' \text{ with } \begin{cases} \mathbf{2} \sim \begin{pmatrix} \alpha_1\beta_1 + \sqrt{2}\alpha_2\beta_2 \\ -\alpha_2\beta_1 + \sqrt{2}\alpha_1\beta_3 \end{pmatrix} \\ \mathbf{2}' \sim \begin{pmatrix} \alpha_1\beta_2 + \sqrt{2}\alpha_2\beta_3 \\ -\alpha_2\beta_2 + \sqrt{2}\alpha_1\beta_1 \end{pmatrix} \\ \mathbf{2}'' \sim \begin{pmatrix} \alpha_1\beta_3 + \sqrt{2}\alpha_2\beta_1 \\ -\alpha_2\beta_3 + \sqrt{2}\alpha_1\beta_2 \end{pmatrix} \end{cases} \tag{A.13}$$

$$\mathbf{2}' \otimes \mathbf{3} = \mathbf{2} \oplus \mathbf{2}' \oplus \mathbf{2}'' \text{ with } \begin{cases} \mathbf{2} \sim \begin{pmatrix} \alpha_1\beta_3 + \sqrt{2}\alpha_2\beta_1 \\ -\alpha_2\beta_3 + \sqrt{2}\alpha_1\beta_2 \end{pmatrix} \\ \mathbf{2}' \sim \begin{pmatrix} \alpha_1\beta_1 + \sqrt{2}\alpha_2\beta_2 \\ -\alpha_2\beta_1 + \sqrt{2}\alpha_1\beta_3 \end{pmatrix} \\ \mathbf{2}'' \sim \begin{pmatrix} \alpha_1\beta_2 + \sqrt{2}\alpha_2\beta_3 \\ -\alpha_2\beta_2 + \sqrt{2}\alpha_1\beta_1 \end{pmatrix} \end{cases} \tag{A.14}$$

$$\mathbf{2}''\otimes\mathbf{3}=\mathbf{2}\oplus\mathbf{2}'\oplus\mathbf{2}''\ \text{with}\ \begin{cases}\mathbf{2}\sim\begin{pmatrix}\alpha_1\beta_2+\sqrt{2}\alpha_2\beta_3\\-\alpha_2\beta_2+\sqrt{2}\alpha_1\beta_1\end{pmatrix}\\ \mathbf{2}'\sim\begin{pmatrix}\alpha_1\beta_3+\sqrt{2}\alpha_2\beta_1\\-\alpha_2\beta_3+\sqrt{2}\alpha_1\beta_2\end{pmatrix}\\ \mathbf{2}''\sim\begin{pmatrix}\alpha_1\beta_1+\sqrt{2}\alpha_2\beta_2\\-\alpha_2\beta_1+\sqrt{2}\alpha_1\beta_3\end{pmatrix}\end{cases}\tag{A.15}$$

$$\mathbf{3}\otimes\mathbf{3}=\mathbf{3}_S\oplus\mathbf{3}_A\oplus\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''\ \text{with}\ \begin{cases}\mathbf{3}_S\sim\begin{pmatrix}2\alpha_1\beta_1-\alpha_2\beta_3-\alpha_3\beta_2\\2\alpha_3\beta_3-\alpha_1\beta_2-\alpha_2\beta_1\\2\alpha_2\beta_2-\alpha_1\beta_3-\alpha_3\beta_1\end{pmatrix}\\ \mathbf{3}_A\sim\begin{pmatrix}\alpha_2\beta_3-\alpha_3\beta_2\\\alpha_1\beta_2-\alpha_2\beta_1\\\alpha_3\beta_1-\alpha_1\beta_3\end{pmatrix}\\ \mathbf{1}\sim\alpha_1\beta_1+\alpha_2\beta_3+\alpha_3\beta_2\\ \mathbf{1}'\sim\alpha_3\beta_3+\alpha_1\beta_2+\alpha_2\beta_1\\ \mathbf{1}''\sim\alpha_2\beta_2+\alpha_1\beta_3+\alpha_3\beta_1\end{cases}\tag{A.16}$$

Note that, in our basis, the representation matrices of S and T are unitary and symmetric in all irreducible representations, and all the CG coefficients of the contractions are real.

B Higher-weight modular forms of level $N = 3$

Higher-weight modular forms can be constructed from tensor product of lower-weight ones. In the following, we will use the weight-1 modular forms $Y^{(1)}_{\mathbf{2}}$ given in eq. (2.20) and the Clebsch-Gordan coefficients of T' presented in appendix A to construct weight 2, 3, 4, 5 and 6 modular forms of T' modular group.

The weight-2 modular forms can be generated from the tensor products of two $Y^{(1)}_{\mathbf{2}}$,

$$Y^{(2)}_{\mathbf{3}}=\left(Y^{(1)}_{\mathbf{2}}Y^{(1)}_{\mathbf{2}}\right)_{\mathbf{3}}=\left(Y_2^2,\quad\sqrt{2}Y_1Y_2,\quad -Y_1^2\right)^T,\tag{B.1}$$

where Y_1 and Y_2 are two components of weight 1 modular forms $Y^{(1)}_{\mathbf{2}}=(Y_1,Y_2)^T$. Then, we can use the weight-1 and weight-2 modular forms to construct the weight-3 modular forms,

$$\begin{aligned}Y^{(3)}_{\mathbf{2}}&=\left(Y^{(1)}_{\mathbf{2}}Y^{(2)}_{\mathbf{3}}\right)_{\mathbf{2}}=\left(3Y_1Y_2^2,\quad -\sqrt{2}Y_1^3-Y_2^3\right)^T,\\ Y^{(3)}_{\mathbf{2}''}&=\left(Y^{(1)}_{\mathbf{2}}Y^{(2)}_{\mathbf{3}}\right)_{\mathbf{2}''}=\left(-Y_1^3+\sqrt{2}Y_2^3,\quad 3Y_2Y_1^2\right)^T.\end{aligned}\tag{B.2}$$

At weight $k = 4$, we find five independent modular forms which can be arranged into two

Modular weight k	Modular form $Y_{\mathbf{r}}^{(k)}$
$k=1$	$Y_{\mathbf{2}}^{(1)}$
$k=2$	$Y_{\mathbf{3}}^{(2)}$
$k=3$	$Y_{\mathbf{2}}^{(3)}, Y_{\mathbf{2}''}^{(3)}$
$k=4$	$Y_{\mathbf{1}}^{(4)}, Y_{\mathbf{1}'}^{(4)}, Y_{\mathbf{3}}^{(4)}$
$k=5$	$Y_{\mathbf{2}}^{(5)}, Y_{\mathbf{2}'}^{(5)}, Y_{\mathbf{2}''}^{(5)}$
$k=6$	$Y_{\mathbf{1}}^{(6)}, Y_{\mathbf{3}I}^{(6)}, Y_{\mathbf{3}II}^{(6)}$

Table 13. Summary of modular forms of level 3 up to weight 6, the subscript $\mathbf{r}$ denotes the transformation property under T' modular symmetry. Here $Y_{\mathbf{3}I}^{(6)}$ and $Y_{\mathbf{3}II}^{(6)}$ stand for two linearly independent weight-6 modular forms transforming in the representation $\mathbf{3}$ of T'.

singlets $\mathbf{1}$ and $\mathbf{1}'$ and a triplet of T',

$$\begin{aligned}
Y_{\mathbf{3}}^{(4)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}}^{(3)}\right)_{\mathbf{3}} = \left(-\sqrt{2}Y_1^3Y_2 - Y_2^4,\ \ -Y_1^4+\sqrt{2}Y_1Y_2^3,\ \ -3Y_1^2Y_2^2\right)^T, \\
Y_{\mathbf{1}'}^{(4)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}}^{(3)}\right)_{\mathbf{1}'} = -\sqrt{2}Y_1^4 - 4Y_1Y_2^3\,, \\
Y_{\mathbf{1}}^{(4)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}''}^{(3)}\right)_{\mathbf{1}} = 4Y_1^3Y_2 - \sqrt{2}Y_2^4\,.
\end{aligned} \tag{B.3}$$

Similarly, the independent weight-5 modular forms can be constructed from the tensor products of weight-1 and weight-4 modular forms as follows,

$$\begin{aligned}
Y_{\mathbf{2}}^{(5)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{2}} = \left[-2\sqrt{2}Y_1^3Y_2+Y_2^4\right]\left(Y_1, \quad Y_2\right)^T, \\
Y_{\mathbf{2}'}^{(5)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{2}'} = \left[-Y_1^4-2\sqrt{2}Y_1Y_2^3\right]\left(Y_1, \quad Y_2\right)^T, \\
Y_{\mathbf{2}''}^{(5)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{2}''} = \left(5Y_1^3Y_2^2-\sqrt{2}Y_2^5,\ \ -\sqrt{2}Y_1^5+5Y_1^2Y_2^3\right)^T.
\end{aligned} \tag{B.4}$$

Finally, the linearly independent weight-6 modular forms of level 3 can be decomposed into one singlet $\mathbf{1}$ and two triplets $\mathbf{3}$ under T',

$$\begin{aligned}
Y_{\mathbf{3},I}^{(6)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}}^{(5)}\right)_{\mathbf{3}} = \left[-2\sqrt{2}Y_1^3Y_2+Y_2^4\right]\left(Y_2^2, \quad \sqrt{2}Y_1Y_2, \quad -Y_1^2\right)^T, \\
Y_{\mathbf{3},II}^{(6)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}'}^{(5)}\right)_{\mathbf{3}} = \left[-Y_1^4-2\sqrt{2}Y_1Y_2^3\right]\left(-Y_1^2, \quad Y_2^2, \quad \sqrt{2}Y_1Y_2\right)^T, \\
Y_{\mathbf{1}}^{(6)} &= \left(Y_{\mathbf{2}}^{(1)}Y_{\mathbf{2}''}^{(5)}\right)_{\mathbf{1}} = \sqrt{2}Y_2^6-\sqrt{2}Y_1^6+10Y_1^3Y_2^3\,.
\end{aligned} \tag{B.5}$$

We summarize the level 3 modular forms up to weight 6 in table 13.

C Representative models of lepton texture zeros

In this section, we provide representative models for the phenomenologically viable patterns of texture zeros in the charged-lepton and neutrino mass matrices. Each of these models is chosen from a set of viable models which give the same texture of lepton mass matrices, and it contains minimum number of free real input parameters. Moreover, we also present the corresponding predictions for lepton masses and mixing parameters in each case.

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C.1 Dirac neutrinos

The representative models of viable textures for the case of Dirac neutrinos are presented in table 8, for which the representation and modular-weight assignments can be found in table 14 for NO neutrino masses. The corresponding predictions for lepton observables are collected in table 15. For IO, the viable textures are given in table 9 and the representative models and the best fitting results can be found in table 16 and table 17 respectively.

Dirac without gCP (NO)								
Combinations	$\#\mathrm{P}_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	24	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,5-x$	$x,-1+x$	$x,-1+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x,3-x$	$2+x,3+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}'$	$2-x,5-x$	$x,-1+x$	$x,-1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	22	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,1-x$	$x,3+x$	$x,1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{3}$	$1-x,2-x$	$2+x,4+x,4+x$	x
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$2-x,1-x$	$-1+x,1+x,3+x$	$x,-1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{3}$	$1-x,2-x$	$2+x,2+x,4+x$	x
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x,3-x$	$1+x,1+x,1+x$	$x,1+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$-1+x,1+x,3+x$	$x,1+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$1-x,2-x$	$x,2+x,2+x$	$x,4+x,4+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$3-x,4-x$	$-2+x,x,x$	$x,x,2+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$3-x,4-x$	$-2+x,x,x$	$x,x,2+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	24	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,-1-x$	$2+x,-1+x$	$x,<x-4$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(5)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$1-x,4-x$	$1+x,-4+x$	x,x,x
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$-1+x,1+x,3+x$	$x,<x-3$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	22	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-1+x,-1+x,1+x$	$x,-1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	20	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,3-x$	$-1+x,-1+x,1+x$	$x,<x-4$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$1+x,1+x,3+x$	$x,<x-3$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$4-x,3-x$	$-3+x,-1+x,1+x$	$x,1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-3+x,-1+x,1+x$	$x,<x-4$
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,3-x$	$1+x,3+x,3+x$	$x,<x-3$
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(5)}$	16	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$-x,-1-x$	$1+x,1+x,3+x$	$x,3+x,5+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-4+x,1+x$	$x,<x-4$

continues on next page

Table 14 — continued from previous page

Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
Dirac with gCP (NO)								
Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	24	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,1-x$	$2+x,-1+x$	$x,-1+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(1)}$	22	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{3}$	$1-x,2-x$	$3+x,2+x$	x
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(2)}$	22	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$2-x,3-x$	$2+x,3+x$	$x,1+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(3)}$	22	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}'$	$3-x,4-x$	$-1+x,-2+x$	$x,2+x,2+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$-1-x,-x$	$3+x,4+x$	$x,4+x,6+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(4)}$	21	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}$	$2-x,1-x$	$4+x,-1+x$	$x,<x-2$
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(5)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,3-x$	$-1+x,1+x,3+x$	$x,3+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x,3-x$	$2+x,3+x$	$x,1+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,1-x$	$4+x,5+x$	$x,5+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(4)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$-x,1-x$	$4+x,5+x$	$x,3+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(5)}$	19	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$1-x,-x$	$5+x,x$	$x,1+x,4+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	17	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,-1-x$	$2+x,-1+x$	$x,<x-4$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$-x,3-x$	$1+x,3+x,3+x$	$x,3+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(3)}$	17	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$3+x,5+x,5+x$	$x,<x-1$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(4)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$3+x,5+x,5+x$	$x,5+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(5)}$	17	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$3-x,4-x$	$1+x,2+x$	$x,x,1+x$
$\mathcal{C}_1^{(1)}-\mathcal{D}_6^{(1)}$	15	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}$	$1-x,-x$	$2+x,4+x,4+x$	$x,2+x,3+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_1^{(1)}$	22	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$4-x,5-x$	$-4+x,-1+x$	$x,-3+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(1)}$	20	8	$\mathbf{3}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$1-x$	$x,5+x$	$x,5+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(2)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,5-x$	$-2+x,1+x$	$x,-5+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(3)}$	20	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$-2+x,3+x$	$x,3+x$

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Table 14 — continued from previous page

Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_2^{(1)}-\mathcal{D}_3^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,1-x$	$x,3+x$	$x,1+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_3^{(4)}$	19	8	$\mathbf{3}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$5-x$	$x-4,1+x$	$x,<x-5$
$\mathcal{C}_2^{(1)}-\mathcal{D}_4^{(4)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}''$	$-x,3-x$	$x,1+x$	$x,3+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_4^{(5)}$	17	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$-x,1-x$	$x,3+x$	$x,5+x,5+x$
$\mathcal{C}_2^{(1)}-\mathcal{D}_5^{(5)}$	15	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$-1-x,2-x$	$1+x,2+x$	$x,2+x,4+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_1^{(1)}$	22	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,5-x$	$x,-1+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{3}$	$1-x,2-x$	$-2+x,2+x,2+x$	x
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	20	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,5-x$	$x,-1+x$	$x,-1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(3)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$2-x,3-x$	$2+x,1+x$	$x,3+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$1-x,-x$	$1+x,x$	$x,x,4+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,1-x$	$4+x,-1+x$	$x,5+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(4)}$	19	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$2+x,1+x$	$x,<x-3$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(5)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}$	$2-x,3-x$	$1+x,1+x,3+x$	$x,3+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$1-x,-x$	$5+x,x$	$x,x,2+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$4+x,-1+x$	$x,-1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$-x,1-x$	$4+x,3+x$	$x,5+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(4)}$	16	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$-x,3-x$	$2+x,1+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(5)}$	17	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$1-x,-x$	$5+x,x$	$x,1+x,4+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(1)}$	15	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$3-x,4-x$	$x,x,2+x$	$x,1+x,2+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(2)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$-x,3-x$	$1+x,1+x,3+x$	$x,3+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(3)}$	15	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$4+x,-1+x$	$x,<x-1$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(4)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$3+x,3+x,5+x$	$x,5+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(5)}$	15	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$1-x,4-x$	$1+x,x$	x,x,x

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Table 14 — continued from previous page

Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_2^{(2)}-\mathcal{D}_6^{(1)}$	13	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$1-x,-x$	$x,4+x,4+x$	$x,1+x,2+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	22	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,1-x$	$x,3+x$	$x,1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(1)}$	20	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{3}$	$1-x,2-x$	$x,2+x,2+x$	x
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(2)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}''$	$2-x,3-x$	$1+x,3+x,3+x$	$x,-3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(3)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,3-x$	$1+x,3+x,3+x$	$x,1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$1-x,-x$	$1+x,4+x$	$x,2+x,4+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	19	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,1-x$	$2+x,3+x$	$x,<x-2$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$2-x,-3-x$	$4+x,3+x$	$x,1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x,3-x$	$-1+x,1+x,1+x$	$x,1+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$-x,1-x$	$1+x,3+x,5+x$	$x,5+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(4)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$-x,1-x$	$5+x,5+x,5+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(5)}$	17	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}''$	$-1-x,-x$	$4+x,6+x,6+x$	$x,2+x,4+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	15	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,3-x$	$x,1+x$	$x,<x-4$
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(4)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,-5-x$	$6+x,5+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(5)}$	15	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$-1-x,-x$	$6+x,6+x,6+x$	$x,4+x,4+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_1^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}''$	$2-x,3-x$	$-3+x,1+x,1+x$	$x,1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{3}$	$1-x,2-x$	$2+x,2+x,4+x$	x
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(2)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,3-x$	$-3+x,3+x,3+x$	$x,-3+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,3-x$	$1+x,1+x,3+x$	$x,1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,3-x$	$1+x,3+x,3+x$	$x,3+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	17	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$1+x,1+x,1+x$	$x,<x-3$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-3+x,-1+x,1+x$	$x,-1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x,3-x$	$1+x,1+x,1+x$	$x,1+x$

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Table 14 — continued from previous page

Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(2)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$-1+x,3+x,5+x$	$x,-1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(3)}$	14	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$-x,1-x$	$3+x,3+x,5+x$	$x,5+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(4)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$-2-x,1-x$	$3+x,3+x,5+x$	$x,5+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(5)}$	15	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}$	$-1-x,-x$	$4+x,4+x,4+x$	$x,4+x,6+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	13	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-3+x,-1+x,1+x$	$x,<x-4$
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(3)}$	13	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$-1+x,3+x,5+x$	$x,<x-1$
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(5)}$	13	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$1-x,4-x$	$x,x,2+x$	$x,x,2+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,3-x$	$-1+x,1+x,3+x$	$x,1+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$4-x,5-x$	$-3+x,-1+x,1+x$	$x,-1+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$1-x,2-x$	$x,2+x,2+x$	$x,4+x,4+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$3-x,4-x$	$-4+x,x,2+x$	$x,x,2+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(4)}$	15	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$3-x,4-x$	$-2+x,x,2+x$	$x,1+x,2+x$
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(5)}$	13	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$3-x,2-x$	$-2+x,-2+x,x$	$x,1+x,2+x$
$\mathcal{C}_4^{(2)}-\mathcal{D}_2^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,3-x$	$x,1+x$	$x,3+x$
$\mathcal{C}_4^{(2)}-\mathcal{D}_3^{(4)}$	15	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$-x,1-x$	$x,-1+x$	$x,3+x,5+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,1-x$	$-2+x,3+x$	$x,1+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_2^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,3-x$	$-4+x,1+x$	$x,1+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$1-x,-x$	$-1+x,4+x$	$x,2+x,4+x$
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	15	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,3-x$	$-4+x,1+x$	$x,<x-4$
$\mathcal{C}_1^{(1)}-\mathcal{D}_6^{(2)}$	14	9	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$-1-x,3-x,3-x$	$2+x,1+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(6)}$	16	9	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}'$	$-1-x,3-x,3-x$	$2+x,3+x$	$x,3+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(6)}$	14	9	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}'$	$1-x,1-x,3-x$	$2+x,-1+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_6^{(2)}$	12	9	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$-1-x,3-x,3-x$	$x,1+x$	$x,1+x$

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Combinations	$\#\mathrm{P}_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_3^{(3)} - \mathcal{D}_3^{(4)}$	17	9	$\mathbf{1} \oplus \mathbf{1''} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1'}$	$3-x, 5-x, 5-x$	$-2+x, 1+x$	$x, -3+x$
$\mathcal{C}_3^{(3)} - \mathcal{D}_4^{(1)}$	14	9	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1''}$	$\mathbf{2'} \oplus \mathbf{1}$	$3-x, 3-x, 5-x$	$x, 1+x$	$x, -3+x$
$\mathcal{C}_3^{(3)} - \mathcal{D}_4^{(6)}$	14	9	$\mathbf{1} \oplus \mathbf{1''} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2'} \oplus \mathbf{1''}$	$1-x, 3-x, 5-x$	$x, -1+x$	$x, -1+x$
$\mathcal{C}_3^{(3)} - \mathcal{D}_5^{(2)}$	12	9	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1''}$	$\mathbf{2'} \oplus \mathbf{1}$	$1-x, 1-x, 3-x$	$2+x, 3+x$	$x, 3+x$
$\mathcal{C}_3^{(3)} - \mathcal{D}_5^{(6)}$	12	9	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1''}$	$\mathbf{2} \oplus \mathbf{1}$	$1-x, 1-x, 3-x$	$2+x, 3+x$	$x, 3+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_3^{(3)}$	14	9	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1''}$	$\mathbf{2'} \oplus \mathbf{1''}$	$3-x, 3-x, 5-x$	$x, -3+x$	$x, 1+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_4^{(4)}$	12	9	$\mathbf{1'} \oplus \mathbf{1''} \oplus \mathbf{1''}$	$\mathbf{2} \oplus \mathbf{1''}$	$\mathbf{2} \oplus \mathbf{1'}$	$-1-x, -1-x, 1-x$	$4+x, 1+x$	$x, 5+x$

Table 14. Representative Dirac-neutrino models of the viable texture-zero patterns in (M_E, M_D) that can accommodate the experimental data at 3σ level for NO neutrino mass spectrum. Both cases with and without gCP are considered. The number of real free parameters "$\#P$" involved in each model is listed in the third column. In the second column, we provide the value of "$\#P_0$" which refers to the number of real free parameters of the lepton mass matrices in the general context of texture zero ansatz where the non-vanishing elements of mass matrices are independent from each other. The parameter x is a generic integer, the modular symmetry would enforce the neutrinos to be Dirac particles if x is chosen to be a negative integer with sufficiently large absolute value.

Dirac without gCP (NO)								
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.452	1.361	44.054	44.888	66.720	0.187
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(1)}$	0.326	0.02262	0.440	1.082	7.927	11.706	50.722	5.844
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	0.286	0.02247	0.442	1.813	0	8.614	50.100	9.709
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	0.304	0.02242	0.455	1.609	29.860	31.078	58.309	2.812
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(1)}$	0.303	0.02246	0.473	1.262	34.122	35.192	60.575	1.479
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	0.304	0.02245	0.451	1.042	35.717	36.741	61.528	2.879
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(1)}$	0.303	0.02246	0.473	1.262	34.122	35.192	60.575	1.479
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	0.304	0.02244	0.451	1.158	39.428	40.358	63.754	0.749
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.450	1.314	27.801	29.105	57.294	0.032
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.451	1.216	38.705	39.652	63.312	0.199
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(1)}$	0.304	0.02245	0.449	1.279	13.327	15.869	51.815	0.007
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	0.304	0.02242	0.454	1.605	29.613	30.840	58.183	2.735
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	0.316	0.02220	0.431	1.000	0	8.614	49.778	6.892
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(5)}$	0.286	0.02247	0.442	1.813	0	8.614	50.100	9.709
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	0.304	0.02246	0.450	1.269	0	8.614	50.105	0.004
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	0.316	0.02220	0.431	1.000	0	8.614	49.776	6.919
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	0.316	0.02220	0.431	1.000	0	8.614	49.776	6.919
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02246	0.450	1.269	0	8.614	50.105	0.005
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	0.316	0.02220	0.431	1	0	8.614	49.775	6.939
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	0.316	0.02220	0.431	1	0	8.614	49.775	6.939
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(3)}$	0.296	0.02171	0.450	1.277	0	8.614	49.292	3.314
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(5)}$	0.275	0.02309	0.550	1.047	0	8.614	49.948	37.499
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	0.275	0.02335	0.522	1.015	0	8.614	51.238	28.241
Dirac with gCP (NO)								
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.452	1.363	44.104	44.938	66.753	0.197
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(1)}$	0.306	0.02249	0.479	1.200	34.763	35.814	60.974	2.737
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02246	0.450	1.266	32.315	33.443	59.617	0.008
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.450	1.281	59.609	60.228	77.867	0.000
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(1)}$	0.304	0.02245	0.449	1.288	13.295	15.841	51.815	0.005
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(4)}$	0.297	0.02181	0.450	1.007	0	8.614	49.391	6.330
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(5)}$	0.304	0.02246	0.450	1.099	0	8.614	50.095	1.649
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(1)}$	0.326	0.02262	0.440	1.082	7.927	11.706	50.723	5.844
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(3)}$	0.304	0.02246	0.450	1.134	11.481	14.353	51.399	1.070

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Table 15 — continued from previous page

Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(4)}$	0.304	0.02246	0.450	1.317	0	8.614	50.100	0.041
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(5)}$	0.302	0.02245	0.453	1.248	0	8.614	50.116	0.112
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	0.316	0.02220	0.431	1.000	0	8.614	49.778	6.892
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(2)}$	0.304	0.02246	0.450	1.038	0	8.614	50.100	2.982
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(3)}$	0.304	0.02246	0.450	1.123	0	8.614	50.094	1.241
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(4)}$	0.304	0.02246	0.451	1.075	0	8.614	50.100	2.131
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(5)}$	0.304	0.02246	0.450	1.287	0	8.614	50.102	0.002
$\mathcal{C}_1^{(1)}-\mathcal{D}_6^{(1)}$	0.304	0.02246	0.450	1.071	0	8.614	50.100	2.222
$\mathcal{C}_2^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	3.998
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(1)}$	0.304	0.02245	0.452	1.296	21.870	23.505	54.657	0.016
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02244	0.451	1.168	38.825	39.769	63.383	0.635
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.450	1	0	8.614	50.100	4.000
$\mathcal{C}_2^{(1)}-\mathcal{D}_3^{(3)}$	0.304	0.02246	0.450	1	0	8.614	50.100	4
$\mathcal{C}_2^{(1)}-\mathcal{D}_3^{(4)}$	0.314	0.02219	0.412	1.000	0	8.614	49.058	12.931
$\mathcal{C}_2^{(1)}-\mathcal{D}_4^{(4)}$	0.305	0.02254	0.448	1.220	0	8.614	50.237	0.244
$\mathcal{C}_2^{(1)}-\mathcal{D}_4^{(5)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	3.996
$\mathcal{C}_2^{(1)}-\mathcal{D}_5^{(5)}$	0.303	0.02246	0.448	1.329	0	8.614	50.063	0.133
$\mathcal{C}_2^{(2)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.450	1.261	14.776	17.103	52.232	0.015
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(1)}$	0.304	0.02246	0.450	1.265	8.163	11.868	50.759	0.009
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	0.273	0.02248	0.464	1.000	0	8.614	50.100	11.456
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.450	1.298	34.446	35.507	60.787	0.012
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(1)}$	0.304	0.02246	0.450	1	28.082	29.374	57.434	4.000
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(3)}$	0.304	0.02246	0.450	1	0	8.614	50.100	4.000
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(4)}$	0.307	0.02263	0.450	1.007	0	8.614	50.318	4.025
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(5)}$	0.304	0.02246	0.450	1.099	0	8.614	50.095	1.649
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(1)}$	0.304	0.02246	0.450	1.213	36.515	37.518	61.995	0.216
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(2)}$	0.296	0.02171	0.450	1	52.849	53.547	72.269	7.307
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(3)}$	0.304	0.02246	0.450	1.017	14.705	17.043	52.214	3.535
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(4)}$	0.273	0.02248	0.464	1.000	0	8.614	50.100	11.456
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(5)}$	0.302	0.02245	0.453	1.246	0	8.614	50.104	0.113
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(1)}$	0.304	0.02246	0.451	1.120	0	8.614	50.091	1.299
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(2)}$	0.304	0.02246	0.450	1.038	0	8.614	50.100	2.982
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(3)}$	0.296	0.02171	0.450	1	0	8.614	49.293	7.307
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(4)}$	0.304	0.02246	0.451	1.075	0	8.614	50.100	2.131
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(5)}$	0.273	0.02248	0.464	1.000	0	8.614	50.100	11.456

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Table 15 — continued from previous page

Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							χ^2_{min}
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_2^{(2)}-\mathcal{D}_6^{(1)}$	0.304	0.02246	0.450	1.071	0	8.614	50.100	2.223
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	0.304	0.02242	0.455	1.610	29.830	31.049	58.293	2.821
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(1)}$	0.305	0.02249	0.465	1.211	34.585	35.641	60.912	0.847
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(2)}$	0.304	0.02245	0.450	1.178	23.347	24.885	55.263	0.519
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.450	1.265	4.720	9.823	50.322	0.009
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(1)}$	0.304	0.02246	0.450	1.150	17.798	19.773	53.170	0.850
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	0.302	0.02229	0.450	1.007	0	8.614	49.908	3.985
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	0.304	0.02246	0.450	1.002	0	8.614	50.100	3.945
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	0.304	0.02244	0.451	1.158	39.422	40.352	63.750	0.748
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(3)}$	0.304	0.02246	0.450	1.134	11.477	14.350	51.398	1.072
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(4)}$	0.304	0.02247	0.450	1.282	0	8.614	50.105	0.001
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(5)}$	0.304	0.02246	0.451	1.237	0	8.614	50.099	0.091
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	0.313	0.02227	0.436	1	0	8.614	49.864	5.564
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(4)}$	0.304	0.02246	0.450	1.002	0	8.614	50.100	3.950
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(5)}$	0.304	0.02246	0.450	1.285	0	8.614	50.102	0.001
$\mathcal{C}_3^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.450	1.259	25.052	26.492	56.014	0.018
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(1)}$	0.303	0.02245	0.474	1.187	34.079	35.151	60.549	2.000
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02246	0.450	1.251	10.322	13.444	51.158	0.038
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.450	1.265	4.720	9.822	50.322	0.009
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(3)}$	0.304	0.02246	0.450	1	0	8.614	50.100	4.000
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	0.307	0.02264	0.450	1.007	0	8.614	50.342	4.049
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	0.316	0.02220	0.431	1	0	8.614	49.775	6.939
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	0.304	0.02244	0.451	1.158	39.428	40.358	63.754	0.749
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(2)}$	0.296	0.02171	0.450	1	27.978	29.274	56.679	7.307
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(3)}$	0.304	0.02246	0.450	1.134	11.478	14.350	51.398	1.072
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(4)}$	0.304	0.02245	0.450	1.297	0	8.614	50.107	0.010
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(5)}$	0.304	0.02246	0.450	1.252	0	8.614	50.100	0.035
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	0.316	0.02220	0.431	1	0	8.614	49.775	6.939
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(3)}$	0.296	0.02171	0.450	1	0	8.614	49.293	7.307
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(5)}$	0.304	0.02247	0.450	1.277	0	8.614	50.116	0.002
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02246	0.450	1.314	27.801	29.105	57.294	0.032
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02246	0.450	1.263	17.398	19.414	53.026	0.012
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.451	1.216	38.707	39.654	63.313	0.199
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(1)}$	0.304	0.02245	0.449	1.280	13.325	15.867	51.815	0.006
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02246	0.451	1.072	0	8.614	50.101	2.202

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Table 15 — continued from previous page

Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(5)}$	0.275	0.02309	0.550	1.047	0	8.614	49.948	37.499
$\mathcal{C}_4^{(2)}-\mathcal{D}_2^{(1)}$	0.304	0.02246	0.450	1	2.597	8.997	50.167	4
$\mathcal{C}_4^{(2)}-\mathcal{D}_3^{(4)}$	0.304	0.02246	0.450	1.014	0	8.614	50.100	3.607
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	0.304	0.02242	0.454	1.606	29.557	30.786	58.155	2.753
$\mathcal{C}_4^{(3)}-\mathcal{D}_2^{(3)}$	0.304	0.02246	0.456	1.282	77.612	78.088	92.378	0.097
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(1)}$	0.304	0.02246	0.450	1.147	17.975	19.932	53.229	0.890
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	0.284	0.02313	0.502	1.941	0	8.614	50.796	23.316
$\mathcal{C}_1^{(1)}-\mathcal{D}_6^{(2)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	4.000
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(6)}$	0.304	0.02246	0.450	1.001	0	8.614	50.100	3.974
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(6)}$	0.304	0.02247	0.450	1.301	0	8.614	50.095	0.016
$\mathcal{C}_2^{(3)}-\mathcal{D}_6^{(2)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	3.998
$\mathcal{C}_3^{(3)}-\mathcal{D}_3^{(4)}$	0.304	0.02245	0.450	1.040	0	8.614	50.092	2.926
$\mathcal{C}_3^{(3)}-\mathcal{D}_4^{(1)}$	0.304	0.02246	0.450	1.001	1.077	8.681	50.111	3.979
$\mathcal{C}_3^{(3)}-\mathcal{D}_4^{(6)}$	0.304	0.02246	0.450	1.001	0	8.614	50.100	3.984
$\mathcal{C}_3^{(3)}-\mathcal{D}_5^{(2)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	3.986
$\mathcal{C}_3^{(3)}-\mathcal{D}_5^{(6)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	3.989
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(3)}$	0.304	0.02246	0.450	1.001	1.143	8.689	50.113	3.978
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(4)}$	0.304	0.02246	0.450	1.000	0	8.614	50.100	4.000

Table 15. Best-fit values of the lepton mixing and neutrino mass parameters for the representative models of table 14. In all representative models, the predictions of the lepton mass ratios are $m_e/m_\mu = 0.00474$, $m_\mu/m_\tau = 0.0586$ and $\Delta m^2_{21}/\Delta m^2_{31} = 0.030$.

Dirac without gCP (IO)								
Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)} - \mathcal{D}_1^{(1)}$	24	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$2-x, 5-x$	$x, 1+x$	$x, -1+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_1^{(1)}$	22	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}'$	$\mathbf{2} \oplus \mathbf{1}''$	$2-x, 5-x$	$x, 1+x$	$x, 1+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_2^{(2)}$	20	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}'$	$\mathbf{2}' \oplus \mathbf{1}'$	$2-x, 5-x$	$x, -1+x$	$x, -1+x$
$\mathcal{C}_2^{(3)} - \mathcal{D}_1^{(1)}$	22	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}''$	$2-x, 1-x$	$x, 3+x$	$x, -1+x$
$\mathcal{C}_2^{(3)} - \mathcal{D}_4^{(1)}$	16	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}'$	$2-x, 3-x$	$-1+x, 1+x, 1+x$	$x, -3+x$
$\mathcal{C}_3^{(1)} - \mathcal{D}_4^{(1)}$	14	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}'$	$\mathbf{2} \oplus \mathbf{1}'$	$2-x, 3-x$	$1+x, 1+x, 1+x$	$x, 1+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_1^{(1)}$	18	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$2-x, 5-x$	$-1+x, -1+x, 1+x$	$x, -1+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_2^{(3)}$	16	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1}'' \oplus \mathbf{1}$	$1-x, 2-x$	$2+x, 2+x, 4+x$	$x, 4+x, 4+x$
$\mathcal{C}_4^{(3)} - \mathcal{D}_1^{(1)}$	18	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}'$	$2-x, 1-x$	$-2+x, 3+x$	$x, 3+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_3^{(4)}$	21	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}$	$4-x, 5-x$	$-2+x, -3+x$	$x, < x-5$
$\mathcal{C}_1^{(1)} - \mathcal{D}_3^{(5)}$	20	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$2-x, 1-x$	$2+x, 1+x$	$x, 1+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_4^{(5)}$	19	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}'$	$3-x, 4-x$	$-1+x, x$	$x, 1+x, 2+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_5^{(1)}$	17	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$1-x, -x$	$3+x, 4+x$	$x, x, 2+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_3^{(4)}$	19	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}$	$4-x, 5-x$	$x, -1+x$	$x, < x-5$
$\mathcal{C}_2^{(2)} - \mathcal{D}_3^{(5)}$	18	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}''$	$2-x, 1-x$	$2+x, -1+x$	$x, 1+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_4^{(5)}$	17	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$1-x, 2-x$	$1+x, 2+x$	$x, 2+x, 3+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_5^{(1)}$	15	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}''$	$1-x, -x$	$3+x, x$	$x, 1+x, 2+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_5^{(5)}$	15	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$1-x, 4-x$	$1+x, x$	x, x, x
$\mathcal{C}_2^{(3)} - \mathcal{D}_3^{(4)}$	19	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}$	$4-x, 3-x$	$-2+x, 1+x$	$x, < x-4$
$\mathcal{C}_2^{(3)} - \mathcal{D}_3^{(5)}$	18	8	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$2-x, -1-x$	$1+x, 1+x, 3+x$	$x, 3+x$
$\mathcal{C}_2^{(3)} - \mathcal{D}_5^{(1)}$	15	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$1-x, 2-x$	$2+x, 2+x, 4+x$	$x, x, 4+x$
$\mathcal{C}_3^{(1)} - \mathcal{D}_3^{(4)}$	17	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}$	$2-x, 5-x$	$-1+x, 1+x, 1+x$	$x, < x-5$
$\mathcal{C}_3^{(1)} - \mathcal{D}_3^{(5)}$	16	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$1-x, 2-x$	$2+x, 2+x, 4+x$	$x, 4+x, 4+x$

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Table 16 — continued from previous page

Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_3^{(1)} - \mathcal{D}_5^{(1)}$	13	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}$	$-1-x, -x$	$4+x, 4+x, 6+x$	$x, 2+x, 6+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_3^{(4)}$	15	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}' \oplus \mathbf{1} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$4-x, 3-x$	$-3+x, -1+x, -1+x$	$x, < x-4$
$\mathcal{C}_4^{(1)} - \mathcal{D}_4^{(5)}$	13	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}$	$-x, -1-x$	$1+x, 1+x, 3+x$	$x, 1+x, 5+x$
$\mathcal{C}_4^{(3)} - \mathcal{D}_3^{(4)}$	15	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$4-x, 3-x$	$-4+x, 1+x$	$x, < x-4$
Dirac with gCP (IO)								
Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)} - \mathcal{D}_1^{(1)}$	24	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$2-x, 5-x$	$x, 1+x$	$x, -1+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_2^{(1)}$	22	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}''$	$\mathbf{3}$	$1-x, 4-x$	$x, 2+x, 2+x$	x
$\mathcal{C}_1^{(1)} - \mathcal{D}_2^{(2)}$	22	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}''$	$4-x, 5-x$	$-2+x, 1+x$	$x, -5+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_2^{(3)}$	22	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}'$	$5-x, 6-x$	$-3+x, -2+x$	x, x, x
$\mathcal{C}_1^{(1)} - \mathcal{D}_3^{(1)}$	20	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1}'' \oplus \mathbf{1}'$	$1-x, 2-x$	$1+x, 2+x$	$x, 2+x, 2+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_3^{(4)}$	21	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}$	$4-x, 5-x$	$-2+x, 1+x$	$x, < x-5$
$\mathcal{C}_1^{(1)} - \mathcal{D}_3^{(5)}$	20	8	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$2-x, 1-x$	$2+x, -1+x$	$x, 1+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_4^{(1)}$	18	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1}'' \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}''$	$2-x, 3-x$	$1+x, 3+x, 3+x$	$x, -3+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_4^{(5)}$	19	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}'$	$3-x, 4-x$	$-1+x, x$	$x, 1+x, 2+x$
$\mathcal{C}_1^{(1)} - \mathcal{D}_5^{(1)}$	17	8	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$2-x, 1-x$	$1+x, 3+x, 3+x$	$x, < x-2$
$\mathcal{C}_1^{(1)} - \mathcal{D}_5^{(3)}$	17	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$-x, 1-x$	$3+x, 5+x, 5+x$	$x, < x-1$
$\mathcal{C}_1^{(1)} - \mathcal{D}_5^{(4)}$	16	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}''$	$-x, -1-x$	$1+x, 5+x, 5+x$	$x, 3+x$
$\mathcal{C}_2^{(1)} - \mathcal{D}_1^{(1)}$	22	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}'' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$2-x, 5-x$	$-2+x, 1+x$	$x, 1+x$
$\mathcal{C}_2^{(1)} - \mathcal{D}_2^{(1)}$	20	8	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$1-x, 6-x$	x	x
$\mathcal{C}_2^{(1)} - \mathcal{D}_2^{(3)}$	20	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$2-x, 3-x$	$-2+x, 1+x$	$x, 3+x$
$\mathcal{C}_2^{(1)} - \mathcal{D}_3^{(4)}$	19	8	$\mathbf{3}$	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}' \oplus \mathbf{1} \oplus \mathbf{1}$	$2-x$	$-1+x, 4+x$	$x, 1+x, 4+x$
$\mathcal{C}_2^{(1)} - \mathcal{D}_4^{(5)}$	17	9	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}''$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}$	$-x, 1-x$	$x, 3+x$	$x, 1+x, 5+x$

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Table 16 — continued from previous page

Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_2^{(2)}-\mathcal{D}_1^{(1)}$	22	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,5-x$	$x,-1+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$4-x,5-x$	$-2+x,-5+x$	$x,-1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(3)}$	20	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}$	$3-x,2-x$	$-1+x,-2+x$	$x,2+x,2+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$1-x,2-x$	$1+x,2+x$	$x,2+x,2+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(4)}$	19	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x,5-x$	$-2+x,-1+x$	$x,<x-5$
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(5)}$	18	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,1-x$	$2+x,-1+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$2-x,3-x$	$1+x,1+x,3+x$	$x,1+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(5)}$	17	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$3-x,4-x$	$-1+x,x$	$x,1+x,2+x$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(1)}$	15	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}$	$2-x,3-x$	$1+x,1+x,3+x$	$x,<x-3$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(3)}$	15	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$-x,1-x$	$-1+x,3+x,5+x$	$x,<x-1$
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(4)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$-x,-1-x$	$1+x,5+x,5+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	22	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$2-x,1-x$	$x,3+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(2)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}''$	$2-x,3-x$	$3+x,3+x,3+x$	$x,-3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(3)}$	20	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$2-x,-3-x$	$2+x,3+x$	$x,3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	19	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x,3-x$	$-2+x,1+x$	$x,<x-4$
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	18	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{3}$	$3-x,-x$	$1+x,x$	x
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$2-x,3-x$	$-1+x,1+x,1+x$	$x,-3+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(5)}$	17	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$-1-x,-x$	$2+x,6+x,6+x$	$x,4+x,4+x$
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	15	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$1-x,2-x$	$x,2+x,4+x$	$x,x,4+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_1^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$4-x,5-x$	$-5+x,-1+x,1+x$	$x,1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(2)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,1-x$	$-1+x,1+x,3+x$	$x,-1+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(3)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$2-x,3-x$	$1+x,1+x,1+x$	$x,3+x$
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	17	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$2-x,5-x$	$-1+x,1+x,1+x$	$x,<x-5$

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Table 16 — continued from previous page

Combinations	$\#P_0$	$\#P$	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_3^{(1)} - \mathcal{D}_3^{(5)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$1-x, 2-x$	$2+x, 2+x, 4+x$	$x, 4+x, 4+x$
$\mathcal{C}_3^{(1)} - \mathcal{D}_4^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}'$	$2-x, 3-x$	$1+x, 1+x, 1+x$	$x, 1+x$
$\mathcal{C}_3^{(1)} - \mathcal{D}_4^{(5)}$	15	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$-1-x, -x$	$4+x, 4+x, 4+x$	$x, 4+x, 6+x$
$\mathcal{C}_3^{(1)} - \mathcal{D}_5^{(1)}$	13	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$-1-x, -x$	$4+x, 4+x, 6+x$	$x, 2+x, 6+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$2-x, 5-x$	$-1+x, -1+x, 1+x$	$x, -1+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_2^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}'$	$4-x, 3-x$	$-3+x, -3+x, -1+x$	$x, -3+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_2^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}$	$3-x, 2-x$	$-2+x, -2+x, x$	$x, 2+x, 2+x$
$\mathcal{C}_4^{(1)} - \mathcal{D}_3^{(4)}$	15	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$4-x, 3-x$	$-3+x, -1+x, -1+x$	$x, < x-4$
$\mathcal{C}_4^{(2)} - \mathcal{D}_2^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{3}$	$1-x, 6-x$	$-1+x, -6+x$	x
$\mathcal{C}_4^{(3)} - \mathcal{D}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}''$	$2-x, 1-x$	$-2+x, 3+x$	$x, -1+x$
$\mathcal{C}_4^{(3)} - \mathcal{D}_2^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}'$	$4-x, 3-x$	$-4+x, 1+x$	$x, -3+x$
$\mathcal{C}_4^{(3)} - \mathcal{D}_2^{(3)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$3-x, 4-x$	$-3+x, 2+x$	$x, 2+x, 2+x$
$\mathcal{C}_4^{(3)} - \mathcal{D}_3^{(4)}$	15	8	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$4-x, 3-x$	$-4+x, 1+x$	$x, < x-4$
$\mathcal{C}_2^{(2)} - \mathcal{D}_4^{(6)}$	16	9	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}'$	$1-x, 3-x, 3-x$	$2+x, -1+x$	$x, 3+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_5^{(2)}$	14	9	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}'$	$1-x, 1-x, 3-x$	$2+x, -1+x$	$x, 3+x$
$\mathcal{C}_2^{(2)} - \mathcal{D}_5^{(6)}$	14	9	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}$	$-1-x, 1-x, 1-x$	$4+x, 3+x$	$x, 3+x$
$\mathcal{C}_3^{(3)} - \mathcal{D}_3^{(4)}$	17	9	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$3-x, 5-x, 5-x$	$-2+x, -1+x$	$x, -3+x$

Table 16. The same as in table 14 but for IO neutrino mass spectrum.

Dirac without gCP (IO)								
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	0.306	0.02241	0.569	1.561	52.681	53.381	18.971	0.033
$\mathcal{C}_2^{(2)}-\mathcal{D}_1^{(1)}$	0.298	0.02244	0.560	1.720	57.720	58.359	30.236	2.537
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	0.287	0.02243	0.577	1.185	49.151	49.900	0	6.894
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	0.306	0.02241	0.570	1.514	52.591	53.292	18.720	0.051
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	0.305	0.02234	0.573	1.750	60.717	61.325	35.648	2.868
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	0.305	0.02234	0.573	1.750	60.718	61.326	35.650	2.869
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	0.307	0.02242	0.569	1.576	52.700	53.399	19.036	0.138
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02241	0.570	1.543	72.865	73.372	53.791	0.001
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	0.307	0.02242	0.571	1.489	52.635	53.335	18.851	0.166
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02241	0.570	1.542	49.150	49.899	0	0.001
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(5)}$	0.304	0.02242	0.570	1.540	49.144	49.893	0	0.001
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(5)}$	0.290	0.02240	0.560	1.774	48.715	49.471	0	5.557
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	0.304	0.02241	0.570	1.544	49.157	49.906	0	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(4)}$	0.306	0.02242	0.572	1.406	49.137	49.887	0	0.727
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(5)}$	0.304	0.02241	0.570	1.542	49.153	49.902	0	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(5)}$	0.290	0.02240	0.560	1.774	48.715	49.471	0	5.557
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(1)}$	0.304	0.02241	0.570	1.546	49.153	49.902	0	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(5)}$	0.287	0.02243	0.577	1.185	49.151	49.900	0	6.894
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	0.329	0.02246	0.569	1.557	49.100	49.850	0	3.770
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	0.303	0.02254	0.549	1.833	49.117	49.867	0	6.581
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	0.303	0.02254	0.549	1.833	49.117	49.867	0	6.581
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	0.307	0.02242	0.570	1.551	49.147	49.896	0	0.042
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	0.303	0.02254	0.549	1.834	49.117	49.867	0	6.590
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	0.303	0.02254	0.549	1.834	49.117	49.867	0	6.590
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02241	0.571	1.470	49.152	49.901	0	0.201
$\mathcal{C}_4^{(1)}-\mathcal{D}_4^{(5)}$	0.300	0.02253	0.585	1.125	49.190	49.939	0	7.410
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	0.321	0.02247	0.571	1.253	49.068	49.818	0	4.873
Dirac with gCP (IO)								
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_1^{(1)}$	0.306	0.02241	0.569	1.562	52.677	53.377	18.961	0.038
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(1)}$	0.304	0.02239	0.571	1.627	49.145	49.894	0	0.458
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02241	0.570	1.548	49.151	49.900	0.001	0.001
$\mathcal{C}_1^{(1)}-\mathcal{D}_2^{(3)}$	0.302	0.02242	0.572	1.380	49.323	50.070	3.473	1.023
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(1)}$	0.290	0.02240	0.560	1.781	48.717	49.473	0.041	5.752
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Table 17 — continued from previous page

Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02241	0.570	1.548	49.151	49.900	0	0.001
$\mathcal{C}_1^{(1)}-\mathcal{D}_3^{(5)}$	0.304	0.02239	0.571	1.367	49.085	49.835	0	1.153
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(1)}$	0.302	0.02242	0.580	1.612	49.161	49.910	0.369	0.699
$\mathcal{C}_1^{(1)}-\mathcal{D}_4^{(5)}$	0.290	0.02240	0.560	1.781	48.717	49.473	0	5.752
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(1)}$	0.304	0.02241	0.572	1.557	49.150	49.899	0	0.023
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(3)}$	0.305	0.02239	0.569	1.405	49.150	49.899	0	0.714
$\mathcal{C}_1^{(1)}-\mathcal{D}_5^{(4)}$	0.304	0.02241	0.571	1.093	49.151	49.900	0	7.346
$\mathcal{C}_2^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02241	0.571	1.800	49.151	49.900	0	4.364
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(1)}$	0.294	0.02242	0.562	1.698	63.955	64.532	40.906	2.447
$\mathcal{C}_2^{(1)}-\mathcal{D}_2^{(3)}$	0.305	0.02241	0.571	1.216	49.154	49.903	0	3.878
$\mathcal{C}_2^{(1)}-\mathcal{D}_3^{(4)}$	0.309	0.02253	0.581	1.136	49.153	49.902	0	6.669
$\mathcal{C}_2^{(1)}-\mathcal{D}_4^{(5)}$	0.305	0.02241	0.571	1.216	49.154	49.903	0	3.878
$\mathcal{C}_2^{(2)}-\mathcal{D}_1^{(1)}$	0.298	0.02244	0.560	1.720	57.726	58.365	30.248	2.542
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(2)}$	0.304	0.02241	0.570	1.548	49.152	49.901	0.277	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_2^{(3)}$	0.303	0.02241	0.571	1.477	49.260	50.007	1.018	0.224
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(1)}$	0.290	0.02240	0.560	1.781	48.717	49.473	0.000	5.752
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(4)}$	0.304	0.02241	0.570	1.548	49.151	49.900	0	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_3^{(5)}$	0.304	0.02239	0.571	1.367	49.085	49.835	0	1.153
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(1)}$	0.302	0.02241	0.569	1.337	56.941	57.589	28.750	1.576
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(5)}$	0.290	0.02240	0.560	1.781	48.717	49.473	0	5.752
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(1)}$	0.304	0.02241	0.572	1.557	49.150	49.899	0	0.023
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(3)}$	0.305	0.02239	0.569	1.405	49.150	49.899	0	0.714
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(4)}$	0.304	0.02241	0.571	1.093	49.151	49.900	0	7.346
$\mathcal{C}_2^{(3)}-\mathcal{D}_1^{(1)}$	0.306	0.02241	0.570	1.514	52.591	53.292	18.720	0.051
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(2)}$	0.303	0.02241	0.573	1.252	49.151	49.900	0	3.134
$\mathcal{C}_2^{(3)}-\mathcal{D}_2^{(3)}$	0.306	0.02239	0.569	1.638	49.148	49.897	0	0.605
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(4)}$	0.329	0.02246	0.569	1.557	49.101	49.851	0	3.771
$\mathcal{C}_2^{(3)}-\mathcal{D}_3^{(5)}$	0.304	0.02242	0.563	1.708	49.049	49.800	0	1.911
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(1)}$	0.305	0.02234	0.573	1.750	60.717	61.325	35.648	2.868
$\mathcal{C}_2^{(3)}-\mathcal{D}_4^{(5)}$	0.304	0.02241	0.570	1.543	49.149	49.898	0	0.001
$\mathcal{C}_2^{(3)}-\mathcal{D}_5^{(1)}$	0.303	0.02254	0.549	1.834	49.117	49.867	0	6.586
$\mathcal{C}_3^{(1)}-\mathcal{D}_1^{(1)}$	0.304	0.02241	0.570	1.542	49.151	49.900	0	0.001
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(2)}$	0.303	0.02242	0.571	1.459	62.173	62.767	38.077	0.280
$\mathcal{C}_3^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02241	0.570	1.540	57.265	57.909	29.383	0.001
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(4)}$	0.307	0.02242	0.570	1.551	49.147	49.896	0	0.042

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Table 17 — continued from previous page

Combinations	Predictions for mixing parameters and neutrino masses at best fitting point							$\chi^2_{\rm min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ^l_{CP}/π	m_1/meV	m_2/meV	m_3/meV	
$\mathcal{C}_3^{(1)}-\mathcal{D}_3^{(5)}$	0.303	0.02254	0.549	1.834	49.117	49.867	0	6.590
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(1)}$	0.305	0.02234	0.573	1.750	60.718	61.326	35.650	2.869
$\mathcal{C}_3^{(1)}-\mathcal{D}_4^{(5)}$	0.304	0.02241	0.570	1.543	49.150	49.899	0	0.001
$\mathcal{C}_3^{(1)}-\mathcal{D}_5^{(1)}$	0.303	0.02254	0.549	1.834	49.117	49.867	0	6.590
$\mathcal{C}_4^{(1)}-\mathcal{D}_1^{(1)}$	0.307	0.02242	0.569	1.576	52.700	53.399	19.036	0.138
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(2)}$	0.304	0.02241	0.570	1.534	72.054	72.567	52.688	0.004
$\mathcal{C}_4^{(1)}-\mathcal{D}_2^{(3)}$	0.304	0.02241	0.570	1.545	50.211	50.945	10.273	0.001
$\mathcal{C}_4^{(1)}-\mathcal{D}_3^{(4)}$	0.304	0.02242	0.573	1.349	49.149	49.898	0	1.405
$\mathcal{C}_4^{(2)}-\mathcal{D}_2^{(1)}$	0.290	0.02243	0.558	1.747	64.015	64.592	40.996	4.468
$\mathcal{C}_4^{(3)}-\mathcal{D}_1^{(1)}$	0.307	0.02242	0.571	1.489	52.635	53.335	18.851	0.167
$\mathcal{C}_4^{(3)}-\mathcal{D}_2^{(2)}$	0.336	0.02247	0.569	1.542	49.087	49.837	0	6.085
$\mathcal{C}_4^{(3)}-\mathcal{D}_2^{(3)}$	0.304	0.02241	0.570	1.532	94.045	94.439	80.178	0.006
$\mathcal{C}_4^{(3)}-\mathcal{D}_3^{(4)}$	0.336	0.02247	0.569	1.543	49.086	49.836	0	6.085
$\mathcal{C}_2^{(2)}-\mathcal{D}_4^{(6)}$	0.304	0.02241	0.570	1.543	49.150	49.899	0	0.001
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(2)}$	0.304	0.02250	0.571	1.622	49.294	50.041	0	0.490
$\mathcal{C}_2^{(2)}-\mathcal{D}_5^{(6)}$	0.304	0.02241	0.570	1.546	49.151	49.900	0	0.001
$\mathcal{C}_3^{(3)}-\mathcal{D}_3^{(4)}$	0.304	0.02241	0.570	1.534	49.151	49.900	0	0.005

Table 17. The same as in table 15 but for IO neutrino mass spectrum and for the representative models of table 16.

C.2 Majorana neutrinos (Weinberg operator)

In case neutrinos are Majorana particles and their masses are generated via Weinberg operator, the viable pairs of texture zeros of the lepton mass matrices are summarized in table 10. Here, we present the corresponding representative models for NO and IO neutrino masses spectrum in tables 18 and 20, respectively. The numerical results of these representative models are given in tables 19 and 21.

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Weinberg operator without gCP (NO)						
Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	k_L	k_{E^c}
$\mathcal{C}_2^{(3)} - \mathcal{W}_1^{(1)}$	18	9	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$1, 2$	$0, 2, 4$
$\mathcal{C}_2^{(3)} - \mathcal{W}_2^{(2)}$	16	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}''$	$1, 2$	$0, 2, 2$
$\mathcal{C}_3^{(1)} - \mathcal{W}_1^{(1)}$	16	9	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{1}' \oplus \mathbf{1}' \oplus \mathbf{1}$	$1, 2$	$-2, 2, 4$
$\mathcal{C}_3^{(1)} - \mathcal{W}_2^{(2)}$	14	9	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1} \oplus \mathbf{1}'' \oplus \mathbf{1}'$	$1, 2$	$2, 2, 2$
$\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$	15	7	$\mathbf{2}' \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}'$	$1, 0$	$0, 0, 2$
Weinberg operator with gCP (NO)						
Combinations	$\#\mathrm{P}_0$	$\#\mathrm{P}$	ρ_L	ρ_{E^c}	k_L	k_{E^c}
$\mathcal{C}_1^{(1)} - \mathcal{W}_1^{(1)}$	20	8	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}''$	$1, 2$	$3, 2$
$\mathcal{C}_1^{(1)} - \mathcal{W}_2^{(2)}$	18	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{2}' \oplus \mathbf{1}$	$1, 2$	$3, 2$
$\mathcal{C}_1^{(1)} - \mathcal{W}_3^{(1)}$	16	9	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}'$	$-1, 2$	$2, 4, 4$
$\mathcal{C}_2^{(2)} - \mathcal{W}_1^{(1)}$	18	8	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{2}' \oplus \mathbf{1}$	$1, 2$	$3, 2$
$\mathcal{C}_2^{(2)} - \mathcal{W}_2^{(2)}$	16	9	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}$	$1, 2$	$2, 2, 4$
$\mathcal{C}_2^{(2)} - \mathcal{W}_3^{(1)}$	14	9	$\mathbf{2}' \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}''$	$-1, 2$	$2, 2, 4$
$\mathcal{C}_2^{(3)} - \mathcal{W}_1^{(1)}$	18	8	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{1}$	$1, 2$	$2, 2, 4$
$\mathcal{C}_2^{(3)} - \mathcal{W}_2^{(2)}$	16	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}''$	$1, 2$	$0, 2, 2$
$\mathcal{C}_3^{(1)} - \mathcal{W}_1^{(1)}$	16	8	$\mathbf{2}' \oplus \mathbf{1}''$	$\mathbf{1}'' \oplus \mathbf{1}' \oplus \mathbf{1}$	$1, 2$	$2, 2, 2$
$\mathcal{C}_3^{(1)} - \mathcal{W}_2^{(2)}$	14	8	$\mathbf{2} \oplus \mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1} \oplus \mathbf{1}'$	$1, 2$	$2, 2, 2$
$\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$	15	7	$\mathbf{2} \oplus \mathbf{1}''$	$\mathbf{2}'' \oplus \mathbf{1}'$	$1, 0$	$3, 0$

Table 18. Representative models of the viable patterns of texture zero in (M_E, M_ν) that can accommodate the experimental data at 3σ level for NO neutrino mass spectrum. Here neutrinos are Majorana particles and neutrino masses are assumed to be described by the Weinberg operator. Models with and without gCP are considered. The same convention as table 14 is adopted.

Weinberg operator without gCP											
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point										$\chi^2_{\min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π	α_{31}/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	0.303	0.02269	0.475	1.495	1.996	0.018	26.818	28.168	56.808	25.325	3.730
$\mathcal{C}_2^{(3)}-\mathcal{W}_2^{(2)}$	0.304	0.02244	0.451	1.158	0.492	1.683	39.422	40.352	63.750	28.608	0.748
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	0.305	0.02246	0.452	0.839	1.595	1.628	20.198	21.958	54.062	18.033	10.017
$\mathcal{C}_3^{(1)}-\mathcal{W}_2^{(2)}$	0.304	0.02244	0.451	1.158	0.584	1.775	39.428	40.358	63.754	25.599	0.749
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	0.304	0.02254	0.601	1.482	1.957	0.970	44.194	45.025	66.800	44.865	64.205
Weinberg operator with gCP											
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point										$\chi^2_{\min}$
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π	α_{31}/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	0.304	0.02246	0.450	1.351	0.308	1.199	18.820	20.698	53.510	17.466	0.133
$\mathcal{C}_1^{(1)}-\mathcal{W}_2^{(2)}$	0.304	0.02244	0.451	1.158	1.594	0.785	39.425	40.355	63.752	32.076	0.749
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	0.304	0.02246	0.450	1.045	0.240	1.330	0	8.614	50.100	1.436	2.807
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	0.304	0.02246	0.450	1.351	0.308	1.199	18.820	20.698	53.510	17.466	0.133
$\mathcal{C}_2^{(2)}-\mathcal{W}_2^{(2)}$	0.304	0.02245	0.449	1.128	0.308	0.125	3.919	9.464	50.243	5.614	1.171
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	0.304	0.02246	0.450	1.045	1.665	0.756	0	8.614	50.100	1.436	2.807
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	0.304	0.02246	0.451	1.412	1.588	0.708	29.686	30.910	58.235	25.667	0.451
$\mathcal{C}_2^{(3)}-\mathcal{W}_2^{(2)}$	0.304	0.02244	0.451	1.158	0.594	1.785	39.422	40.352	63.750	25.259	0.748
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	0.304	0.02246	0.450	1.374	1.594	0.697	20.178	21.940	54.009	18.013	0.231
$\mathcal{C}_3^{(1)}-\mathcal{W}_2^{(2)}$	0.304	0.02244	0.451	1.158	0.594	1.785	39.428	40.358	63.754	25.259	0.749
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	0.304	0.02254	0.601	1.482	1.957	0.970	44.214	45.045	66.813	44.883	64.198

Table 19. Best-fit values of the lepton mixing and neutrino mass parameters for the representative models presented in table 18. For all representative models, the predictions of the lepton mass ratios are $m_e/m_\mu = 0.00474$, $m_\mu/m_\tau = 0.0586$ and $\Delta m^2_{21}/\Delta m^2_{31} = 0.030$.

Weinberg operator without gCP (IO)						
Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	k_L	k_{E^c}
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1''}\oplus\mathbf{1}$	$2,1$	$1,3,3$
$\mathcal{C}_2^{(3)}-\mathcal{W}_2^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1''}\oplus\mathbf{1}$	$1,2$	$0,2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1'}\oplus\mathbf{1''}\oplus\mathbf{1}$	$2,1$	$-1,-1,3$
$\mathcal{C}_3^{(1)}-\mathcal{W}_2^{(2)}$	14	9	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1'}$	$1,2$	$2,2,2$
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	15	9	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1}\oplus\mathbf{1''}$	$3,0$	$0,0,2$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	14	8	$\mathbf{2'}\oplus\mathbf{1'}$	$\mathbf{2''}\oplus\mathbf{1''}$	$-1,2$	$3,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	12	9	$\mathbf{2''}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1'}$	$-1,2$	$2,2,4$
Weinberg operator with gCP (IO)						
Combinations	#P$_0$	#P	ρ_L	ρ_{E^c}	k_L	k_{E^c}
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	20	8	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{2''}\oplus\mathbf{1''}$	$2,3$	$2,1$
$\mathcal{C}_1^{(1)}-\mathcal{W}_2^{(2)}$	18	8	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{2'}\oplus\mathbf{1}$	$1,2$	$3,2$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	16	9	$\mathbf{2'}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1}\oplus\mathbf{1''}$	$2,1$	$3,3,3$
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	18	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2''}\oplus\mathbf{1}$	$2,1$	$2,-1$
$\mathcal{C}_2^{(2)}-\mathcal{W}_2^{(2)}$	16	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1''}$	$1,0$	$0,4,4$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	18	8	$\mathbf{2'}\oplus\mathbf{1'}$	$\mathbf{2}\oplus\mathbf{1''}$	$1,0$	$3,4$
$\mathcal{C}_2^{(3)}-\mathcal{W}_2^{(2)}$	16	8	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1''}\oplus\mathbf{1}$	$1,2$	$0,2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	16	8	$\mathbf{2'}\oplus\mathbf{1'}$	$\mathbf{1'}\oplus\mathbf{1''}\oplus\mathbf{1''}$	$1,0$	$0,0,4$
$\mathcal{C}_3^{(1)}-\mathcal{W}_2^{(2)}$	14	8	$\mathbf{2'}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1}\oplus\mathbf{1''}$	$1,2$	$2,2,2$
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	15	8	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{1'}\oplus\mathbf{1''}\oplus\mathbf{1}$	$3,0$	$0,0,2$

Table 20. The same as in table 18 but for IO neutrino masses.

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Combinations	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π	α_{31}/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	χ^2_{min}
Weinberg operator without gCP											
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point										χ^2_{min}
$\mathcal{C}_2^{(3)} - \mathcal{W}_1^{(1)}$	0.304	0.02241	0.570	1.546	0.663	1.585	49.529	50.272	6.106	29.481	0.001
$\mathcal{C}_2^{(3)} - \mathcal{W}_2^{(2)}$	0.305	0.02234	0.573	1.750	1.604	0.724	60.717	61.325	35.648	49.818	2.868
$\mathcal{C}_3^{(1)} - \mathcal{W}_1^{(1)}$	0.304	0.02241	0.570	1.546	1.695	0.666	49.529	50.272	6.106	44.101	0.001
$\mathcal{C}_3^{(1)} - \mathcal{W}_2^{(2)}$	0.305	0.02234	0.573	1.750	0.869	0.724	60.718	61.326	35.650	24.881	2.869
$\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$	0.299	0.02245	0.508	1.662	0.021	1.004	84.281	84.720	68.435	83.290	9.084
$\mathcal{C}_2^{(2)} - \mathcal{W}_3^{(1)}$	0.287	0.02245	0.566	1.155	1.887	0.881	49.154	49.903	0	47.639	7.521
$\mathcal{C}_3^{(1)} - \mathcal{W}_3^{(1)}$	0.303	0.02241	0.574	1.573	1.465	0.416	49.152	49.901	0	35.101	0.116
Weinberg operator with gCP (IO)											
Combinations	Predictions for mixing parameters and neutrino masses at best fitting point										χ^2_{min}
	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π	α_{31}/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_1^{(1)} - \mathcal{W}_1^{(1)}$	0.304	0.02241	0.568	1.532	0.007	0.957	49.439	50.184	5.327	48.662	0.011
$\mathcal{C}_1^{(1)} - \mathcal{W}_2^{(2)}$	0.305	0.02234	0.573	1.750	1.577	0.724	60.715	61.323	35.644	48.618	2.867
$\mathcal{C}_1^{(1)} - \mathcal{W}_3^{(2)}$	0.304	0.02241	0.572	1.556	0.011	0.958	49.150	49.899	0	48.265	0.020
$\mathcal{C}_2^{(2)} - \mathcal{W}_1^{(1)}$	0.303	0.02242	0.576	1.590	0.023	0.960	49.154	49.903	0.386	48.253	0.308
$\mathcal{C}_2^{(2)} - \mathcal{W}_2^{(2)}$	0.302	0.02241	0.579	1.599	0.014	0.964	56.358	57.013	27.606	55.733	0.515
$\mathcal{C}_2^{(3)} - \mathcal{W}_1^{(1)}$	0.304	0.02241	0.569	1.545	0.016	0.958	57.111	57.757	29.083	56.600	0.001
$\mathcal{C}_2^{(3)} - \mathcal{W}_2^{(2)}$	0.305	0.02234	0.573	1.750	1.578	0.724	60.717	61.325	35.648	48.625	2.868
$\mathcal{C}_3^{(1)} - \mathcal{W}_1^{(1)}$	0.304	0.02241	0.569	1.546	0.016	0.958	56.986	57.633	28.836	56.473	0.003
$\mathcal{C}_3^{(1)} - \mathcal{W}_2^{(2)}$	0.305	0.02234	0.573	1.750	0.578	0.724	60.718	61.326	35.650	40.133	2.869
$\mathcal{C}_4^{(1)} - \mathcal{W}_1^{(1)}$	0.299	0.02241	0.508	1.655	0.021	1.004	85.542	85.975	69.985	84.594	9.085

Table 21. The same as in table 19 but for IO neutrino masses.

C.3 Majorana neutrinos (seesaw mechanism)

If neutrino masses are described by the type-I seesaw mechanism with two right-handed neutrinos, the texture-zero patterns of (M_E, M_ν) that can explain the experimental data on lepton masses and mixing parameters are presented in table 11. Here, we give the examples of lepton models that can realise those texture zeros. For NO neutrino mass spectrum, the representative models and corresponding predictions are provided in table 22 and 23. The same is presented for IO in tables 24 and 25.

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Seesaw mechanism without gCP (NO)									
Combinations		#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{2}$	$2,-1$	$1,1,3$	2
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{2}$	$-3,-2$	$4,6,8$	3
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	12	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}$	$-1,0$	$4,4,4$	3
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}$	$-3,-2$	$4,6,6$	3
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$0,-1$	$1,5,5$	$3,3$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$3,2$	$-2,2,2$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$	$4,3$	$-1,-1,1$	$-1,1$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}$	$4,5$	$-1,-1,-1$	$-1,1$
Seesaw mechanism with gCP (NO)									
Combinations		#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}$	$-1,-2$	$5,6$	3
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}$	$-3,-4$	$6,8,8$	3
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{2}$	$2,1$	$-1,3,3$	2
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{2}$	$-3,-2$	$6,6,8$	3
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}$	$1,-2$	$2,2,4$	3
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		8	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}$	$-3,-4$	$7,8$	3
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	12	8	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}$	$2,1$	$-1,-1,3$	2
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		8	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}$	$-3,-4$	$4,6,8$	3
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}$	$1,2$	$3,4$	$2,2$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$4,3$	$2,-1$	$-1,1$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}''$	$0,1$	$4,1$	$3,3$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	16	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$	$1,-4$	$5,2$	$0,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$0,-1$	$1,5,5$	$3,3$
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	20	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$3,4$	$2,2$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	16	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''$	$1,2$	$3,4$	$2,2$
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$3,2$	$2,2$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''$	$2,3$	$2,1$	$1,3$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_1^{(2)}$	14	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''$	$-1,2$	$3,2$	$2,2$
	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$0,1$	$4,3$	$3,3$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$3,2$	$-2,2,2$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$	$4,3$	$-1,-1,1$	$-1,1$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}$	$2,3$	$1,1,3$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$		9	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$1,-4$	$3,4$	$2,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$	$1,0$	$3,4$	$0,2$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$0,2,4$	$2,2$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	14	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''$	$1,2$	$0,2,4$	$2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$4,5$	$-1,-1,-1$	$-1,1$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	12	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}$	$2,3$	$1,1,3$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}'$	$1,0$	$0,2,4$	$2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	16	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$2,2,4$	$2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	12	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''$	$1,2$	$2,2,4$	$2,2$
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	15	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$-2,0,4$	$2,2$

Table 22. Representative models for the viable patterns of texture zero in (M_E, M_ν) that can accommodate the experimental data at 3σ level for NO neutrino mass spectrum. Here, neutrinos are Majorana particles and their masses are assumed to be generated by the type-I seesaw mechanism. The models are given without and with gCP. The same convention as table 14 is adopted.

Seesaw mechanism without gCP (NO)											
Combinations		Predictions for mixing parameters and neutrino masses at best fitting point									$\chi^2_{\rm min}$
		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}/\mathrm{meV}$	
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02245	0.450	1.288	0.125	0	8.614	50.096	2.101	0.004
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.318	0.02215	0.428	1.136	1.881	0	8.614	49.724	3.692	4.918
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02239	0.453	1.280	0.634	0	8.614	50.078	1.761	0.039
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.318	0.02215	0.428	1.136	1.881	0	8.614	49.724	3.693	4.934
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02245	0.449	1.245	1.350	0	8.614	50.114	3.648	0.064
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.303	0.02248	0.455	1.465	0.438	0	8.614	50.054	2.332	0.961
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02246	0.450	1.304	0.134	0	8.614	50.096	1.961	0.018
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02246	0.453	1.367	0.006	0	8.614	50.081	1.964	0.235
Seesaw mechanism with gCP (NO)											
Combinations		Predictions for mixing parameters and neutrino masses at best fitting point									$\chi^2_{\rm min}$
		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}/\mathrm{meV}$	
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.300	0.02252	0.458	1.003	1.998	0	8.6139	50.169	3.659	4.214
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02246	0.451	1.108	1.493	0	8.6144	50.095	3.425	1.491
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02246	0.450	1.273	1.566	0	8.6139	50.099	3.637	0.001
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02246	0.451	1.070	1.953	0	8.6144	50.093	3.709	2.234
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.305	0.02242	0.452	1.163	0.624	0	8.6139	50.064	1.466	0.704
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.313	0.02227	0.436	1.014	1.987	0	8.6139	49.844	3.748	5.181
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.298	0.02262	0.456	1.097	0.887	0	8.6139	50.157	1.443	2.088
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.316	0.02222	0.430	1.014	1.988	0	8.6139	49.791	3.766	6.575
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.303	0.02246	0.452	1.318	1.028	0	8.614	50.109	3.259	0.051
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02246	0.450	1.302	0.131	0	8.614	50.096	1.986	0.015
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.335	0.02273	0.438	1.045	1.019	0	8.614	50.345	1.784	10.717
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.316	0.02220	0.431	1.015	1.987	0	8.614	49.773	3.769	6.495
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02245	0.449	1.245	1.350	0	8.614	50.114	3.648	0.064
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.305	0.02246	0.449	1.190	0.621	0	8.614	50.095	1.440	0.408
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.305	0.02246	0.449	1.190	0.621	0	8.614	50.095	1.440	0.408
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.303	0.02246	0.452	1.318	1.028	0	8.614	50.109	3.259	0.051
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.299	0.02249	0.448	1.796	1.030	0	8.614	50.261	2.341	6.966
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_1^{(2)}$	0.273	0.02248	0.464	1.000	1.000	0	8.614	50.100	1.170	11.456
	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.335	0.02273	0.438	1.045	1.019	0	8.614	50.346	1.784	10.723
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.303	0.02248	0.455	1.465	0.438	0	8.614	50.054	2.332	0.961
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02246	0.450	1.304	0.134	0	8.614	50.096	1.961	0.018
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.300	0.02266	0.463	1.414	0.103	0	8.614	49.972	1.501	1.208
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$	0.291	0.02275	0.502	1.676	1.680	0	8.614	49.805	1.324	13.186
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.313	0.02226	0.436	1.015	1.986	0	8.614	49.861	3.746	5.146
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.305	0.02246	0.449	1.189	0.621	0	8.614	50.095	1.440	0.411
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.305	0.02246	0.449	1.189	0.621	0	8.614	50.095	1.440	0.411
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02246	0.453	1.367	0.006	0	8.614	50.081	1.964	0.235
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.302	0.02250	0.458	1.648	1.772	0	8.614	50.076	1.462	3.638
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.304	0.02243	0.455	1.738	0.716	0	8.614	50.062	3.544	5.390
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.305	0.02246	0.449	1.189	0.621	0	8.614	50.095	1.440	0.411
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.305	0.02246	0.449	1.189	0.621	0	8.614	50.095	1.440	0.411
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.303	0.02245	0.453	1.386	1.043	0	8.614	50.120	3.544	0.336

Table 23. Best-fit values of the lepton mixing and neutrino mass parameters for the representative models presented in table 22. For all cases, the predictions of the lepton mass ratios are $m_e/m_\mu = 0.00474$, $m_\mu/m_\tau = 0.0586$ and $\Delta m^2_{21}/\Delta m^2_{31} = 0.030$.

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Seesaw mechanism without gCP (IO)									
Combinations		#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	16	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{2}$	$1,2$	$2,2,4$	1
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		8	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{2}$	$-1,0$	$4,4,6$	1
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	14	8	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{2}$	$1,2$	$2,2,4$	1
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		8	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}$	$\mathbf{2}$	$-1,0$	$4,4,6$	1
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}$	$1,-2$	$2,2,4$	3
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{2}$	$-3,-6$	$6,6,8$	3
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	12	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}'$	$\mathbf{2}$	$-1,-2$	$2,4,6$	3
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{2}$	$-3,-2$	$2,6,6$	3
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	20	8	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'$	$5,6$	$-3,-2$	$0,0$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}'\oplus\mathbf{1}''$	$5,6$	$-2,0,0$	$0,0$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}''$	$5,6$	$-2,-2,0$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	18	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$3,4$	$0,0,2$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	14	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$0,1$	$1,3,5$	$3,3$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$0,1$	$3,3,5$	$3,3$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	12	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}''$	$4,5$	$-1,-1,-1$	$-1,1$
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	15	8	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'$	$5,6$	$-6,-2,0$	$0,0$

Seesaw mechanism with gCP (IO)									
Combinations		#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	20	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}$	$1,2$	$1,2$	$2,2$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		8	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'$	$5,6$	$-3,-2$	$0,0$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}''$	$4,5$	$0,-1$	$-1,1$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}$	$0,-1$	$4,5$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$		9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''$	$1,0$	$3,2$	$2,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}$	$1,-2$	$3,4$	$0,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$5,6$	$-2,0,0$	$0,0$
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	20	9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$3,4$	$2,2$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	16	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''$	$1,2$	$3,4$	$2,2$
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}'$	$1,2$	$1,2$	$2,2$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}'$	$0,1$	$4,3$	$1,3$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	14	9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}''\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}$	$0,1$	$4,3$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$		9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{2}'\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''$	$1,0$	$3,0$	$2,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2}''\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}$	$1,2$	$3,2$	$0,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}''\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}$	$\mathbf{1}'\oplus\mathbf{1}''$	$5,6$	$-2,-2,0$	$0,0$
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}''\oplus\mathbf{1}$	$1,2$	$3,2$	$2,2$
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	14	9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}\oplus\mathbf{1}''$	$1,2$	$3,-2$	$2,2$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}$	$3,4$	$-1,0$	$2,2$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{2}'\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1}''$	$2,1$	$2,3$	$1,3$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'$	$3,4$	$-2,0,2$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	14	9	$\mathbf{2}\oplus\mathbf{1}''$	$\mathbf{1}\oplus\mathbf{1}''\oplus\mathbf{1}'$	$\mathbf{1}''\oplus\mathbf{1}$	$0,-1$	$3,5,5$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$		9	$\mathbf{2}'\oplus\mathbf{1}''$	$\mathbf{1}''\oplus\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{1}'\oplus\mathbf{1}''$	$1,0$	$2,4,4$	$2,2$

continues on next page

Table 24 — continued from previous page

Combinations		#P$_0$	#P	ρ_L	ρ_{E^c}	ρ_{N^c}	k_L	k_{E^c}	k_{N^c}
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2''}\oplus\mathbf{1''}$	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{1}\oplus\mathbf{1'}$	$1,0$	$3,4$	$0,2$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$		9	$\mathbf{2''}\oplus\mathbf{1}$	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1''}$	$5,0$	$1,0$	$0,0$
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	18	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1'}$	$1,2$	$2,2,4$	$2,2$
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	14	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1''}$	$1,2$	$2,4,4$	$2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	16	9	$\mathbf{2'}\oplus\mathbf{1''}$	$\mathbf{1}\oplus\mathbf{1'}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1'}$	$0,1$	$3,3,5$	$3,3$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	12	9	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1'}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1''}$	$4,5$	$-1,-1,-1$	$-1,1$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	12	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1}$	$0,-1$	$1,3,5$	$1,3$
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$		9	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1'}\oplus\mathbf{1'}$	$\mathbf{1}\oplus\mathbf{1'}$	$1,0$	$0,2,4$	$0,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	16	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1'}\oplus\mathbf{1}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1'}$	$1,2$	$2,2,4$	$2,2$
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	12	9	$\mathbf{2}\oplus\mathbf{1''}$	$\mathbf{1''}\oplus\mathbf{1'}\oplus\mathbf{1}$	$\mathbf{1'}\oplus\mathbf{1''}$	$1,2$	$0,2,2$	$2,2$
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	15	9	$\mathbf{2}\oplus\mathbf{1}$	$\mathbf{1}\oplus\mathbf{1''}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1}$	$2,1$	$-1,1,3$	$1,3$
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$		8	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1'}$	$5,6$	$-2,-2,0$	$0,0$
$\mathcal{C}_4^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	14	9	$\mathbf{2''}\oplus\mathbf{1'}$	$\mathbf{2}\oplus\mathbf{1'}$	$\mathbf{1''}\oplus\mathbf{1'}$	$3,4$	$-3,0$	$2,2$

Table 24. The same as in table 22 but for IO neutrino mass spectrum.

Seesaw mechanism without gCP (IO)											
Combinations		Predictions for mixing parameters and neutrino masses at best fitting point									$\chi^2_{\rm min}$
		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.572	1.557	0.343	49.150	49.899	0	42.529	0.023
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.572	1.557	0.014	49.150	49.899	0	48.262	0.023
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.572	1.557	0.343	49.150	49.899	0	42.529	0.023
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.572	1.557	0.014	49.150	49.899	0	48.262	0.023
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.570	1.546	1.589	49.151	49.900	0	40.168	0.001
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.570	1.543	1.023	49.151	49.900	0	18.682	0.001
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.570	1.545	0.654	49.154	49.903	0	29.609	0.001
	$\mathfrak{D}_4^{(1)}-\mathfrak{N}_0^{(1)}$	0.304	0.02241	0.570	1.546	1.134	49.151	49.900	0	20.796	0.001
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.586	0.178	49.150	49.899	0	46.687	0.206
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.571	1.552	0.012	49.151	49.900	0	48.264	0.007
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.571	1.552	0.012	49.151	49.900	0	48.264	0.007
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.570	1.542	1.087	49.151	49.900	0	19.568	0.001
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.583	1.544	49.152	49.902	0	38.436	0.222
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.570	1.542	0.580	49.151	49.900	0	33.029	0.001
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.576	1.584	1.213	49.152	49.902	0	23.730	0.235
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.586	1.853	49.151	49.900	0	47.187	0.220
Seesaw mechanism with gCP (IO)											
Combinations		Predictions for mixing parameters and neutrino masses at best fitting point									$\chi^2_{\rm min}$
		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.569	1.540	1.081	49.151	49.900	0	19.454	0.001
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.586	1.745	49.150	49.899	0	45.044	0.206
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.576	1.584	0.632	49.152	49.902	0	30.693	0.234

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Table 25 — continued from previous page

Combinations		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	$\chi^2_{\min}$
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.570	1.547	1.045	49.151	49.900	0	18.883	0.001
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$	0.300	0.02249	0.589	1.740	0.031	49.156	49.905	0	48.222	4.107
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.304	0.02241	0.569	1.414	1.165	49.150	49.900	0	21.824	0.609
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.571	1.552	0.009	49.151	49.900	0	48.267	0.007
$\mathcal{C}_1^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.305	0.02241	0.570	1.234	1.067	49.151	49.900	0	19.080	3.486
$\mathcal{C}_1^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.305	0.02241	0.570	1.233	0.921	49.151	49.900	0	19.303	3.503
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.569	1.540	0.512	49.151	49.900	0	36.081	0.001
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.571	1.549	1.149	49.151	49.900	0	21.302	0.002
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.569	1.536	0.659	49.152	49.901	0	29.383	0.005
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$	0.307	0.02238	0.539	1.911	0.846	49.204	49.952	0	21.219	11.085
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.304	0.02241	0.569	1.415	0.803	49.151	49.900	0	23.025	0.608
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.571	1.552	1.009	49.151	49.900	0	18.641	0.007
$\mathcal{C}_2^{(2)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.303	0.02246	0.506	1.252	1.791	49.140	49.890	0	46.075	11.536
$\mathcal{C}_2^{(2)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.303	0.02246	0.506	1.252	0.912	49.141	49.890	0	19.692	11.536
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.307	0.02243	0.548	1.335	0.798	49.149	49.898	0	23.029	2.675
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.570	1.546	0.800	49.151	49.900	0	23.142	0.001
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.583	0.129	49.152	49.902	0	47.438	0.222
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.570	1.545	1.395	49.150	49.899	0	31.908	0.001
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(2)}$	0.310	0.02243	0.521	1.363	0.986	49.148	49.898	0	18.010	6.357
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.304	0.02241	0.570	1.536	0.003	49.151	49.900	0	48.271	0.002
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_2^{(1)}$	0.304	0.02242	0.571	1.540	1.008	49.142	49.891	0	18.627	0.003
$\mathcal{C}_2^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.572	1.556	1.066	49.150	49.899	0	19.203	0.018

Header: Predictions for mixing parameters and neutrino masses at best fitting point (spanning $\sin^2\theta_{12}$ through $m_{\beta\beta}$/meV)

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Table 25 — continued from previous page

Combinations		Predictions for mixing parameters and neutrino masses at best fitting point									$\chi^2_{\rm min}$
		$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	δ_{CP}/π	ϕ/π	m_1/meV	m_2/meV	m_3/meV	$m_{\beta\beta}$/meV	
$\mathcal{C}_2^{(3)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.304	0.02241	0.572	1.556	1.042	49.150	49.899	0	18.869	0.018
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.304	0.02241	0.570	1.542	1.099	49.151	49.900	0	19.855	0.001
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_2^{(2)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.576	1.584	0.046	49.152	49.902	0	48.164	0.235
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(2)}$	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.570	1.545	1.012	49.151	49.900	0	18.632	0.001
	$\mathfrak{D}_2^{(3)}-\mathfrak{N}_1^{(3)}$	0.304	0.02241	0.570	1.534	1.006	49.151	49.900	0	18.616	0.004
$\mathcal{C}_3^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(1)}$	0.304	0.02241	0.572	1.556	1.553	49.150	49.899	0	38.765	0.018
$\mathcal{C}_3^{(1)}-\mathcal{W}_3^{(1)}$	$\mathfrak{D}_3^{(1)}-\mathfrak{N}_1^{(2)}$	0.304	0.02241	0.572	1.556	1.130	49.150	49.899	0	20.717	0.018
$\mathcal{C}_4^{(1)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.303	0.02242	0.575	1.583	0.983	49.152	49.902	0	18.746	0.219
	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_2^{(1)}$	0.303	0.02242	0.575	1.586	0.231	49.151	49.900	0	45.611	0.220
$\mathcal{C}_4^{(3)}-\mathcal{W}_1^{(1)}$	$\mathfrak{D}_1^{(1)}-\mathfrak{N}_1^{(1)}$	0.307	0.02243	0.547	1.333	1.994	49.149	49.898	0	48.269	2.727

Table 25. The same as in table 23 but for IO neutrino mass spectrum.

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