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Neutrino masses and oscillations

in theories beyond the Standard Model

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Abstract

Neutrinos are very light fermions, which have three flavour states (ν_e, ν_μ, ν_τ) and three mass states (ν_1, ν_2, ν_3). Being neutral leptons, they participate only in the weak and gravitational interactions. Gravitational effects are negligibly small, since neutrinos are light and consequently ultrarelativistic. Until the detection of neutrino oscillations during the turn of the millennium, neutrinos were thought to be massless. Now we know better. Neutrino mixing is a result of a mismatch between neutrino flavour and mass bases, which can be easily implemented to the Standard Model (SM), but their masses cannot. On one hand, insertion of Dirac neutrino mass terms requires the existence of right-handed neutrinos, which are not observed yet. On the other hand, insertion of Majorana neutrino mass terms would imply that neutrino is its own antiparticle. This could be confirmed upon the discovery of neutrinoless double beta decay, which is also unobserved today.

Light neutrino masses lead to lepton flavour violating (LFV) decays and nonzero magnetic moment. These are unobserved due to suppression by small neutrino masses. In contrast, another consequence of neutrino masses - neutrino oscillation - was spectacularly observed, earning the 2015 Nobel Prize in Physics. Neutrino oscillation experiments are approaching precision measurements on mass squared splittings and mixing angles, but these are partially spoiled by degeneracy of θ_{23} mixing angle octant, ambiguousness of neutrino mass ordering and low confidence limit on leptonic CP phase angle. A high-luminosity long-baseline neutrino oscillation experiment is needed to decisively constrain the parameter space and guide neutrino physics to a new era. Studies on non-standard interactions (NSI) allow us to quantify the effects of new physics as a perturbation from the standard three neutrino framework. We have developed a formalism in the case of matter NSI in long baseline neutrino oscillation experiments and derived baseline-dependent bounds for the prospects of a future discovery of NSI. We have also chosen a popular neutrino mass generation model called Type II seesaw and studied how the next-generation long baseline neutrino oscillation experiment, DUNE, can constrain the model parameters.

While the experiments at the Large Hadron Collider (LHC) have found hints of new physics beyond the SM in addition to SM Higgs boson, these hints are far from conclusive evidence. Thus it is increasingly likely that there are no new physics at the TeV energy scale. Most neutrino mass models also induce LFV decays. Constraints from them push the limits of new physics of neutrino mass models to higher energies than center-of-mass energy of LHC. New physics may manifest itself also as nonstandard interactions or nonunitary mixing. We studied the likelyhood of confirming neutrinophilic two Higgs doublet model at the LHC and found out that $\mathbb{Z}_2$ conserving version of the model has been ruled out by current data, and constrained the parameters of the theory further.

Current experimental efforts on neutrino physics are concentrated on the precise measurements of neutrino oscillation parameters. On both short- and long-baseline experiments, neutrino mass models will induce subleading corrections to neutrino flavour transition. Current hints point to a existence of an eV-scale sterile neutrino. Neutrino mass models predict the vanilla seesaw scale to be at $\sim 10^{11}$ GeV or higher, but at such a high scale, Higgs mass must be fine-tuned. Dark matter searches favor keV scale sterile neutrinos. These incompatible mass scales show that the search for neutrino physics beyond the SM is far from straightforward.

Keywords. Neutrino, neutrino oscillation, neutrino mixing, CP violation, neutrino mass, seesaw mechanism, nonstandard interactions, neutrinophilic Higgs, beyond the Standard Model.

Preface

> I see now that the circumstances of one's birth are irrelevant; it is what you do with the gift of life that determines who you are.
>
> *Mewtwo* (1999)

In Lewis Carroll's book, *Alice's Adventures in Wonderland* (1865), Alice falls through a rabbit hole to fantasy world full of strange nonsensical characters and events. During the Master's studies I stumbled on elementary particle physics and quantum field theory, finding that our present knowledge of Nature is clearly deficient. There was a huge rabbit hole of unknown ahead. I didn't fall into it until I started my doctoral studies. I felt like Alice, emerging to particle wonderland, where I met invisible schizophrenic neutrinos, loyal hard-working Higgses, familiar charged leptons with differently tasting flavours and looking up and down, from top to bottom, many more unbelievably strange and charming particles.

Acknowledgements

First and foremost, I thank my supervisor, Katri Huitu for her patience, guidance, knowledge and support in every step on the ladder leading to the crown jewel, the graduation. My collaborators, Jukka Maalampi, Subhadeep Mondal, Santosh Kumar Rai and Sampsa Vihonen have my heartfelt gratitude for their enlightening help and expertise. I also thank my monitoring group members Venus Keus and Mikko Voutilainen for their guidance, optimism and support.

Kari Rummukainen first introduced me to neutrino physics. Without his introduction, my MSc thesis and this thesis would have completely different title. Also, Ilpo Vattulainen gave a hypermotivational talk at Pecha Kucha event in 29.9.2017. Due to his talk on that particular day and event, I firmly decided I will continue my academic career after I finish my doctoral degree. Therefore I give my special thanks to Kari and Ilpo.

I would like to thank my fellow collagues of Department of Physics in University of Helsinki, who assisted and cheered me up during the more pessimistic phases. My sincere thanks to George Bulmer, Jaana Heikkilä, Sofia Patomäki and Marco Zatta.

I am especially grateful to Magnus Ehrnrooth foundation, whose support provided financial stability for the duration of my graduate studies. University of Helsinki and Helsinki Institute of Physics provided much appreciated financial support for travel and opportunities for teaching undergraduate courses.

Finally, I thank my cherished wife Salla for always standing up with me, even during the agonizing eras. Lastly, I thank my father Kullervo, who is a materialization of an ideal, responsible and diplomatic parent and grandparent and whom I admire and respect more and more every year that passes.

Outline of thesis

Chapter 1 contains the history of neutrino physics and paints an overall view of the Standard Model. It also briefly displays the shortcomings of the Standard Model. Chapter 2 demonstrates the consequences of neutrino masses and demonstrates the weakness of these effects. Chapter 3 describes the phenomenology of three-neutrino oscillations and moves beyond it to nonstandard and nonunitary approaches. Chapter 4 showcases the most important ways to generate mass for neutrinos via the Seesaw mechanism.

List of papers

[1] K. Huitu, T. J. Kärkkäinen, J. Maalampi and S. Vihonen
Constraining the nonstandard interaction parameters in long baseline neutrino experiments,
Phys. Rev. D **93**, 053016, arXiv:1601.07730 [hep-ph].

[2] K. Huitu, T. J. Kärkkäinen, J. Maalampi and S. Vihonen
The effects of triplet Higgs bosons in long baseline neutrino experiments,
Phys. Rev. D **97**, 095037, arXiv:1711.02971 [hep-ph].

[3] K. Huitu, T. J. Kärkkäinen, S. Mondal and S. K. Rai
Searching for a neutrinophilic scalar sector in multilepton channels,
Phys. Rev. D **97**, 035026, arXiv:1712.00338 [hep-ph].

Contribution to papers

Paper [1]. I wrote the first draft. I wrote C programs for GLoBES and carried data analysis jointly with Mr. Vihonen.

Paper [2]. I produced all the plots and did most of the code and data analysis with MATLAB.

Paper [3]. I utilized Mathematica, SARAH, MadGraph, Pythia, Delphes and MadAnalysis jointly with Dr. Mondal. I carried data analysis with MATLAB for the $\mathbb{Z}_2$ conserving case of the model we considered.

Interpretation and writing of the results were done jointly in all the publications.

Notations and conventions

I abuse the terminology slightly by calling Lagrangian density simply Lagrangian. Einstein summation convention is in use. There repeated Latin indices are summed over from one to three and repeated Greek indices are summed over from zero to three or over lepton flavours (e, μ, τ). Complex conjugation, Hermitian conjugation, charge conjugation and matrix transpose are denoted by star (*), dagger (†), lower case letter c and capital letter T in superscript, respectively. Three- and eight-component vectors are in **bold**. Imaginary unit is denoted by i, which is also sometimes a summation index. Spatial indices are Latin and spacetime indices Greek. Minkowski spacetime metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is used. Spacetime point is denoted $x = (t, \mathbf{x})$.

Unit and zero matrices are denoted by I and 0, respectively. If the dimensionality is specified, it is placed in the subscript and $I_n \equiv I_{n\times n}$. Diagonal matrix is denoted by $\text{diag}(a, \cdots)$ with the diagonal elements placed inside parenthesis, separated by commas. If A and B are matrices, then $A < B$ means that the matrix elements of A are smaller than corresponding matrix elements of B. Placement of a matrix in a denominator signifies matrix inverse. When a sum of an expression and its Hermitian conjugate is confronted, sometimes only the first part is explicitly shown, the second term being shortened to abbreviation "h.c.". Adjoint spinor is defined as $\overline{\Psi} \equiv \Psi^\dagger \gamma^0$. Feynman slash notation is adopted: $\not{a} \equiv \gamma^\mu a_\mu$.

Natural unit system is in place, where reduced Planck constant $\hbar$, speed of light in vacuum c and Boltzmann constant k_B are defined to be dimensionless and one. Equal by definition and identically true is denoted by $\equiv$. Order-of-magnitude estimation is denoted by $\sim$ and big $\mathcal{O}$ notation. Base-10 logarithm is abbreviated by lg and natural logarithm by ln. Napier's constant is denoted e, while elementary charge and electron flavour by e.

Chapter 1

Introduction

> The only way of discovering the limits of the possible is to venture a little way past them into the impossible.
>
> *Arthur C. Clarke*
> Profiles of the Future (1962)

Standard Model (SM) is the pinnacle of the human knowledge of elementary particle physics. This chapter begins with a historical tour on the world of neutrino physics. As neutrinos are Standard Model (SM) particles, history of neutrino physics naturally coincides with the history of the SM. The Lagrangian density of the SM is discussed in detail, concentrating on electroweak sector. In this chapter, neutrinos are treated as massless particles. Currently we know this is not true. Massive neutrino phenomenology will be discussed in the next chapter. This chapter concludes with a short review of existing problems in the SM.

1.1 History

1.1.1 History of non-oscillation neutrino physics

Chadwick [4] discovered in 1914 that β decay energy spectra were continuous. The decay was back then believed to be a two-body decay, resulting in a Dirac delta-distribution smeared by detector. Unbeknownst to physicists, the missing energy was stolen by a neutrino, a third particle ejected in β decay. This was proposed by Pauli [5] in 1930, who called the proposal a desperate remedy, since in those times it was unusual to postulate the existence of a new particle in order to make an anomaly disappear. Indeed, Bohr had even suggested that conservation of energy does not hold in weak interactions.

In 1934, Fermi [6] formulated the theory of weak interactions, which made it possible to calculate the neutrino cross section. Pauli, Bethe and Peierls concluded that there is no possible way of observing the neutrino. After the development of nuclear weaponry, the detection of a neutrino was possible [7]. As a matter of fact, one of the first proposals to detect a neutrino by Pontecorvo [8] involved detonating a small nuclear bomb. The plan didn't come to fruition, but a nuclear reactor instead was used as an intense $\overline{\nu}_e$ source. Using inverse β decay, a neutrino was detected in 1956 by Cowan, Reines *et al* [9].

Weak interaction was predicted to contain parity violation by Lee and Yang [10] in 1956 and confirmed by Wu *et al* [11] in 1957. Fermi's theory conserved parity, so Feynman, Gell-Mann, Marshak and Sudarshan [12, 13] upgraded it to V−A theory in 1958, where the inclusion of axial vectors results in the desired parity violation. In the same year, Goldhaber *et al* [14] showed that neutrinos are exclusively left-handed particles, demonstrating maximal parity violation in weak interactions. Later, the theory was combined with QED by Glashow, Weir, Weinberg and Salam [15–19] by combining QED and weak interactions to electroweak theory in the 1960s. Removal of the four-fermion vertex meant that the electroweak interaction could possibly be renormalizable. This was proven by 't Hooft and Veltman [20–22] in 1972.

Muon neutrino was discovered by Lederman *et al* [23] in 1962. The experiment observed neutrinos which were produced from charged pion decay together with muons. These muon-associated neutrinos upon interaction with target material produced only muons, proving the existence of a second type of neutrino. Three generations of matter was proposed in 1973 by Kobayashi and Maskawa [24], prompting a search for a tau neutrino. Indirect evidence for three light active neutrino flavours surfaced in 1989, upon the analysis of Z boson decay in ALEPH experiment. Tau neutrino was eventually detected in 2000 by DONUT collaboration [25].

After the first detection of supernova neutrinos in 1987 by various experiments and geoneutrinos in 2005 by KamLAND [26], some experimental efforts are underway to detect the cosmic neutrino background, first proposed by Weinberg [27] in 1962. In fact, astrophysics and cosmology have brought new insight to neutrino physics. The newest PLANCK upper limits for neutrino masses [28] ($\sum m_\nu \lesssim 0.12$ eV) are stricter than the limits imposed by terrestial experiments by one order of magnitude.

1.1.2 History of neutrino oscillations

Neutrino oscillation was first proposed in 1957 by Pontecorvo [29, 30]. By then, only $\overline{\nu}_e$ had been detected, and he proposed neutrino-antineutrino oscillation $\nu_e \leftrightarrow \overline{\nu}_e$, inspired by kaon-antikaon oscillations $K^0 \leftrightarrow \overline{K}^0$. After the ν_μ was detected in 1962, Maki, Nakagawa and Sakata published in the same year their proposal [31] on flavor oscillations, like $\nu_e \leftrightarrow \nu_\mu$.

First hint of neutrino oscillations appeared during the late 1960s, when Davis *et al.* detected a deficit of electron neutrino flux from the sun [32]. Only 1/3 of electron neutrinos from the sun were detected. This could have meant that the rest of electron neutrinos had oscillated to different flavors invisible to the detector or the stellar nucleosynthesis theory developed by Bethe [33] in 1939 had to be altered. This discrepancy was known as **solar neutrino problem**.

In the absence of confirmation of neutrino oscillations, the efforts on the theory side marched on. In 1968 Pontecorvo discovered [34] that the existence of at least one neutrino flavor transition channel implies that at least one neutrino mass state has nonzero mass - the oscillation frequency for $\nu_i \leftrightarrow \nu_j$ transition is proportional to $|m_{\nu_i}^2 - m_{\nu_j}^2|$.

As the neutrinos have extremely tiny cross sections, they smash through matter almost without any interactions, as Bethe calculated in 1930s. For this reason the discovery of large effects of matter potential to oscillation parameters by Wolfenstein [35] in 1979 was very surprising. In 1985 Mikheyev and Smirnov [36] noticed that slow decrease of matter density can enhance neutrino mixing to even a maximal case. This effect is currently known as MSW effect by the initials of the physicists. It can be used to determine the average electron density of matter through which the neutrinos travel.

Neutrino oscillation was first detected in 1998 by Super-Kamiokande collaboration [37], where a deficit of muon neutrinos was detected. The detector was designed to detect atmospheric neutrinos. When high-energy cosmic rays collide with air molecules in the upper atmosphere, a large number of unstable elementary and composite particles are produced, and the most relevant to neutrino studies are pions. Lion's share of the neutrinos are produced from pion decay chain, producing twice as many muon-flavored neutrinos than ν_e. An equal amount of both neutrino flavors was seen instead. As the average oscillation length was much longer than the distance from upper atmosphere to the ground, the deficit was seen to increase as a function of arrival angle (and therefore, baseline length), confirming atmospheric neutrino oscillation. The experiment was also the first to measure atmospheric mixing angle θ_{23} and mass splitting $|m_3^2 - m_2^2|$.

The measurements by Super-Kamiokande were followed by Sudbury Neutrino Observatory (SNO), which confirmed [38] solar neutrino oscillation in 2001, complemented the Super-Kamiokande experiment and provided the first measurement of solar mixing angle θ_{12} and mass splitting $|m_2^2 - m_1^2|$, solved the solar neutrino problem and confirmed MSW effect, which is very important in the sun, where matter density is high.

Afterwards the efforts have been concentrated to precision measurements of the already measured oscillation parameters. In the neutrino community there were high hopes of discovering a symmetry within the neutrino mixing matrix and several proposals were soon published, where the matrix elements were some simple irrational numbers. In the beginning stages, **trimaximal mixing matrix** [39]

$$U_{\text{Tri}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ z & 1 & z^* \\ z^* & 1 & z \end{pmatrix} \tag{1.1}$$

where $z = e^{i2\pi/3}$ was very popular, but was ruled out during the first decade of 2000s. Next, a **tribimaximal mixing matrix** [40] devoid of CP violation was proposed:

$$U_{\text{Tribi}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1.2}$$

In 2011, Daya bay, RENO and Double Chooz collaborations measured [41] the reactor angle θ_{13} to be nonzero, meaning that the top-right element of the mixing matrix could not be zero. Currently there are no comparable proposals for exact values for mixing matrix elements. However, the tribimaximal case remains to be a good approximation if one is willing to ignore the effect of θ_{13} and CP violation.

Current objective is to find out the sign of $m_3^2 - m_2^2$ and the value of CP violating phase δ. In addition θ_{23} angle is degenerate, meaning that there are two almost equally probable values the angle might be at $\approx$ (45$\pm$ few) degrees. These can be probed with next generation long baseline neutrino oscillation experiments, of which there are several proposals, for example DUNE, Hyper-Kamiokande, JUNO and INO.

1.1.3 History of speculative neutrino physics

In 1937, Majorana proposed the existence of Majorana fermions [42], which would be their own antiparticles. Neutrino, being electrically neutral, was the only possible candidate. Furry noted in 1939 that if neutrinos are Majorana fermions, a neutrinoless double beta ($0\nu\beta\beta$) decay is possible but did not prove that $0\nu\beta\beta$ decay implies that neutrinos are Majorana fermions [43]. That was done in 1982 by Schechter and Valle [44], and today goes by the name of Schechter-Valle theorem. To this day, $0\nu\beta\beta$ is unobserved.

Weinberg noted [45] in 1979 that the only effective dimension-five operator allowed by SM gauge symmetries is Majorana neutrino mass term $\frac{1}{\Lambda}LLHH$, which explicitly breaks the lepton number symmetry by two units. This effective operator appears as a low-energy limit of seesaw theories, proposed by Minkowski [46], Yanagida [47], Gell-Mann [48], Mohapatra and Senjanović [49], Schechter and Valle [50] and Glashow [51]. They do not aim to achieve grand unification, but increase the field content. In the simplest version (Type I) proposed in 1977-1979, the SM is extended by three heavy sterile right-handed Majorana neutrinos. Instead of inclusion of right-handed neutrinos, neutrino masses could be generated by extending the scalar sector (Type II) by one or more doublet or triplet, the triplet case being more popular. Scalar extension approach was proposed in 1980-1981. Third way to generate tree-level neutrino mass terms (Type III) was proposed [52] in 1989 by Foot, Lew, He and Joshi. Their model extends the SM by three singlet fermions, causing a rich collider phenomenology at lepton sector.

Alternative solution to neutrino mass problem is to generate neutrino mass at one- or many-loop level, called radiative seesaw or Zee-Babu model. One-loop version was proposed by Zee [53] in 1980, and has been ruled out after the discovery of solar neutrino oscillations in the beginning of 2000s. Two-loop version was proposed by Babu [54] in 1988. More exotic seesaw mechanisms have been proposed. Wyler and Wolfenstein [55], Mohapatra and Valle [56] and Ma [57] considered a model in 1983-87 where in addition to three right-handed neutrinos, there are also three SM gauge-singlet neutrinos. In addition, the lightness of Dirac type neutrinos can be explained by assuming the existence of extra dimensions, proposed by Arkani-Hamed, Dimopoulos and Dvali [58] in 1998. Also, the possibility of attributing the lightness of neutrinos to small VEV was considered by Ma [59] in 2001, and the model is called neutrinophilic model.

1.2 Standard Model

Standard Model (SM) is a theory, which is the pinnacle of the work done by several particle physicists during the second third of the 1900s. All the known particle interactions governed by electromagnetism, weak nuclear force and strong nuclear force are described in a gauge theory called the SM. Its gauge symmetry group is

$$G_{\mathrm{SM}} \equiv \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y, \tag{1.3}$$

with $\mathrm{SU}(3)_C$ group responsible for strong nuclear force[1] and $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ for the electroweak interaction. Being a Yang-Mills theory, the SM has a non-Abelian gauge group G_{SM}. It is a Lie group, consisting of three Lie groups $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, having eight, three and one group generators, respectively. Each generator corresponds to a gauge boson: eight gluons of $\mathrm{SU}(3)_C$ and the $W^\pm$ and Z bosons and photon γ of $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The field content of SM can be seen from Table 1.1.

In addition to the gauge bosons, SM contains also a Higgs boson, which via the Higgs mechanism generates a vacuum expectation value and mass to quarks, leptons, W and Z bosons and to Higgs itself. The rest of the particles in SM are fermions, which are divided to quarks and leptons. Quarks are sensitive to strong interactions, and leptons are not.

All SM phenomena can be encapsulated to its Lagrangian density[2]. The equations of motion may be derived from the Lagrangian with the use of Euler-

[1] $\mathrm{SU}(3)_C$ group interactions are commonly denoted as quantum chromodynamics (QCD), strong force or color force, with the subscript C referring to color charge, which is conserved in SM.

[2] Simply called Lagrangian afterwards.

Field	**Notation**	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	$Q_{1L} = \begin{pmatrix} u \\ d \end{pmatrix}_L, Q_{2L} = \begin{pmatrix} c \\ s \end{pmatrix}_L, Q_{3L} = \begin{pmatrix} t \\ b \end{pmatrix}_L$	3	2	$\frac{1}{3}$
	$u_{1R} = u_R, u_{2R} = c_R, u_{3R} = t_R$	3	1	$-\frac{4}{3}$
	$d_{1R} = d_R, d_{2R} = s_R, d_{3R} = b_R$	3	1	$\frac{2}{3}$
Leptons	$L_{eL} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, L_{\mu L} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, L_{\tau L} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1	2	-1
	$\ell_{eR} = e_R, \ell_{\mu R} = \mu_R, \ell_{\tau R} = \tau_R,$	1	1	2
Hypercharge boson	B^μ	1	1	0
$SU(2)_L$ bosons	$W_1^\mu, W_2^\mu, W_3^\mu$	1	3	0
Gluons	$G_1^\mu \cdots G_8^\mu$	8	1	0
Higgs boson	$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1

Table 1.1: SM field content. Quark numerical and lepton flavor indices are equivalent with generational indices, with e, μ and τ belonging to I, II and III generation, respectively. L and R in subscript refer to left- and right-handed chiralities (see definition in Appendix A.1). The group columns refer to quantum numbers attributed to the fields with respect to the gauge group.

Lagrange equations. For clarity, I split the Lagrangian to four parts:

$$\mathscr{L}_{\text{SM}} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{Yukawa}} + \mathscr{L}_{\text{kinetic}} - V(H) \tag{1.4}$$

1.2.1 Gauge sector

Local gauge transformations induce extra terms in the Lagrangian. To retain gauge invariance, SM is formulated with covariant derivatives. Terms included in the covariant derivatives will cancel the gauge-induced terms. Consider a field ψ transformed via a Lie gauge group G with dimension n, having generators $t_1, \cdots, t_n$ and thus general transformation matrices are $U = \exp\left(i \sum_{j=1}^{n} t_j \theta_j(x)\right)$. The field and its derivative transform as follows:

$$\psi \mapsto U\psi \tag{1.5}$$

$$\partial_\mu \psi \mapsto U\left(\partial_\mu + i \sum_{j=1}^{n} t_j \partial_\mu \theta_j(x)\right)\psi \tag{1.6}$$

The extra term is seen above in Eq. (1.6). To cancel it, one needs to postulate the existence of n gauge boson fields $K_1^\mu(x), \cdots K_n^\mu(x)$ and define a covariant derivative as

$$D^\mu = I_n \partial^\mu + iC \sum_{j=1}^{n} t_j K_j^\mu(x) \equiv I_n \partial^\mu + iCK^\mu(x) \tag{1.7}$$

where C is a constant and I defined $K^\mu(x) \equiv \sum_{j=1}^{n} t_j K_j^\mu(x)$. To fulfill local gauge invariance, it is imperative for ψ and $D^\mu \psi$ transform the same way. This can be achieved by requiring

$$D^\mu \mapsto UD^\mu U^\dagger \tag{1.8}$$

which implies gauge boson field transformation rule

$$K^\mu \mapsto UK^\mu U^\dagger + \frac{i}{C}(\partial^\mu U)U^\dagger. \tag{1.9}$$

Now the field strength tensor corresponding to these gauge bosons can be defined as

$$K^{\mu\nu}(x) \equiv -\frac{i}{C}[D^\mu, D^\nu], \tag{1.10}$$

and the trace $\text{Tr}(K^{\mu\nu}K_{\mu\nu})$ will be invariant under local gauge transformation. **Gauge principle** is a crucial ingredient in SM, which states that Lagrangian must be invariant when the fields transform in their corresponding gauge group transformations, both global and local. Due to local gauge invariance requirement, the gauge fields must be massless as long as the symmetry group is unbroken.

Let's return to SM. The gauge part of the SM Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{gauge}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\text{Tr}\left(W_{\mu\nu}W^{\mu\nu}\right) - \frac{1}{2}\text{Tr}\left(G_{\mu\nu}G^{\mu\nu}\right) \\ &= -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{3} W_{i\mu\nu}W_i^{\mu\nu} + \sum_{j=1}^{8} G_{j\mu\nu}G_j^{\mu\nu}\right) \end{aligned} \tag{1.11}$$

contains the field strength tensors $F_{\mu\nu}$, $W_{i\mu\nu}$ and $G_{j\mu\nu}$ for gauge groups SU(3)$_C$, SU(2)$_L$ and U(1)$_Y$, respectively, defined as

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{1.12}$$

$$W_{i\mu\nu} = \partial_\mu W_{i\nu} - \partial_\nu W_{i\mu} + g_2 \varepsilon_{ijk} W_\mu^j W_\nu^k \tag{1.13}$$

$$G_{j\mu\nu} = \partial_\mu G_{j\nu} - \partial_\nu G_{j\mu} + g_3 f_{jmn} G_\mu^m G_\nu^n \tag{1.14}$$

where g_2 and g_3 are the gauge coupling constants of SU(2)$_L$ and SU(3)$_C$ gauge groups, respectively. The elements of the totally antisymmetric tensors ε_{ijk} and f_{jmn} are the structure constants of the groups SU(2)$_L$ and SU(3)$_C$, respectively (see Appendix A.1). While the Abelian B^μ field doesn't have any self-interactions, the W_i^μ and G_j^μ fields interact with themselves due to the existence of the nonzero structure constants of the corresponding gauge groups.

1.2.2 Kinetic sector

The formalism above is now applied to SM kinetic Lagrangian, which reads as

$$\begin{aligned}\mathscr{L}_{\text{kinetic}} = &\sum_{i=1}^{3}\left(\overline{Q}_{iL} i\gamma^\mu D'_\mu Q_{iL} + \overline{u}_{iR} i\gamma^\mu D'_\mu u_{iR} + \overline{d}_{iR} i\gamma^\mu D'_\mu d_{iR}\right) \\ &+ \sum_{\alpha=e,\mu,\tau}\left(\overline{L}_{\alpha L} i\gamma^\mu D_\mu L_{\alpha L} + \overline{\ell}_{\alpha R} i\gamma^\mu D_\mu \ell_{\alpha R}\right) + (D^\mu H)^\dagger (D_\mu H)\end{aligned} \tag{1.15}$$

where the covariant derivatives are defined as follows:

$$D_\mu = \partial_\mu - \frac{i}{2}g_1 Y B_\mu - \frac{i}{2}g_2 \boldsymbol{\sigma}\cdot\mathbf{W}_\mu \tag{1.16}$$

$$D'_\mu = D_\mu - \frac{i}{2}g_3 \boldsymbol{\lambda}\cdot\mathbf{G}_\mu \tag{1.17}$$

Here $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$ vectors are assumed to be in their standard three- and eight-dimensional spaces. σ^i and λ^i are the three Pauli and eight Gell-Mann matrices, and the generators of gauge groups $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_C$, respectively (for explicit forms, see Appendix A.1). Hypercharge operator Y is the generator of $\mathrm{U}(1)_Y$ gauge group. Higgs, leptons and quarks couple to $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge fields, but $\mathrm{SU}(3)_C$ case is exclusive to quarks, which is the reason for extended covariant derivative D'_μ for quark fields. The kinetic Lagrangian therefore consists of the interactions between the fermions and gauge fields and also the interactions between Higgs field and gauge fields.

1.2.3 Brout-Englert-Higgs mechanism

The mass terms in the SM Lagrangian are not gauge invariant. Clearly quarks, leptons and $W^\pm$ and Z bosons are massive, so a mass generation mechanism is required. Therefore the existence of a scalar field giving masses to elementary particles is crucial. The phenomenon of spontaneous symmetry breaking was migrated from solid state physics to particle physics by Nambu, although Anderson was the first to apply it in a nonrelativistic context. Relativistic version was done by Brout and Englert [60], Higgs [61] and Hagen, Guralnik and Kibble [62] in 1964, and this is now known as **Brout-Englert-Higgs (BEH) mechanism**[3].

In SM, Higgs potential is defined as

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \tag{1.18}$$

where H is an SU(2) doublet containing two complex scalar fields. Assuming $\lambda > 0$ leads to the existence of global minimum of Higgs potential. If $\mu^2 > 0$, the minimum is trivial: $H_{\mathrm{min}} = 0$. In the case of $\mu^2 < 0$, defining $v \equiv \sqrt{-\mu^2/\lambda}$ and completing the square in the potential one gets

$$\begin{aligned} V(H) &= \lambda\left((H^\dagger H)^2 + 2\frac{\mu^2}{2\lambda}H^\dagger H + \left(\frac{\mu^2}{2\lambda}\right)^2 - \left(\frac{\mu^2}{2\lambda}\right)^2\right) \\ &= \lambda\left((H^\dagger H)^2 - 2\frac{v^2}{2}H^\dagger H + \left(\frac{-v^2}{2}\right)^2 - \left(\frac{-v^2}{2}\right)^2\right) = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2 \end{aligned} \tag{1.19}$$

where a constant term in the potential is ignored. From this form it is clear that at Higgs potential minimum $H^\dagger H = v^2/2$, or $|H| = v/\sqrt{2}$. The minimum energy is

[3] For conciseness, I simply call it **Higgs mechanism**.

shared by a continuous set of degenerate vacua, $\phi_0(x) = \frac{v}{\sqrt{2}}e^{i\theta}$, where $\theta \in [0, 2\pi[$. Moving along this degree of freedom does not require any energy, so it will correspond to a massless field, which is called the **Nambu-Goldstone boson** (NGB). The vacuum state must still choose a particular value from the set, which breaks the symmetry spontaneously, destroying the degrees of freedom of the NGB [63–65].

This potential minimum corresponds to vacuum state, in which it contains a nonzero energy, called **vacuum expectation value** (VEV). Since only neutral Higgs fields may develop a VEV, the Higgs doublet gains a VEV

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{1.20}$$

where $v \approx 246$ GeV. At tree-level it is advantageous to build the theory and calculations using unitary gauge fixing ($\xi \to \infty$), where the phase of the neutral complex Higgs field is set to zero. The Higgs doublet can be rotated with an SU(2) matrix, which has three degrees of freedom. In the unitary gauge they are fixed by requiring $\text{Re}(\phi^+) = \text{Im}(\phi^+) = \text{Im}(\phi^0) = 0$ and NGB has been destroyed. The Higgs doublet now reads as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.21}$$

where $h(x) \sim \text{Re}(\phi^0)$ is a real scalar field, the physical Higgs boson. We may now generate the masses for $W^\pm$ and Z gauge bosons by looking at the covariant derivative term of Higgs doublet in unitary gauge:

$$(D^\mu H)^\dagger (D_\mu H)_{\substack{\text{mass terms} \\ \text{of } W/Z}} = \frac{1}{2} v^2 \left(\frac{1}{2} g_2^2 W_\mu^+ W^{\mu -} + \frac{1}{4}(g_1^2 + g_2^2) Z_\mu Z^\mu \right). \tag{1.22}$$

Here I have defined new fields after spontaneous symmetry breaking (SSB):

$$Z_\mu = \frac{g_2 W_{3\mu} - g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}}, \qquad A_\mu = \frac{g_1 W_{3\mu} + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}}, \qquad W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \mp i W_{2\mu}). \tag{1.23}$$

Here A_μ denotes the photon field. The mass terms are proportional to v^2. Inspecting these terms from Eq. (1.22), the gauge boson masses are recognized as

$$M_{W^\pm} = \frac{1}{2} g_2 v, \qquad M_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\, v, \qquad m_A = 0. \tag{1.24}$$

Photon stays massless, as it should. Defining **Weinberg angle** $\theta_W = \arctan \frac{g_1}{g_2}$, it is evident that the physical fields Z_μ and A_μ are related to the fields $W_{3\mu}$ and B_μ by a simple rotation:

$$\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix} = R(\theta_W) \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{1.25}$$

Here $R(\theta_W)$ denotes two-dimensional rotation matrix (see Appendix A.1), with rotation angle θ_W. There remains a residual $\mathrm{U}(1)_Q$ symmetry group, which has a generator

$$Q = I_3 + \frac{Y}{2}, \tag{1.26}$$

where $I_3 = \tau_3$ is the third generator of the SU(2) group (third component of weak isospin **I**) and Y is the hypercharge operator. This formula is known as Gell-Mann–Nishijima -formula [66, 67]. Q is the electric charge operator, which has the eigenvalues of electric charge of the target field, in the units of elementary charge (e).

1.2.4 Yukawa sector

Since in the unbroken phase all the SM particles are massless, and in the broken phase $W^\pm$ and Z bosons become massive, it is natural to ask if it is possible to generate masses to fermions the same way. The answer turns out to be yes, and it utilizes the same Higgs mechanism I described in previous section. Consider the Yukawa terms in the Lagrangian:

$$\mathscr{L}_{\text{Yukawa}} = -\sum_{j,k=1}^{3} \overline{Q_{jL}} Y^d_{jk} H d_{kR} - \sum_{j,k=1}^{3} \overline{Q_{jL}} Y^u_{jk} H' u_{kR} - \sum_{\alpha,\beta=e,\mu,\tau} \overline{L_{\alpha L}} Y^\ell_{\alpha\beta} H \ell_{\beta R} + \text{h.c.} \tag{1.27}$$

where $H' = i\sigma_2 H^*$ and Yukawa matrices Y^u, Y^d and Y^ℓ are dimensionless 3×3 matrices. Note that H and H' have opposite hypercharges, otherwise the Lagrangian wouldn't be gauge invariant. After SSB, mass terms emerge in the Yukawa Lagrangian $\mathscr{L}_{\text{Y}} \equiv \mathscr{L}_{\text{Yukawa}}$:

$$\begin{aligned} \mathscr{L}_{\text{Y}} &= -\frac{v}{\sqrt{2}} \left(\sum_{j,k=1}^{3} \left(Y^d_{jk} \overline{d_{jL}} d_{kR} + Y^u_{jk} \overline{u_{jL}} u_{kR} \right) - \sum_{\alpha,\beta=e,\mu,\tau} Y^\ell_{\alpha\beta} \overline{\ell_{\alpha L}} \ell_{\beta R} \right) + \text{h.c.} + \cdots \\ &= -\frac{v}{\sqrt{2}} \left(\sum_{jk=1}^{3} Y^d_{jk} \overline{d_j} d_k + \sum_{j,k=1}^{3} Y^u_{jk} \overline{u_j} u_k + \sum_{\alpha,\beta=e,\mu,\tau} Y^\ell_{\alpha\beta} \overline{\ell_\alpha} \ell_\beta \right) + \cdots \end{aligned} \tag{1.28}$$

From this expression, fermion mass matrices for down-type quarks, up-type quarks and charged leptons, respectively, can be read:

$$M_d = \frac{Y_d v}{\sqrt{2}}, \qquad M_u = \frac{Y_u v}{\sqrt{2}}, \qquad M_\ell = \frac{Y_\ell v}{\sqrt{2}} \tag{1.29}$$

Together with $W^\pm$ and Z bosons, this accounts all the massive fields in the SM. The remaining terms describe Higgs-fermion interactions. I will discuss the Higgs-lepton interactions in more detail in the next chapter.

1.3 Some problems in the Standard Model

Even though the SM has an impressive track record, describing nearly all the non-gravitational particle interactions, it is far from complete. It is presently understood that the SM is an effective theory, which holds on until new interactions kick in at a large energy scale. This is expected to happen even below the **Planck scale** 10^{19} GeV. Any theory replacing the SM should reduce back to SM at low-energy limit. Unfortunately it is not evident which of the various proposed SM extensions describes reality. Here I briefly describe some of the most serious issues.

1.3.1 Flavour problem

Masses of quarks and leptons are generated by the Higgs mechanism, and the dimensionless proportionality constants are the Yukawa couplings, which scale the base value of electroweak VEV $v \approx 246$ GeV. The coupling for the top quark is $\mathcal{O}(1)$, but for the lightest charged lepton it is $\mathcal{O}(10^{-6})$. It is unclear why the couplings would have numerical values over six orders of magnitude.

1.3.2 Neutrino masses

Neutrino oscillations were experimentally confirmed during the turn of the millennium by Super-K and SNO collaborations. Therefore at least two of the three SM neutrinos are massive, but SM is missing a neutrino mass term. Direct inclusion of the mass term implies the existence of a right-handed neutrino and if there are no new active scalar fields, neutrino Yukawa matrix elements are so tiny, they would make the flavour problem even worse. A very popular solution to this is the seesaw mechanism, which will be discussed in Chapter 4.

1.3.3 Hierarchy problem

Loop corrections to SM Higgs mass are proportional to the SM cutoff scale which is the energy scale where the SM is expected to break down. The cutoff scale can be very large, even the Planck scale. Since the SM Higgs is 17 orders of magnitude lighter than the Planck scale, there must be a Planck scale order cancellation to tame the large value. The cancellation should be fine-tuned to provide the observed Higgs mass, and for this reason it is also known as fine-tuning problem. One proposed solution to this problem is supersymmetry, as it would induce cancellations to Higgs mass of supersymmetry breaking scale, which is regarded to be much lighter than the Planck scale.

1.3.4 Cosmological issues

Majority of matter content of the universe is now known to be **dark matter**, which is non-electromagnetically interacting elementary particles not included in the SM. Neutrinos are too light to account for suitable dark matter, but right-handed neutrinos (among many other proposals) might be the necessary ingredients.

All efforts to quantize **gravity** have failed, and gravity is not included in the SM. Gravity itself is weak compared to other interactions, a fact which is also unexplained. Also, the accelerating expansion of the universe can be associated with the vacuum energy, but the expansion rate predicted by the SM is too large. In addition, **cosmic inflation** is currently understood to be driven by one or more scalar fields. The only scalar field in the SM (Higgs) is a candidate, but such a Higgs inflation suffers from breakdown of perturbative unitarity below the energy scale of inflation, and a more elegant beyond-the-SM inflation theory is usually preferred.

Lastly, **baryonic asymmetry in the universe** (BAU) is larger than the SM expectation by factor of 10^{10}. The amount of BAU must be generated during the early universe and fulfilling Sakharov's conditions. The crux of the issue is that in the SM the amount of CP violation is too small. This might be solved in leptogenesis theories, where heavy Majorana neutrinos produce a lepton asymmetry, which is converted to baryon asymmetry during the early universe.

1.3.5 Strong CP problem

It is possible to construct a θ-term in the QCD Lagrangian which is proportional to $\theta\varepsilon^{\mu\nu\rho\sigma}G^{j}_{\mu\nu}G_{j\rho\sigma}$. The coupling constant θ must be extraordinarily small ($\lesssim 10^{-10}$) due to nonobservation of electric dipole moment of neutron. θ-term also breaks CP invariance, which gives the name for the problem. The source of the problem is not the expected breaking of CP symmetry itself but the suppression mechanism which renders the symmetry breaking feeble with tiny θ. A possible solution to this would be the existence of invisible axion, which promotes the θ constant to a field.

CHAPTER 2

Phenomenology of massive light neutrinos

Neutrino physics is largely an art of learning a great deal by observing nothing.

Haim Harari (1988)

At least two of the three active neutrinos definitely are massive due to the observation of neutrino oscillations. Ergo, it is enlightening to consider the implications of a minimal extension of the SM, where right-handed neutrinos are added to the SM and where the neutrinos are massive. Neutrino oscillations is a vast topic, which is why Ch. 3 is dedicated to it. I will discuss the case where SM Higgs mechanism generates Dirac mass terms for the light neutrinos. Majorana mass terms and the mass generation mechanism itself are discussed in Ch. 4. Massive neutrinos open an avenue for many new phenomena, which are however notoriously hard to detect, including lepton number violating decays and neutrino electromagnetic interactions.

Since neutrino oscillations probe only the absolute values of the neutrino mass squared differences, $|m_i^2 - m_j^2|$, the values of individual neutrino masses and their ordering are unknown. The two possible mass ordering schemes are normal hierarchy (NH: $m_1 < m_2 < m_3$) and invered hierarchy (IH: $m_3 < m_1 < m_2$). See Fig. 2.1 for details. In both the NH and IH cases neutrinos are nearly mass degenerate if the lightest neutrino is $\gtrsim 0.1$ eV. In NH case m_1 and m_2 are nearly

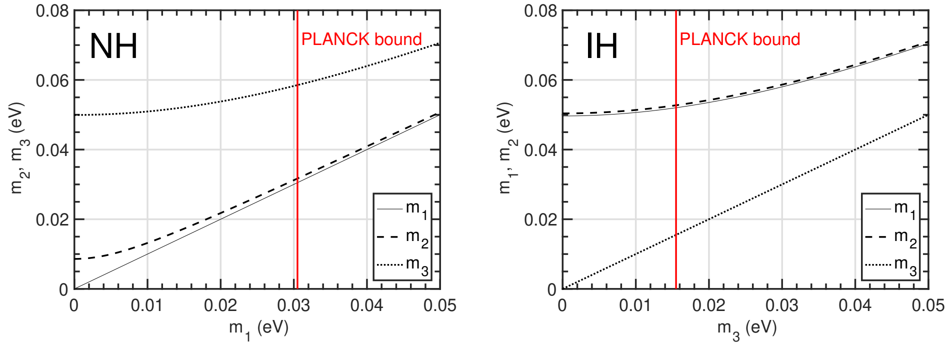


Figure 2.1: Neutrino masses in normal (left) and inverse (right) ordering. If the lightest neutrino mass exceeds ~ 0.1 eV, neutrinos can be considered to be nearly mass degenerate. PLANCK limits require $\sum m_\nu < 0.12$ eV (95 % CL). Red line corresponds to the limit, and right side of the red line is excluded.

mass degenerate if $m_1 \gtrsim 0.02$ eV. Conversely, in IH case m_1 and m_2 are almost mass degenerate regardless of the magnitude of m_3.

PLANCK limits impose stringent restrictions on the neutrino masses. In NH case, $m_1 \lesssim 0.03$ eV and 0.05 eV $\lesssim m_3 \lesssim 0.06$ eV. IH case is possible only if $\sum m_\nu > 0.1$ eV, giving even stricter constraints: $m_3 \lesssim 0.016$ eV and 0.050 eV $\lesssim m_1 \approx m_2 \lesssim 0.055$ eV.

However the case for effective neutrino masses is different (see Fig. 2.2). Even if one neutrino state is massless, all the effective masses of flavour neutrinos are nonzero, since they are linear combinations of all three physical neutrino masses m_i. It is now known that all the coefficients of the linear combinations are nonzero. With similar deduction, lightest effective neutrino mass can be 0.004 – 0.031 eV (NH) or 0.021 – 0.031 eV (IH), and the heaviest 0.031 – 0.046 eV (NH) or ≈ 0.05 eV (IH).

2.1 Dirac mass term

Consider the existence of a right-handed neutrino field ν_R, with zero hypercharge. At first, I assume only the existence of one generation. Before spontaneous symmetry breaking, the Higgs-lepton (HL) part of Yukawa Lagrangian would then include an extra term containing ν_R.

$$\mathscr{L}_{\mathrm{HL}} = -Y_e \overline{L_L} H e_R - Y_\nu \overline{L_L} H' \nu_R + \text{h.c.} + \cdots \tag{2.1}$$

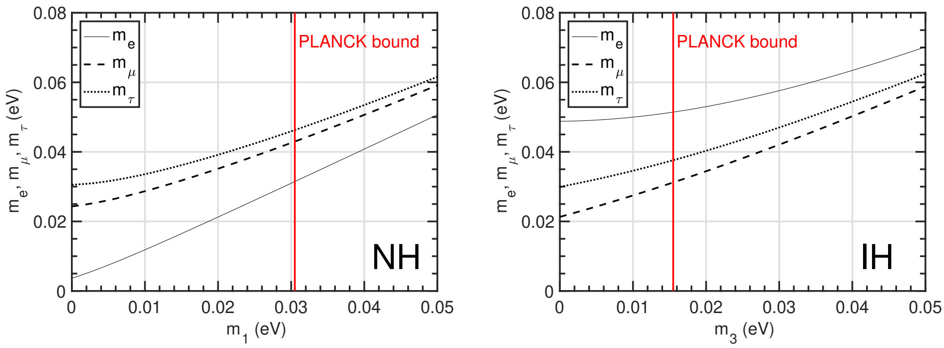


Figure 2.2: Flavour neutrino masses in normal (left) and inverse (right) ordering.

Then after spontaneous symmetry breaking, the extra term produces neutrino-Higgs interaction term and neutrino mass term

$$-\frac{Y_\nu v}{\sqrt{2}}\overline{\nu_L}\nu_R + \text{h.c.} = -m_\nu \overline{\nu}\nu, \tag{2.2}$$

where neutrino mass is $m_\nu = Y_\nu v/\sqrt{2}$ and $\nu = \nu_L + \nu_R$. The mass term together with the derivative term gives

$$\overline{\nu}(i\partial\!\!\!/ - m_\nu)\nu, \tag{2.3}$$

which produces Dirac equation for neutrinos. The next step is to generalize the treatment to three generations.

I now add an additional object to the theory: flavour. Yukawa couplings will be promoted to matrices, and they must be diagonalized. Flavour states will be marked with additional index and the original untransformed fields and Yukawa matrices will be denoted with prime ($'$). The three lepton doublets are denoted as follows:

$$L'_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, L'_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}, L'_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}.$$

In addition there are six SU(2) singlet fields $e'_R, \mu'_R, \tau'_R, \nu'_{eR}, \nu'_{\mu R}$ and $\nu'_{\tau R}$. Yukawa Lagrangian in lepton sector with three generations before SSB is straightforwardly generalized:

$$\mathscr{L}_{\text{HL}} = -\sum_{\alpha,\beta=e,\mu,\tau}\left(Y'^{\ell}_{\alpha\beta}\overline{L'_{\alpha L}}H\ell'_{\beta R} + Y'^{\nu}_{\alpha\beta}\overline{L'_{\alpha L}}H'\nu'_{\beta R}\right) + \text{h.c.} \tag{2.4}$$

After the SSB, it can be written as

$$\begin{aligned}\mathscr{L}_{\mathrm{HL}} &= -\frac{v+h}{\sqrt{2}} \sum_{\alpha,\beta=e,\mu,\tau} \left(Y'^{\ell}_{\alpha\beta} \overline{\ell'}_{\alpha L} \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \overline{\nu'}_{\alpha L} \nu'_{\beta R} \right) + \mathrm{h.c.} \\ &= -\frac{v+h}{\sqrt{2}} \left(\overline{\ell'_L} Y'^{\ell} \ell'_R + \overline{\nu'_L} Y'^{\nu} \nu'_R \right) + \mathrm{h.c.}, \end{aligned} \tag{2.5}$$

where I have defined flavour triplets

$$\ell'_L = \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}, \ell'_R = \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}, \nu'_L = \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}, \nu'_R = \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix},$$

From the Lagrangian one can now peel off the charged lepton and neutrino mass matrices, which are proportional to corresponding Yukawa matrices: $m'^{\ell} = vY'^{\ell}/\sqrt{2}$ and $m'^{\nu} = vY'^{\nu}/\sqrt{2}$.

As far as I have now become, all has been analogous to one generation case. I will proceed by diagonalizing the Yukawa matrices with 3×3 unitary matrices $V^{\ell}_L, V^{\ell}_R, V^{\nu}_L$ and V^{ν}_R. This procedure - biunitary transformation - works for any matrix. The transformed fields are thus

$$\begin{aligned} \ell_L \equiv V^{\ell\dagger}_L \ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \ell_R \equiv V^{\ell\dagger}_R \ell'_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}, \\ \nu_L \equiv V^{\nu\dagger}_L \nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \nu_R \equiv V^{\nu\dagger}_R \nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}. \end{aligned}$$

Note that the neutrinos lose their flavour index. It is easily seen that kinetic terms in the Lagrangian are invariant under this transformation. Looking at Higgs-lepton Lagrangian, I get

$$\begin{aligned}\mathscr{L}_{\mathrm{HL}} &= -\frac{v+h}{\sqrt{2}} \left(\overline{\ell_L} V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \ell_R + \overline{\nu_L} V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \nu_R \right) + \mathrm{h.c.} \\ &= -\frac{v+h}{\sqrt{2}} \left(\overline{\ell_L} Y^{\ell} \ell_R + \overline{\nu_L} Y^{\nu} \nu_R \right) + \mathrm{h.c.} \end{aligned} \tag{2.6}$$

where I have defined $V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R = Y^{\ell}$ and $V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R = Y^{\nu}$. The matrices Y^{ℓ} ja Y^{ν} are diagonal, and the diagonal entries are real and positive. That allows us to write them in the index notation as $Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha} \delta_{\alpha\beta}$ and $Y^{\nu}_{ij} = y^{\nu}_i \delta_{ij}$. Reverting the

Lagrangian back to index notation one arrives at

$$\mathscr{L}_{\mathrm{HL}} = -\frac{v+h}{\sqrt{2}}\left(\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\overline{\ell_{\alpha L}}\ell_{\alpha R} + \sum_{i=1}^{3} y^{\nu}_{i}\overline{\nu_{iL}}\nu_{iR}\right) + \mathrm{h.c.} \tag{2.7}$$

We can now work backwards from chiral decomposition, resulting in

$$\mathscr{L}_{\mathrm{HL}} = -\sum_{\alpha=e,\mu,\tau}\frac{v y^{\ell}_{\alpha}}{\sqrt{2}}\overline{\ell_{\alpha}}\ell_{\alpha} - \sum_{i=1}^{3}\frac{v y^{\nu}_{i}}{\sqrt{2}}\overline{\nu_{i}}\nu_{i} - \sum_{\alpha=e,\mu,\tau}\frac{y^{\ell}_{\alpha}}{\sqrt{2}}\overline{\ell_{\alpha}}\ell_{\alpha}h - \sum_{i=1}^{3}\frac{y^{\nu}_{i}}{\sqrt{2}}\overline{\nu_{i}}\nu_{i}h \tag{2.8}$$

where the first two sums are the charged lepton and neutrino mass terms, respectively, with masses $m_{\alpha} = v y^{\ell}_{\alpha}/\sqrt{2}$ and $m_{\nu_i} = v y^{\nu}_{i}/\sqrt{2}$, where $\alpha = e,\mu,\tau$ and $i = 1,2,3$. Note that the mass of charged lepton is defined by its flavor, unlike mass of neutrinos. The latter two sums describe the lepton-Higgs interactions.

2.2 Weak lepton current

In the previous Chapter it was found out that interactions between fermions and gauge bosons reside in kinetic sector of the Lagrangian. The interactions are categorized to charged current (CC) and neutral current (NC) Lagrangians:

$$\begin{aligned}\mathscr{L}_{\mathrm{CC}} + \mathscr{L}_{\mathrm{NC}} = {}& -g_2\sum_{\ell=e,\mu,\tau}\overline{L'_{\ell L}}\gamma^{\mu}\boldsymbol{\tau}\cdot\mathbf{W}_{\mu}L'_{\ell L} - g_1 B_{\mu}\sum_{\ell=e,\mu,\tau}Y(L'_{\ell L})\overline{L'_{\ell L}}\gamma^{\mu}L'_{\ell L} \\ & - g_1 B_{\mu}\sum_{\ell=e,\mu,\tau}Y(\ell'_R)\overline{\ell'_R}\gamma^{\mu}\ell'_R + (\text{quark - gauge boson interactions})\end{aligned} \tag{2.9}$$

Here $Y(\psi)$ denotes the hypercharge of field ψ. Right-handed neutrino ν_R is absent, since it is hyperchargeless. Terms containing $W_{1\mu}$ and $W_{2\mu}$ (and therefore, the $W^{\pm}$ fields) form the CC part and the rest, containing $W_{3\mu}$ and B_{μ} (which lead to Z and photon fields), form the NC part.

The CC part Lagrangian can be written compactly:

$$\mathscr{L}_{\mathrm{CC}} = -\frac{g}{2\sqrt{2}} j^{\rho}_{W} W_{\rho} + \mathrm{h.c.}$$

where $j^{\rho}_{W} = j^{\rho}_{L} + j^{\rho}_{Q}$ is the weak CC, which is the sum of the weak lepton (L) and weak quark (Q) currents. The CC term is part of the SM kinetic term $\mathscr{L}_{\mathrm{kinetic}}$. The lepton current

$$j^{\rho}_{L} = \sum_{\alpha=e,\mu,\tau}\overline{\nu'_{\alpha}}\gamma^{\rho}(I-\gamma_5)\ell'_{\alpha} = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu'_{\alpha L}}\gamma^{\rho}\ell'_{\alpha L} = 2\overline{\nu'_{L}}\gamma^{\rho}\ell'_{L} \tag{2.10}$$

is relevant for neutrinos. Above I used the property $\gamma^\rho(I-\gamma_5) = 2\gamma^\rho P_L = 2\gamma^\rho P_L P_L = 2P_R\gamma^\rho P_L$. Changing to primeless fields by defining $\ell_L \equiv V_L^{\ell\dagger}\ell'_L$ and $\nu_L \equiv V_L^{\nu\dagger}\nu'_L$ as before, the current is then

$$j_L^\rho = 2\overline{\nu_L}V_L^{\nu\dagger}\gamma^\rho V_L^\ell \ell_L = 2\overline{\nu_L}V_L^{\nu\dagger}V_L^\ell\gamma^\rho\ell_L = 2\overline{\nu_L}U^\dagger\gamma^\rho\ell_L \tag{2.11}$$

where I defined **neutrino mixing matrix** $U \equiv V_L^{\ell\dagger}V_L^\nu$. Now it's possible to define neutrino flavour fields

$$\nu \equiv U\nu_L = V_L^{\ell\dagger}\nu'_L \equiv \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

and write the lepton current in the final form:

$$j_L^\rho = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\gamma^\rho\ell_{\alpha L} \tag{2.12}$$

In this form, it is seen that each flavour neutrino appears alongside its corresponding charged lepton flavour field. During neutrino creation in $W^\pm$ boson decay, a same-flavoured charged lepton must be present in the interaction vertex. Similarly when a neutrino is detected, a corresponding charged lepton must be seen. Once its mass is identified, the flavour of the lepton - and therefore the flavour of the neutrino is clear.

2.3 Lepton flavour violation

In SM one can assign **lepton flavour numbers** L_e,L_μ and L_τ to all leptons, assigned as in Table 2.1. Lepton flavour numbers and (total) lepton number $L = L_e + L_\mu + L_\tau$ are conserved quantities, which corresponds to U(1) gauge invariance. Indeed, for U(1) transformation

$$\ell_{\alpha L} \mapsto \mathrm{e}^{i\phi_\alpha}\ell_{\alpha L},\ \nu_{\alpha L} \mapsto \mathrm{e}^{i\phi_\alpha}\nu_{\alpha L} \tag{2.13}$$

where $\alpha = e,\mu,\tau$, the lepton current is invariant. Similarly the Higgs-lepton Lagrangian, Eq. (2.8) is invariant in such a transformation. However, the right-handed neutrino ν_R kinetic part of the Lagrangian is not invariant, which leads to nonconservation of flavor lepton number. If one postulates right-handed neutrinos and adds a neutrino mass term, this is the inevitable conclusion.

Fields	L_e	L_μ	L_τ	L
ν_e, e^-	1	0	0	1
ν_μ, μ^-	0	1	0	1
ν_τ, τ^-	0	0	1	1
$\overline{\nu_e}, e^+$	−1	0	0	−1
$\overline{\nu_\mu}, \mu^+$	0	−1	0	−1
$\overline{\nu_\tau}, \tau^+$	0	0	−1	−1

Table 2.1: Leptons and the corresponding lepton numbers in SM.

However, total lepton number L is still conserved. If neutrinos are Majorana fermions, L will be broken by two units.

One important consequence of massive neutrinos, which has been widely studied, is the rise of a new set of decays which violate lepton flavor, for example $\mu^- \to e^-\gamma$, $\mu^- \to e^-e^-e^+$ and $\mu^-e^+ \to \mu^+e^-$ (see Fig. 2.3). Bounds for these processes are quite strict. Given neutrino masses $m_i = \mathcal{O}(1)$ eV, for example the branching ratio

$$\mathrm{BR}(\mu^- \to e^-\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=1}^{3} U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2}\right|^2 \lesssim 10^{-50} \tag{2.14}$$

is heavily suppressed by $\sim (m_i/m_W)^4$. Fine structure constant is defined as $\alpha \equiv e^2/4\pi$. MEG collaboration has reported $\mathrm{BR}(\mu \to e\gamma)$ to have experimental upper limit of $\mathcal{O}(10^{-13})$ [68], while for other lepton flavor violating processes the limit can be relaxed. The large suppression can be understood by considering the typical neutrino oscillation length. The baseline length for a neutrino in lepton flavour violating decay is of the order of the characteristic length scale of weak interaction, $\mathcal{O}(10^{-18})$ m, while the average oscillation lengths are $\mathcal{O}(1)$ m and longer.

2.4 Neutrino electromagnetic interactions

In the SM, even though neutral and massless, neutrino has loop-level electromagnetic interactions (for a recent review, see [69]), resulting in nonzero neutrino

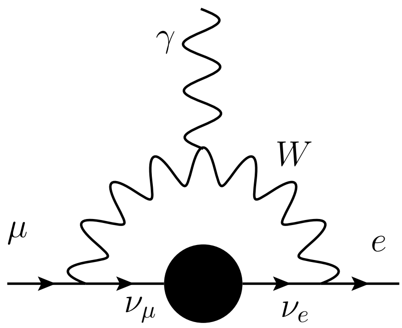


Figure 2.3: An example of a lepton flavour violation process $\mu \to e\gamma$. In this case, muon neutrino oscillates to electron neutrino in-flight. The blob corresponds to neutrino oscillation.

charge radius. In one-loop approximation, Bernabéu *et al* [70–72] calculated the following expression for the square of charge radius:

$$\langle r^2_{\nu_\ell}\rangle_{\rm SM} = \frac{G_F}{4\pi^2\sqrt{2}}\left(3-2\ln\frac{m_\ell^2}{m_W^2}\right) = \begin{cases} \langle r^2_{\nu_e}\rangle_{\rm SM} \approx 4.1\times 10^{-33}\ {\rm cm}^2 \\ \langle r^2_{\nu_\mu}\rangle_{\rm SM} \approx 2.4\times 10^{-33}\ {\rm cm}^2 \\ \langle r^2_{\nu_\tau}\rangle_{\rm SM} \approx 1.5\times 10^{-33}\ {\rm cm}^2 \end{cases} \tag{2.15}$$

Here $G_F = \sqrt{2}g_2^2/8M_W^2$ in Fermi coupling constant. This shifts the neutrino vector coupling constant [73] by an amount

$$\Delta g_V^{\nu_\ell} = \frac{2}{3}M_W^2\langle r^2_{\nu_\ell}\rangle\sin^2\theta_W \approx 2.47\cdot 10^{-3}\cdot\frac{\langle r^2_{\nu_\ell}\rangle}{10^{-33}\ {\rm cm}^2}, \tag{2.16}$$

which may be observable with the next-generation neutrino-electron scattering experiments. The current experimental constraints of scattering cross section measurements infer $\langle r^2_{\nu_\ell}\rangle \lesssim \mathcal{O}(10^{-32})\ {\rm cm}^2$, only one order of magnitude from the SM prediction [74, 75]. It is possible that the experimental accuracy will in near future improve to confirm the SM prediction of the neutrino charge radius.

Since neutrinos are massive, there are other electromagnetic properties available for neutrinos. Massive neutrinos imply a nonzero magnetic moment for neutrinos. Assuming Dirac masses (m_1, m_2 and m_3) for neutrinos, the magnetic moment can be calculated via the one-loop radiative Feynman diagrams, resulting in [76]

$$\mu_{kj} = \frac{eG_F}{8\sqrt{2}\pi^2}(m_k+m_j)\sum_{\ell=e,\mu,\tau} f\left(\frac{m_\ell^2}{M_W^2}\right)U^*_{\ell k}U_{\ell j}, \tag{2.17}$$

where $j,k = 1,2,3$ and the loop function is

$$f(x) = \frac{3}{4}\left(1 + \frac{1}{1-x} - \frac{2x}{(1-x)^2} - \frac{2x^2 \ln x}{(1-x)^3}\right). \tag{2.18}$$

In leading order, taking m_ℓ^2/M_W^2 small, the neutrino diagonal magnetic moment is

$$\mu_{kk} \approx \frac{3eG_F m_k}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \frac{m_k}{\text{eV}} \mu_B. \tag{2.19}$$

It is intriguing to see that in leading order, the magnitude is insensitive to charged lepton masses and neutrino mixing matrix elements. The current experimental efforts are frustratingly insufficient to measure the neutrino magnetic moment, devoid of any additional new physics contributions, lagging eight or more orders of magnitude behind [77–79]. Nondiagonal magnetic moment is further suppressed by ratio $m_\tau^2/M_W^2 \sim 10^{-4}$.

It is natural to investigate whether it is possible that a theory beyond the SM produces a large enough magnetic moment for the neutrinos so that it is feasible to be measured in the near future. It would also have a role of constraining classes of such theories. Assuming such a theory exists at scale Λ, producing a contribution μ_ν to neutrino magnetic moment, it will provide a correction to neutrino mass [80]:

$$\Delta m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} = \frac{\mu_\nu}{10^{-18}\mu_B}\left(\frac{\Lambda}{\text{TeV}}\right)^2 \text{eV}. \tag{2.20}$$

Therefore theories, which produce large neutrino magnetic moments, additionally induce large m_ν corrections. However, in some theories it is possible to suppress neutrino mass without suppressing its magnetic moment [81].

CHAPTER 3

Neutrino oscillation

Earth is the cradle of humanity, but one cannot live in a cradle forever.

Konstantin Tsiolkovsky

Leptons mix, just like quarks. In this chapter, I derive the transition probability for a general neutrino three-flavour oscillation[1] (see Fig. 3.1). In neutrino oscillation, the neutrino flavour changes during the propagation. I will cover the most important approximations, matter effects and nonstandard effects. At the end of this chapter, I consider neutrino oscillations beyond the three-neutrino (3ν) framework, where the neutrino flavour might differ from the corresponding lepton during the creation and detection processes and where matter potential is modified.

3.1 Derivation of transition probability

Neutrino eigenstates of weak interaction are superpositions of the physical mass states: the neutrino states mix. The mixing is described with a Pontecorvo-Maki-

[1] Neutrino flavor oscillations are sometimes called oscillations of the first kind, to distinguish them from neutrino-antineutrino oscillations, which are called oscillations of the second kind. Since Majorana neutrinos are their own antiparticles, it does not make sense to define oscillations of the second kind for them.

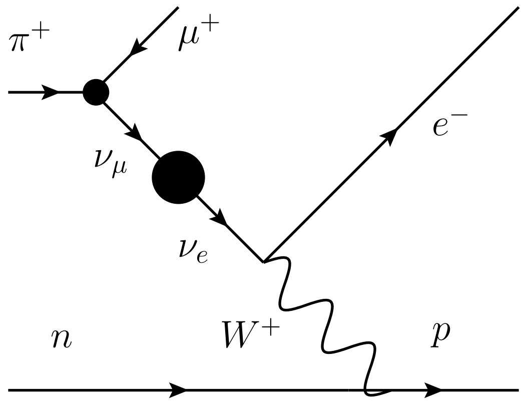


Figure 3.1: A Feynman diagram of vacuum neutrino oscillation. A muon neutrino is created in pion decay. This oscillates to electron neutrino, which interacts via virtual W^+ boson with a nucleus, producing an electron, which is subsequently seen in a detector. This process violates lepton flavour and is therefore forbidden in the SM. Lepton number is conserved. Time flows from left to right.

Nakagawa-Sakata (PMNS) neutrino mixing matrix [29–31, 82]

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \equiv R_{23}(\theta_{23})P_3(\delta)R_{13}(\theta_{13})P_3(-\delta)R_{12}(\theta_{12})$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.1}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and by convention $\theta_{ij} \in [0, \pi/2[$ $(i, j = 1, 2, 3)$ and $\delta \in [0, 2\pi[$. θ_{12} is called the **solar angle**, θ_{13} the **reactor angle** and θ_{23} the **atmospheric angle**. This matrix was encountered already upon examining the weak lepton current in Eq. (2.12). For alternate parameterisation, see [83]. If neutrinos are Majorana fermions, the mixing matrix gains two additional phases:

$$U_M = UP_1(\alpha)P_2(\beta), \tag{3.2}$$

where $\alpha, \beta \in [0, 2\pi[$ are Majorana phases. The currently known experimental values are listed in Table 3.1. With this notation, neutrino oscillations can be written in a compact form:

Parameter	Normal hierarchy	Inverted hierarchy
θ_{12} (°)	$33.63^{+0.78}_{-0.75}$	$33.63^{+0.78}_{-0.75}$
θ_{23} (°)	$48.7^{+1.4}_{-3.1}$	$49.1^{+1.2}_{-1.6}$
θ_{13} (°)	8.52 ± 0.15	8.55 ± 0.14
δ (°)	228^{+51}_{-33}	281^{+30}_{-33}
Δm^2_{21} (eV^2)	$7.40^{+0.21}_{-0.20} \cdot 10^{-5}$	$7.40^{+0.21}_{-0.20} \cdot 10^{-5}$
$\Delta m^2_{3\ell}$ (eV^2)	$2.515 \pm 0.035 \cdot 10^{-3}$	$-2.483^{+0.034}_{-0.035} \cdot 10^{-3}$

Table 3.1: Currently known neutrino oscillation parameters with their corresponding 1σ errors. For normal hierarchy $\ell = 2$ and for inverted hierarchy $\ell = 1$. [84]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{3.3}$$

Treating neutrino mass and weak interaction states as state vectors in three-dimensional quantum mechanical Hilbert spaces, one can define

$$|\nu_\ell\rangle \equiv \sum_{i=1}^{3} U_{\ell i} |\nu_i\rangle, \qquad |\nu_i\rangle = \sum_{\ell=e,\mu,\tau} U^*_{\ell i} |\nu_\ell\rangle. \tag{3.4}$$

From this one can define the **effective mass of flavour neutrinos**:

$$m_{\nu_\ell} = \sum_{i=1}^{3} \left| U^2_{\ell i} \right| m_i. \tag{3.5}$$

Flavour neutrino masses are sensitive to Majorana phases. Considering that only mass eigenstates are eigenvalues of Hamiltonian operator, I may write the Schrödinger equation for the state:

$$H|\nu_i\rangle = E|\nu_i\rangle.$$

Time evolution is plane wave solution of it:.

$$|\nu_i(t)\rangle = \mathrm{e}^{-iE_i t} |\nu_i(0)\rangle \equiv \mathrm{e}^{-iE_i t} |\nu_i\rangle \tag{3.6}$$

Using Eq. (3.4),

$$|\nu_\ell(t)\rangle = \sum_{i=1}^{3} U_{\ell i} \mathrm{e}^{-iE_i t} |\nu_i\rangle \tag{3.7}$$

it can be seen that the time evolution of the flavor state is just a superposition of the time evolutions of the mass states. However it should be noted that the neutrinos in this Chapter are considered to be ultrarelativistic, which is an excellent approximation[2]. This is important, because at the ultrarelativistic limit the momentum and energy of both the flavor and mass eigenstates of neutrinos are the same. Using again Eq. (3.4), the time evolution of flavor state becomes apparent:

$$|\nu_\ell(t)\rangle = \sum_{\ell'=e,\mu,\tau} \left(\sum_{i=1}^{3} U_{\ell i} \mathrm{e}^{-iE_i t} U^*_{\ell' i} \right) |\nu_{\ell'}\rangle \equiv \sum_{\ell'=e,\mu,\tau} A_{\ell\ell'} |\nu_{\ell'}\rangle. \tag{3.8}$$

Here $A_{\ell\ell'}$ is the transition probability amplitude for the case, where a neutrino of flavour ℓ at time 0 has transformed to flavour ℓ' at time t. If a neutrino is at pure state at $t = 0$, the time evolution causes the state to immediately evolve to mixed state for $t > 0$. The corresponding transition probability is then

$$P_{\ell\ell'} \equiv P(\nu_\ell \to \nu_{\ell'}) \equiv |A_{\ell\ell'}|^2 = \sum_{i,j=1}^{3} U^*_{\ell i} U_{\ell' i} U_{\ell j} U^*_{\ell' j} \mathrm{e}^{-i(E_i - E_j)t}. \tag{3.9}$$

These probabilities should obey unitarity conditions

$$\sum_{\ell} P_{\ell\ell'} = \sum_{\ell'} P_{\ell\ell'} = 1. \tag{3.10}$$

Violation of these conditions leads to nonunitary mixing, which is discussed in Section 3.6. From Eq. (3.9), it can be shown that inclusion of the Majorana phases does not affect the transition probability (see Appendix). Consequently, oscillation experiments are insensitive to the Majorana phases, and one must turn to, for example, neutrinoless double beta decay to determine them.

Since I assumed neutrinos are ultrarelativistic, the energy difference translates to mass difference. If E is the energy and $\mathbf{p} \approx \mathbf{p}_i \approx \mathbf{p}_j$ is the momentum of a neutrino, then

$$E_i - E_j = \sqrt{\mathbf{p}^2 + m_i^2} - \sqrt{\mathbf{p}^2 + m_j^2} \approx |\mathbf{p}| \left(1 + \frac{m_i^2}{2\mathbf{p}^2} - 1 - \frac{m_j^2}{2\mathbf{p}^2} \right) \equiv \frac{\Delta m^2_{ij}}{2E} \tag{3.11}$$

where I use notation $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ for the difference of squared neutrino masses. Therefore it can be also negative. Since neutrinos are ultrarelativistic,

[2]This does not hold with the cosmic neutrino background, but we do not consider it anyway in this Chapter.

after time t they have travelled a distance $L \approx t$, called a **baseline**. The transition probability now depends only on constants of nature, neutrino energy and baseline, and defining a dimensionless quantity $\Delta_{ij} = L\Delta m^2_{ij}/4E$, it can be written as follows:

$$P_{\ell\ell'} \equiv |A_{\ell\ell'}|^2 = \sum_{i,j=1}^{3} U^*_{\ell i} U_{\ell' i} U_{\ell j} U^*_{\ell' j} e^{-2i\Delta_{ij}} \tag{3.12}$$

From this expression it is clear that the existence of neutrino oscillations requires that there are at least two neutrinos with different masses, as the oscillation is driven by the squared mass differences Δm^2_{ij}. This was first discovered by Pontecorvo in [34]. See Table 3.1 for experimental values.

The current knowledge of parameters is clearly inadequate, since the ordering of neutrino masses is ambiguous. First, the neutrino mass hierarchy is unknown. Also, it is unclear which octant the θ_{23} mixing angles does belong to: higher ($\theta_{23} > \pi/4$) or lower ($\theta_{23} < \pi/4$)? Optimal baseline for the octant determination was investigated in [85], where longer baseline was found to be more sensitive for low luminosity neutrino beams. Interference from new physics effects to octant determination was considered in [86]. Running behaviour of the mixing angles and CP phase were studied in [87–89]. The CP violating phase and its implications are discussed in Section 3.4.

Even though the oscillation parameters still have inaccuracies with varying degree, the tribimaximal mixing (TBM) approximation is shown to be false, since θ_{13} angle has been known to be nonzero since 2012 [41]. The angle is small, and the TBM pattern approximation is useful in circumstances where the effect of θ_{13} can be ignored (see next Section for examples).

Instead of hoping that PMNS matrix exhibits clear patterns corrsponding to a particular symmetry group, it can be considered that the TBM is created by a pattern in neutrino mass matrix and that the deviations of TBM are produced by subleading effects. A neutrino mass matrix invariant under $\mathbb{Z}_2 \times \mathbb{Z}_2^{\text{magic}} \times \mathbb{Z}_2^{\mu-\tau}$ symmetry can produce the TBM pattern. Here the magic symmetry group refers to the second column of TBM matrix, where all the values are equal. The $\mu-\tau$-symmetry (or $2-3$-symmetry) refers to the third column, where the U_{e3} element is zero and $U_{\mu 3} + U_{\tau 3} = 0$. Invocation of these symmetries allows one or more elements of neutrino mass and Yukawa coupling matrices to vanish. These elements are called **texture zeros**. Implementing these to a neutrino mass model will decrease the amount of free parameters, and therefore increase the predictability of the model.

3.2 Two flavour approximation

From an experimentalist standpoint, it is possible to tune the baseline L and neutrino energy E. This is used to gain sensitivity for a particular oscillation channel. With a proper choice of L and E, one can suppress all but one oscillation channel, resulting in effective two flavour neutrino oscillation approximation. Depending of the assumptions, the transition matrix elements may coincide with exact two-neutrino (2ν) scenario, where the mixing matrix is $R(\theta)$ and mass difference is $\Delta m \equiv m_2^2 - m_1^2$. Assuming neutrino flavours ℓ and ℓ', the transition probability matrix would be

$$\begin{pmatrix} P_{\ell\ell} & P_{\ell\ell'} \\ P_{\ell'\ell} & P_{\ell'\ell'} \end{pmatrix} = \begin{pmatrix} 1-\sin^2(2\theta)\sin^2\Delta & \sin^2(2\theta)\sin^2\Delta \\ \sin^2(2\theta)\sin^2\Delta & 1-\sin^2(2\theta)\sin^2\Delta \end{pmatrix} \tag{3.13}$$

where $\Delta \equiv \frac{L\Delta m^2}{4E}$ is dimensionless. Returning back to 3ν scenario and looking at Eq. (3.12), the periodic exponential function can be written as $\mathrm{e}^{-2\pi i L/L_{ij}}$, where

$$L_{ij} \equiv L_{ij}(E) \equiv \frac{4\pi E}{\Delta m_{ij}^2} \tag{3.14}$$

is the **oscillation length**, providing an estimate for any neutrino oscillation experiment baseline aiming to be sensitive to Δm_{ij}^2. Defining a shorthand notation

$$J_{\ell\ell'}^{jk} \equiv U_{\ell j}^* U_{\ell' j} U_{\ell k} U_{\ell' k}^* \tag{3.15}$$

one can write Eq. (3.12) in the following forms:

$$P_{\ell\ell'} = \sum_{i=1}^{3} J_{\ell\ell'}^{ii} + 2\mathrm{Re}\sum_{i>j} J_{\ell\ell'}^{ij}\mathrm{e}^{-2\pi i L/L_{ij}} \tag{3.16}$$

$$= \delta_{\ell\ell'} - \sum_{i>j}\left(4\sin^2(\Delta_{ij})\mathrm{Re}(J_{\ell\ell'}^{ij}) - 2\sin(2\Delta_{ij})\mathrm{Im}(J_{\ell\ell'}^{ij})\right) \tag{3.17}$$

In most oscillation experiments, the neutrino energy can be tuned to a narrow interval. For simplicity, here E is regarded as constant. Considering $\Delta m_{21}^2 \ll \Delta m_{31}^2$, then $\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2 \approx \Delta m_{32}^2$. Therefore there are two relevant oscillation lengths: L_{21} and $L_{31} \approx L_{32} \ll L_{21}$. In the case $L \ll L_{32}$, baseline is too short for the neutrino oscillation to be observable (all Δ_{ij} are zero), resulting in a trivial case $P_{\ell\ell'} \approx \delta_{\ell\ell'}$. On very long baseline $L \gg L_{21}$ one is able to gain only the averaged-out effects. Most experiments utilize GeV energy scale, resulting in oscillation lengths ranging from a few kilometers to a couple thousand kilometers.

3.2.1 Small L/E region

Consider a case where L/E is small and it can be approximated that $\Delta_{21} \approx 0$. This is the case for atmospheric neutrinos produced by high-energy cosmic ray collisions in the upper atmosphere. Disappearance of flavour ℓ is driven purely by Δ_{32} terms. Tau neutrino cannot be detected from atmospheric channel, since neutrino energies are too low. Assuming $s_{13} \approx 0.02$ to be negligible, the relevant transition probability matrix block is

$$P_{e\times\mu} = \begin{pmatrix} P_{ee} & P_{e\mu} \\ P_{\mu e} & P_{\mu\mu} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \sin^2(2\theta_{23})\sin^2\Delta_{32} \end{pmatrix} \tag{3.18}$$

To maximize the prospects of detecting the muon neutrino disappearance, it is necessary to set $\sin^2\Delta_{32} = 1$, corresponding to a distance $L = L_{32}\left(n + \frac{1}{2}\right)$, where $n \in \mathbb{N}$. For such cases $P_{\mu\mu} = \cos^2(2\theta_{23})$, allowing a measurement of mixing angle θ_{23}. Note that the matrix block in Eq. (3.18) is nonunitary, which is the consequence of ignoring the tau neutrino contribution. Atmospheric neutrino oscillations were first discovered in 1998 by Super-Kamiokande collaboration [37], which measured θ_{23} and $|\Delta m^2_{32}|$.

3.2.2 Large L/E region

Consider next a case where L/E is large. Now the effect by Δ_{21} cannot be ignored. For low-energy neutrinos on keV and MeV scale, it is advantageous to measure the electron neutrino disappearance. At such a low energy, ν_μ and ν_τ cannot be directly seen, as the incoming neutrinos can't produce on-shell muon or tau leptons. Assuming $\sin\theta_{13} \approx 0$, the survival probability for electron neutrino is

$$P_{ee} = 1 - \sin^2(2\theta_{12})\sin^2\Delta_{21} \tag{3.19}$$

Measuring the amount of detected electron neutrinos allows then the measurement of mixing angle θ_{12}. Analogously to atmospheric neutrino case, the effect is largest at baselines $L = L_{21}\left(n + \frac{1}{2}\right)$ ($n \in \mathbb{N}$). In 2002, KAMLand collaboration

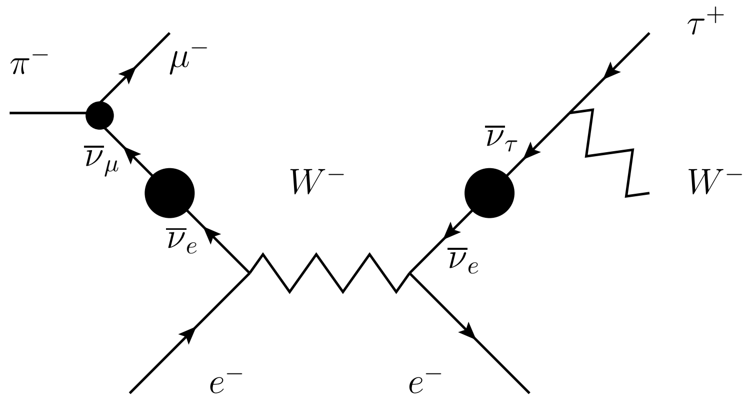


Figure 3.2: An example of neutrino charged current matter interactions. Electrons in medium interact with the electron flavour state neutrinos in-flight via neutrino-electron scattering. This is the origin of index of refraction for neutrinos.

discovered that antielectron neutrinos produced in nuclear reactors oscillate [90, 91], and measured θ_{12} and Δm^2_{12}.

3.2.3 θ_{13} measurement

If the goal is measure θ_{13} angle, small L/E is favored in order to remove interference from Δ_{21} term. Electron antineutrino survival probability is then

$$P(\overline{\nu}_e \to \overline{\nu}_e) = P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_{13})\sin^2\Delta_{32} \tag{3.20}$$

Since $\theta_{13} \sim 9^\circ$ is a small angle, confirming nonzero value of it took significantly longer than for $\theta_{12} \sim 34^\circ$ and $\theta_{23} \sim 45^\circ$. Similarly to atmospheric oscillation case, the disappearance is largest, when the baseline is $L = L_{32}\left(n + \frac{1}{2}\right)$. The value of θ_{13} was discovered in 2011 by Daya bay, RENO and Double Chooz collaborations [41].

3.3 Matter effects

In previous Chapter, I covered the neutrino CC and NC interactions, where the weak current couples to gauge bosons. From the SM point of view, neutrino oscillations are in most cases a low energy process. At the low energy limit, it is

Fermion	g_V	g_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

Table 3.2: Vector and axial vector coupling constants of fermions in the SM.

possible to invoke effective field theory and integrate out the $W^\pm$ gauge bosons, resulting in current-current interactions[3]:

$$\mathscr{L}_{\text{eff}}^{\text{CC}} = -\frac{G_F}{\sqrt{2}} j_L^\mu j_{L\mu} = -2\sqrt{2} G_F \sum_{\ell=e,\mu.\tau} (\overline{\nu_{L\ell}} \gamma^\mu \ell_L)(\overline{\ell_L} \gamma_\mu \nu_{L\ell}) \tag{3.21}$$

For a derivation of above result, see Appendix A.5. Via Fierz transform, this can be written as

$$\mathscr{L}_{\text{eff}}^{\text{CC}} = -2\sqrt{2} G_F \sum_{\ell=e,\mu,\tau} \left(\overline{\nu_{L\ell}} \gamma^\mu \nu_{L\ell}\right) \left(\overline{\ell_L} \gamma_\mu \ell_L\right) \tag{3.22}$$

Integrating out the Z boson, one acquires effective NC interaction:

$$\mathscr{L}_{\text{eff}}^{\text{NC}} = -2\sqrt{2} G_F \sum_{\ell=e,\mu.\tau} \left(\overline{\nu_{L\ell}} \gamma^\mu \nu_{L\ell}\right) \sum_f \left(\overline{f} \gamma_\mu \left(g_V^f - g_A^f \gamma_5\right) f\right). \tag{3.23}$$

Here g_A^f and g_V^f are axial and vector coupling constants of a fermion f (see Table 3.2).

In terrestrial experiments, the medium consists of electrons and u and d quarks. For CC interactions, see Fig. 3.2. In 1977, Wolfenstein [35] calculated the refractive index for neutrinos:

$$n = 1 + \frac{2\pi N}{|\mathbf{p}|^3} \times \frac{E^2 - M^2}{16\pi E^2} \times \mathscr{M}(\theta = 0) \tag{3.24}$$

Here N is the number density of potential scatterers, $\mathbf{p}$ is the momentum of neutrino, E is the center-of-mass energy of the scattering and $\mathscr{M}(\theta)$ is the Feynman amplitude of the scattering as a function of scattering angle θ. Calculating this for elastic neutrino-electron scattering leads to

$$n = 1 - \frac{\sqrt{2} G_F N_e}{E}. \tag{3.25}$$

[3] Also known as four fermion interaction or Fermi interaction.

Neutrino momentum is $|\mathbf{p}| = nE$, leading to effective matter potential

$$V = E - |\mathbf{p}| = \sqrt{2}G_F N_e, \tag{3.26}$$

resulting in Hamiltonian operators

$$H_{\mathrm{CC}} = \sqrt{2}G_F N_e \mathrm{diag}(1,0,0), \qquad H_{\mathrm{NC}} = \sqrt{2}G_F I_3 \sum_f N_f g_V^f, \tag{3.27}$$

where the NC operator is obtained similar to CC case. Eigenvalues of these operators with respect to neutrino state $|\nu_\ell\rangle$ are

$$V_{\mathrm{CC}}(x) \equiv \sqrt{2}\delta_{e\ell}G_F N_e, \quad V_{\mathrm{NC}}(x) \equiv \sqrt{2}G_F \sum_f N_f g_V^f = -\frac{\sqrt{2}}{2}G_F N_n \tag{3.28}$$

where $N_e \equiv N_e(x)$ and $N_f \equiv N_f(x)$ are electron and fermion f number densities of the medium, respectively, at spacetime point x. In general, the CC operator would contain also muon and tau lepton number densities, but in terrestrial neutrino oscillation experiments, their background is negligible. Due to neutrality of matter, proton and electron number densities are the same. Consequently the g_V contributions by protons and electrons cancel, leaving the NC operator to be dependent only on the neutron density $N_n \equiv N_n(x)$.

Now it is possible to construct the effective Hamiltonian in medium. At the ultrarelativistic limit, energy of neutrino (with mass m_i) can be approximated (see Eq. (3.11)) as

$$E \approx |\mathbf{p}| + \frac{m_i^2}{2|\mathbf{p}|}. \tag{3.29}$$

Defining neutrino mass matrix $M \equiv \mathrm{diag}(m_1, m_2, m_3)$, the effective Hamiltonian in flavour space can be constructed:

$$H = EI_3 + \frac{1}{2E}UM^2U^\dagger + \mathrm{diag}(V_{\mathrm{CC}} + V_{\mathrm{NC}}, V_{\mathrm{NC}}, V_{\mathrm{NC}}) \tag{3.30}$$

Since during propagation, neutrino energy is constant, the EI_3 and $V_{\mathrm{NC}}I_3$ terms may be removed as unphysical phase factors, since neutrino propagator has the form e^{-iHL}. In addition, phasing the Hamiltonian with $-m_1^2UU^\dagger/2E$, the Hamiltonian can be written as

$$H_{\mathrm{SI}} = \frac{1}{2E}U\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}U^\dagger + \begin{pmatrix} V_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.31}$$

where the subscript SI stands for standard (neutrino) interactions in medium. It is interesting to note that the disappearance of NC potential reflects the irrelevance of a flavour universal potential. Now the transition probability from Hamiltonian formalism can be written as

$$P_{\ell\ell'} = \left| \langle \nu_{\ell'} | e^{-iH_{\mathrm{SI}}L} | \nu_\ell \rangle \right|^2 . \tag{3.32}$$

3.3.1 Mikheyev-Smirnov-Wolfenstein effect

Large density in Sun's core, where solar neutrinos are produced, indicates that matter effects are important with solar neutrinos. Solar neutrino disapprearance was first seen in 1968 by Homestake experiment [32], which found only 1/3 of the expected solar ν_e flux. Pontecorvo [34] considered this as a hint of neutrino oscillations. However, naively calculating the survival probability of solar electron neutrinos in vacuum approximation gives $P_{ee} \approx 0.58$ [92], a clearly incompatible result with Homestake experiment. It turns out that effective neutrino masses and mixing angles differ significantly in medium, contributing corrections to oscillation probabilities. These matter effects were first described by Wolfenstein [35] in 1978 and elaborated by Mikheyev and Smirnov [36] in 1985. This resonance enhancement of neutrino oscillation probabilities is presently known as **Mikheyev-Smirnov-Wolfenstein** (MSW) effect, which was confirmed in 2001 by Sudbury Neutrino Observatory (SNO) collaboration [38]. The solar ν_μ and ν_τ fluxes participate in NC interactions, but not in CC interactions. SNO discovered the CC reaction rate to be one third of NC rate.

Since in matter the transition probability is dependent on electron density of matter, this opens new avenues for neutrino tomography. It is possible to measure the density of Earth's mantle with next-generation atmospheric neutrino oscillation experiments [93] or even search oil [94]. In the case of solar neutrinos, the day-night effect [95] refers to a case where one observes more solar neutrinos during the night than during the day due to the matter effects.

For simplicity, consider 2ν scenario, with electron and non-electron flavours (ν_e, ν_ℓ) and neutrino mass eigenvalues m_1 and m_2. This can be encoded to mass matrix $M' = \mathrm{diag}(m_1, m_2)$. Consider Hamiltonian of Eq. (3.31) in 2 dimensions with NC current intact:

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{\mathrm{CC}} + V_{\mathrm{NC}} & 0 \\ 0 & V_{\mathrm{NC}} \end{pmatrix} \tag{3.33}$$

Here $U = R(\theta)$ is the neutrino mixing matrix (in vacuum) with two generations and mixing angle θ. Mass squared difference is $\Delta m^2 = m_2^2 - m_1^2$. Performing a phase shift, the Hamiltonian can be simplified to

$$
\begin{aligned}
H &= \frac{\Delta m^2}{4E} \underbrace{\begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}}_{=R(2\theta)P_1(\pi)} + \begin{pmatrix} V_{\text{CC}} - \frac{1}{2}V_{\text{CC}} & 0 \\ 0 & -\frac{1}{2}V_{\text{CC}} \end{pmatrix} \\
&= \frac{\Delta m^2}{4E} R(2\theta)P_1(\pi) + \frac{V_{\text{CC}}}{2}\text{diag}(1,-1) \\
&\equiv \frac{\Delta m_m^2}{4E} R(2\theta_m)P_1(\pi) \equiv \frac{1}{2E}M^2,
\end{aligned} \tag{3.34}
$$

where I approximated $N_e \approx N_n$, resulting in $V_{\text{NC}} = -\frac{1}{2}V_{\text{CC}}$. M is the effective mass matrix in medium. On the last row the Hamiltonian is reparameterised with θ_m and Δm_m^2, which are the effective matter mixing angle and effective matter mass squared difference, respectively. Ergo, it is possible to treat the matter oscillations as vacuum oscillations with different oscillation parameters. The changes to vacuum oscillation parameters are driven by the V_{CC} term. First, I solve Δm_m^2. The effective mass matrix

$$
M^2 = \frac{1}{2} \begin{pmatrix} -\Delta m^2 \cos(2\theta) + EV_{\text{CC}} & \Delta m^2 \sin(2\theta) \\ \Delta m^2 \sin(2\theta) & \Delta m^2 \cos(2\theta) - EV_{\text{CC}} \end{pmatrix} \tag{3.35}
$$

has eigenvalues

$$
\lambda_\pm = \pm\frac{1}{2}\sqrt{(\Delta m^2 \cos(2\theta) - EV_{\text{CC}})^2 + (\Delta m^2)^2 \sin^2(2\theta)} \tag{3.36}
$$

and therefore the effective mass squared difference in medium is

$$
\Delta m_m^2 \equiv \lambda_+ - \lambda_- = \sqrt{(\Delta m^2 \cos(2\theta) - EV_{\text{CC}})^2 + (\Delta m^2)^2 \sin^2(2\theta)}, \tag{3.37}
$$

and dependent of the CC potential. In the vacuum limit $V_{\text{CC}} \to 0$, $\Delta m_m^2 \to \Delta m^2$, as expected. The next stage is to find the matter mixing angle θ_m. It can be solved from the last line of Eq. (3.34), resulting in [35]

$$
\tan(2\theta_m) = \frac{\sin(2\theta)}{\cos(2\theta) - \dfrac{EV_{\text{CC}}}{\Delta m^2}}, \tag{3.38}
$$

from where one can see that $\theta_m \to \theta$, when $V_{\text{CC}} \to 0$, as it should. The matter angle can gain a value which maximizes the oscillation effects in 2ν probability matrix in Eq. (3.13). This maximal value is $\pi/4$, and it is achieved when

$$
\cos(2\theta) = \frac{EV_{\text{CC}}}{\Delta m^2} \tag{3.39}
$$

This is the **MSW resonance condition** [35, 36].

3.4 Leptonic CP violation

The existence of CP violation in the lepton sector can be verified by counting the available free parameters of the neutrino mixing matrix. A general complex $n \times n$ matrix has $2n^2$ free parameters. Requiring unitarity reduces the number of parameters to n^2. Number of mixing angles of a unitary matrix is equal to the number of parameters of a orthogonal matrix of same dimension, that is $n(n-1)/2$. The remaining $n(n+1)/2$ parameters are phase angles. However, the amount of physical phases is lower, since $2n-1$ of them can be removed by redefining the neutrino fields. Therefore the number of physical Dirac phases is $(n-1)(n-2)/2$. Since the existence of CP violation requires physical phases, at least three generations are required. Indeed, in two generations, the mixing matrix is simply two-dimensional rotation matrix. If neutrinos are Majorana fermions, number of physical phases is increased by $n-1$. However, as discussed earlier, these Majorana phases have no effect on neutrino oscillations. Summarising these remarks, number of mixing angles (n_A) and physical phases (n_P) are

$$n_A = \frac{n(n-1)}{2} = n_P^{\text{Majorana}}, \qquad n_P^{\text{Dirac}} = \frac{(n-1)(n-2)}{2}. \tag{3.40}$$

The amount of CP violation can be quantified using **Jarlskog invariant** [96]

$$J = \text{Im}\, J_{\mu e}^{23} = \text{Im}\,(U_{\mu 3} U_{e2} U_{\mu 2}^* U_{e3}^*), \tag{3.41}$$

which is parameterization-free up to sign. In the standard parameterization,

$$|J| = \frac{1}{8}\left|\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23}) c_{13} \sin\delta\right| \leq \frac{1}{6\sqrt{3}}, \tag{3.42}$$

where the upper limit is realized by trimaximal mixing (see Eq. (1.1), where $\theta_{12} = \theta_{23} = \frac{\pi}{4}$, $s_{13} = \frac{1}{\sqrt{3}}$ and $|\sin\delta| = 1$). Since trimaximal mixing is decisively ruled out by neutrino oscillation experiments, CP violation cannot be maximal even if the Dirac phase is $\pm\frac{\pi}{2}$. See Table 3.1 for experimental limits. If $\delta = 0$ or π, there is no CP violation in the lepton sector. However, this has been ruled out within 2σ confidence limit [84]. Jarlskog invariant would vanish also if any mixing angle was zero. Conversely, the angle $-\frac{\pi}{2}$ is favored by the current data, with IH favoring it slightly more than NH.

3.4.1 Contribution to baryon asymmetry of the universe

The existence of matter particles today and nonobservation of antimatter suggests that a larger amount of matter than antimatter was produced in the early universe. The amount of difference is denoted by the ratio of number densities of baryons and photons today, $\eta_{\text{obs}} \sim 10^{-10}$. In the context of baryon-to-photon ratio, or baryon asymmetry of the universe (BAU), there exists a dimension-12 Jarlskog-like invariant [97–99]:

$$\begin{aligned} J' &\sim s_{12}s_{13}s_{23}\sin\delta(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2)(m_\mu^2 - m_e^2)\Delta m_{31}^2\Delta m_{32}^2\Delta m_{21}^2 \\ &\approx s_{12}s_{13}s_{23}m_\tau^4 m_\mu^2 \Delta m_{31}^2\Delta m_{32}^2\Delta m_{21}^2 \approx 1.7\times 10^{-29}\ \text{MeV}^{12} \end{aligned} \tag{3.43}$$

where I approximated $m_\tau \gg m_\mu \gg m_e$ and assumed $\delta = \pi/2$. If the energy scale where the generation of BAU is relevant is $E \sim 100$ GeV, then a rough estimate of it is

$$\eta_{\text{PMNS}} \sim \frac{|J'|}{E^{12}} \sim 10^{-89} \ll \eta_{\text{CKM}} \sim 10^{-20} \ll \eta_{\text{obs}} \tag{3.44}$$

from where it is seen that the contribution by massive light neutrinos to BAU is highly suppressed by the lightness of neutrinos, and completely negligible. η_{CKM} refers to the expected BAU from the quark sector. The estimation is analogous in the CKM matrix case. Introducing extra massive neutrinos may produce the correct value for η given a suitable mass hierarchy for them. This approach is called leptogenesis [100], and it has been widely studied in the literature, see for example [101–103].

3.4.2 Optimal experimental setup for CP phase

Measurement of leptonic CP phase is currently ongoing by several collaborations, and the discovery of it might be confirmed by next-generation long baseline neutrino oscillation experiments. The most useful channel for the CP phase detection would be the electron-to-muon neutrino oscillation.

Leptonic CP violation can be confirmed by measuring nonzero value of difference of two probabilities:

$$P(\nu_e \to \nu_\mu) - P(\overline{\nu}_\mu \to \overline{\nu_e}) = 16\text{Im}\,(U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^*)\sin\Delta_{12}\sin\Delta_{13}\sin\Delta_{23}. \tag{3.45}$$

If CP is conserved, this will yield zero. Note that if the mixing matrix is real, there is no CP violation. The goal is to plan the experiment so that the expression

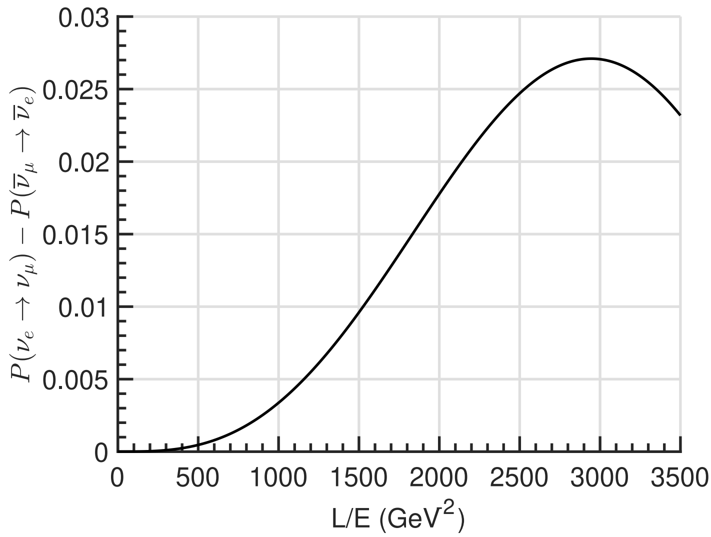


Figure 3.3: Contribution to $P_{e\mu} - P_{\overline{\mu e}}$ by CP violation as a function of L/E assuming $|\sin\delta| = 1$. Note the use of natural units.

in Eq. (3.45) is extremized. With the exception of trivial solution $L/E = 0$, this problem can be solved only numerically, the first nontrivial extremum being at

$$\frac{L}{E} \approx 580 \, \frac{\text{km}}{\text{GeV}}. \tag{3.46}$$

The neutrino energies which can be produced for oscillation experiments are on MeV and GeV scale. For this reason a long baseline experiment is obligatory. If $|\sin\delta| = 1$, then the coefficient corresponds to approximately 2.7 % change to probability $P_{e\mu}$ compared to case $\sin\delta = 0$. See Fig. 3.3 for illustration.

3.5 Nonstandard interactions

The present 3ν oscillation framework has thus far been very successful in describing the neutrino flavor transition. While the oscillations must be the dominant mechanism behind it, there might be subleading effects contributing to it. This would cause the future neutrino flavor transition data to be unfittable to 3ν framework. This is the case in most extensions of the SM, since they often alter the neutrino sector. This might result in new interactions which change the standard oscillation probabilities. Invoking effective field theory, heavy degrees of freedom may be integrated out from the SM extension, leading to nonstandard interaction (NSI) formalism described in this section. For recent reviews, see [104–110].

In the low energy limit, nonstandard interactions manifest themselves as the following Lagrangians [111, 112]

$$\begin{aligned} \mathscr{L}^{\mathrm{CC}}_{\mathrm{NSI}} &= -2\sqrt{2}G_F\varepsilon^{ff',C}_{\ell\ell'}(\overline{\nu}_{L\alpha}\gamma^{\mu}\nu_{L\beta})(\overline{f}\gamma^{\mu}P_C f'), \\ \mathscr{L}^{\mathrm{NC}}_{\mathrm{NSI}} &= -2\sqrt{2}G_F\varepsilon^{f,C}_{\ell\ell'}(\overline{\nu}_{L\alpha}\gamma^{\mu}\nu_{L\beta})(\overline{f}\gamma^{\mu}P_C f). \end{aligned} \tag{3.47}$$

It should be noted that NSI Lagrangians are nonrenormalizable dimension-6 operators and that they break $\mathrm{SU}(2)_L$ gauge symmetry explicitly. Here summing over chiralities ($C = L, R$), fermions (f, f') and flavours ($\ell, \ell' = e, \mu, \tau$) is implied. The Lagrangians are reminiscent of Eq. (3.22) and (3.23). Indeed, the NSI Lagrangians provide correction terms to standard 3ν oscillation paradigm (the CC NSI Lagrangian), and to creation and detection processes of neutrinos (the NC NSI Lagrangian). The NSI parameters $\varepsilon^{f,C}_{\ell\ell'} \equiv \varepsilon^{ff,C}_{\ell\ell'}$ and $\varepsilon^{ff',C}_{\ell\ell'}$ are dimensionless complex numbers. Possible nonvanishing imaginary parts of these parameters induce additional CP violation. Absolute values of the NSI parameters describe the strength of the interaction with respect to weak interaction. As the weak coupling $G_F \sim M_W^{-2}$, then $|\varepsilon| \sim \left(\frac{M_W}{\Lambda}\right)^2$, where Λ is the scale of the high-energy theory responsible for nonstandard interactions. Λ is expected to be at TeV scale or higher.

In the presence of NSI in matter, effective neutrino Hamiltonian in medium is

$$H = H_{\mathrm{SI}} + \underbrace{\sum_{f=e,u,d} V_f \begin{pmatrix} \varepsilon^f_{ee} & \varepsilon^f_{e\mu} & \varepsilon^f_{e\tau} \\ \varepsilon^{f*}_{e\mu} & \varepsilon^f_{\mu\mu} & \varepsilon^f_{\mu\tau} \\ \varepsilon^{f*}_{e\tau} & \varepsilon^{f*}_{\mu\tau} & \varepsilon^f_{\tau\tau} \end{pmatrix}}_{=H_{\mathrm{NSI}}}, \tag{3.48}$$

where $V_f \equiv \sqrt{2}G_F N_f(x)$ is the matter potential of fermion f and $\varepsilon^f_{\ell\ell'} \equiv \varepsilon^{f,L}_{\ell\ell'} + \varepsilon^{f,R}_{\ell\ell'}$ is the corresponding NSI parameter summed over chiralities. The $\varepsilon^f = (\varepsilon^f_{\ell\ell'})$ matrices are Hermitian. The total amount of matter NSI is defined as a sum over the fermions, too:

$$\varepsilon^m_{\ell\ell'} \equiv \sum_f \varepsilon^f_{\ell\ell'}\frac{N_f}{N_e} = \varepsilon^e_{\ell\ell'} + 2\varepsilon^u_{\ell\ell'} + \varepsilon^d_{\ell\ell'} + \frac{N_n}{N_e}(\varepsilon^u_{\ell\ell'} + 2\varepsilon^d_{\ell\ell'}). \tag{3.49}$$

Here neutrality of matter was presumed, resulting in $N_p = N_e$. Note that ε^m is unitary: $\varepsilon^m_{\ell\ell'} = \varepsilon^{m*}_{\ell'\ell}$. For current experimental limits for ε^m matrix elements, see

Constraint on	**Upper bound**
$\lvert\varepsilon^m_{ee} - \varepsilon^m_{\mu\mu}\rvert$	4.2
$\lvert\varepsilon^m_{e\mu}\rvert$	0.3
$\lvert\varepsilon^m_{e\tau}\rvert$	3.0
$\lvert\varepsilon^m_{\mu\tau}\rvert$	0.04
$\lvert\varepsilon^m_{\tau\tau} - \varepsilon^m_{\mu\mu}\rvert$	0.15

Table 3.3: 90 % confidence limit upper bounds to absolute values of matter NSI matrix elements and their differences, from [113].

Constraint on	**Upper bound**
$\lvert\varepsilon^{s/d}_{ee}\rvert$	0.041
$\lvert\varepsilon^{s/d}_{e\mu}\rvert$	0.025
$\lvert\varepsilon^{s/d}_{e\tau}\rvert$	0.041
$\lvert\varepsilon^{s/d}_{\mu e}\rvert$	0.026
$\lvert\varepsilon^{s/d}_{\mu\mu}\rvert$	0.078
$\lvert\varepsilon^{s/d}_{\mu\tau}\rvert$	0.013
$\lvert\varepsilon^{s/d}_{\tau e}\rvert$	0.120
$\lvert\varepsilon^{s/d}_{\tau\mu}\rvert$	0.018
$\lvert\varepsilon^{s/d}_{\tau\tau}\rvert$	0.130

Table 3.4: 90 % confidence limit upper bounds to absolute values of source and detector NSI matrix elements, from [110].

Table 3.3. Taking NSI effects into account also at the neutrino vertices, the state vectors should be altered accordingly:

$$|\nu^s_\alpha\rangle = |\nu_\alpha\rangle + \varepsilon^s_{\alpha\beta}|\nu_\beta\rangle, \qquad \langle\nu^d_\beta| = \langle\nu_\beta| + \varepsilon^d_{\alpha\beta}\langle\nu_\alpha|, \tag{3.50}$$

where the superscripts s and d refer to neutrino states at source and detector, respectively. Source and detector NSIs are related by $\varepsilon^s_{\alpha\beta} = \varepsilon^{d*}_{\beta\alpha}$. See Table 3.4 for current experimental bounds for ε^s and ε^d matrix elements.

Neutrino flavor transition probability is therefore

$$P_{\alpha\beta} = |A_{\alpha\beta}|^2 = \left|\langle\nu^d_\beta|e^{-iHL}|\nu^s_\alpha\rangle\right|^2. \tag{3.51}$$

3.5.1 Zero-distance flavour transition

In the presence of NSI at source and detector, neutrino flavour in the vertex is not determined by the corresponding charged lepton. Also, the modified neutrino flavour states will not form an orthonormal set of vectors in the three-dimensional flavour space. This results in **zero-distance flavour transition** (zero-distance effect), since the inner product of two does not any more simply result in Kronecker delta. Instead, the transition amplitude for $\nu_\alpha \to \nu_\beta$ is

$$\begin{aligned} A_{\alpha\beta} &= (\langle\nu_\beta| + \varepsilon^d_{\sigma\beta}\langle\nu_\sigma|)(|\nu_\alpha\rangle + \varepsilon^s_{\alpha\gamma}|\nu_\gamma\rangle) \\ &= \delta_{\alpha\beta} + \varepsilon^s_{\alpha\beta} + \varepsilon^d_{\alpha\beta} + \varepsilon^d_{\sigma\beta}\varepsilon^s_{\alpha\sigma} \\ &= (I + \varepsilon^s + \varepsilon^d + \varepsilon^s\varepsilon^d)_{\alpha\beta}, \end{aligned} \tag{3.52}$$

from which the zero-distance transition matrix can be read:

$$A = I + \varepsilon^s + \varepsilon^d + \varepsilon^s\varepsilon^d = I + \varepsilon^s + \varepsilon^{s\dagger} + \varepsilon^s\varepsilon^{s\dagger} \tag{3.53}$$

This results in transition probability

$$\begin{aligned} P_{\alpha\beta} &= (\delta_{\alpha\beta} + \varepsilon^s_{\alpha\beta} + \varepsilon^d_{\alpha\beta} + \varepsilon^d_{\sigma\beta}\varepsilon^s_{\alpha\sigma})(\delta_{\alpha\beta} + \varepsilon^s_{\alpha\beta} + \varepsilon^d_{\alpha\beta} + \varepsilon^d_{\sigma\beta}\varepsilon^s_{\alpha\sigma})^* \\ &\approx \delta_{\alpha\beta}(1 + 2\mathrm{Re}\,(\varepsilon^s_{\alpha\beta} + \varepsilon^d_{\alpha\beta} + \varepsilon^d_{\sigma\beta}\varepsilon^s_{\alpha\sigma})) + |\varepsilon^s_{\alpha\beta}|^2 + |\varepsilon^d_{\alpha\beta}|^2 + 2\mathrm{Re}\,(\varepsilon^s_{\alpha\beta}\varepsilon^{d*}_{\alpha\beta}), \end{aligned} \tag{3.54}$$

where $\mathcal{O}(|\varepsilon|^3)$ terms are ignored. From this form it is apparent that the zero-distance disappearance effect ($\alpha = \beta$) to oscillation probabilities is of order $\mathcal{O}(|\varepsilon|)$ and appearance effect ($\alpha \neq \beta$) of order $\mathcal{O}(|\varepsilon|^2)$. To confirm this effect, a neutrino oscillation expirement with very short baseline would be needed.

It should be noted that the transition probability may exceed 1 with the definition of the transition amplitude given in Eq. (3.52). To get the physical probability, proper normalisation must be done. Assuming that N is the nonunitary 3×3 light neutrino mixing matrix, where the interference of NSI parameters is included (see Section 3.6 for details), the physical transition probability is

$$\overline{P}_{\alpha\beta} = \frac{P_{\alpha\beta}}{(NN^\dagger)^2_{\alpha\alpha}}. \tag{3.55}$$

3.5.2 Interference from CP angle to matter NSI determination

Since the value of δ is unknown, it will severely interfere the efforts to determine the limits for some of the matter NSI parameters. This was investigated in Paper [1] and expanded in [114]. For earlier studies, see [115, 116]. I expand the transition amplitude in Eq. (3.51), resulting in

$$A_{\ell\ell'} = \sum_{j,k=1}^{3} \frac{L}{2E_\nu} U_{\ell j}(M^2)_{jk}(U^\dagger)_{k\ell'} + LV_{\ell\ell'}. \tag{3.56}$$

Next, I parameterise the amplitude in the following way to isolate the dependence on the CP phase:

$$\frac{2E}{L} A_{\ell\ell'} = \begin{cases} N_{\ell\ell'} + K_{\ell\ell'} e^{-i\delta}, \text{ when } \ell \neq \ell' \\ N_{\ell\ell} + K_{\ell\ell} \cos\delta, \text{ when } \ell = \ell' \\ N_{\mu\tau} + K_{\mu\tau}^{(-)} e^{-i\delta} + K_{\mu\tau}^{(c)} \cos\delta, \text{ for } \nu_\mu \to \nu_\tau \text{ transition} \end{cases} \tag{3.57}$$

See Paper [1] for complete expressions of N and K functions. It is interesting to note that since $K_{ee} = 0$, the ν_e survival probability is independent of the CP phase at first order of approximation. Note that the appearance and disappearance channels have different parameterisations. The $N_{\ell\ell'}$ and $K_{\ell\ell'}$ functions are linear functions in $\varepsilon^m_{\ell\ell'}$ matrix elements, one at a time. The CP phase factors are multiplied by the $K_{\ell\ell'}$ functions. The absolute value of the K function reflects the CP phase measurement sensitivity. The transition probabilities for the appearance and disappearance channels are therefore

$$P_{\ell\ell'} = \frac{L^2}{4E^2} \times \begin{cases} N^2_{\ell\ell'} + K^2_{\ell\ell'} + 2N_{\ell\ell'}K_{\ell\ell'}\cos\delta, \text{ when } \ell \neq \ell' \\ N^2_{\ell\ell} + K^2_{\ell\ell}\cos^2\delta - 2N_{\ell\ell}K_{\ell\ell}\cos\delta, \text{ when } \ell = \ell' \end{cases} \tag{3.58}$$

The $\nu_\mu \to \nu_\tau$ transition does not follow the above rule. CP interference is strong in the $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ transitions but not in the other transitions. See Fig. 3.4 for examples of theis interference. Until δ is measured, experiments attempting to discover precise upper bounds for matter NSI should concentrate on oscillation channel(s) with small interference. The magnitude of interference can be quantified by considering relative variation of transition probability

$$R \equiv \frac{P^{\max}_{\ell\ell'} - P^{\min}_{\ell\ell'}}{P^{\min}_{\ell\ell'}}, \tag{3.59}$$

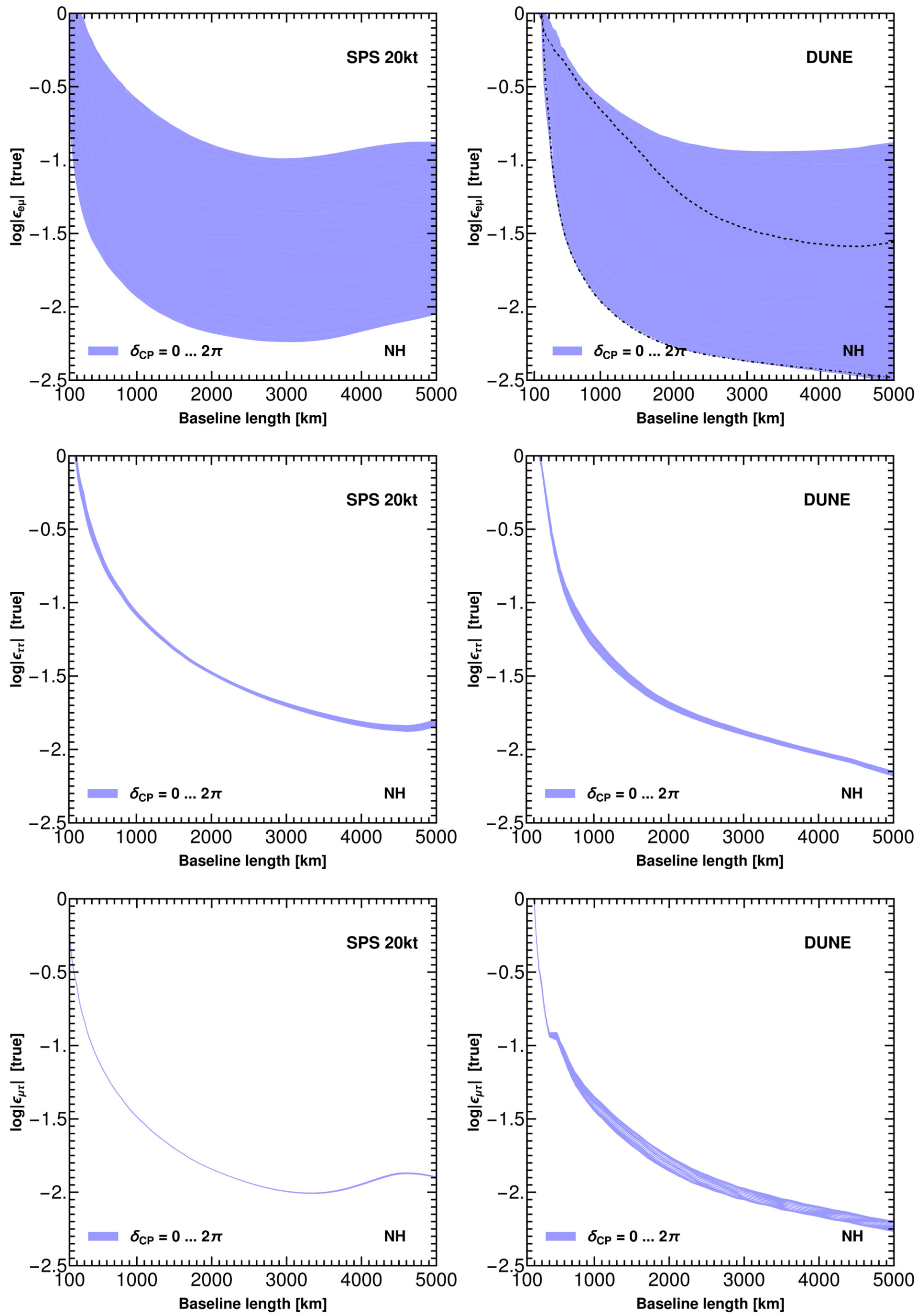


Figure 3.4: Upper plots: 90 % CL discovery reach of $|\varepsilon^m_{e\mu}|$ as a function of baseline length for SPS (left) and DUNE (right) setups. Band thickness results from the ambiguity of δ_{CP}, which visibly interferes with constraining NSI. Absolute exclusion area is above the band. Dashed line in DUNE plot represents the case $\delta_{CP} = 0$ and dot-dashed line the case $\delta_{CP} = \pi/2$. Middle plots are for $|\varepsilon^m_{\tau\tau}|$ and lower plots for $|\varepsilon^m_{\mu\tau}|$. Plots are from [114].

Constraint on	Upper bound
α_{ee}	0.02
$\alpha_{\mu\mu}$	0.01
$\alpha_{\tau\tau}$	0.07
$\|\alpha_{\mu e}\|$	0.010
$\|\alpha_{\tau e}\|$	0.042
$\|\alpha_{\tau\mu}\|$	0.0098

Table 3.5: Current experimental upper limits of the nonunitarity of the light neutrino mixing matrix [119]. All limits are given at 90 % C. L. confidence limit.

where $P_{\ell\ell'}^{\max}$ and $P_{\ell\ell'}^{\min}$ are, respectively, the largest and the smallest value the transition probability achieves when δ_{CP} varies in the range 0 to 2π for a given value of $\varepsilon_{\ell\ell'}^{m}$. If $R \ll 1$, the interference is small, if $R \approx 1$, it is intermediate and if $R > 1$, it is large.

3.6 Nonunitary mixing

If the PMNS matrix turns out to be nonunitary in the future, it could be an indication that it would be a constituent of a larger mixing matrix, of type $(3+s, 3+s)$, where s is the amount of extra neutrinos. These neutrinos would be considered sterile, that is, being singlets with respect to the SM gauge group. In such a case, the PMNS matrix should be modified. I will discuss such extensions in the next Chapter. Within a reasonable degree of accuracy, PMNS matrix is unitary, and any corrections to it will be small. Therefore it is advantageous to parameterise the nonunitarities as a small perturbation. Consider a lower triangular matrix [117]

$$\alpha \equiv \begin{pmatrix} \alpha_{ee} & 0 & 0 \\ \alpha_{\mu e} & \alpha_{\mu\mu} & 0 \\ \alpha_{\tau e} & \alpha_{\tau\mu} & \alpha_{\tau\tau} \end{pmatrix}. \tag{3.60}$$

For alternate parameterisation, see [118]. For all the α matrix elements the condition $|\alpha_{\ell\ell'}| \ll 1$ is valid (see experimental constraint from Table 3.5). Now the new nonunitary light neutrino mixing matrix is defined $N \equiv (I_3 - \alpha)U$. In the context of long baseline neutrino oscillations, the effects on neutrino propagation by the nonunitarity neutrino mixing matrix are similar to effects induced by nonstandard interactions in matter, when matter effects are significant. Comparing the Hamiltonian operator of nonstandard matter interactions (Eq. (3.48)) to nonunitary oscillations in matter (Eq. (3.31) with the replacement $U \mapsto N$) yields the following condition

$$N^\dagger \begin{pmatrix} V_{\mathrm{CC}} + V_{\mathrm{NC}} & 0 & 0 \\ 0 & V_{\mathrm{NC}} & 0 \\ 0 & 0 & V_{\mathrm{NC}} \end{pmatrix} N = V_{\mathrm{CC}} U^\dagger \begin{pmatrix} \varepsilon^m_{ee} & \varepsilon^m_{e\mu} & \varepsilon^m_{e\tau} \\ \varepsilon^{m*}_{e\mu} & \varepsilon^m_{\mu\mu} & \varepsilon^m_{\mu\tau} \\ \varepsilon^{m*}_{e\tau} & \varepsilon^{m*}_{\mu\tau} & \varepsilon^m_{\tau\tau} \end{pmatrix} U \tag{3.61}$$

One finds that the same effect can be described in these two different ways. The ε^m and α parameters can be related to each other. Approximating $N_p \approx N_e \approx N_n$ and eliminating an irrelevant phase, within the first order, the relation between them is

$$\varepsilon^m_{ee} = -\alpha_{ee}, \qquad \varepsilon^m_{e\mu} = \frac{1}{2}\alpha^*_{\mu e}, \qquad \varepsilon^m_{e\tau} = \frac{1}{2}\alpha^*_{\tau e} \tag{3.62}$$

$$\varepsilon^m_{\mu\mu} = \alpha_{\mu\mu}, \qquad \varepsilon^m_{\mu\tau} = \frac{1}{2}\alpha^*_{\tau\mu}, \qquad \varepsilon^m_{\tau\tau} = \alpha_{\tau\tau} \tag{3.63}$$

This interpretation should not be taken too far. One must be aware that this approximation is valid only at long-baseline experiments with substantial matter effects. Since nonunitarity constraints are significantly stricter than bounds on nonstandard interaction in matter, any nonunitarity in such an experiment wears a mask. This mask hides the identification of the source of the deviation of the standard neutrino interactions. In contrast, any signal suggesting the existence of matter NSI matrix elements exceeding the nonunitarity bounds cannot be fully attributed to nonunitarity of neutrino mixing matrix. In such a case, to obtain a true minimal amount of NSI which could be considered one should reduce the nonunitarity bound from the detected amount of NSI.

3.7 Charged lepton oscillation

Since the flavour of charged lepton is defined by its mass, the flavour of a charged lepton can not change. Therefore there are no charged lepton *flavour* oscillations. However, charged leptons do oscillate analogous to neutrino oscillations [120]. In neutrino oscillations, a neutrino with a definite unchanging mass propagates. In the creation and detection processes, different charged leptons are associated in the interaction vertices (see Fig. 3.1). Analogously in charged lepton oscillations, a charged lepton with a definite unchanging mass (and therefore, flavour) propagates. In the creation and detection processes, different neutrino mass states are associated in the interaction vertices.

There are significant obstacles in the way of a detection of charged lepton oscillations. In order to identify the change in neutrino mass at the charged lepton creation and detection vertices, one would need to measure neutrino masses precisely enough to distinguish from the three mass states. While such a feat might not be unrealistic in the future, there is another, awful problem. Consider the longest possible oscillation length [121],

$$L_{\mu e} = \frac{4\pi E}{m_\mu^2 - m_e^2} \approx 2 \cdot 10^{-11} \frac{E}{\text{GeV}} \text{ cm.} \tag{3.64}$$

In order to have an oscillation length of ~ 1 m, the charged leptons should have ultra-high energies, $\mathcal{O}(10^{12})$ GeV.

CHAPTER 4

Neutrino mass models

It doesn't matter how beautiful your theory is. It doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

Richard Feynman

Long before the discovery of neutrino oscillations, neutrinos were generally thought to be massless. Today neutrino mass is one of the biggest mysteries of particle physics, since the SM is unable to accomodate them. There are dozens of competing models on the generation of neutrino mass. This Chapter does not aim to list all of them. The most widely studied models belong to seesaw playground, and I will present detailed descriptions of the three main types of seesaw models, inverse and linear seesaw and neutrinophilic two Higgs doublet model.

4.1 Neutrino mass terms

So far the topic of neutrino mass has repeatedly come up, and the phenomenology of it has been thoroughly reviewed. Nevertheless I have omitted one crucial chunk: where the neutrino Majorana mass term

$$-\frac{1}{2}m_{LL}\overline{\nu_L}\nu_L^c + \text{h.c.} \tag{4.1}$$

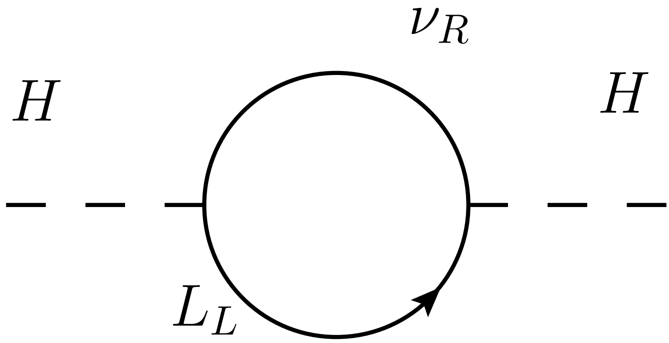


Figure 4.1: A Feynman diagram of a loop correction to SM Higgs mass induced by the inclusion of right-handed neutrinos ν_R in Type I seesaw theory. A similar correction results also in Type III seesaw.

originates? A direct inclusion of the mass term is forbidden due to $SU(2)_L$ gauge invariance. **Seesaw mechanism** assumes that at first such a term doesn't exist, but it can be generated at the effective field theory level. Using only the SM fields, there is one possible gauge invariant dimension-5 operator (Weinberg operator) [45],

$$\frac{f}{\Lambda_{\rm NP}}(L^T C^\dagger \varepsilon H)(H^T \varepsilon L), \tag{4.2}$$

where f is a dimensionless constant[1] and C is the charge conjugation operator. Eq. (4.2) corresponds to Majorana neutrino mass, when H acquires its VEV. At tree-level, there are three distinct ways to generate the Weinberg operator. This works by introducing additional heavy degrees of freedom: right-handed neutrinos (Type I) [46–49, 51, 124], scalars (Type II) [50, 125–128] or fermions triplets (Type III) [52]. All these seesaw types suffer from a **seesaw-induced fine-tuning problem** if the new fields are too heavy: a large loop-correction (see Fig. 4.1) for SM Higgs is induced if the seesaw scale exceeds $\sim 10^7$ GeV [129–131]. See Appendix A.4 for a detailed calculation. For a high-energy scale seesaw, additional new physics must be assumed. A possible way out would be to assume that seesaw scale resides below the scale where the level of fine-tuning becomes unacceptable, or to invoke supersymmetry, where sneutrino loop corrections produce cancellations, stabilising Higgs mass.

[1]The running of f was considered in [122, 123].

In contrast to the left-handed neutrino mass term, Eq. (4.1), the mass term for right-handed Majorana neutrinos

$$-\frac{1}{2}M_{RR}\overline{\nu_R}\nu_R^c + \text{h.c.} \tag{4.3}$$

does not break gauge invariance, since right-handed neutrinos are assumed to be **sterile**, i.e. singlets with respect to the SM gauge symmetries. Indeed, they have only three interaction types: gravitational interactions, mixing to active flavours and decays. In addition, 'left-right' -mass terms

$$m_{LR}\overline{\nu_L}\nu_R + m_{LR}^T\overline{\nu_R^c}\nu_L^c \tag{4.4}$$

may be generated by the SM Higgs mechanism when right-handed neutrinos are included. These mass terms are of Dirac type: $m_{LR} = Y^\nu v$.

Seesaw mechanism combines two mass scales to explain the tiny neutrino masses: the electroweak scale $\Lambda_{\text{EW}} \sim \mathcal{O}(100)$ GeV where the SM Higgs operates and new physics scale Λ_{NP}, here also known as seesaw scale. Requiring the Dirac type Yukawa coupling of neutrino mass term to be $\mathcal{O}(1)$, the scale of new physics is usually considered to be very high: $\Lambda_{\text{NP}} \gtrsim \mathcal{O}(10^{10})$ GeV. This scale can be pushed up to the Grand Unified Theory (GUT) scale, but not at Planck scale $\mathcal{O}(10^{19})$ GeV, since Planck scale seesaw would generate too small active neutrino masses. Assuming lightest neutrino mass to be m_ν, the Yukawa coupling will be perturbative if

$$M_R \lesssim 1.1\times10^{16}\ \text{GeV}\times\frac{0.07\ \text{eV}}{m_\nu}. \tag{4.5}$$

As the Dirac and Majorana mass terms are generated by different mechanisms, the magnitude of their matrix elements is expected to be proportional to the electroweak and seesaw scale, respectively. Since $\Lambda_{\text{EW}} \ll \Lambda_{\text{NP}}$, then $m_{LR} \ll M_{RR}$.

4.2 Type I seesaw

Equations (4.3) and (4.4) can be combined to represent the neutrino mass terms in the following matrix form:

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix}\begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \tag{4.6}$$

Here one can read the symmetric mass matrix in the middle. It is in block form, where 0 and M_{RR} are square matrices, with dimension corresponding to the

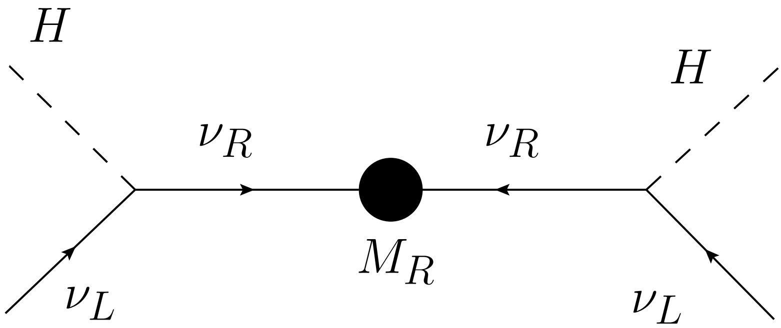


Figure 4.2: Mass of light neutrinos is generated at tree-level. Integrating out the heavy neutrino degrees of freedom, one obtains the Weinberg operator for light Majorana neutrinos.

amount of active (three) and sterile ($s \geq 2$) neutrinos, respectively[2]. To get the mass matrices of the corresponding neutrinos, the mass matrix must be block diagonalized [132] with a unitary transformation matrix U:

$$M_\nu = U^T \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} U = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}. \tag{4.7}$$

Mass matrix type is then $(s+3)\times(s+3)$, subblock m_{LR} is $3 \times s$, M_{RR} and M_N is $s \times s$ and m_ν is 3×3. Light and heavy neutrino mass matrices are m_ν and M_N, respectively. Without losing generality, the block structure of U can be written as follows:

$$U = \begin{pmatrix} A & D^\dagger \\ -C & B^\dagger \end{pmatrix}, \tag{4.8}$$

where A is of type 3×3, B is $s \times s$ and C and D is $s \times 3$. From unitarity condition $U^\dagger U = UU^\dagger = I$ the following matrix equations appear:

$$\begin{aligned} A^\dagger A + C^\dagger C &= I & \qquad AA^\dagger + D^\dagger D &= I \\ BB^\dagger + DD^\dagger &= I & \qquad B^\dagger B + CC^\dagger &= I \\ DA - BC &= 0 & \qquad B^\dagger D - CA^\dagger &= 0 \end{aligned} \tag{4.9}$$

[2]Case $s = 2$ is called the **minimal seesaw**.

Substituting, additional constraints appear.

$$\begin{pmatrix} -A^T m_{LR} C - C^T m_{LR}^T A + C^T M_{RR} C & A^T M_{LR} B^\dagger - C^T m_{LR}^T D^\dagger - C^T M_{RR} B^\dagger \\ -D^* m_{LR} C + B^* m_{LR}^T A - B^* M_{RR} C & D^* m_{LR} B^\dagger + B^* m_{LR}^T D^\dagger + B^* M_{RR} B^\dagger \end{pmatrix}$$
$$= \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix} \tag{4.10}$$

The matrix elements of C and D are assumed to be small, so the quadratic terms involving these matrices may be ignored. Equating the off-diagonal block matrices, one arrives at

$$C = M_{RR}^{-1} m_{LR}^T A$$
$$C = (M_{RR})^{-1T} m_{LR}^T A$$

from which $M_{RR}^{-1} = M_{RR}^{-1T} \Rightarrow M_{RR} = M_{RR}^T$, so M_{RR} is symmetric. Substituting C into eq. (4.9) we get

$$D = B M_{RR}^{-1} m_{LR}^T \tag{4.11}$$

Once C and D are entered to the diagonalized mass matrix we obtain

$$M_\nu = \begin{pmatrix} -A^T m_{LR} M_{RR}^{-1} m_{LR}^T A & 0 \\ 0 & B^*(M_{RR} + M_{RR}^{-1} m_{LR}^\dagger m_{LR} + m_{LR}^T m_{LR}^\dagger M_{RR}^{-1\dagger}) B^\dagger \end{pmatrix} \tag{4.12}$$

In the lower right block $M_{RR} \gg m_{LR}^2 M_{RR}^{-1}$, so it is justified to approximate

$$M_\nu = \begin{pmatrix} -A^T m_{LR} M_{RR}^{-1} m_{LR}^T A & 0 \\ 0 & B^* M_{RR} B^\dagger \end{pmatrix} \tag{4.13}$$

This procedure works for any matrices A and B of the given type, as long as for the matrix elements $A,B \gg C,D$ is valid, too. Choosing A and B as unit matrices will keep the hierarchy between block matrix elements valid, since then $C = D = M_{RR}^{-1} m_{LR}^T$. With this choice, the final form of the neutrino mass matrix is

$$M_\nu = \begin{pmatrix} -m_{LR} M_{RR}^{-1} m_{LR}^T & 0 \\ 0 & M_{RR} \end{pmatrix}. \tag{4.14}$$

This is usually shown as a result of **Type I seesaw mechanism** (see Fig. 4.2), also known as canonical seesaw. Negative sign of the upper diagonal element can be removed by performing a phase rotation with respect to the neutrino doublet. To wit, consider the following Majorana phase transformation matrix:

$$T = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, \underbrace{e^{i\alpha_4}, \cdots, e^{i\alpha_{s+3}}}_{s}) = \begin{pmatrix} iI_3 & 0 \\ 0 & I_s \end{pmatrix}, \tag{4.15}$$

where $\alpha_1 = \alpha_2 = \alpha_3 = \pi/2$ and $\alpha_j = 0$ for $j > 3$. Clearly T is unitary: $TT^\dagger = T^\dagger T = I_{s+3}$. Inserting the unit matrix in Eq. (4.6), the neutrino mass Lagrangian then reads:

$$\begin{aligned}
\mathscr{L}_\nu^m &= -\frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} T \left[T^\dagger \begin{pmatrix} -m_{LR}M_{RR}^{-1}m_{LR}^T & 0 \\ 0 & M_{RR} \end{pmatrix} T^\dagger \right] T \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \\
&= -\frac{1}{2}\begin{pmatrix} i\nu_L^c & \nu_R \end{pmatrix} C \begin{pmatrix} m_{LR}M_{RR}^{-1}m_{LR}^T & 0 \\ 0 & M_{RR} \end{pmatrix} \begin{pmatrix} i\nu_L^c \\ \nu_R \end{pmatrix} && (4.16) \\
&= -\frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} m_{LR}M_{RR}^{-1}m_{LR}^T & 0 \\ 0 & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} && (4.17)
\end{aligned}$$

where C is charge conjugation operator. On the last row, the left-handed neutrino field was redefined as $\nu_L^c \mapsto -i\nu_L^c$. The seesaw mechanism elegantly relates the lightness of active neutrinos to heaviness of sterile neutrinos, as the light neutrino mass matrix is

$$m_\nu = m_{LR}M_{RR}^{-1}m_{LR}^T = v^2 Y^\nu M_{RR}^{-1} Y^{\nu T} \tag{4.18}$$

The mechanism is named seesaw, since the product of light and heavy neutrino masses is expected to be $\sim \Lambda_{\mathrm{EW}}^2$. Decreasing light neutrino mass increases the heavy neutrino mass and vice versa, resembling a seesaw-like dynamic. Invocation of seesaw mechanism leads to nonunitarity of light neutrino mixing matrix via active-sterile mixing, which must be small due to nonobservation. Indeed, the active-sterile mixing block element is proportional to $M_N^{-1/2}$, providing a large suppression.

4.3 Inverse and linear seesaw

In addition to adding the right-handed neutrinos ν_R, in **inverse seesaw** theories [55–57] additional heavy singlet neutrinos $(S_1, S_2, ...)$ are added to the theory.

The mass matrix in flavor space spanned by (ν_L, ν_R, S) is then

$$\begin{pmatrix} 0 & m_{LR} & 0 \\ m_{LR}^T & 0 & M_{RS} \\ 0 & M_{RS}^T & M_{SS} \end{pmatrix}. \tag{4.19}$$

The goal is to diagonalize this mass matrix as was done previously. This diagonalization problem can be reduced to familiar 2×2 block case by looking only at the lower right 2×2 block in above matrix. Assuming $M_{SS} \gg M_{RS}$ and performing the block diagonalization, the resulting partially block diagonalized form is

$$\begin{pmatrix} 0 & m_{LR} & 0 \\ m_{LR}^T & M_{RS} M_{SS}^{-1} M_{RS}^T & 0 \\ 0 & 0 & M_{SS} \end{pmatrix} \tag{4.20}$$

Repeating this procedure for the upper left 2×2 block in above matrix, the final mass matrix is obtained:

$$\begin{pmatrix} m_{LR} M_{RS}^{-1} M_{SS} M_{RS}^{-1T} m_{LR}^T & 0 & 0 \\ 0 & M_{RS} M_{SS}^{-1} M_{RS}^T & 0 \\ 0 & 0 & M_{SS} \end{pmatrix} \tag{4.21}$$

This mechanism is called *inverse* seesaw[3], because instead of postulating just heavy fields, the inverse seesaw assumes the existence of additional light degrees of freedom. The M_{SS} matrix can be naturally small in 't Hooft sense [133], if the right-handed neutrinos have lepton number +1 and the singlet neutrinos −1. Then the only lepton number violating term is proportional to M_{SS}, and the symmetry of the theory is enhanced at the limit $M_{SS} \to 0$.

Light neutrinos are light by double suppression (light singlets and heavy right-handed neutrinos), reducing the preferred energy scale for sterile neutrinos compared to the standard type I seesaw. With inverse seesaw, the seesaw scale can be naturally at TeV scale along with keV scale singlet neutrinos, which are favoured by present cosmological bounds [134].

Instead of postulating the existence of a small lepton number violating term, one could assume direct mixing of active and singlet neutrinos. **Linear seesaw**

[3]In the literature, this mechanism occasionally goes by the name **double seesaw**, since the neutrino masses are doubly suppressed by heavy right-handed neutrinos and light singlet neutrinos. On the other hand, the name may refer also to amount of times needed to block diagonalize the flavor space mass matrix to arrive at the effective light neutrino mass matrix.

[135] was first proposed embedded in a SO(10) grand unified theory, resulting in mass matrix

$$\begin{pmatrix} 0 & m_{LR} & m_{LS} \\ m_{LR}^T & 0 & M_{RS} \\ m_{LS}^T & M_{RS}^T & 0 \end{pmatrix}. \tag{4.22}$$

The inclusion of the m_{LS} term induces the breakdown of $\mathrm{U}(1)_{B-L}$ symmetry. The key feature of linear seesaw is that the neutrino mass is suppressed by SO(10) breaking scale regardless of the $B-L$ breaking scale, which can lie at the TeV scale, or higher.

4.4 Neutrinophilic two Higgs doublet model

Many extensions of the SM assume an enlarged Higgs sector. The most popular of them add an extra singlet or doublet. Singlet extension has been widely studied in the literature due to its simplicity. On the other hand, doublet extension is very well motivated by supersymmetric models and due to its rich phenomenology. Here I review the doublet extension (commonly denoted 2HDM, Two Higgs Doublet Model) with a twist.

Neutrinophilic 2HDM (νHDM) [136] aims to explain the tiny neutrino masses without a need to use an experimentally unfeasible mass scale the standard seesaw theories operate in. After spontaneous symmetry breaking, the light neutrino Majorana and Dirac mass terms are often written as

$$m_M^\nu = \frac{Y_\nu^2 v^2}{M}, \qquad m_D^\nu = Y_\nu v, \tag{4.23}$$

respectively, where Y_ν is the neutrino Yukawa coupling and M is the mass scale of new physics responsible for Majorana masses. If one simply adds Dirac type neutrino masses to SM utilizing SM Higgs VEV, Yukawa coupling turns out to be unnaturally small. Majorana type cases usually utilize a huge seesaw mass scale, rendering the new physics phenomena unobservable. A third option is to generate neutrino mass via an enlarged Higgs sector, here the neutrinophilic Higgs doublet, with a tiny VEV v_ν of eV or keV scale.

νHDM extends the SM by a second Higgs doublet H_ν and three right-handed sterile neutrinos ($\nu_{R1}, \nu_{R2}, \nu_{R3}$). However, just inserting these fields would implicate a SM Higgs coupling to right-handed neutrinos and the new Higgs coupling to charged leptons. These couplings can be forbidden by inserting an extra symmetry in the model. The simplest alternatives are either a discrete $\mathbb{Z}_2$ parity symmetry [137] or a global U(1) symmetry [138]. For the $\mathbb{Z}_2$ case, the new fields are odd with respect to $\mathbb{Z}_2$ and all SM fields even. Similarly for the U(1) case, the new fields carry a U(1) charge and all SM fields are chargeless with respect to the new U(1) charge.

4.4.1 Higgs sector

Upon enlarging the Higgs sector, the Higgs potential takes the form

$$V(H,H_\nu) = -m_1^2 H^\dagger H + m_2^2 H_\nu^\dagger H_\nu - m_3^2 (H^\dagger H_\nu + H_\nu^\dagger H) + \frac{\lambda_1}{2}(H^\dagger H)^2 + \frac{\lambda_2}{2}(H_\nu^\dagger H_\nu)^2 + \lambda_3 (H^\dagger H)(H_\nu^\dagger H_\nu) + \lambda_4 (H^\dagger H_\nu)(H_\nu^\dagger H) + \frac{\lambda_5}{2}\left((H^\dagger H_\nu)^2 + (H_\nu^\dagger H)^2\right). \quad (4.24)$$

This is the most general possible Higgs potential for any 2HDM theory. It has been shown that the vacuum is stable with this Higgs potential in νHDM against radiative corrections [139]. The term containing factor m_3^2 explicitly breaks the extra symmetry.

It is possible to consider a case where $m_3^2 = 0$, where the extra symmetry is broken spontaneously. In the case of discrete symmetry, spontaneous symmetry breaking will produce domain walls [140]. The surface energy density of this domain wall is proportional to v_ν^3. This will contribute to the temperature anisotropies of cosmic microwave background,

$$\frac{\Delta T}{T} \sim \frac{G v_\nu^3}{H_0}, \quad (4.25)$$

where H_0 is Hubble constant and G is Newton's gravitational constant. Since the observed temperature anisotropies by PLANCK are $\sim 10^{-5}$ [141], domain walls will not contradict cosmological data if the neutrinophilic VEV is at most $\mathcal{O}$(MeV), as can be seen from Eq. (4.25). However, to maintain consistency with CLFV decays and neutrino oscillation parameter data it has been shown that $v_\nu \gtrsim \mathcal{O}(10)$ MeV [142], which contradicts the PLANCK constraint. Therefore a $\mathbb{Z}_2$ conserving neutrinophilic model is ruled out. For this reason I consider the case $m_3^2 \neq 0$, removing the CMB limitations.

There are other constraints which need to be considered. The right-handed neutrinos must be heavy enough to fall off of thermal equilibrium during primordial nucleosynthesis in the early universe. This follows from PLANCK constraints for effective number of neutrino degrees of freedom $N_\nu = 3.15 \pm 0.23$ [141], giving a constraint $M_{\nu_R} > 100$ MeV.

The mixing of the Higgs bosons is heavily suppressed by the ratio of the VEVs, $v_\nu/v \ll 1$, which means that the mixing can be ignored altogether. The Higgs sector consists of charged Higgs boson $H^\pm$, neutral pseudoscalar A, and two neutral scalars h and H, where h denotes the SM Higgs. The squared masses of the Higgs bosons are

$$m_h^2 = \lambda_1 v^2 \qquad m_H^2 = m_3^2 \frac{v}{v_\nu} + \lambda_2 v_\nu^2$$
$$m_A^2 = m_3^2 \frac{v}{v_\nu} - \lambda_5(v^2 + v_\nu^2) \qquad m_{H^\pm}^2 = m_3^2 \frac{v}{v_\nu} - \frac{v^2 + v_\nu^2}{2}(\lambda_4 + \lambda_5) \tag{4.26}$$

In general, an extended Higgs sector usually couples to electroweak gauge bosons, providing self-energy diagram loop corrections, denoted oblique corrections. These loop corrections contribute to the self-energy diagrams of electroweak gauge bosons and the $\gamma - Z$ mixing loop diagram, and are parameterised by dimensionless real-valued **Peskin-Takeuchi parameters** (also known as STU formalism) [143, 144]. The absolute values of these parameters mirror the amount of deviations from SM by a BSM theory if there are no BSM gauge bosons sensitive to electroweak interactions and that the nonoblique corrections are negligible. The experimental constraints of Peskin-Takeuchi parameters translate to constraints in νHDM Higgs sector. This was studied in Paper [3], where it was found out that only a relatively small mass gap $\Delta m \equiv |m_{H^\pm} - m_{H/A}| \lesssim 100$ GeV is consistent with the current bounds [145] for the Peskin-Takeuchi parameters.

4.4.2 Neutrino sector

Once the extra symmetry imposed by νHDM is broken by neutrinophilic VEV at TeV energy scale or higher, neutrinos acquire their mass. The relevant Lagrangian is

$$\Delta\mathscr{L} = Y_{ij}^\nu \overline{L}_i H_\nu \nu_{Rj} + \frac{1}{2} M_{ij}^R \overline{\nu_R}_i \nu_{Rj} + \text{h.c.}, \tag{4.27}$$

where the Yukawa terms are responsible for Dirac neutrino mass term after the spontaneous symmetry breaking. νHDM does not predict texture for neutrino Yukawa coupling matrices. Nonstandard interactions arising from low-energy limit were studied in [146]. The second term in Eq. (4.27) is the Majorana mass term. From equation (4.27) it can be seen that after spontaneous symmetry breaking, the Dirac mass is given by

$$m_D \equiv m_{LR} = Y^\nu v_\nu. \tag{4.28}$$

The notation introduced should be reminiscent of Type I seesaw framework. Following the usual seesaw procedure, the 6×6 neutrino mass matrix is then block diagonalized, giving the light neutrino 3 × 3 mass matrix:

$$m_\nu = m_{LR} M_{RR}^{-1} m_{LR}^T \equiv U^T m_\nu^{\text{diag}} U \tag{4.29}$$

where U is the PMNS mixing matrix and $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ is the diagonal light neutrino mass matrix. Going beyond tree level, it is seen that one-loop H and A corrections to light neutrino masses may be large enough to contradict present experimental data [147, 148]. The loop contributions cancel out exactly if $m_H = m_A$ [149], which can be accomplished by setting $\lambda_5 = 0$ in the Higgs potential.

4.4.3 Collider phenomenology

Since νHDM contains charged Higgs bosons, they can be produced and detected with LHC. There are extensive studies constraining the mass of charged Higgses [150–152], but these studies assume that they couple to quarks. Since the νHDM extended Higgs sector is leptophilic, these constraints do not apply. Possible decay modes of $H^\pm$ are

$$H^\pm \to \begin{cases} \nu_R \ell^\pm \\ HW^\pm \\ AW^\pm, \end{cases} \tag{4.30}$$

where $\ell = e, \mu, \tau$. Under the assumption that νHDM Higgs sector is observable at the LHC, the Higgs boson and right-handed neutrino masses should lie at 100 — 1000 GeV scale. This forces the neutrino Yukawa coupling to be small, but it will not worsen the flavour problem. If $\Delta m > M_W$, the branching ratio of $H^\pm$ to $HW^\pm$ and $AW^\pm$ decay modes dominates, since H^+ can decay to on-shell W boson. Nonetheless, decay via virtual W boson dominates even if 50 GeV $\lesssim \Delta m < M_W$, since the decay via $\nu_R \ell^\pm$ is suppressed by small Yukawa couplings. At $\Delta m < 50$ GeV, $\nu_R \ell^\pm$ is the dominant decay mode [3].

Neutrino mass hierarchy	Number of events 6L	5L	SS3L
Normal	8	84	83
Inverted	25	199	85
SM background	0	2	1

Table 4.1: Adapted from Paper [3]. Number of six-lepton (6L), five-lepton (5L) and same sign trilepton (SS3L) states seen in 13 TeV center-of-mass energy LHC with 1000 fb^{-1} integrated luminosity with different neutrino mass hierarchies. The contribution by SM background is on the last row. For corresponding neutrinophilic scalar masses, see text.

Resulting right-handed neutrinos, neutral Higgs bosons and W bosons are unstable, and produce more leptons, which results in an excess of multilepton states: particularly six-lepton, five-lepton and same-sign trilepton (SS3L) signals would be seen significantly more than the SM predicts at the LHC. Of these signal regions, same-sign trilepton signal has the largest cross section, and it is therefore the most promising signal to search for. In all decay modes, the amount of multilepton states is larger for inverted neutrino mass hierarchy, since the corresponding branching ratio for $H^{\pm} \to e^{\pm}\nu_R$ is larger for IH. This provides an indirect hint on neutrino mass hierarchy, since the high electron multiplicity events are more frequent for IH [3].

To obtain useful benchmark points, we scanned through the parameter space for the masses of neutrinophilic scalars in different signal regions for LHC with 13 TeV center-of-mass energy. The five- and six-lepton signals are largest at small mass gaps. In contrast, SS3L signal cross section is lower for small Δm, while obtaining largest values at $\Delta m \geq M_W$, with on-shell W boson. As expected, the signal cross sections drop, when the scalar masses increase or end-state lepton multiplicity increases. We performed the same analysis for a 1 TeV center-of-mass energy electron-positron collider, and obtained similar results. With latter collider analysis, the cross section sinks significantly slower at low scalar masses, and significantly faster at high scalar masses. All signal regions have tiny SM background contamination, which turns out to be small enough to be safely neglected.

In [3], we have chosen one particular benchmark point such that $m_{H^{\pm}} =$

188.5 GeV and $m_{H/A} = 187.5$ GeV. To promote the decay mode via heavy neutrinos, we chose the right-handed neutrino to be much lighter than the charged neutrinophilic scalars ($m_{\nu_R} = 100$ GeV) and a small mass gap Δm. With 1000 fb^{-1} integrated luminosity, we obtained a significant amount of events, see Table 4.1, which elucidates the cleanliness of the chosen signal regions for this benchmark point.

4.5 Type II seesaw

Instead of postulating heavy right-handed neutrinos and integrating them out as was done in the previous section, an extended Higgs sector may be presumed. Since neutrino masses are so light, one may conjecture that vastly different mass scales are generated by different Higgs bosons. Seesaw models with new Higgs fields are called **Type II seesaws** [50, 125–128].

The Weinberg operator can be produced also by integrating out a BSM scalar field. Since the combination $\overline{\nu}\nu^c$ has hypercharge 2, one could instead couple the neutrino to a scalar with hypercharge -2. In Type II theories by default one scalar SU(2) triplet is added, but there are also proposals which extend the Higgs sector with one or more SU(2) doublets. The inclusion of extra scalars leads to

1. contribution to the SM Higgs self-interaction
2. contribution to masses of electroweak gauge bosons and SM Higgs
3. contribution to charged lepton flavor violating decays
4. nonstandard neutrino interactions
5. neutrino masses

4.5.1 Higgs sector

I will present the default case, where the SM is extended with only a scalar triplet $\Delta = (\Delta_1, \Delta_2, \Delta_3) \sim (\mathbf{3}, 2)$. In some cases, a lepton number $L = -2$ is assigned to Δ. This would preserve the lepton number symmetry in Majorana mass term. The scalar sector of the Lagrangian is then significantly more complicated than in SM:

$$\mathscr{L}_{\text{scalar}} = (D_\mu H)^\dagger (D^\mu H) + \text{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] + Y^\nu L_L^T C i\sigma_2 \Delta L_L - V(H,\Delta) \quad (4.31)$$

Here V is the Higgs potential, Y^ν is neutrino Yukawa matrix, C is charge conjugation operator, and the triplet is presented as a bidoublet form

$$\Delta = \frac{1}{\sqrt{2}}\sigma_i \Delta_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix} \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} \quad (4.32)$$

The covariant derivative of Δ is

$$D_\mu \Delta = \partial_\mu \Delta + i g_2 \left[\boldsymbol{\tau} \cdot \mathbf{W}_\mu, \Delta\right] + i g_1 Y_\Delta B_\mu \Delta/2 \quad (4.33)$$

where $Y_\Delta = 2$ is the hypercharge of the triplet. The Higgs potential in Eq. (4.31) is

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \left(\lambda_\phi H^T i\sigma_2 \Delta^\dagger H + \text{h.c.}\right), \quad (4.34)$$

where λ_ϕ is dimension-1 coupling. Corresponding term contributes to SM Higgs quartic self-interaction (see Fig. 4.3.d). The next stage is to find out the electric charges of the triplet using Gell-Mann–Nishijima -formula (Eq. 1.26):

$$Q\Delta = [\tau^3, \Delta] + \frac{Y}{2}\Delta = \begin{pmatrix} +\Delta_{11} & +2\Delta_{12} \\ 0 \cdot \Delta_{21} & +\Delta_{22} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \odot \Delta \quad (4.35)$$

Here $\odot$ refers the Hadamard product for matrices. The charges of corresponding fields in units of e are easily read from the decomposed form. Hence I define

$$\Delta_1 - i\Delta_2 = \sqrt{2}\Delta^{++}, \qquad \Delta_1 + i\Delta_2 = \sqrt{2}\Delta^0, \qquad \Delta_3 = \Delta^+, \quad (4.36)$$

arriving at the following bidoublet form

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \xrightarrow{\text{VEV}} \begin{pmatrix} 0 & 0 \\ v' & 0 \end{pmatrix}, \quad (4.37)$$

where v' is the VEV of the triplet Higgs. The explicit form of it can be found by considering the effective Higgs potential, where H has acquired a VEV but Δ has not:

$$\begin{aligned} V(v,\Delta) = &-\frac{1}{2}m_H^2 v^2 + \frac{\lambda v^4}{16} + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) \\ &+ \left(\lambda_\phi \begin{pmatrix} 0 & v \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}^\dagger \begin{pmatrix} 0 \\ v \end{pmatrix} + \text{h.c.}\right) \quad (4.38) \\ = &\, M_\Delta^2 \left(|\Delta^{++}|^2 + |\Delta^+|^2 + |\Delta^0|^2\right) - v^2 \lambda_\phi \Delta^{0*} - v^2 \lambda_\phi \Delta^0 + \text{constant} \quad (4.39) \end{aligned}$$

Minimizing the potential, the VEV of Δ^0 is solved:

$$\frac{\partial V(v,\Delta)}{\partial \Delta^{0*}} = 0 \Rightarrow \langle \Delta^0 \rangle \equiv v' = \lambda_\phi \frac{v^2}{M_\Delta^2} \tag{4.40}$$

The triplet VEV v' will contribute to electroweak gauge boson masses. The SM ρ parameter is then shifted (see Appendix A.3 for details):

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1+2\left(\frac{v'}{v}\right)^2}{1+4\left(\frac{v'}{v}\right)^2}. \tag{4.41}$$

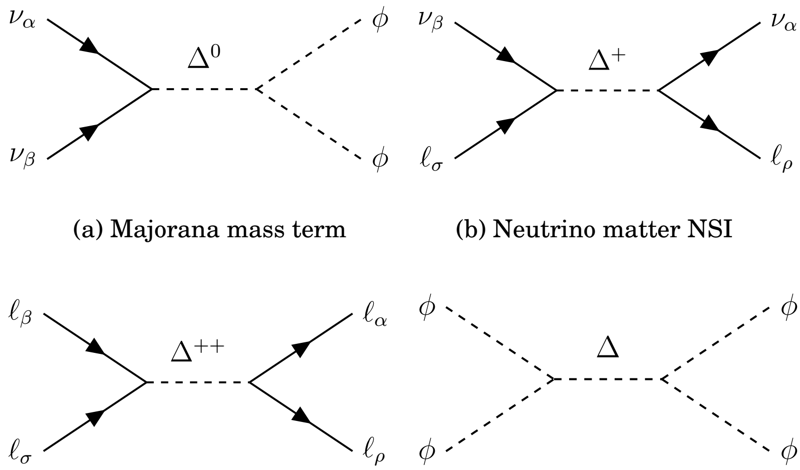
In the SM $\rho = 1$ exactly at the tree level. The experimental limits are quite strict, which implicates a low VEV of the neutral component of the scalar triplet with respect to SM Higgs VEV $v \approx 246$ GeV. Indeed, for $\rho = 1.00037 \pm 0.00023$ [153] the triplet VEV is at most 4.3 GeV. The measured value corresponds to 1.6σ deviation from the SM prediction. The triplet VEV might not lie at the GeV scale, but could be naturally small to account for tiny neutrino masses, which are given by light neutrino mass matrix

$$m_\nu = -Y^\nu v' = -Y^\nu \lambda_\phi \frac{v^2}{M_\Delta^2}. \tag{4.42}$$

Similar to Type I seesaw, the troubling negative sign may be removed by phase redefinition. Neutrino masses would be suppressed by a small coupling Y^ν or λ_ϕ or heaviness of triplet Higgs, or by a combination of these. If the triplet is assigned with $L = -2$, the only term in the Lagrangian which would violate lepton number conservation would be proportional to the coupling λ_ϕ. Since at the limit $\lambda_\phi \to 0$ the symmetry of the theory would be enhanced, it could be naturally small in 't Hooft sense [133], providing a natural reason for lightness of neutrinos. Direct searches for Δ in colliders have produced a lower bound $M_\Delta \gtrsim 750$ GeV [154]. Additional constraints for parameters M_Δ and λ_ϕ were considered in Paper [2].

In this Section, the triplet scalars were assumed to be mass degenerate. This turns out to be a good approximation, since the mass gap $|\Delta m| \equiv |m_{\Delta^{++}} - m_{\Delta^+}|$ is not allowed to be larger than ~ 40 GeV due to the constraints given by Peskin-Takeuchi parameters [155, 156].

Even though Type II seesaw solves many problems and provides exciting opportunities for collider and neutrino oscillation phenomenology, it is not perfect.



(a) Majorana mass term (b) Neutrino matter NSI

(c) Four-lepton NSI (d) SM Higgs self-coupling

Figure 4.3: Tree-level Feynman diagrams for interactions between neutrinos ν, leptons ℓ and the Standard Model Higgs scalar ϕ, and are mediated by the triplet Higgs fields Δ.

The tree-level correction to the SM Higgs mass is small in Type II seesaw:

$$m_H^2 = \frac{\lambda}{4}v^2 - \sqrt{2}\lambda_\phi v' = \left(\frac{\lambda}{4} - \frac{\sqrt{2}\lambda_\phi^2}{M_\Delta^2}\right)v^2 \tag{4.43}$$

However, loop-level corrections can be large, as was mentioned in Sec. 4.1.

4.5.2 Yukawa sector

Written in terms of component fields, the Yukawa terms from Eq. (4.31) take the form

$$\begin{aligned}\mathscr{L} &= Y^\nu_{\alpha\beta} L^T_{\alpha L} C\, i\sigma_2 \Delta L_{\beta L} + \text{h.c.}\\ &= Y^\nu_{\alpha\beta}\left[\Delta^0\, \overline{\nu^C_{\alpha R}}\,\nu_{\beta L} - \frac{1}{\sqrt{2}}\Delta^+\left(\overline{\ell^C_{\alpha R}}\,\nu_{\beta L} + \overline{\nu^C_{\alpha R}}\,\ell_{\beta L}\right) - \Delta^{++}\,\overline{\ell^C_{\alpha R}}\,\ell_{\beta L}\right] + \text{h.c.},\end{aligned} \tag{4.44}$$

which correspond to the Feynman diagrams shown in Fig. 4.3.a, 4.3.b and 4.3.c.

Currently neutrino flavor transition is in excellent agreement with the 3ν oscillation framework, but there is still room for moderately large subleading effects. Unfortunately Type II -model does not provide any hints for texture or patterns on neutrino mixing matrix. This stems from elements of Yukawa coupling matrix Y^{ν}, which are arbitrary. It is possible to integrate out the heavy scalar to produce the low-energy Lagrangians, resulting in

$$\mathscr{L}_{\nu}^{m} = \frac{Y_{\alpha\beta}\,\lambda_{\phi}\,v^2}{M_{\Delta}^2}\left(\overline{\nu_{\alpha R}^{C}}\,\nu_{\beta L}\right) = -(m_{\nu})_{\alpha\beta}\,\overline{\nu_{\alpha R}^{C}}\,\nu_{\beta L}, \tag{4.45}$$

$$\mathscr{L}_{\text{NSI}} = \frac{Y_{\sigma\beta}\,Y_{\alpha\rho}^{\dagger}}{M_{\Delta}^2}\left(\overline{\nu_{\alpha L}}\,\gamma_{\mu}\,\nu_{\beta L}\right)\left(\overline{\ell_{\rho L}}\,\gamma^{\mu}\,\ell_{\sigma L}\right), \tag{4.46}$$

$$\mathscr{L}_{4\ell} = \frac{Y_{\sigma\beta}\,Y_{\alpha\rho}^{\dagger}}{M_{\Delta}^2}\left(\overline{\ell_{\alpha L}}\,\gamma_{\mu}\,\ell_{\beta L}\right)\left(\overline{\ell_{\rho L}}\,\gamma^{\mu}\,\ell_{\sigma L}\right), \tag{4.47}$$

which are low-energy versions of Fig. 4.3.a, Fig. 4.3.b and Fig. 4.3.c, respectively [157–160]. Running of the model parameters was studied in [158]. CLFV decays induced by four-lepton NSI were studied in [161]. Solving Yukawa matrix element $Y_{\alpha\beta}$ from Eq. (4.45), inserting it to Eq. (4.46) and comparing it to Eq. (3.47), the NSI coupling constant can be solved for left-handed leptons:

$$\varepsilon_{\alpha\beta}^{\rho\sigma} = -\frac{M_{\Delta}^2}{2\sqrt{2}\,G_F\,v^4\,\lambda_{\phi}^2}\,(m_{\nu})_{\sigma\beta}\,(m_{\nu}^{\dagger})_{\alpha\rho}, \tag{4.48}$$

This relation allows to interpret the neutrino NSI bounds as bounds on $\frac{M_{\Delta}}{\lambda_{\phi}}$, which is dimensionless. See Fig. 4.4 for such bounds. Neutrino NSI gives an upper bound $\mathcal{O}(10^{12})$. Combining the lower bound for M_{Δ} by CMS collaboration[154], lower bound for trilinear coupling from neutrino NSI is $|\lambda_{\phi}| > 31$ meV [2].

4.6 Type III seesaw

It turns out that there is a third way of accommodating tiny neutrino masses at tree-level. As in Type I and II cases, **Type III seesaw** [52] is a simple extension: add a hyperchargeless fermion triplet field $\Sigma = (\Sigma^1, \Sigma^2, \Sigma^3) \sim (\mathbf{3}, 0)$ to the SM. The relevant Lagrangian of Type III seesaw mechanism is

$$\mathscr{L} = \text{Tr}(\overline{\Sigma} i \not{D} \Sigma) - \frac{1}{2}\text{Tr}\underbrace{(\overline{\Sigma} M_{\Sigma} \Sigma^c + \overline{\Sigma^c} M_{\Sigma}^* \Sigma)}_{\text{Majorana mass}} - \underbrace{H^{'\dagger}\overline{\Sigma}\sqrt{2}\,Y_{\Sigma} L}_{\text{Yukawa}} + \text{h.c.}, \tag{4.49}$$

where similar to Eq. (4.32), Σ is presented as a bidoublet:

$$\Sigma = \frac{1}{\sqrt{2}}\sigma_i \Sigma_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \Sigma_3 & \Sigma_1 - i\Sigma_2 \\ \Sigma_1 + \Sigma_2 & -\Sigma_3 \end{pmatrix} \equiv \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \tag{4.50}$$

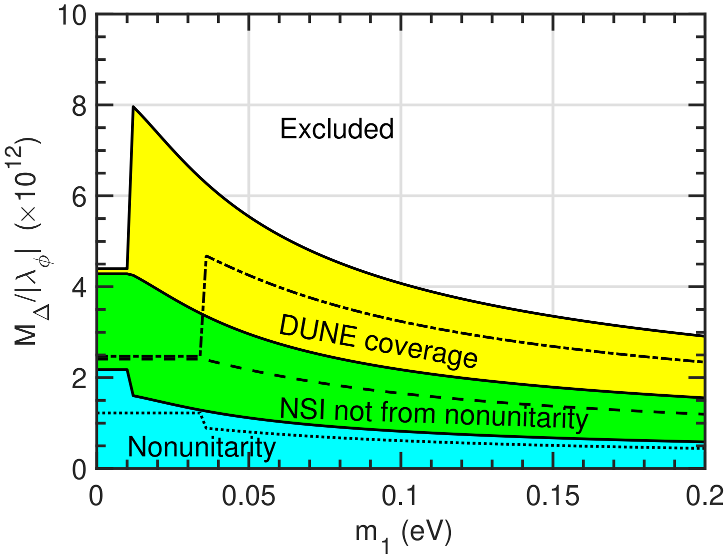


Figure 4.4: Bounds for $M_\Delta/|\lambda_\phi|$ as function of the lightest neutrino mass for NH (solid lines) and IH (dashed lines). The white region is excluded at 90% CL when m_1. The yellow region shows the values which are sensitive to DUNE. The green region is insensitive to DUNE but is distinguishable from nonunitarity effects. A signal from the blue region could be misinterpreted as matter NSI effect. This plot is from Paper [2].

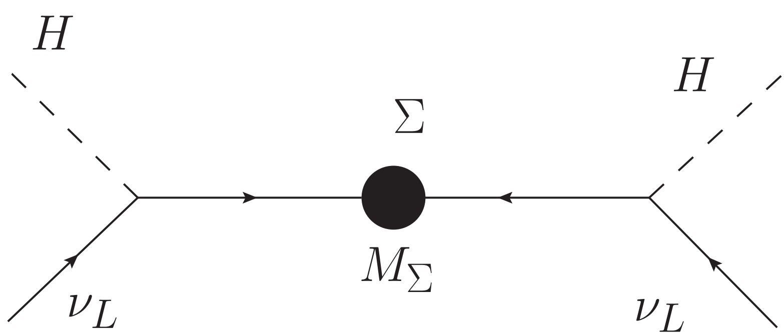


Figure 4.5: Tree-level realization of light Majorana neutrino mass term in Type III seesaw mechanism. At low-energy limit this Feynman diagram produces Weinberg operator.

Using Gell-Mann–Nishijima -formula, the electric charges can be determined:

$$Q\Sigma = [\tau^3, \Sigma] + \frac{Y}{2}\Sigma = \begin{pmatrix} 0\cdot\Sigma_{11} & +\Sigma_{12} \\ -\Sigma_{21} & 0\cdot\Sigma_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \odot \Sigma \tag{4.51}$$

Now the Σ fields can be defined by their charge:

$$\Sigma_3 = \Sigma^0, \qquad \Sigma_1 \mp i\Sigma_2 = \sqrt{2}\Sigma^{\pm} \tag{4.52}$$

The first (kinetic) term in Eq. (4.49) contains interactions, which couple Σ to W and Z bosons. Ergo, Σ fermions can be produced by colliders. Current direct search bounds from LHC imply $M_\Sigma > 840$ GeV [162].

The Yukawa term in Eq. (4.49) induces mixing between $\Sigma^{\pm}$ and charged leptons $\ell^{\pm}$ and also between Σ^0 and neutrinos ν, with mixing strength $Y_\Sigma v/M_\Sigma$. This will generate flavour violating vertices, producing CLFV decays, like $\mu \to e\gamma$. The effect corresponds to nonstandard interactions at source and detector,

$$\varepsilon^s = \frac{v^2}{2} Y_\Sigma^\dagger (M_\Sigma^\dagger M_\Sigma)^{-1} Y_\Sigma \ll 1. \tag{4.53}$$

CLFV decays provide stricter limits than direct search bounds. Model-independent CLFV limits can be found from Table 3.4. These constraints provide most stringent limit for triplet mass: $M_\Sigma > 200$ TeV.

Neutrino mass generation is similar to Type I case (see Fig. 4.5). The neutrino mass Lagrangian is identical to Eq. (4.6), with $M_{RR} \mapsto M_\Sigma$. Then following the derivation of light neutrino mass matrix in Section 4.2 results in

$$m_\nu \sim \frac{Y_\Sigma^2 v^2}{M_\Sigma}, \tag{4.54}$$

which is analogous to Eq. (4.18). As in Type I case, the mass term breaks lepton number and light neutrino mixing matrix will be nonunitary due to $\nu - \Sigma^0$ mixing.

CHAPTER 5

CONCLUSIONS AND OUTLOOK

> The saddest aspect of life right now is that science gathers knowledge faster than society gathers wisdom.
>
> *Isaac Asimov*
> Isaac Asimov's Book of Science and Nature Questions (1988)

Arguably, this is the **dark age of particle physics**. Quantum field theory and the SM have been rigorously tested and no significant deviations have been measured. However, as stated in Section 1.3, black clouds have been gathering on top of our heads, namely dark matter, inflation, strong CP problem, hierarchy problem, Higgs mass instability and so on. There might be a desert between the electroweak scale and GUT scale, meaning that we will find new interactions only in the distant future, when we go up 14 degrees of magnitude on the energy scale. BSM particle theorists are increasingly cornered. They have retreated to speculate an endless amount of increasingly exotic theories, adressing a small amount of anomalies of the SM. There is not a theory in sight which would smash all the problems of the SM altogether. But is everything so hopeless?

Albert Michelson, known from Michelson-Morley experiment, famously said in 1894:

"...*it seems probable that most of the grand underlying principles*

> *have been firmly established [...] the future truths of physical science are to be looked for in the sixth place of decimals.*"

This mirrors the situation today in 2018. Many predictions of QFT and the SM have been tested to tenth place of decimals. Soon after Michelson's speech, Röntgen rays, radioactivity, quantization of light, Zeeman effect and many other phenomena were discovered, and it would take several decades of work of several physicists to unravel the mysteries of these phenomena in the modern language of quantum mechanics and QFT. I expect the same thing happen here in the next two decades, and I expect the neutrino sector to play a crucial role due to several unsolved problems related to it.

It is almost certain that a powerful long-baseline neutrino experiment will begin operation in near future. Such an experiment will be able to distinguish the neutrino mass hierarchies and atmospheric octant angle and measure leptonic CP violating angle. $0\nu2\beta$ decay experiments will be able to distinguish neutrino Dirac and Majorana masses. Also, next-generation tritium beta decay electron energy measurements will be able to measure the absolute masses of neutrinos. Neutrino charge radius can also be measured in the near future. For some oscillation parameters, the experimental errors are already small enough to characterize neutrino physics entering a new era - a precision era.

This thesis is a tribute for all the hard work that has been done. I have extensively considered the possibilities of measuring matter NSI in long baseline neutrino oscillation experiments and using it as a tool for deriving constraints from Type II seesaw framework. I have also analyzed the properties and constraints of neutrinophilic two Higgs doublet model.

After measuring phenomena predicted by the SM extended by three light massive neutrinos with increasing precision, the road is open for a more speculative territory of measuring possible anomalies of neutrino oscillation, measuring a large value of magnetic moment of neutrinos, detecting cosmic neutrino background and perhaps finding hints of leptogenesis or right-handed neutrinos. In fact, there are several tantalizing oscillation anomalies hinting at the existence of sterile neutrinos. For example, see [163]. It is too early to say whether these anomalies are statistical fluctuations or the winds of new beginnings.

APPENDIX A

APPENDIX

A.1 Definitions

- Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.1}$$

  In addition I define $\tau_j = \sigma_j/2$ and $\sigma_0 = \overline{\sigma}_0 = I_2$, $\overline{\sigma}_j = -\sigma_j$ $(j = 1,2,3)$. The τ matrices are SU(2) group generators.

- Gamma matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{A.2}$$

- Projection operators:

$$P_L = \frac{1}{2}(I - \gamma_5), \qquad P_R = \frac{1}{2}(I + \gamma_5) \tag{A.3}$$

- SU(2) group generators in 3×3-representation:

$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{A.4}$$

- Gell-mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{A.5}$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

  Gell-Mann matrices are SU(3) group generators.

- Levi-Civita-symbol of $n > 1$ dimensions has n indices, defined with $\varepsilon_{1\cdots n} = 1$ and antisymmetricity of the indices. In the case where at least two indices are the same, the symbol is zero. In two dimensions, it has a matrix form

$$(\varepsilon)_{jk} = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{A.6}$$

  In three dimensions, they form the structure constants of SU(2) group. Contracting Levi-Civita with a symmetric tensor yields zero.

- SU(3) structure constants:

$$f_{123} = 1 \tag{A.7}$$
$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}$$
$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

  All the other nonzero f_{ijk} $(i,j,k = 1,\cdots,8)$ can be inferred from these by permutation and utilizing antisymmetricity.

- Rotation matrix in two dimensions in counterclockwise direction through and angle θ reads

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{A.8}$$

  These matrices form the rotation group SO(2). Notably $R(\theta)^{-1} = R(-\theta)$.

- Rotation matrices in three dimensions are defined

$$R_1(\theta) \equiv R_{23}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \tag{A.9}$$

$$R_2(\theta) \equiv R_{13}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{A.10}$$

$$R_3(\theta) \equiv R_{12}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A.11}$$

  In the one-index notation the subscript defines the rotation axis (with 1, 2 and 3 corresponding to x_1, x_2 and x_3, respectively). Two-index notation defines the rotation plane: with R_{ij} the rotation is confined to $x_i x_j$-plane. These matrices form the rotation group SO(3).

- Phase matrices in three dimensions are defined

$$P_1(\theta) = \mathrm{diag}(e^{i\theta}, 1, 1), \qquad P_2(\theta) = \mathrm{diag}(1, e^{i\theta}, 1), \qquad P_3(\theta) = \mathrm{diag}(1, 1, e^{i\theta}), \tag{A.12}$$

  where the index corresponds to the placement of the phase factor on the diagonal. Note also that for any j, $P_j(0) = I$ and $P_j(\theta)^{-1} = P_j(-\theta)$.

A.2 Oscillation probability insensitivity to Majorana phases

Let U be neutrino mixing matrix without Majorana phases. Transition probability for oscillation $\nu_\ell \to \nu_{\ell'}$ is

$$P_{\ell\ell'} = \left| \sum_{i=1}^{3} U_{\ell i}^* U_{\ell' i} e^{-iE_i L} \right|^2 \tag{A.13}$$

where E_i is neutrino energy and L its baseline. Majorana phases can be added by performing **Majorana transform**

$$U \mapsto U \cdot \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}), \tag{A.14}$$

where $\mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}) \equiv D$. Clearly $|D_{ij}| = \delta_{ij}$. On element-level Majorana transform is

$$U_{ij} \mapsto U_{ik} D_{kj} = U_{ik} \delta_{jk} D_{kj} = U_{ij} D_{jj}. \tag{A.15}$$

Therefore the matrix element product in transition probability transforms as

$$U^*_{\ell i} U_{\ell' i} \mapsto U^*_{\ell i} D^*_{ii} U_{\ell' i} D_{ii} = U^*_{\ell i} U_{\ell' i} \tag{A.16}$$

where $D_{ii} D^*_{ii} = |D_{ii}|^2 = \delta^2_{ii} = 1$. Therefore $P_{\ell\ell'}$ is invariant in Majorana transform and Majorana phases do not enter to transition probability expressions of neutrino oscillation.

A.3 Electroweak gauge boson masses in Type II seesaw

Covariant derivative of $\Delta = (\Delta^{++}\ \Delta^{+}\ \Delta^{0})^T$ is

$$D_\mu \Delta = \left(I_3 \partial_\mu - i g_2 \mathbf{T} \cdot \mathbf{W}_\mu - i g_1 B_\mu Y_\Delta \right) \Delta \tag{A.17}$$

$$= \partial_\mu \Delta - \begin{pmatrix} g_2 W^3_\mu + g_1 B_\mu & g W^-_\mu & 0 \\ g_2 W^+ & g_1 B_\mu & g W^-_\mu \\ 0 & g_2 W^+_\mu & -g_2 W^3_\mu + g_1 B_\mu \end{pmatrix} \Delta, \tag{A.18}$$

where T_j are the SU(2) group generators in 3×3 representation. Since the charged fields do not gain a VEV, they will not affect the masses of W and Z bosons. Therefore I replace Δ by the form of the triplet after it has acquired its VEV,

$$\langle \Delta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v' + \Delta^{0'} \end{pmatrix}. \tag{A.19}$$

Now looking at kinetic term and considering only terms relevant mass corrections of W and Z bosons,

$$(D_\mu\Delta)^\dagger(D^\mu\Delta) = (0\ 0\ 1)\begin{pmatrix} g_2W^3_\mu + g_1B_\mu & gW^-_\mu & 0 \\ g_2W^+ & g_1B_\mu & g_2W^-_\mu \\ 0 & g_2W^+_\mu & -g_2W^3_\mu + g_1B_\mu \end{pmatrix}$$

$$\times \begin{pmatrix} g_2W^3_\mu + g_1B_\mu & gW^-_\mu & 0 \\ g_2W^+ & g_1B_\mu & gW^-_\mu \\ 0 & g_2W^+_\mu & -g_2W^3_\mu + g_1B_\mu \end{pmatrix}^\dagger \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{v' + \Delta^{0'}}{\sqrt{2}}\right) + \cdots \tag{A.20}$$

$$= \frac{v'^2}{2}\left(g^2 W^+_\mu W^-_\mu + (g_1^2 + g_2^2) Z_\mu Z^\mu\right) + \cdots, \tag{A.21}$$

where the corrections for mass terms can be read:

$$\Delta m_W^2 = \frac{1}{2} g_2^2 v'^2 \qquad \Delta m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2) v'^2 \tag{A.22}$$

$$\Rightarrow m_W^2 = g_2^2\left(\frac{1}{4}v^2 + \frac{1}{2}v'^2\right) \qquad m_Z^2 = (g_1^2 + g_2^2)\left(\frac{1}{4}v^2 + \frac{1}{2}v'^2\right). \tag{A.23}$$

The ρ parameter in Type II seesaw is therefore (note: $\cos^2\theta_W = 1 + (g_1/g_2)^2$):

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{1 + 2\left(\frac{v'}{v}\right)^2}{1 + 4\left(\frac{v'}{v}\right)^2}. \tag{A.24}$$

Solving this with respect to the fraction of VEVs, one obtains

$$\frac{v'}{v} = \sqrt{\frac{\rho - 1}{4 - 2\rho}} = \sqrt{\frac{\rho - 1}{2}}\left(1 + \frac{1}{2}(\rho - 1) + \frac{3}{8}(\rho - 1)^2 + \cdots\right) \tag{A.25}$$

where on the last stage the expression is expanded as binomial series, centered on $\rho_{SM} = 1$.

A.4 Fine-tuning of Higgs mass in seesaw theories

The correction to Higgs mass can be calculated from a Feynman loop diagram, Fig. 4.1, where a SM lepton with mass m and heavy neutrino (Type I) or fermion (Type III) with mass $M \gg m$ produce it. I approximate Higgs momentum p to be low and SM lepton mass negligible. Relevant Yukawa coupling is Y. The amplitude of loop diagram is then calculated. Symmetry factor of the diagram is $S = -1$ (negative sign from fermion loop).

$$A = -\frac{Y^2}{2} S^{-1} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\frac{(\not{p} - \not{k}) + M}{(p-k)^2 - M^2} \cdot \frac{\not{k} + m}{k^2 - m^2} \right) \tag{A.26}$$

Performing Feynman parameterization to the above integral and changing variables to $\ell = k - xp$, the integral is transformed to

$$A = \frac{Y^2}{2} \int_0^1 dx \int \frac{d^4\ell}{(2\pi)^4} \left(\frac{\ell^2}{(\ell^2 + \Delta)^2} + \frac{N}{(\ell^2 + \Delta)^2} \right) = \frac{Y^2}{2} \int_0^1 dx (I_1 + I_2) \tag{A.27}$$

where $\Delta = -x^2p^2 + xp^2 + (m^2 - M^2)x - m^2$ and $N = x^2p^2 - xp^2 - Mm$. Using cutoff regularization (with Λ being the seesaw scale), the integrals I_1 and I_2 can be calculated:

$$I_1 \equiv \int \frac{d^4\ell}{(2\pi)^4} \frac{\ell^2}{(\ell^2 + \Delta)^2} = \int d\Omega_4 \int \frac{d\ell}{(2\pi)^4} \left(\ell + \frac{2\Delta}{\ell} \right) \tag{A.28}$$

$$I_2 \equiv \int \frac{d^4\ell}{(2\pi)^4} \frac{N}{(\ell^2 + \Delta)^2} = N \int d\Omega_4 \int \frac{d\ell}{(2\pi)^4} \frac{1}{\ell} \tag{A.29}$$

Here I grabbed only the divergent part of the integrals. $\int d\Omega_4 = 2\pi^2$ is the area of a four-dimensional unit sphere. Combining the momentum integrals I_1 and I_2, one gets

$$I_1 + I_2 = \frac{1}{8\pi^2} \int_M^\Lambda d\ell \left(\ell + \frac{2\Delta + N}{\ell} \right) = \frac{1}{8\pi^2} \left(\Lambda^2 + (2\Delta + N) \ln \frac{\Lambda}{M} \right) \tag{A.30}$$

Performing the parameterization integral, one arrives at

$$A = \frac{Y^2}{2} \int_0^1 dx \frac{1}{8\pi^2} \left(\Lambda^2 + (2\Delta + N) \ln \frac{\Lambda}{M} \right) \tag{A.31}$$

$$= \frac{Y^2}{16\pi^2} \left(\Lambda^2 + \left(\frac{p^2}{6} - m^2 - M^2 - mM \right) \ln \frac{\Lambda}{M} \right) \tag{A.32}$$

$$\approx \frac{Y^2}{16\pi^2} \left(\Lambda^2 + M^2 \ln \frac{M}{\Lambda} \right), \tag{A.33}$$

where on the last step I neglected terms involving p and m, being small compared to M.

A.5 Effective field theory

Any theory proclaiming to describe reality at high-energy scale must reduce to SM interactions at low-energy limit. Taking a low-energy limit of a high-energy theory produces a corresponding **effective field theory** (EFT), which describes the phenomenology at an energy scale easier to access. This means that the SM itself is considered an effective theory. Here I briefly present some basic tools of EFT and a couple of illuminating examples. Low-energy Lagrangians of Type II seesaw and seesaw mass terms introduced in Chapter 4 can be derived using EFT. For reviews of EFT, see [164–167].

Upon applying an EFT, one must be aware of the validity region. Every EFT breaks down at sufficiently large energy scale. Effective operator of dimension $d > 4$ with cutoff scale Λ will predict scattering amplitudes proportional to $\left(\frac{E}{\Lambda}\right)^{d-4}$, where E is the center-of-mass energy. At high enough E, the cross sections will eventually break unitarity bounds, rendering the predictive power of an EFT worthless. Once $\frac{E}{\Lambda} = \mathcal{O}(1)$, one must abandon the corresponding EFT.

From a BSM perspective the total EFT Lagrangian contains operators with $d > 4$:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{O_5}{\Lambda} + \frac{O_6}{\Lambda^2} + \cdots \tag{A.34}$$

It should be noted that such high-dimensional operators are nonrenormalizable, even if the original high-energy operators are not.

A.5.1 Propagator expansion

The first step in propagator expansion technique is to write a full amplitude of tree-level scattering involving a heavy field mediator, with mass M. This heavy field provides momentum transfer, which must be small at $E \ll M$. The momentum transfer is put to zero, producing a point interaction amplitude corresponding to an effective Lagrangian.

The best way to elucidate propagator expansion is via a concrete example. Consider the μ^- decay amplitude in the SM, Fig. A.1. It has the form

$$A(\mu^- \to \nu_\mu \overline{\nu}_e e^-) = \left(\frac{ig_2}{\sqrt{2}}\right)^2 (\overline{e}\gamma^\mu P_L \nu_e) \frac{-ig_{\mu\nu}}{p^2 - M_W^2} (\overline{\nu_\mu}\gamma^\nu P_L \mu). \tag{A.35}$$

At the low energy limit, the momentum transfer p by the W boson is negligible, and the denominator of the W propagator can be replaced by simply $-M_W^2$. In

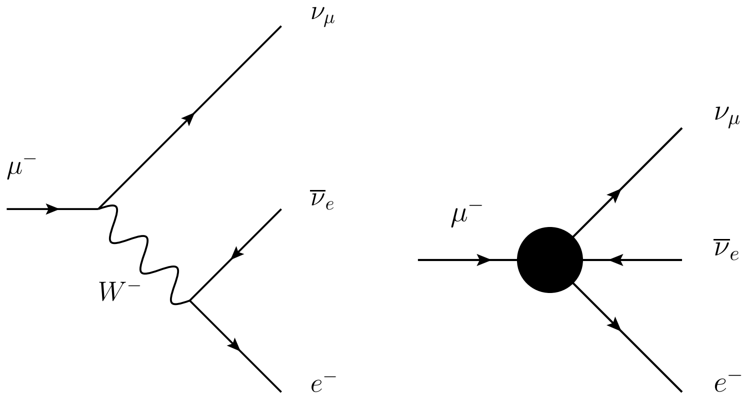


Figure A.1: Left diagram represents the μ^- decay in the SM. At low energy, the scattering amplitude and muon decay width can be calculated within zero momentum transfer approximation, without considering the W boson, in the right diagram.

the leading order, the amplitude is then

$$A(\mu^- \to \nu_\mu \overline{\nu}_e e^-) \approx \frac{-ig_2^2}{2M_W^2}(\overline{e}\gamma^\mu P_L \nu_e)(\overline{\nu_\mu}\gamma^\nu P_L \mu) \tag{A.36}$$

First subleading term is proportional to M_W^{-4}, which produces an even smaller constribution provided that the center-of-mass energy is not at the same scale as M_W. At EFT level μ^- decay is a point interaction, which corresponds to a vertex factor. This leading order amplitude can be produced from an EFT Lagrangian

$$\mathscr{L}_{\text{eff}} = -C(\overline{e}\gamma^\mu P_L \nu_e)(\overline{\nu_\mu}\gamma^\nu P_L \mu), \tag{A.37}$$

where C is an effective coupling constant. Comparing the expressions in Eq. (A.36) and Eq. (A.37) one arrives at relation

$$C = \frac{g_2^2}{2M_W^2} \equiv 2\sqrt{2}G_F, \tag{A.38}$$

where $G_F = \sqrt{2}g_2^2/8M_W^2$ is the Fermi coupling, the coupling constant of effective Fermi theory of weak interactions. Eq. (A.37) is the EFT realization of Eq. (A.36).

A.5.2 Heavy field substitution

Consider a rather generic Lagrangian, which contains the following terms:

$$\mathscr{L} = \mathscr{L}_{\text{kinetic}} + M^2 V_\mu^\dagger V^\mu + \sum_i g_i V_\mu J_i^\mu + \sum_i g_i J_i^{\mu\dagger} V_\mu^\dagger \tag{A.39}$$

The Lagrangian includes conserved currents J_i^μ, real coupling constants g_i, a heavy field V^μ and mass of the field M. If V^μ is self-adjoint, the mass term has an additional factor of 1/2. Simple dimension analysis indicates that V^μ field must be bosonic. Nevertheless the procedure described here is applicable to fermions too, having a mass term $m_f \overline{f} f$.

Since the validity of EFT is exclusively at low energy domain, where kinetic energy of heavy particle is insignificant with respect to its rest mass the kinectic Lagrangian may be neglected. The first step is to solve V^μ from the Euler-Lagrange equation with respect to its conjugate field $V_\mu^\dagger$:

$$\frac{\partial L}{\partial V_\mu^\dagger} - \partial_\mu \left(\frac{\partial L}{\partial(\partial V_\mu^\dagger)} \right) = M^2 V^\mu + \sum_i g_i J_i^{\mu\dagger} = 0 \Rightarrow V^\mu = -\frac{1}{M^2} \sum_i g_i J_i^{\mu\dagger} \tag{A.40}$$

After plugging the solution back to the original Lagrangian, Eq. (A.39), the heavy degrees of freedom have been effectively integrated out, and a current-current interaction emerges.

$$\mathscr{L} = -\frac{1}{M^2} \sum_i g_i J_i^{\mu\dagger} \sum_j g_j J_j^\mu . \tag{A.41}$$

Example: Fermi theory

The power of this formalism is now illustrated by deriving the Fermi Lagrangian from lepton sector of electroweak theory. In this case, there is only one relevant coupling constant $g_2/2\sqrt{2}$ and one relevant current - the leptonic current, Eq. (2.12).

Using the technique developed above, W boson is integrated out, resulting in the effective Fermi Lagrangian

$$\mathscr{L} = -\frac{g_2^2}{8M_W^2} J_l^\mu J_{l\mu}^\dagger \equiv -\frac{G_F}{\sqrt{2}} J_l^\mu J_{l\mu}^\dagger \tag{A.42}$$

Historically, after measuring G_F, the energy scale of the high-energy theory (which would be electroweak theory) was accurately estimated to be $\mathscr{O}(10^2)$ GeV, assuming $g \sim \mathscr{O}(1)$.

A.5.3 Path integration

The integration referred in the phrase *integrating out the heavy degrees of freedom* is just a path integral. Consider a (full) Lagrangian $\mathscr{L}(\psi_H, \psi_L)$ containing both heavy and light fields, ψ_H and ψ_L, respectively. The effective Lagrangian $\mathscr{L}_{\text{eff}}(\psi_L)$ containing only light fields can be obtained by performing a path integration [168, 169]:

$$e^{iS_{\text{eff}}} = \exp\left(i\int \mathscr{L}_{\text{eff}}(\psi_L) d^4x\right) = \int (D\psi_H)(D\psi_H^\dagger) \exp\left(i\int \mathscr{L}(\psi_H, \psi_L) d^4x\right) \tag{A.43}$$

$$= \exp\left(i\int \left(\mathscr{L}(0, \psi_L) + \mathscr{O}(\psi_L)\right) d^4x\right). \tag{A.44}$$

Here S_{eff} is effective action corresponding to the effective Lagrangian and $O(\psi_L)$ is an effective operator consisting of light fields ψ_L, and not necessarily having dimension 4

Next, I demonstrate how the path integral approach applied to Type I seesaw model results in the effective Weinberg operator. The relevant part of the Lagrangian, containing the right-handed neutrinos $N = (N_1, N_2, N_3)$ reads as

$$\mathscr{L}_N = \frac{1}{2}\overline{N_i} i\partial\!\!\!/ N_i - \frac{1}{2} M_i \overline{N_i} N_i - \left(\overline{\ell_{\alpha L}} Y'^{\nu}_{\alpha i} H' N_i + \text{h.c.}\right) \tag{A.45}$$

$$= \frac{1}{2}\overline{N} K N - \frac{1}{2}\overline{N} J - \frac{1}{2}\overline{J} N \tag{A.46}$$

where I defined

$$K \equiv i\partial\!\!\!/ - M, \qquad J \equiv Y'^{\nu T} H^T \ell_L^c + Y'^{\nu\dagger} H'^{\dagger} \ell_L \tag{A.47}$$

The next stage is to literally integrate out the heavy neutrino.

$$S = \int (DN)(D\overline{N}) \exp\left(i\int \mathscr{L}_N d^4x\right) \tag{A.48}$$

$$= \int (DN)(D\overline{N}) \exp\left(i\int \left(\frac{1}{2}\overline{N} K N - \frac{1}{2}\overline{N} J - \frac{1}{2}\overline{J} N\right) d^4x\right) \tag{A.49}$$

$$= \det(K) \exp\left(i\int \underbrace{\frac{1}{2}\overline{J} K^{-1} J}_{=\mathscr{L}_{\text{eff}}} d^4x\right) \tag{A.50}$$

The path integral is Gaussian, which allows a quick evaluation of the resulting effective Lagrangian. The determinant factor contributes only a constant in the

Lagrangian, so its effects can be ignored. The result is

$$\begin{aligned}\mathscr{L}_{\text{eff}} &= \frac{1}{2}(\overline{\ell}_L Y^{'\nu} H' + \overline{\ell^c_L} Y^{'\nu *} H^{'*})(i\partial\!\!\!/ - M)^{-1}(Y^{'\nu T} H^T \ell^c_L + Y^{'\nu\dagger} H^{'\dagger} \ell_L) \\ &= \frac{1}{2}\overline{\ell_L} H' Y^{'\nu} M^{-1} Y^{'\nu T} H^{'T} \ell^c_L + \text{h.c.} \\ &= \frac{1}{2}\overline{\nu}_\alpha (v+h)^2 (Y^{'\nu} M^{-1} Y^{'\nu T})_{\alpha\beta} \nu_\beta + \text{h.c.}\end{aligned} \tag{A.51}$$

where on the second row the propagator was approximated by $-M^{-1}$, since the right-handed neutrino is assumed to be very massive. The last row is the situation after SSB. Picking up the term proportional to v^2, the effective mass term for light neutrinos emerges:

$$\mathscr{L}^{\nu}_{\text{mass}} = \overline{\nu}_\alpha \left[\underbrace{(Y^{'\nu} v)}_{=m_{LR}} M^{-1} \underbrace{(v Y^{'\nu})^T}_{=m_{LR}}\right]_{\alpha\beta} \nu_\beta = \overline{\nu}\underbrace{m_{LR} M^{-1} m^T_{LR}}_{=m_\nu} \nu \tag{A.52}$$

See Eq. (4.18) for comparison.

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