Three-body-model calculations for ^{22}C

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Abstract. There are now several well-studied instances where very neutron-rich light nuclei at or near the neutron drip-line, such as ^{6}He, ^{11}Li and ^{14}Be, have been found to have a Borromean three-body structure. Such systems are modelled effectively as a well-bound core nucleus plus two weakly-bound valence neutrons, where none of the two-body subsystems forms a bound state. It is now known that the heaviest particle-bound carbon isotope, ^{22}C, shares these properties. We discuss a development of four-body reaction model calculations, using the fast adiabatic approximation, that is particularly well-suited for a quantitative analysis of reactions of such neutron-rich nuclei with a target nucleus at beam energies of order 100-300 MeV per nucleon; energies available at new and future radioactive ion beam (RIB) facilities. The ^{22}C projectile wave function is calculated using the ^{20}C core plus two-valence neutron three-body description.

1. Introduction

Calculations of the elastic scattering, reaction and breakup observables of weakly-bound two-neutron-halo nuclei, the ground states of which can be modelled using three-body-model calculations, is a four-body reaction problem. As was shown in previous three- and four-body analyses of the reaction cross sections for halo nuclei, at energies of order 800 MeV/u where Glauber theory was employed [1, 2], the importance of breakup degrees of freedom meant that the few-body structure of the projectile must be treated explicitly to obtain a quantitative description of the reaction observables. Counter to intuition, the inclusion of breakup channels resulted in calculated reaction cross sections that were smaller than those obtained using simpler (no-breakup) one-body density-based models [1, 2]. This is also expected to be the case for reactions of the heaviest particle-bound carbon isotope, ^{22}C, for which first measurements are now possible, see e.g. [3]. This data set, for the ^{22}C reaction cross section on a proton target at relatively low energy, was used to infer a very large root-mean-squared (rms) matter radius for ^{22}C.

We discuss the first application of the coupled-channels adiabatic approach to four-body model calculations of reaction cross sections, σ_R. Our objective is to clarify the sensitivity of the elastic scattering S-matrix and reaction observables to the structure assumed for this weakly-bound dripline (and Borromean) system. We explore the ^{22}C nucleus ground-state wave function within a core plus two-valence neutron (^{20}C+n+n) three-body framework [4] and the reaction dynamics using the four-body adiabatic model, see [5].

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2. Projectile three-body model

The nucleus ^{22}C and its neutron-unbound subsystem ^{21}C remain poorly understood. Both the two-neutron separation energy from ^{22}C and the ground-state energy of ^{21}C are poorly determined. The 2003 mass evaluation [6] gives $S_{2n}(22)= 0.4(8)$ MeV and $S_{1n}(21)=-0.3(6)$ MeV, both with large uncertainties. Thus, ^{22}C is thought to have a Borromean, three-body character and is interesting structurally. The shell-model suggests that this $N =16$ nucleus will, predominantly, be described by a $\nu[1d_{5/2}]^6 [2s_{1/2}]^2$ closed neutron sub-shell configuration. The expectation therefore is that ^{22}C will have an extended, predominantly s-wave two-neutron-halo wave function leading to large reaction, nuclear and Coulomb dissociation cross sections in its collisions with a target nucleus. The latter has already been observed in a recent experiment at RIKEN [7].

A very recent analysis of new measurements of (inclusive) neutron removal reactions from the most neutron-rich carbon isotopes [8], also made at RIKEN at 240 MeV per nucleon, is consistent with this shell-model picture. The ^{22}C(-1n) data suggest a large spectroscopic factor (of ≈ 1.4) for $2s_{1/2}$ neutron removal to an unbound ^{21}C($1/2^+$) ground state, which subsequently decays by neutron emission to ^{20}C. This set of measurements also identifies a non-negligible $\nu[2s_{1/2}]^2$ component in the ^{20}C(0^+) ground-state, manifest as population of the ^{19}C($1/2^+$) ground-state in the neutron removal from ^{20}C. This complication will not be considered further in this paper.

2.1. Model wave function

Here we treat ^{22}C using the three-body model, shown schematically in Figure 1, as a ^{20}C(0^+) core+n+n system. Related ^{22}C structure studies can be found in Refs. [9, 10], on which we comment. The core is assumed to have a filled $\nu[1d_{5/2}]^6$ sub-shell. The n-n and nn-core relative orbital angular momenta are ℓ_1 and ℓ_2 in our chosen (T-basis) Jacobi coordinate set $\vec{r}$ and $\vec{\rho}$, respectively. The projectile's ground state wave function is written, in general, as a sum of
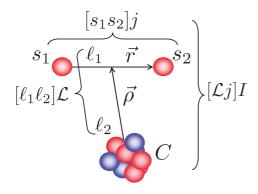

Figure 1. Schematic representation of the angular momentum decomposition and the angular momentum couplings (in the Jacobi T-basis) used for the description of the ^{20}C(0^+) core+n+n Borromean three-body projectile in its ground and (continuum) excited states. In the case of the ^{22}C($I^\pi = 0^+$) ground state, then $\mathcal{L} = j$ with $[s_1 s_2]j = 0, 1$, and thus $[\ell_1\ell_2]\mathcal{L}^\pi = 0^+, 1^+$.

individual angular momentum components

$$\Phi_0^{IM_I}(\vec{\rho},\vec{r}) = \sum_{\ell_1\ell_2\mathcal{L}j} [[\ell_1 \otimes \ell_2]_\mathcal{L} \otimes j]_{IM_I} \times \frac{U_{[[\ell_1\ell_2]\mathcal{L}j]I}(\rho,r)}{\rho r} \tag{1}$$

only a small number of which are expected to have significant probabilities, as discussed below. The two-neutron configurations are thus $^{(2j+1)}\ell_1$. For ^{22}C, $I^\pi = 0^+$ and thus $\mathcal{L} = j$ with $[\ell_1\ell_2]\mathcal{L}^\pi = 0^+, 1^+$.

2.2. Three-body model parameters and results

To solve for the three-body wave function we use the Gogny, Pires and De Tourreil (GPT) interaction [11] for V_{nn}. The neutron-^{20}C core interactions V_{n20}^ℓ are described by Woods-Saxon

plus spin-orbit interactions. For all ℓ_j-states we use radius and diffuseness parameters 1.25 fm and 0.65 fm, respectively, and a spin-orbit strength V_{so}=6.3 MeV. The depth of the central d-wave neutron-core interaction, $V_0^{\ell=2}$=42.0 MeV, was chosen to bind the neutron+^{20}C $1d_{5/2}$ state (by 2.3 MeV) while the $1d_{3/2}$ state is unbound by 1.9 MeV. This fixed potential was used for all $n\ell_j$ neutron+^{20}C configurations other than the s-wave states. For the s-states this depth is too strong, binding the $2s_{1/2}$ state. The s-state well depth, $V_0^{\ell=0}$, was thus adjusted (reduced) so that the n+^{20}C $2s_{1/2}$ state is unbound, as expected empirically. This two-body s-state potential depth and the strength of an added attractive central hyperradial three-body force, $V_{3B}(\varrho) = -V_0/(1 + [\varrho/5]^3)$, were then used as parameters to generate bound ^{22}C three-body wave functions with a range of three-body energies, allowed by the uncertainty on the evaluated ^{22}C two-nucleon separation energy. We use $V_0 = 1.6$ MeV. Each wave function is thus characterised by (a) the position of the n+^{20}C $s_{1/2}$ virtual state pole (located using a complex k-plane S-matrix search [12]) and its associated scattering length a_0 and (b) its bound three-body energy eigenvalue, $E_{3B} = -S_{2n}$. Even in the absence of the added three-body force V_{3B}, ^{22}C is found to be bound with the potential set used, provided that $a_0 \leq -46$ fm. With $V_{3B} = 0$ and the largest a_0 value, model K5 in Table 1, $S_{2n}(22) = 134$ keV.

The parameters of our calculated wave functions and their probabilities for their most important two-neutron configurations are shown in Table 1. These are all dominated by the neutron $[\nu 2s_{1/2}]^2$ configuration. These calculations use a maximum hypermomentum $K_{max} = 45$. The ^{22}C point-nucleon rms matter radii $\langle r^2 \rangle_{22}^{1/2}$ are also shown, computed using $\langle r^2 \rangle_{22} = (20/22)\langle r^2 \rangle_{20} + \langle \varrho^2 \rangle/22$, where $\langle \varrho^2 \rangle$ is the mean-squared hyperradius [13]. The rms radius of the ^{20}C core, $\langle r^2 \rangle_{20}^{1/2}$, was taken to be 2.913 fm, deduced from the neutron and proton point-particle densities of a Skyrme (SkX interaction) Hartree-Fock (HF) calculation.

A different set of potential choices was used in Ref. [9], in which the unbound $s_{1/2}$ virtual state was placed very close to the ^{21}C threshold and the $1d_{3/2}$ state was located at a significantly higher energy. In Ref. [10], similarly to here, the position of the $s_{1/2}$ virtual state was varied (there between 0 and 100 keV), however a simplified contact (δ-function) n-n interaction was used, acting only in n-n relative s-waves.

Table 1. Three-body model wave functions for the ground state of ^{22}C($I^\pi = 0^+$) calculated using the code EFADDY [4]. The probabilities associated with the dominant two-neutron components $^{(2j+1)}\ell_1$ of each wave function are shown (see also Fig. 1) as are the calculated ^{22}C matter rms radii, the computed three-body binding energy E_{3B} and the n+^{20}C $2s_{1/2}$ virtual state scattering length a_0 for each case. Wave functions K1–K4 include a three-body force with V_0=1.6 MeV. For wave function K5, $V_0 = 0$. The evaluated empirical $S_{2n}(22)$ value is 0.4(8) MeV [6].

Model	$V_0^{\ell=0}$ (MeV)	E_{3B} (MeV)	a_0 (fm)	^1S(nn)	^1D(nn)	rms (fm)	$[2s_{1/2}]^2$	$[1d_{3/2}]^2$
K1	33.5	−0.441	−333.3	0.69	0.23	5.68	0.931	0.016
K2	33.0	−0.294	−45.5	0.71	0.21	5.99	0.927	0.018
K3	32.5	−0.162	−24.4	0.73	0.19	6.32	0.923	0.020
K4	32.0	−0.045	−16.1	0.75	0.17	6.67	0.919	0.022
K5	33.5	−0.134	−333.3	0.73	0.19	6.37	0.939	0.020

3. Four-body reaction approach

We now outline the coupled channels methodology used for the four-body calculations of elastic scattering and reaction cross sections of ^{22}C, and like-projectiles described by three-body model wave functions, incident upon a nuclear target. We use and generalise the adiabatic four-body approach as has been discussed previously by Christley *et al.* [5], applied there to study ^{11}Li elastic scattering.

3.1. Adiabatic four-body model formalism

The neutrons and ^{20}C core are assumed to interact with the target (here assumed to be carbon) through (spin-independent) complex optical potentials, with the result that the total neutron spin $[s_1 s_2]j$ is a constant of the motion. The relative orbital angular momentum between the centres of mass of the projectile and the target is L and their separation is $\vec{R}$.

For each frozen configuration $(\vec{\rho}, \vec{r})$ between the neutrons and core the collision and excitation of the system is computed by the solution of an adiabatic (single, fixed energy) radial coupled channels set for the wave functions $\chi^J_{\alpha'\alpha}(R, \rho, r)$ for the chosen set of internal and orbital configurations $\alpha = \{[\ell_1 \ell_2]\mathcal{L}, L\}$. The coupled equations have total orbital angular momentum J, with couplings $[\mathcal{L}L]J$. The channel couplings are the matrix elements of the summed neutron- and core-target interactions $V(\vec{R}, \vec{\rho}, \vec{r}) \equiv V_{n1} + V_{n2} + V_{20}$ for the given $(\vec{\rho}, \vec{r})$, that is $V^J_{\alpha'\alpha}(R, \rho, r) \equiv \langle \alpha'; J|V(\vec{R}, \vec{\rho}, \vec{r})|\alpha; J \rangle$. Details of the computation of these couplings, using a generalised multipole expansion, can be found in Ref. [5]. The result of the coupled channels calculations is a (ρ, r)-dependent matrix of amplitudes, $M^J_{\alpha';\alpha}(\rho, r)$, for the outgoing waves in each final-state configuration α', computed from the asymptotic (in R) forms of the $\chi^J_{\alpha'\alpha}$.

Elastic scattering is now determined by the matrix elements of these configuration-dependent amplitudes with the projectile ground state wave function, Eq. (1), and its components. Specifically, we require the wave function component-weighted ρ- and r-integrated amplitudes

$$\mathcal{M}^{jJI}_{\alpha'\alpha} = \int d\rho \int dr\, U^*_{[\ell'_1 \ell'_2]\mathcal{L}'j]I}(\rho, r) M^J_{\alpha';\alpha}(\rho, r) U_{[[\ell_1 \ell_2]\mathcal{L}j]I}(\rho, r)\ . \tag{2}$$

The formalism simplifies significantly in our ^{22}C case, since $I^\pi = 0^+$. Now the physical total angular momentum of the system $\mathcal{J}$, with couplings $[L\,I]\mathcal{J}$, is equal to L and, as noted earlier, $\mathcal{L} = j$. In this case the physical elastic partial-wave T-matrix elements are a weighted sum of those coupled channels amplitudes that are in common with configurations in the projectile ground state wave function. Explicitly,

$$T_{\mathcal{J}} = \sum_{\ell_1 \ell_2 \ell'_1 \ell'_2 Jj} \frac{(2J+1)}{(2j+1)(2\mathcal{J}+1)} \mathcal{M}^{jJ0}_{[\ell'_1 \ell'_2]j,\mathcal{J}\,[\ell_1 \ell_2]j,\mathcal{J}}\ , \tag{3}$$

from which the elastic S-matrix elements and the reaction cross section follow.

3.2. Reaction model inputs

We perform calculations for ^{22}C incident on a ^{12}C target at 300 MeV per nucleon. The neutron- and ^{20}C core-target interactions were calculated using the single-folding $t\rho_t$ model (for nucleon-target) and double-folding $t\rho_c\rho_c$ models (for core-target). The inputs needed were the core (c) and target (t) point neutron and proton one-body densities and an effective nucleon-nucleon (NN) interaction. The ^{20}C densities were taken from a spherical Skyrme (SkX interaction) HF calculation [14]. The density of the carbon target was taken to be of Gaussian form with a point-nucleon root-mean-squared radius of 2.32 fm. A zero-range NN effective interaction was used, its strength calculated from the free neutron-neutron and neutron-proton cross sections at the beam energy and from the real-to-imaginary ratios of the NN forward scattering amplitudes

at 300 MeV, interpolated (using a polynomial fit) from the values tabulated by Ray [15]. A recent analysis by Bertulani and De Conti [16] confirms that corrections to this procedure due to Pauli blocking corrections to the NN effective interaction are very small at the energies of interest here.

3.3. Reaction cross section results

The coupled-channels adiabatic calculations were performed including s-wave and both dipole p-wave ($\ell_1 = 0$, $\ell_2 = 1$) and d-wave ($\ell_1 = 0$, $\ell_2 = 2$ and $\ell_1 = 2$, $\ell_2 = 0$) breakup channels, referred to as *spdd* calculations in Ref. [5]. The ^{22}C reaction cross sections were then calculated for the wave functions of subsection 2.2, as are summarised in Table 1. Given the dominance of the ^1S(nn) and $(2s_{1/2})^2$ configurations, these first calculations used only the $[[\ell_1 \ell_2]\mathcal{L} j]I = [[0\,0]0\,0]0$ component of the ^{22}C ground-state wave functions, renormalised to unity. Full calculations will be presented in a future, more extended publication. The calculated cross sections are shown in Table 2.

Table 2. Calculated reaction cross sections for ^{22}C, incident on a carbon target at 300 MeV per nucleon, for the three-body model wave functions K1–K4 listed in Table 1.

Model wave function	K1	K2	K3	K4
E_{3B} (MeV)	-0.441	-0.294	-0.162	-0.045
σ_R (mb)	1308.66	1319.23	1330.11	1341.01

4. Summary

We have presented first calculations (using a three-body structure model and a four-body reaction treatment) of the reaction cross section for the last particle-bound carbon isotope ^{22}C. A set of model three-body wave functions were obtained, all of which have plausible S_{2n}, and which have different values for the scattering length a_0 for the virtual $2s_{1/2}$-state in the unbound n+^{20}C two-body subsystem. We have also outlined the formulation of the ^{22}C elastic scattering S-matrix from which we calculate the reaction cross section. The calculations include the $s-$, $p-$ and $d-$wave four-body breakup channels of these very weakly-bound projectile systems using the coupled-channels adiabatic approach. This approach is well-suited to the energy of the present study (300 MeV per nucleon) and to current and future experimental programmes at the new generation of fragmentation-based RIB facilities, such as RIBF, FRIB and GSI/FAIR.

The initial calculations presented already reveal the sensitivity of the reaction cross sections to the assumed structure of the projectile, which encourage further study. Full details of the method, the features of the ^{22}C wave function and the reaction calculations, that include a full treatment of the smaller components of the ground state wave functions, will be presented in a forthcoming publication.

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