IMPLICATIONS OF CPT VIOLATION IN NEUTRAL KAON DECAYS

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From our present knowledge of weak interactions, the three discrete symmetries C, P and T are not exact symmetries in our universe, and the same applies to the combined symmetries CP, CT and PT. This fact is well accommodated in the Standard Model (SM) and is well established experimentally. However, in the framework of a local field theory of Lorentz invariance, and of the usual spin-statistics requirement, any order of the triple product CPT of these discrete symmetries automatically represents an exact symmetry expressed by the CPT theorem [1]. In quantum field theory, the CPT operation implies that when we change particles to antiparticles, keeping their momentum fixed and reversing their helicity, all matrix elements of the interaction should become their appropriate complex conjugates. The most accurate CPT tests, such as lifetime and mass equalities between particles and antiparticles, do not verify the full extent of CPT invariance [2] since they do not involve helicity or complex conjugation. The $K^0\bar{K}^0$ complex is a superior tool for making detailed tests of CPT symmetry. Such tests are almost inconceivable at external beams and will hopefully be made possible by the existence of high-intensity, well-defined strangeness kaons [3] at the CERN Low-Energy Antiproton Ring (LEAR).

The parameters of CP and CPT invariance in the kaon system can be found in many textbooks and reviews [4], and only a brief outline will be presented here for clarity and completeness. In the $\Delta S = 2$ transitions, CP violation is described by the non-vanishing parameter ε, whilst a possible CPT non-conservation would be expressed through the parameter δ. Although ε is measured [5] to be $\mathrm{Re}(\varepsilon) = (1.621 \pm 0.088) \times 10^{-3}$, there is no evidence for a non-vanishing value of δ. Assuming CPT invariance in the strong interactions one can introduce for the $\Delta S = 1$ current, the amplitudes A_I and B_I [6,7]. These are the CPT-conserving and CPT-violating amplitudes of isospin I in the decay amplitudes of the neutral kaon, for the two-pionic final state into the two possible isospin states $I = 0$ and $I = 2$ (table 1).

Table 1
The amplitudes of neutral kaon decays as a function of CPT, CP and T.

	Violation of CPT	Violation of CP	Violation of T
$\mathrm{Re}(A_I) \neq 0$	No	No	No
$\mathrm{Im}(A_I) \neq 0$	No	Yes	Yes
$\mathrm{Re}(B_I) \neq 0$	Yes	Yes	No
$\mathrm{Im}(B_I) \neq 0$	Yes	No	Yes

$$\left\langle I \middle| H_w \middle| K^0 \right\rangle = (A_I + B_I)e^{i\delta_I} , \tag{1}$$

$$\left\langle I \middle| H_w \middle| \bar{K}^0 \right\rangle = \left(A_I^* - B_I^*\right)e^{i\delta_I} , \tag{2}$$

where H_w is the Hamiltonian of the weak interaction.

A measure of CP and CPT violation in the decay amplitudes are the phenomenological parameters ε' and a, defined [6] as

$$\varepsilon' = \frac{1}{\sqrt{2}} \frac{\langle 2|H_w|K_2\rangle}{\langle 0|H_w|K_1\rangle} ; \quad a = \frac{\langle 0|H_w|K_2\rangle}{\langle 0|H_w|K_1\rangle} . \tag{3}$$

In the following we will assume that

$$\left|\mathrm{Im}(B_I)\right| << \left|\mathrm{Re}(A_I)\right| , \tag{4}$$

because a large value of $\mathrm{Im}(B_I)$ would contradict the unitarity of the S matrix and the hermeticity of the Hamiltonian, which seems very unlikely [6]. Also, the generalised Wu-Yang phase convention [8] is adopted, which sets

$$\mathrm{Im}(a) = 0 . \tag{5}$$

The parameters ε' and a can be expressed through A_I and B_I as follows:

$$\varepsilon' = \frac{1}{\sqrt{2}} \frac{\mathrm{Re}(B_2) + i\,\mathrm{Im}(A_2)}{\mathrm{Re}(A_0) + i\,\mathrm{Im}(B_0)} e^{i(\delta_2 - \delta_0)} \tag{6}$$

$$\approx \frac{1}{\sqrt{2}} \left[\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_0)} e^{i(\delta_2-\delta_0)} + \frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_0)} e^{i[\delta_2 - \delta_0 + (\pi/2)]} \right] , \tag{7}$$

$$a = \frac{\mathrm{Re}(B_0) + i\,\mathrm{Im}(A_0)}{\mathrm{Re}(A_0) + i\,\mathrm{Im}(B_0)} \approx \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} . \tag{8}$$

The relative strength of $\Delta I = 1/2$ on $\Delta I = 3/2$ transitions is given by the parameter ω

$$\omega = \frac{1}{\sqrt{2}} \frac{\langle 2|H_w|K_1\rangle}{\langle 0|H_w|K_1\rangle} ,$$

$$\approx \frac{1}{\sqrt{2}} \frac{\mathrm{Re}(A_2) + i\,\mathrm{Im}(B_2)}{\mathrm{Re}(A_0) + i\,\mathrm{Im}(B_0)} e^{i(\delta_2 - \delta_0)} , \tag{9}$$

$$\approx \frac{1}{\sqrt{2}} \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} e^{i(\delta_2 - \delta_0)} , \tag{10}$$

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with [6]

$$|\omega| \approx 3.19 \times 10^{-2} \quad ; \quad \varphi_{\mathrm{w}} = (\delta_2 - \delta_0) \quad , \qquad (11)$$

where δ_0 and δ_2 are the phase shifts due to the final-state interactions. The measurements of $(\delta_2 - \delta_0)$ vary drastically with the experimental method. The significant inconsistencies among the apparently precisely measured values are discussed in ref. [9]. The world average [5] is

$$(\delta_2 - \delta_0) = (-45 \pm 15)^\circ \ . \qquad (12)$$

The parameter a is a measure of CPT non-conservation. The parameter ε' describes both CPT and CP violation in the K^0 decay, with a phase difference of 90° between the CPT and CP-violating components. It is therefore convenient to separate the CP-violating (but CPT-conserving) part of ε' from the CPT-violating (and CP-violating) part, by decomposing ε' into $\varepsilon'_{\parallel}$ and $\varepsilon'_{\perp}$, where the indices refer to the direction determined by the direction perpendicular to $(\delta_2 - \delta_0)$. One can see from eq. (7) that a non-vanishing value of $\varepsilon'_{\parallel}$ would imply CP violation [$\mathrm{Im}(A_2) \neq 0$], whilst a non-zero value of $\varepsilon'_{\perp}$ would be a signal for CPT violation in the decay [$\mathrm{Re}(B_2) \neq 0$].

From above properties of the parameters a, ε' and ω (eqs (6)–(10)) we distinguish the two cases:

- if the $\Delta I = 1/2$ rule is also valid for CPT-violating amplitudes, i.e.

$$\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(B_0)} = \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \ ,$$

then $\varepsilon'_{\perp} = a\omega$;

- if the $\Delta I = 1/2$ rule is not valid for the CPT-violating amplitudes, then no prediction of the magnitude of $(\varepsilon'_{\perp} - a\omega)$ is possible.

The measurable quantities are the ratios of the amplitudes of the physical eigenstates K_S and K_L into two pions,

$$\eta_f = \frac{\langle f|H|K_L\rangle}{\langle f|H|K_S\rangle} = |\eta_f| e^{i\varphi_f} \ , \qquad (13)$$

$$f = \pi^+\pi^-, \ \pi^0\pi^0 \ .$$

They depend on the phenomenological parameters as follows (neglecting ε^2 and δ^2 compared with 1):

$$\eta_{+-} = \frac{a+\varepsilon'}{1+\omega} + \varepsilon - \delta \ ,$$

$$\eta_{00} = \frac{a-2\varepsilon'}{1-2\omega} + \varepsilon - \delta \ . \qquad (14)$$

All experiments with K_S and K_L beams, aiming to detect CP violation in the decay of the neutral kaon system [10,11], measure the parameter

$$R = \frac{\Gamma(K_L \to 2\pi^0)/\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to 2\pi^0)/\Gamma(K_S \to \pi^+\pi^-)}$$

$$= \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \ , \qquad (15)$$

which from eq. (14) can be expressed as

$$R \cong 1 - 6\,\mathrm{Re}\left[\frac{\varepsilon' - a\omega}{(\varepsilon-\delta)(1-\omega)}\right]$$

$$= 1 - 6\frac{|(\varepsilon' - a\omega)_{\parallel}|}{|\varepsilon - \delta|} \ , \qquad (16)$$

and is a measure of the projection of $(\varepsilon' - a\omega)$ on $(\varepsilon - \delta)$. Let us assume that δ is small compared with ε, which means that the angle of ε is approximately equal to the superweak angle

$$\varphi_\varepsilon \approx \varphi_{\mathrm{sw}} = \mathrm{arctg}\frac{2(m_L - m_S)}{\gamma_S - \gamma_L}$$

$$= (43.72 \pm 0.14)^\circ \ . \qquad (17)$$

The equality would hold for $\delta = 0$ [6]. Since the value of φ_{sw} and the phase shift $(\delta_2 - \delta_0)$ (eq. (12)) differ just by approximately 90° $(\delta_2 - \delta_0 + \pi/2 \approx \varphi_{\mathrm{sw}})$, the direction perpendicular to $(\delta_2 - \delta_0)$ coincides with the direction given by $\varphi_{\mathrm{sw}} \approx \varphi_\varepsilon$. Therefore, the projection of $(\varepsilon' - a\omega)$ on $(\varepsilon - \delta)$ is proportional to $(\varepsilon' - a\omega)_{\parallel}$. Since $\varepsilon'_{\perp}$ and ω have the same phase $e^{i(\delta_2 - \delta_0)}$ (eqs (7) and (10)), and since a is real (eq. (5)), the component $(a\omega)_{\parallel}$ is vanishing and the quantity $(\varepsilon' - a\omega)_{\parallel}$ is equal to $(\varepsilon')_{\parallel}$ and is nearly independent of $a\omega$. Taking into account eq. (4), $(\varepsilon' - a\omega)_{\parallel}$ is a measure of $\mathrm{Im}(A_2)$. Therefore, R is a measure of CP violation in the decay and is very insensitive to a possible CPT violation.

A more recent approach to the measurement of CP violation is the method of measuring the interference of neutral kaons with tagged K^0 and $\bar{K}^0$ [3]. One way of determining ε' is to measure the integral asymmetry for the $\pi^+\pi^-$ and $\pi^0\pi^0$ final states, defined as

$$I_f(t_0) = \frac{\int_0^{t_0}\bar{R}_f(t)dt - \int_0^{t_0}R_f(t)d}{\int_0^{t_0}\bar{R}_f(t)dt + \int_0^{t_0}R_f(t)d}$$

$$= 2\,\mathrm{Re}(\varepsilon) - 4\,\mathrm{Re}(\eta_f) \ . \qquad (18)$$

Using eq. (14), we can show that

$$R_I = \frac{I_{00}}{I_{+-}} = 1 - 6\frac{\mathrm{Re}[\varepsilon'(1+\omega) - a\omega]}{\mathrm{Re}(\varepsilon)} \ . \qquad (19)$$

Here we measure $\mathrm{Re}(\varepsilon' - a\omega)$ and not, as in the case of R (eq. (16)), the projection of $(\varepsilon' - a\omega)$ on ε, i.e. $\mathrm{Re}[(\varepsilon' - a\omega)_{\parallel}]$. So

R_I is sensitive not only to $(\varepsilon' - a\omega)_{\parallel}$ but also to $(\varepsilon' - a\omega)_{\perp}$ and therefore to a possible CPT violation in the decay. Depending on whether the $\Delta I = 1/2$ rule in the CPT-violating decay amplitudes is assumed to be valid, two cases have to be distinguished:

– if this rule holds, then $(\varepsilon'_{\perp} - a\omega)$ is equal to zero (see above) and therefore R_I is only a measure of CP violation, since

$$R_I = 1 - 6\frac{\mathrm{Re}\left[\varepsilon'_{\parallel} + (\varepsilon'_{\perp} - a\omega) + (\varepsilon'\omega)\right]}{\mathrm{Re}(\varepsilon)} = \frac{\mathrm{Re}(\varepsilon'_{\parallel})}{\mathrm{Re}(\varepsilon)}, \qquad (20)$$

if one neglects $\varepsilon'\omega$;

– if this rule does not hold, then $a\omega$ can be estimated, using eq. (14), from the experimental values of $\mathrm{Re}(\varepsilon)$, $\mathrm{Re}(\eta_{+-})$, and $\mathrm{Re}(\eta_{00})$ [5]: $\eta_{+-} - \varepsilon - 1/3(\eta_{+-} - \eta_{00}) = a(1 + \omega) - \delta \approx a - \delta$, which yields $\mathrm{Re}(a - \delta) = (-1.2 \pm 0.8) \times 10^{-4}$. If $\delta = 0$, then this leads to $\mathrm{Re}(a\omega) \leq -0.6 \times 10^{-5}$, which is of the order of the theoretically predicted value for $\mathrm{Re}(\varepsilon') \leq O(10^{-6})$.

As discussed in ref. [6], a way to test CPT invariance is to measure the phase difference between η_{+-} and η_{00}, i.e. $\Delta\varphi = (\varphi_{+-} - \varphi_{00})$, which represents a measure of $(\varepsilon' - a\omega)_{\perp}$. If we assume that the $\Delta I = 1/2$ rule is valid also for the CPT-violating amplitudes and therefore $\varepsilon'_{\perp} = a\omega$, the phase difference $(\varphi_{+-} - \varphi_{00})$ is zero even though CPT might be violated. If the $\Delta I = 1/2$ rule is not valid for the CPT-violating amplitudes, then $\Delta\varphi$ is sensitive to $\varepsilon'_{\perp}$ but not to a, since a is suppressed by ω.

An unambiguous test of CPT conservation can be achieved by measuring the phase φ_{+-} relative to φ_{sw}

$$\varphi_{+-} = \varphi_{sw} - \frac{\left[\varepsilon'_{\perp}\sin(\varphi_{sw}) + a\right]\cos(\varphi_{sw})}{|\eta_{+-}|} . \qquad (21)$$

The phase of η_{+-} depends directly on the parameter a, without the ω suppression occurring in the measurement of $(\varphi_{+-} - \varphi_{00})$. Therefore, the absolute value of φ_{+-}, which in the case of CPT invariance should be equal to the superweak angle, measures CPT violation directly in the kaon decays via both a and $\varepsilon'_{\perp}$.

We conclude that the experimental quantities R (from K_S/K_L beams) and R_I (from tagged $K^0/\bar{K}^0$) do not depend in the same way on the phenomenological parameters describing CP and CPT violation. If the experimental results of measuring R and R_I were to show a significant difference, this could be explained either by CPT violation in the decay, assuming simultaneous violation of the $\Delta I = 1/2$ rule in the CPT-violating amplitudes or by a value $(\delta_2 - \delta_0) \neq -45°$, which would lead to a component of ε' orthogonal to the direction of ε (assuming $\varphi_{\varepsilon} \approx \varphi_{sw}$) even though CPT holds. Therefore, a precise value of $(\delta_2 - \delta_0)$ would be important for the understanding of the neutral kaon system.

The quantity of $(\varphi_{+-} - \varphi_{00})$ is sensitive to $(\varepsilon' - a\omega)_{\perp}$. In the case of CPT violation, the validity of the $\Delta I = 1/2$ rule in the CPT-conserving and violating amplitudes implies a vanishing value of $(\varepsilon' - a\omega)_{\perp}$. Therefore, even if $(\varphi_{+-} - \varphi_{00})$ could be measured to be exactly zero, this would not allow any conclusion to be drawn regarding CPT invariance.

The significant test of CPT violation in the decay of the neutral kaon can only be achieved by measuring experimental quantities that depend directly on the parameter a, without any ω suppression. Since the phase φ_{+-} depends directly on a, the comparison of its measured value with the superweak angle would be a good test of CPT invariance.

By measuring integral asymmetries in the K^0–$\bar{K}^0$ system, one is sensitive not only to $\mathrm{Re}(\varepsilon'/\varepsilon)$ but also to the parameter a, allowing direct conclusions regarding the values of ε' and a.

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References

[1] J.S. Bell, proc. Royal Soc. A231 (1955) 479;
R. Jost, Helv. Phys. Acta 31 (1958) 263;
G.L. Lüders, Ann. Phys. 2 (1957) 1.

[2] T.D. Lee, Particle physics and introduction to field theory, Harwood Academic Publishers, New York (1981).

[3] L. Adiels et al., proposals CERN/PSCC 85–6 P82 (1985) and CERN/PSCC 85–3 P82 Add.1 (1985); Memorandum CERN/PSCC 86–34 M263 (1986).

[4] T.D. Lee and C.S. Wu, Ann. Rev. Nucl. Sci. 16 (1966) 511;
E.D. Commins and Ph.H. Bucksbaum, Weak interactions of leptons and quarks, Cambridge Univ. Press, Cambridge (1983).

[5] Particle Data Group, Phys. Lett. B170 (1986).

[6] V.V. Barmin et al., Nucl. Phys. B247 (1984) 293.

[7] C. Jarlskog, private communications.

[8] T.T. Wu and C.N. Yang, Phys. Rev. Lett. 13 (1964) 380.

[9] H. Wahl, proc. Rare Decay Symposium, Vancouver, World Scientific, Singapore (1988) 281.

[10] H. Burckhardt et al., Phys. Lett. B206 (1988) 169.

[11] M. Karlsson et al., Phys. Rev. Lett. 64 (1990) 2976.

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