Magnetic field contribution to black-hole-hedgehog's solution in GraviWeak unification

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Abstract. In the framework of Multiple Point Principle (MPP), where the existence of the two degenerate vacua of the Universe, the first, ElectroWeak vacuum with ≈ 246 GeV ("true vacuum"), and the second at Planck scale $\sim 10^{18}$ GeV ("false vacuum"); we investigated the gravitational black-hole-hedgehog's solution with magnetic field contribution in the GraviWeak unification model described by $f(R)$ gravity. We have considered the phase transition from the "false vacuum" to the "true vacuum" and confirmed the stability of the ElectroWeak vacuum. The "false vacuum" defect configurations for the black-hole-hedgehog have given a global monopole and this monopole has been "swallowed" by the black-hole with core mass $M_{BH} \approx 3.65 \times 10^{18}$ GeV and radius $\delta \approx 6 \times 10^{-21}$ GeV^{-1}. The horizon radius of the black-hole-hedgehog is around $r_h \approx 1.14\,\delta$.

1. Introduction

The birth of our Universe is a Big Bang since it represents the point of time when the Universe entered into a regime where the laws of physics began to work. Big Bang is not an explosion in space, but rather an expansion of space. After the initial expansion, the early Universe underwent a series of phase transitions. During these phase transitions, the breakdown of local or global gauge symmetries produces the vacuum topological defects (point, line and sheet defects). The cosmological model developed in Refs. [1, 2, 3] assumes the existence of two degenerate vacua of the Universe, the first ("true") Electroweak (EW) vacuum with VEV $v_1 \approx 246$ GeV and the second ("false") Planck scale vacuum with VEV $v_2 \sim 10^{18}$ GeV, see Fig. 1. In these papers, we investigated hedgehog's configurations as defects of "the false vacuum". In the previous papers, [1, 2, 3] devoted to studying of topological defects of the universal vacua we gave the investigation of hedgehog's configurations [4, 5] neglecting the contribution of magnetic fields. But, recently we have done the full estimation, see [6].

2. Gravi-Weak unification, the action and field equations

In Refs. [7, 8, 9], using results of Refs. [10, 11], we have constructed the Gravi-Weak unification (GWU), considering a $Spin(4, 4)$-group of GWU spontaneously broken into the $SL(2, C)^{(grav)} \times$
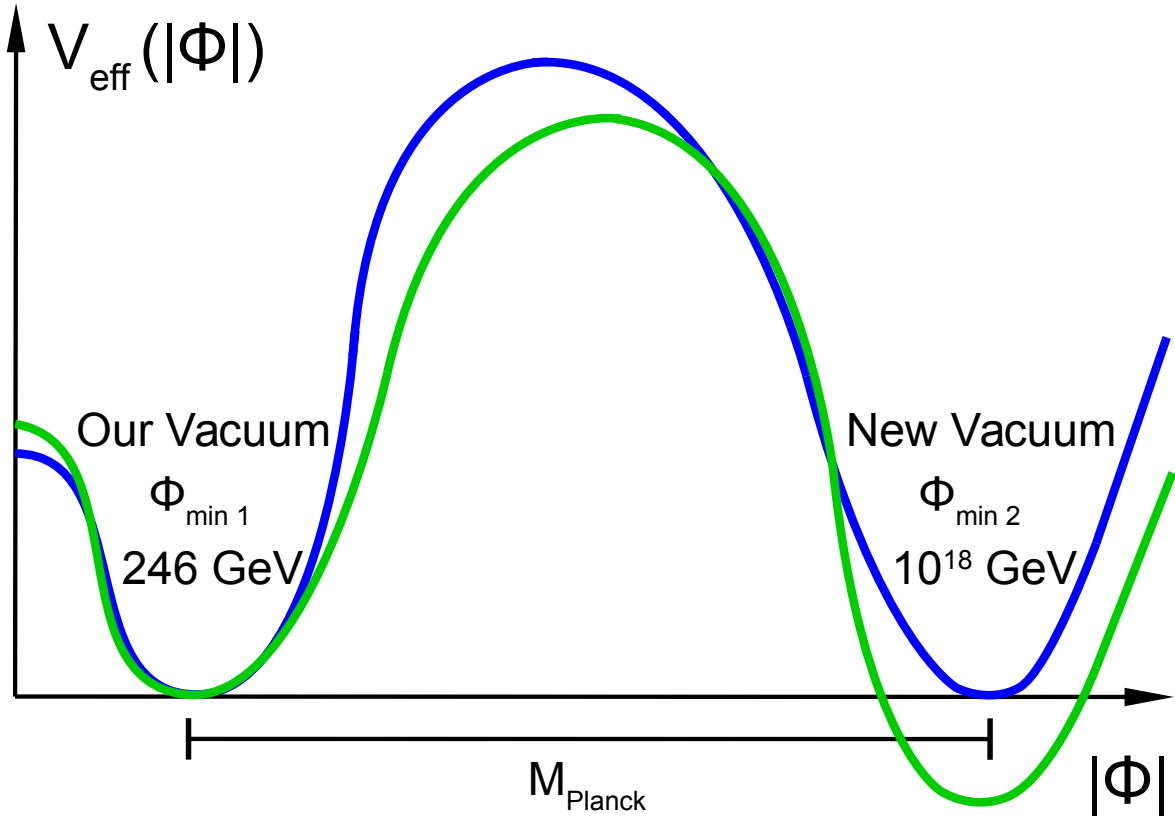
Figure 1. Minima of the effective Higgs potential in the pure Standard Model, which correspond to the first Electroweak "true vacuum", and to the second Planck scale "false vacuum".

$SU(2)^{(weak)}$ group of symmetry. In agreement with experimental and astrophysical results, we assumed that after the Big Bang, there came into being the unification group G_{TOE} of the **Theory of the Everything (TOE)** which was rapidly broken down to the direct product of series of gauge groups (see Ref. [1]) ended by the Standard Model group $G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$. The action $S_{(GW)}$ of the Gravi-Weak unification obtained in Refs. [7, 8, 9] is given by the following expression:

$$
\begin{aligned}
S_{(GW)} &= -\frac{1}{g_{uni}} \int_{\mathfrak{M}} d^4x \sqrt{-g} \left[\frac{1}{16} \left(R|\Phi|^2 - \frac{3}{2}|\Phi|^4 \right) \right. \\
&\left. + \frac{1}{16} \left(aR_{\mu\nu}R^{\mu\nu} + bR^2 \right) + \frac{1}{2}\mathcal{D}_\mu\Phi^\dagger\mathcal{D}^\mu\Phi + \frac{1}{4}F^i_{\mu\nu}F^{i\,\mu\nu} \right],
\end{aligned}
\tag{1}
$$

where g_{uni} is a parameter of the GWU, parameters a, b (with $a + b = 1$) are "bare" coupling constants of the higher derivative gravity, R is the Riemann curvature scalar, $R_{\mu\nu}$ is the Ricci tensor, $|\Phi|^2 = \Phi^a\Phi^a$ is a squared triplet Higgs field, where Φ^a (with $a = 1, 2, 3$) is an isovector scalar belonging to the adjoint representation of the $SU(2)$ gauge group of symmetry. In Eq.(1) the $\mathcal{D}_\mu\Phi^a = \partial_\mu\Phi^a + g_2\epsilon^{abc}A^b_\mu\Phi^c$ is a covariant derivative, and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_2\epsilon^{abc}A^b_\mu A^c_\nu$ is a curvature of the gauge field A^a_μ of the $SU(2)$ Yang-Mills theory with a coupling constant g_2 as a "bare" coupling constant of the $SU(2)$ weak interaction. The action (1) is a special case of the $f(R)$ gravity [12, 13, 14]. From the action (1), using the metric formalism, we obtain the

following field equations:

$$\mathcal{F}(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu\mathcal{F}(R) + g_{\mu\nu}\Box\mathcal{F}(R) = \kappa T^m_{\mu\nu}, \quad \text{where} \quad \mathcal{F}(R) \equiv \left.\frac{df(r)}{dr}\right|_{r=R}, \quad (2)$$

$\kappa = 8\pi G_N$, G_N is the gravitational constant, and T^m is the energy-momentum tensor derived from the matter action S_m.

3. De-Sitter solutions at the early time of the Universe

It is well known that at the early time, the Universe is described by the de-Sitter solutions (see for example Refs. [15, 16]). Our model is a special case of the more general $SU(N)$ model [10], where authors assumed that the Universe is inherently de-Sitter. Then, the 4-spacetime is a hyperboloid in a 5-dimensional Minkowski space under the constraint $x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = r_{dS}^2$, where r_{dS} is a radius of the curvature of the de-Sitter space, or simply "the de-Sitter radius".

Vacuum energy density of our Universe is the Dark Energy (DE). The cosmological constant Λ describes the DE substance, which is dominant in the Universe at later times:

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit}} \simeq 0.75, \tag{3}$$

where ρ_{DE} is the dark energy density and the critical density is:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G_N} \simeq 1.88 \times 10^{-29} H_0^2. \tag{4}$$

Here H_0 is the Hubble constant $H_0 \simeq 1.5 \times 10^{-42}$ GeV. Dark Energy (DE) is related with cosmological constant Λ by the following way $\rho_{DE} = \rho_{vac} = (M_{Pl}^{red})^2\Lambda$, where M_{Pl}^{red} is the reduced Planck mass, $M_{Pl}^{red} \simeq 2.43 \times 10^{18}$ GeV. At present, cosmological measurements give (see [17]) $\rho_{DE} \simeq (2 \times 10^{-3} \text{ eV})^4$, which means a tiny value of the cosmological constant, $\Lambda \simeq 10^{-84}$ GeV^2. This tiny value of ρ_{DE} was first predicted by B.G. Sidharth in 1997 year [18, 19]. In the 1998 year S. Perlmutter, B. Schmidt and A. Riess [20] were awarded the Nobel Prize for the discovery of the accelerating expansion of the Universe.

Having an extremely small cosmological constant of our Universe, Bennett, Froggatt and Nielsen [21, 22, 23] assumed to consider only zero, or almost zero, cosmological constants for all vacua existing in Nature. They formulated a new law of Nature named the Multiple Point Principle (MPP), which means: *There exist in Nature several degenerate vacua with very small energy density or cosmological constants.* The model developed in this article considers the existence of the two degenerate vacua of the Universe, the first ("true") Electroweak (EW) vacuum, and the second ("false") Planck scale vacuum. From experimental results, cosmological constants - minima of the Higgs effective potentials $V_{eff}(\phi_H)$ - are not exactly equal to zero. Nevertheless, they are extremely small. By this reason, Bennett, Froggatt and Nielsen [21, 22, 23] assumed to consider zero cosmological constants as a good approximation. Then according to the MPP, we have a model of pure SM being fine-tuned in such a way that these two vacua proposed have just zero energy density (see also Ref. [24]).

We note that as a fundamental constant under the de-Sitter symmetry, r_{dS} is not a subject to quantum corrections. Local dynamics exist as fluctuations with respect to this cosmological background. In general, the de-Sitter space may be inherently unstable. The quantum instability of the de-Sitter space was investigated by various authors. Abbott and Deser [25] have shown that de-Sitter space is stable under a restricted class of classical gravitational perturbations. So any instability of the de-Sitter space may likely have a quantum origin. Ref. [26] demonstrated through the expectation value of the energy-momentum tensor for a system with a quantum

field in a de-Sitter background space, that in general, it contains a term that is proportional to the metric tensor and grows in time. As a result, the curvature of the spacetime would decrease and the de-Sitter space tends to decay into the flat space (see Ref. [27]). The decay time of this process is of the order of the de-Sitter radius i.e., $\tau \sim r_{dS} \simeq 1.33\,H_0^{-1}$. Since the age of our universe is smaller than τ_{dS}, we are still observing the accelerating expansion of the Universe. Of course, we also can consider the perturbation de-Sitter solutions but these perturbations are very small [15, 16].

4. The solution for the gravitational black-holes-hedgehogs with magnetic field contribution

The field configurations describing a monopole-hedgehog [4, 5] are:

$$\Phi^a = vw(r)\frac{x^a}{r} \quad \text{and} \quad A_\mu^a = a(r)\epsilon_{\mu ab}\frac{x^b}{r}, \tag{5}$$

where $x^a x^a = r^2$ with $(a = 1, 2, 3)$, $w(r)$ and $a(r)$ are some structural functions. This solution is pointing radially. Here Φ^a is parallel to $\hat{r}$ - the unit vector in the radial, and we have a "hedgehog" solution of Refs. [4, 5]. The terminology "hedgehog" was first suggested by Alexander Polyakov in Ref. [5]. The functions $w(r)$ and $a(r)$ are constrained by the following conditions:

$$w(0) = 0, \quad \text{and} \quad w(r) \to 1 \quad \text{when} \quad r \to \infty, \tag{6}$$

$$a(0) = 0, \quad \text{and} \quad a(r) \sim -\frac{g}{r} \quad \text{when} \quad r \to \infty. \tag{7}$$

4.1. The metric in the vicinity of the global monopole

The most general static metric in the vicinity of the global monopole is a metric with a spherical symmetry, $ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2)$. We can calculate the global monopole energy-momentum tensor components:

$$T_t^t = v^2\frac{w'^2}{2A} + v^2\frac{w^2}{r^2} + \frac{1}{4}\lambda v^4\left(w^2 - 1\right)^2 - \frac{a'^2}{A} + \frac{a^2}{r^2},$$

$$T_r^r = -v^2\frac{w'^2}{2A} + v^2\frac{w^2}{r^2} + \frac{1}{4}\lambda v^4\left(w^2 - 1\right)^2 - \frac{a'^2}{A} + \frac{a^2}{r^2},$$

$$T_\theta^\theta = T_\varphi^\varphi = v^2\frac{w'^2}{2A} + \frac{1}{4}\lambda v^4\left(w^2 - 1\right)^2. \tag{8}$$

4.2. The hedgehog's structure functions

As an example we can use the following expressions for monopole structure functions $w(r)$ and $a(r)$, which satisfy the conditions (6) and (7):

$$w(r) = 1 - \exp\left(-\frac{r^2}{\delta^2}\right) \quad \text{and} \quad a(r) = -\frac{g}{r}\left(1 - \exp\left(-\frac{r^2}{\delta^2}\right)\right), \tag{9}$$

The components (8) of the monopole energy-momentum tensor using the result (9). But for simple estimation we can be limited by an approximation:

$$w(r) = 0 \quad \text{for} \quad 0 \leq r \leq \delta, \quad \text{and} \quad w(r) = 1 \quad \text{for} \quad \delta < r < \infty \quad \text{and} \tag{10}$$

$$a(r) = 0, \quad g = 0 \quad \text{for} \quad 0 \leq r \leq \delta, \quad \text{and} \quad a(r) = -\frac{g}{r}, \quad g = g_2 \quad \text{for} \quad \delta < r < \infty, \tag{11}$$

where δ is a radius of the hedgehog.

4.3. The solution with magnetic field contribution

Considering the approximation (10) and (11), in agreement with a solution used by Barriola and Vilenkin in Ref. [28], we can obtain a simple approximate solution for the monopole-hedgehog taking $w = 1$ out the core of the hedgehog [1]. In the case of Refs. [1, 29, 30, 31, 32, 33] scalar curvature R is constant, and the Einstein's field equation from (1):

$$\frac{1}{A}\left(\frac{1}{r^2} - \frac{1}{r}\frac{A'}{A}\right) - \frac{1}{r^2} = \frac{1}{\kappa v^2}T_t^t \quad \text{and} \quad \frac{1}{A}\left(\frac{1}{r^2} + \frac{1}{r}\frac{B'}{B}\right) - \frac{1}{r^2} = \frac{1}{\kappa v^2}T_r^r. \tag{12}$$

Here $\kappa = 8\pi G_N$. In approximation (10) and (11), the energy-momentum tensor components are given by the following approximations:

$$T_t^t = T_r^r \approx \frac{\lambda \kappa^2 v^4}{4} \quad \text{for} \quad 0 \le r \le \delta, \quad T_t^t = T_r^r \approx \frac{\kappa v^2}{r^2} + \frac{g^2}{r^4}\left(-\frac{1}{A} + 1\right) \quad \text{for} \quad \delta < r < \infty. \tag{13}$$

According to Refs. [1, 34, 35], the Schwarzschild type metric fora black-hole is given by the expression:

$$A^{-1} \approx 1 - \kappa v^2 - \frac{2G_N M}{r} + ..., \tag{14}$$

where M is a black-hole's mass parameter, which in our theory is given by the following expression $M \approx -4\pi v^2 \delta$ and the mass parameter is negative. There is a repulsive gravitational potential due to this negative mass parameter given by the metric parameter $A(r)$. But this parameter M is not a mass of the hedgehog, the black-hole-hedgehog has a positive mass i.e., $M_{BH} = -M = 4\pi v^2 \delta$. If we take the space integral of the hedgehog energy density as given by (13), say - this would be total energy of the hedgehog in the ignoring gravity approximation - the integration over the radius r will diverge for large r, because of the term $\kappa v^2/r^2$ in (13). So with the approximations done the a priori "mass" = "energy" of the hedgehog is $+\infty$. The *positivity* is what one expects for a disturbance in a background vacuum, which has minimum energy density and the *divergence* comes from the kinetic term due to the variation of the Φ^a field because of having different directions in a component space essentially following the direction in space from the centre out. Indeed such a variation leads to a gradient square term behaving $\propto 1/r^2$. When this is integrated over space - meaning an integral $\int ...4\pi r^2 dr$ one gets a term proportional to upper end r and thus divergence. It is a kind of infrared divergence in the sense that it comes from large distance scales. This is a priori looking like a hedgehog-"soliton" having an infinite energy or mass.

5. Lattice-like a structure of the false vacuum

Now we can construct the lattice-like topological contribution with negative vacuum energy density. Assuming that black-holes with mass parameter $M = -M_{BH}$ form a hypercubic lattice with lattice parameter $l = \lambda_{Pl}$, we have the negative energy density (and negative cosmological constant Λ_{lat}) of such a lattice equal to $\rho_{lat} \simeq -M_{BH}M_{Pl}^3 = \Lambda_{lat}M_{Pl}^2$. If this energy density of the hedgehogs lattice compensates the Einstein's vacuum energy, we have the following equation:

$$\frac{\lambda}{4}v^4 \approx M_{BH}M_{Pl}^3 \quad \text{and} \quad \frac{3}{2}M_{Pl}^4 \approx M_{BH}M_{Pl}^3 \quad \text{or} \quad M_{BH} = \frac{3}{2}M_{Pl} \approx 3.65 \times 10^{18}\,\text{GeV}. \tag{15}$$

Therefore black-holes-hedgehogs have a huge mass of order of the Planck mass. The radius δ of the hedgehog's core:

$$\delta \approx \frac{M_{BH}}{4\pi v^2} \approx \left(\frac{64\pi}{3}M_{Pl}\right)^{-1} \approx 6 \times 10^{-21}\,\text{GeV}^{-1}. \tag{16}$$

5.1. The hedgehog's horizon radius

We have obtained a global monopole with a huge mass (15). This is a black-hole solution, which corresponds to a global monopole-hedgehog that has been "swallowed" by a black-hole. Indeed, we have obtained the metric result by M. Barriola et al. [28] like:

$$ds^2 = \left(1 - \kappa v^2 + \frac{2 G_N M_{BH}}{r} + ...\right) dt^2 - \frac{dr^2}{\left(1 - \kappa v^2 + \frac{2 G_N M_{BH}}{r} + ...\right)} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

(17)

The $\kappa v^2 = 8$, and the black-hole-hedgehog's horizon radius is equal to:

$$r_h \approx \frac{\frac{\kappa}{4\pi} \times M_{BH}}{1 - \kappa v^2} = \frac{\kappa v^2 \delta}{1 - \kappa v^2} \approx \frac{8}{7}\delta \approx 1.14\,\delta.$$

(18)

We see that the horizon radius r_h is larger than the hedgehog radius δ $(r_h > \delta)$, and our concept that "a black hole contains the hedgehog" is justified.

6. The phase transition from the "false vacuum" to the "true vacuum"

In the present model, we investigated the evolution of the two bubbles of the Universe, considering two phases of the universal vacua (i) one being a "false vacuum" (Planck scale vacuum), and (ii) the other is a "true vacuum" (EW-vacuum). The cosmological model predicts that the Universe exists in the Planck scale phase for an extremely short time. For this reason, the Planck scale phase was called "the false vacuum". The presence of hedgehogs as vacuum defects is responsible for the destabilization of the false vacuum. The decay of the false vacuum is accompanied by the decay of the black-holes-hedgehogs. These configurations are unstable, and at some finite cosmic temperature which is called the critical temperature T_c, a system exhibits a spontaneous symmetry breakdown, and we observe a phase transition from the bubble with the false vacuum to the bubble with the true vacuum. After the phase transition, the Universe begins its evolution toward the low energy Electroweak (EW) phase. Here the Universe underwent the inflation, which led to the phase having the VEV $v_1 \approx 246$ GeV. This is a "true" vacuum, in which we live. Ref. [29] also allowed a possibility to consider an arbitrary domain wall between these two phases. During the inflation, domain wall annihilates, producing gravitational waves and a lot of the SM particles, having masses.

The Electroweak spontaneous breakdown of symmetry $SU(2)_L \times U(1)_Y \to U(1)_{el.mag}$ leads to the creation of the topological defects of the EW-vacuum. They are the Abrikosov-Nielsen-Olesen closed magnetic vortices ("ANO strings") of the Abelian Higgs model [36, 37], and Sidharth's Compton phase objects [38, 39]. Then the Electroweak vacuum and high-field "false vacuum" both present the non-differentiable manifold, described by the non-commutative geometry, giving almost zero cosmological constants Λ_1 and Λ_2 (see [1]).

At the early stage, the Universe was very hot, but then it began to cool down. Black-holes-monopoles (as bubbles of the vapour in the boiling water) began to disappear. The temperature dependent part of the energy density died away. In that case, only the vacuum energy will survive. Since this is a constant, the Universe expands exponentially, and an exponentially expanding Universe leads to the inflation (see reviews [40, 41]). While the Universe was expanding exponentially, so it was cooling exponentially. This scenario was called "supercooling in the false vacuum". When the temperature reached the critical value T_c, the Higgs mechanism of the SM created a new condensate ϕ_{min1}, and the vacuum became similar to a superconductor, in which the topological defects are magnetic vortices. The energy of black-holes is released as particles, which were created during the radiation era of the Universe, and all these particles (quarks, leptons, vector bosons) acquired their masses m_i through the Yukawa coupling mechanism $Y_f \bar\psi_f \psi_f \phi$. Therefore, they acquired the Compton wavelength, $\lambda_i = \hbar/m_i c$.

Then according to the Sidharth's theory of the cosmological constant, in the EW-vacuum we again have lattice-like structures formed by bosons and fermions, and the lattice parameters "l_i" are equal to the Compton wavelengths i.e., $l_i = \lambda_i = \hbar/m_i c$.

7. Stability of the EW-vacuum

Here we emphasize that due to the energy conservation law, the vacuum density before the phase transition (for $T > T_c$) is equal to the vacuum density after the phase transition (for $T < T_c$), therefore we have ρ_{vac}(at Planck scale) = ρ_{vac}(at EW scale). The position of the second minimum depends on the SM parameters, especially on the top and Higgs masses, M_t and M_H. The red solid line of Fig. 2 shows the running of the $\lambda_{H,eff}(\phi)$ for $M_H \simeq 125.7$ GeV and $M_t \simeq 171.43$ GeV, which just corresponds to the stability line, that is, to the stable EW-vacuum. In this case the minimum of the $V_{eff}(H)$ exists at the 10^{18} GeV.
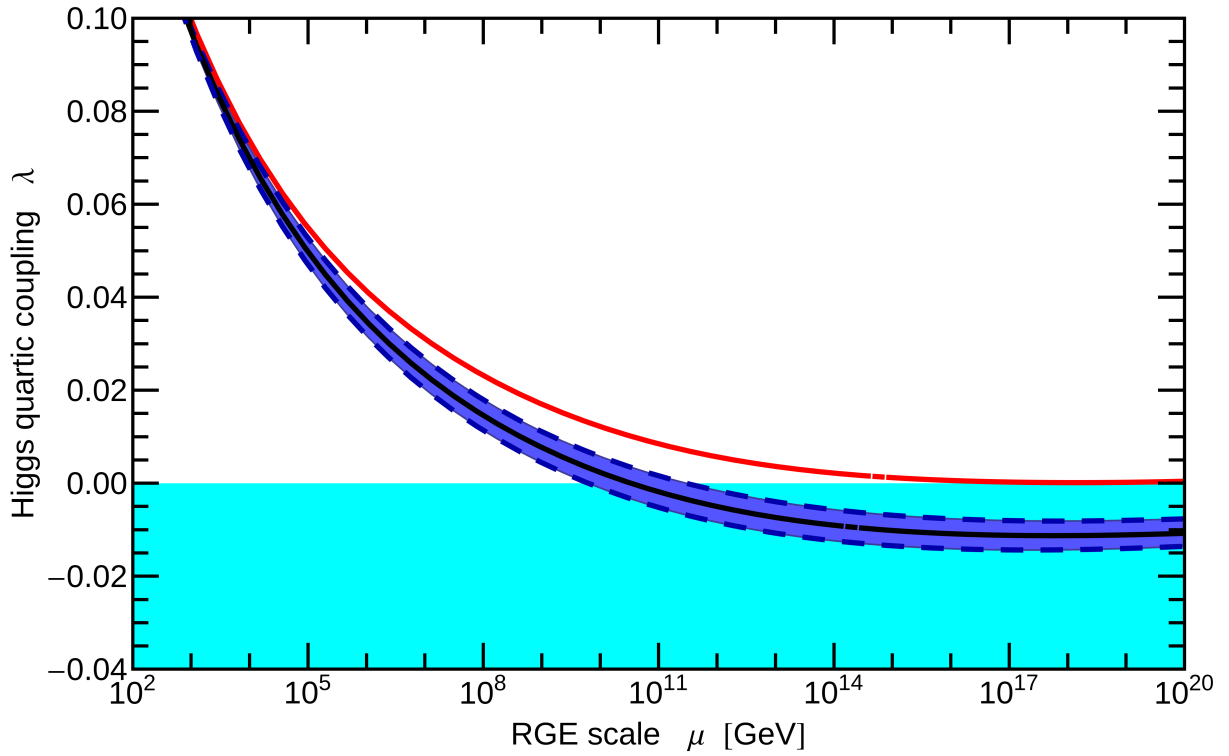

Figure 2. The RG evolution of the Higgs selfcoupling λ for $\alpha_s = 0.1184$ given by $\pm 3\sigma$. Blue lines present metastability for current experimental data, red (thick) line corresponds to the stability of the EW vacuum.

Also, the analogous link between the Planck scale phase and EW phase was considered in the paper [38]. It was shown that the vacuum energy density (DE) is described by the different contributions to the Planck and EW scale phases. This difference is a result of the phase transition. However, the vacuum energy densities (DE) of both vacua are equal, and we have a link between gravitation and electromagnetism via the Dark Energy. We see that if ρ_{vac} (at the Planck scale) is almost zero, then ρ_{vac} (at EW scale) also is almost zero, and we have a triumph of the Multiple Point Principle, two degenerate vacua with almost zero vacuum energy density. Almost zero cosmological constants are equal $\Lambda_1 = \Lambda_2 \approx 0$. Now we have obtained that the EW-vacuum, in which we live, is stable. The Planck scale vacuum cannot be negative, $V_{eff}(min_1) = V_{eff}(min_2)$.

Acknowledgments

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