Relativistic theory of particles with arbitrary intrinsic angular momentum(*)

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Summary. — The author establishes wave equations for particles having arbitrary given intrinsic angular momentum. Such equations are linear in the energy and relativistically invariant.

As is well known, DIRAC's theory of the electron makes use of a four-component wave function. When slow movements are considered, two of the components acquire negligible values, while the remaining two, at least in a first approximation, satisfy the SCHRÖDINGER equation.

In a similar fashion, a particle having intrinsic angular momentum $s\frac{h}{2\pi}$ ($s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$) is described in quantum mechanics by a set of $2s+1$ wave functions which separately satisfy the SCHRÖDINGER equation. Of course, such a representation is valid as long as the relativistic effects are neglected, and this is allowed for a particle moving with speed much smaller than the speed of light. A different case in which the elementary theory retains its validity is, of course, the one in which the velocity of the particle is comparable with c but remains approximately constant in direction and magnitude. In fact, this case can be reduced to the study of slow movements by means of a suitable choice of the frame of reference.

(*) Translated from "Il Nuovo Cimento", vol. 9, 1932, pp. 335-344, by C. A. Orzalesi in Technical Report no. 792, 1968, University of Maryland. (Courtesy of E. Recami.)

On the other hand, the following situation cannot be easily dealt with by means of the nonrelativistic SCHRÖDINGER equation: a particle with speed which retains an almost constant value within fairly large regions in the space-time continuum, but for which the speed is also slowly varying between very different extremum values from one such region to another, as an effect of weak external fields.

A relativistic generalization of the preceding theory should satisfy the following hierarchy of conditions, as it becomes more and more accurate:

(*a*) The theory should allow the study of particles having an almost constant velocity (in direction and magnitude), and the results should be equivalent to those given by the nonrelativistic theory; however, there should be no need to specify any particular reference frame.

(*b*) The theory should allow the study of processes where the speed of the particles is slowly varying, but within arbitrarily separated limits, under the effect of weak external fields.

(*c*) The theory should retain its validity in general, even when the velocities of the particles vary arbitrarily.

It is likely that a rigorous theory satisfying condition (*c*) might be incompatible with the present-day quantum scheme. [For instance,] DIRAC's theory of the electron has largely proved its fruitfulness in the study of genuine relativistic phenomena, *e.g.*, scattering of hard γ-rays; however, this theory certainly satisfies condition (*c*) only incompletely as is shown by the well-known difficulties coming from the transitions to states having negative energy. On the contrary, it is probably true that a theory satisfying condition (*b*), and only partially satisfying condition (*c*), should not meet essential difficulties since its physical content might be essentially the same as that which justifies the SCHRÖDINGER equation, The most remarkable example of this type of generalization is provided precisely by Dirac's theory. However, since this theory can be applied to particles with intrinsic [angular] momentum $s = 1/2$, I have investigated equations formally similar to the ones by DIRAC, although considerably more involved; these equations allow us to consider particles with arbitrary (and, in particular, zero) angular momentum.

According to DIRAC, the wave equation of a material particle in the absence of external fields must have the following form:

$$\left[\frac{W}{c} + (\alpha, p) - \beta mc\right] \psi = 0. \tag{1}$$

Equations of this kind present a difficulty in principle. Indeed, the operator β(*) has to transform as the time component of a four-vector, and thus β cannot be simply a

(*) In "Il Nuovo Cimento" it is erroneously printed $^{-1}$ instead of β. (Note of the Editor, see also E. AMALDI, *op. cit.*)

multiple of the unit matrix, but must have at least two different eigenvalues, say β_1 and β_2. However, this implies that the energy of the particle at rest, obtained from Eq. (1) by taking $p = 0$, shall have at least two different values, *i.e.* $\beta_1 mc^2$ and $\beta_2 mc^2$. According to DIRAC's equations, the allowed values of the mass at rest are, as well known, $+m$ and $-m$; from this it follows by relativistic invariance that for each value of p the energy can acquire two values differing in sign: $W = \pm\sqrt{m^2c^4 + c^2p^2}$.

As a matter of fact, the indeterminacy in the sign of the energy can be eliminated by using equations of the type (1) , only if the wave function has infinitely many components that cannot be split into finite tensors or spinors.

1. Equation (1) can be derived from the following variational principle:

$$\delta \int \tilde{\psi} \left[\frac{W}{c} + (\alpha, p) - \beta mc\right] \psi dV dt = 0 \tag{2}$$

(one of the conditions imposed by relativistic invariance is, of course, that the form $\tilde{\psi}\beta\psi$ has to be invariant).

If now we require the energy at rest to be always positive, all the eigenvalues of β have to be positive so that the form $\tilde{\psi}\beta\psi$ will be positive definite. By means of a *nonunitarity* transformation $\psi \to \varphi$ it is then possible to reduce the expression at hand to the form unity:

$$\tilde{\psi}\beta\psi = \tilde{\varphi}\varphi. \tag{3}$$

By substituting in Eq. (2) ψ with its expression in terms of φ one obtains:

$$\delta \int \tilde{\varphi} \left[\gamma_0 \frac{W}{c} + (\gamma, p) - mc\right] \varphi dV dt = 0, \tag{4}$$

from which the equations equivalent to Eq. (1): follow:

$$\left[\gamma_0 \frac{W}{c} + (\gamma, p) - mc\right] \varphi = 0. \tag{5}$$

We have now to determine the transformation law of φ under a LORENTZ rotation, as well as the expressions for the matrices γ_0, γ_x, γ_y, γ_z, in such a way as to respect the relativistic invariance of the variational principle (4), and thus to have an invariant integrand function in Eq. (4).

We begin by establishing the transformation law of φ and we note, first of all, that the invariance of $\tilde{\varphi}\varphi$ implies that we must restrict ourselves to unitary transformations. Moreover, in order to avoid exaggerated complications, we will give the transformation law only for infinitesimal LORENTZ transformations, since any finite transformation can be obtained by integration of the former ones. We introduce the infinitesimal transformations in the variables ct, x, y, z;

$$
(6) \quad \begin{cases}
S_x = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix}; \;
S_y = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}; \;
S_z = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}; \\
T_x = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}; \;
T_y = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}; \;
T_z = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}.
\end{cases}
$$

We also define

$$
(7) \quad \begin{cases}
a_x = iS_x; & a_y = iS_y; & a_z = iS_z; \\
b_x = -iT_x; & b_y = -iT_y; & b_z = -iT_z.
\end{cases}
$$

The operators a and b must be Hermitian operators in a unitary representation, and *vice versa*; furthermore, in order for the infinitesimal transformations to be integrable, they must satisfy certain relationships under commutation, as can be deduced from Eqs. (6) and (7):

$$
(8) \quad \begin{cases}
\big(a_x, a_y\big) = ia_z \\
\big(a_x, b_x\big) = 0 \\
\big(a_x, b_y\big) = ib_z \\
\big(a_x, b_z\big) = -ib_y \\
\big(b_x, b_y\big) = -ia_z
\end{cases}
$$

the remaining relations can be obtained by cyclic permutations of x, y, z.

The simplest solution of Eqs. (8) by means of Hermitian operators is given by the following infinite matrices, where the diagonal elements are labelled by two indices j and m; we have to distinguish two possibilities according to the assumption $j = 1/2, 3/2, 5/2, \ldots$;

$m = j, j-1, \ldots, -j$, or $j = 0, 1, 2, \ldots; m = j, j-1, \ldots, -j$:

$$\text{(9)} \quad \begin{cases} \big(j,m\big|a_x - ia_y\big|j,m+1\big) = \sqrt{(j+m+1)(j-m)} \\ \big(j,m\big|a_x + ia_y\big|j,m-1\big) = \sqrt{(j+m)(j-m+1)} \\ \big(j,m|a_z|j,m\big) = m \\ \big(j,m\big|b_x - ib_y\big|j+1,m+1\big) = -\frac{1}{2}\sqrt{(j+m+1)(j+m+2)} \\ \big(j,m\big|b_x - ib_y\big|j-1,m+1\big) = \frac{1}{2}\sqrt{(j-m)(j-m-1)} \\ \big(j,m\big|b_x + ib_y\big|j+1,m-1\big) = \frac{1}{2}\sqrt{(j-m+1)(j-m+2)} \\ \big(j,m\big|b_x + ib_y\big|j-1,m-1\big) = -\frac{1}{2}\sqrt{(j+m)(j+m-1)} \\ \big(j,m|b_z|j+1,m\big) = \frac{1}{2}\sqrt{(j+m+1)(j-m+1)} \\ \big(j,m|b_z|j-1,m\big) = \frac{1}{2}\sqrt{(j+m)(j-m)}. \end{cases}$$

If we assume that, by reflection with respect to the origin, the $\varphi_{j,m}$ either remain unchanged or change in sign as j varies, b turns out to be a polar vector while a has axial properties.

The entities to which a and b apply will be called infinite tensors (or spinors) of zero index, for integer (respectively, half-integer) j. The nomenclature of "zero index" comes from the fact that the invariant

$$\text{(10)} \qquad Z = a_x b_x + a_y b_y + a_z b_z$$

vanishes.

More general infinite spinors or tensors can be introduced for any value of Z. A simple way to obtain the spinors is as follows. Let us consider a general solution $\psi(q,t)$ of the DIRAC equation with no external field and transform it relativistically:

$$\text{(11)} \qquad \psi(q,t) \to \psi'(q,t).$$

Thus, the transformation in the space variables:

$$\text{(12)} \qquad \psi(q,0) \to \psi'(q,0)$$

is unitary. Now, if instead of general functions $\psi(q,0)$ we consider only those belonging to a fixed eigenvalue z_0 so of the operator (10), which has a spectrum extending from $-\infty$ to $+\infty$, we obtain functions which transform under (12) as infinite spinors, each function appearing twice.

In the representation (12), the operators a_x and b_x have the following form:

$$a_x = \frac{2\pi}{h}\big(yp_z - zp_y\big) + \frac{1}{2}\sigma_x$$

$$b_x = \frac{2\pi}{h}x\frac{H}{c} + \frac{i}{2}\alpha_x$$

and similarly for a_y, a_z, b_y, b_z.

2. We have now to determine the operators γ_0, γ_x, γ_y, γ_z in such a way as to make Eq. (4) invariant. Since we consider only unitary transformations, these operators transform in the same way as the Hermitian forms related to them; thus, in order for the integrand fraction in (4) to be invariant, it is necessary that the operators in question form a covariant vector $(\gamma_0, \gamma_x, \gamma_y, \gamma_z \sim ct, -x, -y, -z)$.

The interpretation of $\tilde{\varphi}\gamma_0\varphi$ and $-\tilde{\varphi}\gamma\varphi$ as charge and current densities is immediate. The γ operators must satisfy the following commutation relations:

$$(13)\qquad \begin{cases} (\gamma_0, a_x) = 0 \\ (\gamma_0, b_x) = i\gamma_x \\ (\gamma_x, a_x) = 0 \\ \big(\gamma_x, a_y\big) = i\gamma_z \\ (\gamma_x, a_z) = -i\gamma_y \\ (\gamma_x, b_x) = i\gamma_0 \\ \big(\gamma_x, b_y\big) = 0 \\ (\gamma_z, b_z) = 0 \end{cases}$$

and the others obtained by cyclic permutation of x, y z. As can be easily checked, the commutation relations (13), determine γ_0, γ_x, γ_y, γ_z to within a constant factor. One finds:

$$(14)\qquad \begin{cases} \gamma_0 = j + \dfrac{1}{2} \\ \big(j, m\big|\gamma_x - i\gamma_y\big|j+1, m+1\big) = -\dfrac{i}{2}\sqrt{(j+m+1)(j+m+2)} \\ \big(j, m\big|\gamma_x - i\gamma_y\big|j-1, m+1\big) = -\dfrac{i}{2}\sqrt{(j-m)(j-m-1)} \\ \big(j, m\big|\gamma_x + i\gamma_y\big|j+1, m-1\big) = \dfrac{i}{2}\sqrt{(j-m+1)(j-m+2)} \\ \big(j, m\big|\gamma_x + i\gamma_y\big|j-1, m-1\big) = \dfrac{i}{2}\sqrt{(j+m)(j+m-1)} \\ \big(j, m|\gamma_z|j+1, m\big) = \dfrac{i}{2}\sqrt{(j+m+1)(j-m+1)} \\ \big(j, m|\gamma_z|j-1, m\big) = -\dfrac{i}{2}\sqrt{(j+m)(j-m)}. \end{cases}$$

The omitted matrix elements of γ_x, γ_y, γ_z vanish. It should be noticed that the Hermitian form $\varphi\gamma_0\varphi$ is positive definite, as the physical interpretation requires.

We now want to translate the equations written in the form (5) into the form of Eq. (5). For this, it suffices to write

$$\varphi_{j,m} = \frac{\psi_{j,m}}{\sqrt{j+\frac{1}{2}}}, \tag{15}$$

since then the form related to γ_0 reduces to the unit form. In this way we obtain equations having the desired form

$$\left[\frac{W}{c} + (\alpha, p) - \beta mc\right]\psi = 0, \tag{16}$$

where $\beta = \frac{1}{j+\frac{1}{2}}$ and the non-vanishing components of α_x, α_y, α_z are, given as follows:

$$\begin{cases} \big(j,m\big|\alpha_x - i\alpha_y\big|j+1,m+1\big) = -\frac{i}{2}\sqrt{\frac{(j+m+1)(j+m+2)}{\left(j+\frac{1}{2}\right)\left(j+\frac{3}{2}\right)}} \\ \big(j,m\big|\alpha_x - i\alpha_y\big|j-1,m+1\big) = -\frac{i}{2}\sqrt{\frac{(j-m)(j-m-1)}{\left(j-\frac{1}{2}\right)\left(j+\frac{1}{2}\right)}} \\ \big(j,m\big|a_x + i\alpha_y\big|j+1,m-1\big) = \frac{i}{2}\sqrt{\frac{(j-m+1)(j-m+2)}{\left(j+\frac{1}{2}\right)\left(j+\frac{3}{2}\right)}} \\ \big(j,m\big|\alpha_x + i\alpha_y\big|j-1,m-1\big) = \frac{i}{2}\sqrt{\frac{(j+m)(j+m-1)}{\left(j-\frac{1}{2}\right)\left(j+\frac{1}{2}\right)}} \\ \big(j,m|\alpha_z|j+1,m\big) = \frac{i}{2}\sqrt{\frac{(j+m+1)(j-m+1)}{\left(j+\frac{1}{2}\right)\left(j+\frac{3}{2}\right)}} \\ \big(j,m|\alpha_z|j-1,m\big) = -\frac{i}{2}\sqrt{\frac{(j+m)(j-m)}{\left(j-\frac{1}{2}\right)\left(j+\frac{1}{2}\right)}}\,. \end{cases} \tag{17}$$

In looking for solutions of Eq. (16) corresponding to plane waves with positive mass, one finds all those which can be derived by means of a relativistic transformation from a zero-momentum plane wave. For these, the energy is given by

$$W_0 = \frac{mc^2}{j+\frac{1}{2}}\,. \tag{18}$$

For half-integer values of j we thus obtain states corresponding to the values $m, m/2, m/3, \ldots,$ of the mass, while for integer j one has $2m, 2m/3, 2m/5, \ldots$.

It should be emphasized that particles having different masses also have different intrinsic angular momentum, the latter having a determined value only in the system where the particle is at rest.

If we consider the set of all states belonging to the value $\frac{m}{s+\frac{1}{2}}$ of the rest mass, as is realized in nature, all other states having no significance, we obtain an invariant theory for particles of angular momentum s; in the absence of an external field, this theory can be regarded as satisfactory. One can easily verify that in the case of slow movements and for particles having intrinsic angular momentum s, only the functions $\psi_{s,m}$ are appreciably different from zero and satisfy the SCHRÖDINGER equation with mass $M = \frac{m}{s+\frac{1}{2}}$; the functions $\psi_{s+1,m}$ and $\psi_{s-1,m}$ are then of order v/c, while $\psi_{s+2,m}$, and $\psi_{s-2,m}$ are of order v^2/c^2, and so on.

In this way we obtain only two equations; the one suitable for the description of particles with noninteger angular momentum and the other pertinent to zero or integer angular momentum.

Besides the states pertinent to positive values of the mass, there are other states for which the energy is related to the momentum by a relation of the following type:

$$W = \pm\sqrt{c^2p^2 - k^2c^4}; \tag{19}$$

such states exist for all positive values of k but only for $p \geq kc$, and can be regarded as pertaining to the imaginary value ik of the mass.

The "spin" functions belonging to plane waves with $p \neq 0$ have a particularly simple expression in the case of particles with no intrinsic angular momentum if $p_x = p_y = 0$, $p_z = p$. Apart from a normalization factor for these functions, one finds

$$\begin{cases} \psi_{j,0} = \sqrt{\left(j+\frac{1}{2}\right)}\left(i\dfrac{\eta - j}{\varepsilon}\right)^j & (j = 0, 1, 2, \ldots) \\ \psi_{j,m} = 0 \qquad \text{for} \quad m \neq 0, \end{cases} \tag{20}$$

where

$$\varepsilon = \frac{p}{Mc}, \qquad \eta = \frac{\sqrt{M^2c^2 + p^2}}{Mc} \tag{21}$$

and $M = 2m$ is the mass at rest.

3. We want now to discuss briefly the introduction of the electromagnetic field into Eq. (16).

The simplest way to perform the transition from the field equations without external field to those with an external field is to substitute for W and p, the quantities $W - e\varphi$ and $p - \frac{e}{c}A$, respectively, e being the charge of the particle and φ and A being the scalar and vector potentials. However, other possibilities are also open. For instance, one can add other invariant terms, analogous to those introduced by PAULI([2]) in the theory of the magnetic neutron. Those additional terms contain as a factor the field forces instead of the electromagnetic potentials and thus do not destroy the invariance of the field equations coming from the indeterminacy of the potentials.

This artifice allow us to ascribe an arbitrarily fixed magnetic moment to particles having a non-vanishing angular momentum. For instance, in the case of the electron, by means of the simple substitutions W, $p \to W - e\varphi$, $p - \frac{e}{c}A$, one finds a magnetic moment equal to $+\frac{1}{2}\mu_0$, instead of $-\mu_0$.

Thus, if we want to specialize our theory to a theory for the electron and maintain as far as possible good agreement with the experimental data, we have to modify the magnetic moment by introducing additional terms. However, the electron theory obtained in this way is a useless copy of DIRAC's theory, the latter remaining completely preferable thanks to its simplicity and to the wide support from experiment. On the other hand, the advantage of the present theory lies in its applicability to particles with angular momentum different from 1/2.

The equations, including both the external field and the additional terms which modify the intrinsic magnetic moment, have the following form:

$$\left[\left(\frac{W}{c} - \frac{e}{c}\varphi\right) + \left(a, p - \frac{e}{c}A\right) - \beta mc + \lambda(a', H) + \lambda(b', E)\right]\psi = 0, \tag{22}$$

where a' stands for (a'_x, a'_y, a'_z) and b' for (b'_x, b'_y, b'_z), while E and H represent the electric and magnetic field.

The matrix a'_x can be deduced from a_x of Eq. (9) by means of the rule

$$(j, m|a'_x|j', m') = \frac{1}{\sqrt{\left(j + \frac{1}{2}\right)\left(j' + \frac{1}{2}\right)}}(j, m|a_x|j', m') \tag{23}$$

and similarly for a'_y, a'_z, b'_y, b'_z.

For particles having intrinsic angular momentum $s = 1/2$ the choice $\lambda = \frac{2}{c}\mu$ should be made if μ is the magnetic moment which one wants to add to the one which naturally arises from the introduction of the electromagnetic potentials into the wave equation. As seen before, the latter magnetic moment has in this case, the value $-\frac{1}{2}\frac{eh}{4\pi mc}$. For particles having no intrinsic magnetic moment it is natural to choose $\lambda = 0$.

Regarding the practical solutions of the wave equations, we recall that for slow movements they are finite and that those $\psi_{j,m}$ which satisfy to the SCHRÖDINGER equations are only those which have j equal to the intrinsic angular momentum in units $h/2\pi$.

For instance, for particles having no intrinsic momentum, one is left with one component only, namely $\psi_{0,0}$, while $\psi_{1,m}$ are of order v/c, v being the speed of the particle,

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([2]) Quoted by OPPENHEIMER, "Phys. Rev.", *41*, 763 (1932).

$\psi_{2,m}$ are of order v^2/c^2, and so on. In this way one succeeds in eliminating, by successive approximations, the small components and in particular one arrives at very simple expansions for the calculation of the first relativistic corrections.

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I particularly thank Prof. E. Fermi for discussions of the present theory.

Comment on the Scientific Paper no. 7*: "Relativistic theory of particles with arbitrary intrinsic angular momentum."*

The central problem in constructing a purely wave-mechanical relativistic generalization of Schrödinger's equation is the emergence of negative-energy solutions. Even if we assume that only positive-energy states are physically meaningful, the resulting theory would be unstable with respect to transitions to negative-energy states. Dirac tried to avoid this problem by writing a first order-wave equation

$$i\frac{d\psi}{dt} = \left(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m\right)\psi, \qquad \text{or:} \qquad (i\gamma^\mu \frac{\partial}{\partial x_\mu} - m)\psi \; = \; 0. \tag{1}$$

If, following Dirac we assume anticommutation relations, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, this equation describes a particle of mass m, but admits both positive and negative energy solutions. Dirac's solution for this problem was to assume that all negative-energy states are occupied, so that thanks to the Pauli esclusion principle a positive-energy particle cannot jump into a negative-energy state. In this version Dirac's theory goes far beyond wave mechanics, and is essentially equivalent to the modern field-theoretical formulation, admirably presented in Majorana's "Teoria simmetrica dell'elettrone e del positrone".

In his paper on particles of arbitrary spin, Majorana tried to construct a fully relativistic *wave mechanics* that completely avoids the negative-energy states. He was able to show that a *necessary* condition for the absence of negative-energy states is that the operator β has only positive eigenvalues, and in turn this implies that $\phi = \beta^{1/2}\psi$ transforms according to a unitary representation of the Lorentz group. In this paper Majorana displays a complete mastery of the theory of groups. E. Amaldi(1) recalls Majorana's admiration for the work of H. Weyl and E. Wigner on the application of group theory to quantum mechanics. In this paper we can find traces of Weyl's discussion of the Lie

(1) E. AMALDI, *op. cit.*

algebra of the rotation group, extended here to the case of the Lorentz group, in particular when he introduces the commutation rules of the group generators as “integrability conditions”. Starting from the commutation relations, Majorana builds the “simplest” infinite-dimensional unitary representations of the Lorentz group, one for integer angular momentum and one for semi-integer angular momentum, and constructs explicitly the corresponding infinite-dimensional representations of the $\vec{\alpha}$ and β matrices that appear in Dirac’s equation.

The Majorana versions of equation (1) have solutions that respectively describe particles of arbitrary integer or semi-integer spin angular momentum j , with mass

$$M_j = \frac{m}{j + \frac{1}{2}}. \tag{2}$$

The avoidance of negative-energy solutions has a very high price: the existence of an infinite sequence of states with increasing spin and decreasing mass. Majorana could not accept this conclusion and leaves open the posssibility that his equation could describe a single particle of arbitrary spin j_0 by declaring that all solutions with $j \neq j_0$ are to be considered as “unphysical”. He mentions in particular the possibility of using the integer-spin version of the equation to describe a spinless particle. He, however, realizes that in the presence of interactions it would be difficult to ensure the absence of transitions to states of different spin. In a different context this is the same problem that plagued the wave mechanics interpretation of Dirac’s equation. The disease is made worse here by the presence, briefly mentioned in Majorana’s paper, of what we now call “tachyonic” solutions, that would correspond to states with imaginary mass ik, with $E = \pm\sqrt{c^2\vec{p}^2 - k^2c^4}$, that exist for $|\vec{p}| > k$ and for any value of k.

Majorana’s paper was written in the early summer of 1932, just before Anderson announced the discovery of the positive electron, thus sealing the triumph of Dirac’s electron theory. It received little attention, in spite of the brilliant and original results on the unitary representations of the Lorentz group that were rediscovered years later by Wigner. Extensive references to the successive developments in this field can be found in the review of Majorana’s paper by D. M. Fradkin([2]).

If Majorana’s paper on particles with arbitrary spin had not been totally forgotten, it could be considered a precursor of some of the most actively pursued recent developments in theoretical physics. In the sixties the study of Regge-poles and the discovery of high-spin hadrons briefly rekindled the interest for theories, based on fields that transform according to infinite-dimensional representations of the Lorentz group, that describe a sequence of particles with increasing spin and increasing mass. These attempts had their highest expression in the dual models of hadrons but, as in the case of Majorana’s theory, were plagued by the existence of tachyonic states that could only be avoided

([2]) D. M. FRADKIN, *Am. J. Phys.* **34** (1966) 314.

in a space-time with a large number of dimensions. In view of these difficulties and of the emergence of Quantum Chromo Dynamics the application of these ideas to hadron physics was abandoned, but they resurfaced later in modern string theories.

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