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OVERFLOW RATE FOR EVENTS INPUT INTO A FINITE BUFFER

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ABSTRACT

The overflow rate for events input into a finite buffer is calculated for the following problem. A Poissonian distribution of events with rate λ is input onto a finite buffer of size N. The single server servicing the events has a Poissonian distribution of service times with rate μ. If the buffer is filled, incoming events are lost. Let $\lambda/\mu = R$. Then we find that the fraction of events lost is:

$$\frac{R^N - R^{N+1}}{1 - R^{N+1}} \;\; if \; R \neq 1 \;\; and \; \frac{1}{N+1} \;\; if \; R = 1$$

The question of overflow of buffers is very important for SSC experiments. We will be storing large quantities of information coming it at extremely high rates. If we want high efficiency, we must design the system to minimize information lost. In this note we consider a queueing problem, related to a simple model for this process. This is a standard problem in queueing theory and, presumably, can be found in a number of texts. However, it is easy to work out and the answer can be of considerable use in developing our intuition for these processes.

We assume:

1. A Poissonian distribution of incoming events with rate λ.
2. A single server with a Poissonian distribution of service times with rate μ.
3. A finite buffer of size N. A new event immediately enters the buffer. The server works on the oldest event in the buffer. If the buffer is filled, a new incoming event is lost.

Let $P(j)$ be the probability of j events in the buffer $j = 0, 1, 2, \ldots, N$. For $j \neq 0, N$ we have:

$$dP_j(t) = P_{j+1}(t)\mu dt - P_j(t)\mu dt - P_j(t)\lambda dt + P_{j-1}(t)\lambda dt$$

$$dP_j(t) = (P_{j+1} - P_j)\mu dt - (P_j - P_{j-1})\lambda$$

$$\frac{dP_j(t)}{dt} = (P_{j+1} - P_j)\mu - (P_j - P_{j-1})\lambda$$

or

$$\frac{dP_j(t)}{d(\mu t)} = (P_{j+1} - P_j) - (P_j - P_{j-1})R$$

where $R = \lambda/\mu$.

The $j = 0$ term is similar, but there is no $j-1$ term and no $P_j\mu$ term for it:

$$\frac{dP_0(t)}{d(\mu t)} = P_1 - P_0 R$$

For $j = N$, there is no $j+1$ term and no $P_j\lambda$ term:

$$\frac{dP_N(t)}{d(\mu t)} = -P_N + P_{N-1}R$$

Finally we must have:

$$\Sigma_{j=0}^{N} P_j(t) = 1$$

If we are interested in the transient situation, the above equations allow us to find the evolution of the probabilities numerically. (There is probably an analytic solution here also, although I have not tried to find it.) However, we are usually interested in the equilibrium situation for which the $P_j(t)$ are constant in time and $\frac{dP_j(t)}{d(\mu t)} = 0$. From the $j = 0$ equation we see that:

$$P_1 = P_0 R$$

From the $j = 1$ equation we find (after using the above relation):

$$P_2 = P_1 R = P_0 R^2$$

and generally we easily see:

$$P_j = R^j P_0 \quad j = 0, \ldots, N$$

Since we have noted the sum of the probabilities must be 1 we have:

$$\Sigma_{j=0}^{N} R^j P_0 = 1 \quad or \quad P_0 = \frac{1}{\Sigma_{j=0}^{N} R^j}$$

The probability of losing an event is just the probability of an event coming in when the buffer is filled. The fraction of time that the buffer is filled is P_N and this is the probability of losing an event.

$$P_N = \frac{R^N}{\Sigma_{j=0}^{N} R^j} \quad where \quad R = \lambda/\mu \neq 1$$

For the critical case $R = 1$:

$$P_N = \frac{1}{N+1} \quad for \quad R = 1$$

Note that:

$$\Sigma_{j=0}^{N} R^j = \frac{1 - R^{N+1}}{1 - R}$$

(To see that consider $(1 - R)(1 + R + R^2 + \ldots + R^N)$.) Thus:

$$P_N = \frac{R^N - R^{N+1}}{1 - R^{N+1}} \quad for \quad R \neq 1$$

$$P_j = \frac{R^j - R^{j+1}}{1 - R^{N+1}} \quad for \quad R \neq 1$$

Finally we note that if $R << 1$, then $P_N \approx R^N$ or:

$$N \approx \ln P_N / \ln R \quad for \quad R << 1$$

This note was motivated by a very nice talk by H. Pasha, Yale at the electronics seminar held at SSC lab by C. Milner on April 23-25, 1992.