Action-reaction in mass-charge quaternions

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Abstract. Dirac's relativistic quantum mechanical equation has been rewritten in such a way that both the Dirac equation and the structure of the fermion can be derived from a novel mathematical arrangement of the four fundamental parameters of physics, space, time, mass and charge, using quaternions. Mass and charge are conserved quantities and their products are found in the inverse-square laws of Newton, $F = Gm_1m_2 /r^2$ and Coulomb, $kQ_1Q_2/r2$. Peter Rowlands gives this product the name 'interaction' and interprets the product, a squaring, to mean that it is universal and nonlocal. This paper will investigate the relationship between zero-point energy, quaternion operations and mass-charge interactions in relation to Newton's third law of motion.

1. Introduction
From Newton's second and third laws of motion in absolute space and time the laws of conservation of linear momentum and energy can be derived. In the late 1700s and early 1800s more systematic methods for discovering conserved quantities were developed. Lagrangian mechanics, and the principle of least action, uses generalized coordinates and the laws of motion are not expressed in a Cartesian coordinate system as was standard in Newtonian mechanics. The action is defined as the time integral of a function which is unaffected by changes or transformations. The Lagrangian being invariant is said to exhibit a symmetry and is generalized in Noether's theorem. Rowlands has reformulated Dirac's relativistic quantum mechanical equation and combined four fundamental parameters used in physics; two are continuous (time, mass) and two are discrete (mass, charge) [1,2]. He has shown how the dualities of being conserved / unconserved, discrete / continuous, imaginary / real, dimensional / nondimensional, commutative / noncommutative, etc. are ontologically and symmetrically, in a generalized Noether sense, linked in all physical laws. Using quaternions Rowlands links mass and charge in a manner analogous to the 4-vector linkup between space and time ultimately leading to a description called Lorentz-invariance.

This paper will look at how the operations defined in conserved quaternions (1, i, j, k) and nonconserved 4-vector of space-time (*x, y, z, ict*) relate to the mass-charge (strong, electric, weak) in the Dirac nilpotent equation. There are well-defined physical differences between mass and charge and especially the mechanisms of their interactions. Masses are elements defined by continuous fields. Charges, on the other hand, are singularities; another duality. The nature of their differences can only be resolved when we have a true picture of the concept of inertia which is also investigated in this paper. Ultimately the requirement of combining dual variables and/or dual properties is an essential ingredient in any mathematical theory that models the physical world [3].

2. Euclid's points, Newton's lines, Minkowski's space
Newton's 1687 laws of motion rested on principles dependent on a notion of absolute space and time.

Some 200 years later Poincaré showed that by taking time to be an imaginary 4th space-time coordinate *ict*, *c* = speed of light and *i* = imaginary unit, a Lorentz transformation can be treated as a rotation of coordinates in 4-D space. Minkowski rewrote Einstein's 1905 special theory of relativity and showed that properties of Lorentz transformations, such as proper time and length contraction, provided a geometrical interpretation and generalized Newtonian mechanics to relativistic mechanics.

The Minkowski metric and the Poincaré symmetry group of spacetime are consequences of the two postulates (physical laws are invariant in all inertial frames of reference and the speed of light is constant in a vacuum for all observers) of special relativity providing the background setting for all relativistic theories. Space-time became a dynamic fabric as forces morphed into fields. Gravity became space-time curvature and inertial lines became geodesics. Gravity, no longer considered as an attractive force, became nothing more than the warping of the space-time fabric. Putting a man on the moon still depends on classical concepts of forces and requires Newton's laws of motion but precision time communication with astronauts in outer space relies on general relativity.

Hamilton's 1843 *mathematical* creation generalized 2-D complex numbers to 3-D space and his poetical talents captured this new structure of space-time in a 14-line sonnet. In lines 6-7 of *THE TETRACTYS*, a "Pythagorean lore", Hamilton wrote: *And how the One in Time, of Space the Three, Might, in the Chain of Symbol, girdled be.*" Minkowski's linkage of space- and time was not theoretically based but rather, he insisted, "sprung from the soil of *experimental physics*." But, to be sure, it was a Hamiltonian notion he invoked in 1908 at the 80th Assembly of German Natural Scientists and Physicians in an address called "Space and Time" saying, Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality. Minkowski differentiated Hamilton's chain of *mathematical* symbols from his *physical reality* of unified space and time. Mathematically, there is an obvious advantage of quaternion space-time over Minkowski space-time in that the former is a division algebra with a positive quadratic norm allowing for a description of both translational and rotational motion of particles, eliminating the need for complex numbers, vectors and tensors. We also note that the Minkowski space-time 4-vectors, three real parts and one imaginary pseudoscalar, are in an order that reverses the quaternion component structure.

3. Newton's laws of motion

Newton's three laws of motion are identified by ordinal numbers: first, second, and third. The order obviously follows a historical one: Galilean inertia, impressed and gravitational forces, and action-(inertial?) reaction. Understanding 15th century Newtonian laws, in relation to 20th century quantum and relativistic theories, parallels an appreciation of Euclid's postulates in relation to non-Euclidean geometries. If we let m refer to an object's non-varying mass and *v* the velocity then the momentum is *p* = *mv* then we can reorder Newton's laws in terms of momentum.

Second law: the *rate of change of momentum* of an object is equal to the force applied to that object.
First law: the *momentum* of an object is unchanged if there are no forces acting on the object. (It is clear that this is a special case of the second law.)
Third law: the *momentum* of an isolated system of objects is conserved.

In Newton's explanatory comments to this law, the Scholium, *motion* is Newton's name for momentum, and he makes a careful distinction between motion and velocity. The first use of "momentum" in its proper sense is not known but five years before the final edition of Newton's *Principia Mathematica*, in 1721, momentum M or "*quantity of motion*" was being defined in student textbooks as the product of Q and V, where Q is "*quantity of material*" and V is "velocity." Modern physical laws of conservation of energy, momentum and angular momentum have more general validity than Newton's laws applying to both light and matter and can be derived via Noether's theorem from Galilean invariance. The advent of new geometries, non-Euclidean, Minkowski, quaternion and octonion also provide new ways at looking at Newton's laws.

Rueda and Haisch's 1999 concluded that "Newton's third law is fundamental, whereas Newton's second law, $F = ma$, appears to be derivable from the third law together with the laws of electrodynamics [4]." This allows us to view Newtonian laws in a Euclidean order. Newton's third law is fundamental to classical mechanics referring to the measurement of forces at *points* in space.

Another possible reordering of the three laws would be the order in which Euclidean space is built: *points* (at which the equal and opposite force applied), to straight (geodesic) *lines* along inertial paths, to *space-time* structures in which forces operate. The following are translations from the Latin of Newton's laws.

Third law: To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts (*points*).
First law: Every body persists in its state of being at rest or of moving uniformly *straight* forward, except insofar as it is compelled to change its state by force *impressed* (emphasis added by author).
Second law: The alteration of motion is ever proportional to the motive force *impress'd*; and is made in the direction of the *right line* in which that force is *impress'd* (emphasis added by author).

Newton's third law, from a Euclidean perspective, is more fundamental referencing points in space and the continuous vacuum. Zero-point energy (ZPE), also called vacuum fluctuations discussed below, gives mass (energy) a unidimensional and unipolar dimension and is represented by the real part of the quaternion. Newton's *impress'd motive force* is interpreted as a "reaction half of the system" in vacuum fluctuations [1, p. 160].

4. Action-reaction principle
In his pre-Principia manuscript *De Gravitatione et Aequipondio Fluidorum* (called *De Grav* for short), Newton distinguished between two types of action, one mental and the other physical. The former stimulates the insights of thinking creatures and the latter between material bodies. Later he would extend the latter kind even to action at a distance. The distinction today seems unimportant, except to philosophers of science. Harvey and Lehmkuhl [5] argue that "*Newton is clearly appealing to a principle in the De Grav that is more fundamental and general than what he would later designate as his third law of motion in the Principia – ... and space, for Newton, is a kind of exception to this fundamental principle.*" The authors continue with a potent argument that, "... *Newtonian space is assigned a causal role, it is usually to account for inertia, i.e. the privileged existence of inertial frames, or equivalently the special motions of force-free bodies. In the De Grav, Newton explicitly renounced such a role. He stated that the reason why projectiles that are not being acted upon by other bodies move in straight lines and uniform speed is precisely that space has no ability to help or hinder any change in the motion of bodies*!"

Einstein's fundamental requirement of the 1915 field equations was that it safeguarded the Newtonian limit in the case of gravitational fields. These included the equivalence principle connecting gravitational effects with inertia as well as Mach's ideas concerning the origins of inertia. He also required that Newton's first law, the principle of the relativity of motion and third law, the conservation of energy-momentum, be satisfied. Mach's principle did not survive the several rewrites of general relativity (GR) and the action-reaction principle (ARP) appeared in Einstein's assortment of arguments in favor of GR.

In 1918, Einstein's formulation of "Mach's Principle" became equivalent to saying the $g_{\mu\nu}$ field must be "conditioned and determined" by the mass-energy-momentum $T_{\mu\nu}$ of matter [6]. It became clear later that $T_{\mu\nu}$ cannot be defined independently of $g_{\mu\nu}$. Einstein only acknowledged this point in 1954, the year before his death, in a letter to Felix Pirani. But grounds for doubting the validity of the principle, however, already existed in solutions of Einstein's 1915 field equations is a static metric field corresponding to flat, matter-free Minkowski space-time and was the reason why in 1917 Einstein modified the field equations and added the cosmological constant in an attempt to rescue Mach's principle. In his famous 1922 Princeton lectures Einstein wrote about the connection between the

Machian idea of absolute space and ARP saying, "[I]t is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum) which acts itself but which cannot be acted upon. This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. [...] In order to develop this idea within the limits of the modern theory of action through a medium, the properties of the space-time continuum which determine inertia must be regarded as field properties of space, analogous to the electromagnetic field." This was truly a major shift in Einstein's thinking about the nature of spacetime in relation to both Newtonian mechanics and special relativity. Proponents of a Machian origin of inertia would allow for a universe to evolve if quantized inertial mass were created at the same time as its space-time structure, with the creation process also generating the force driving its evolution.

Sakharov [7] regarded scalar curvature as the elasticity of space-time caused by vacuum fluctuations. Zeldovich [8] presents an early attempt to understand the cosmological constant, introduced by Einstein in 1917, in terms of fundamental physical ideas such as vacuum energy Planck-size forces are collectively called gauge fields and include the electric, weak, and strong interactions. Gravity, according to Sakharov, can be viewed as a dynamic elasticity of space arising from particle interactions. He identified the action term of Einstein's Geometrodynamics with the "*change in the action of quantum fluctuations of the vacuum… and the dependence of the action of the quantum fluctuations on the curvature of space.*" This idea was expanded upon and microscopic "Planck cells" [9] with zero total charge, mass and energy emerge producing a discrete lattice structure of the vacuum. There is a common thread that begins with Newton's concept of absolute space and time to Einstein (who told Mach in 1913, "*For me it is an absurdity to ascribe physical properties to 'space'*") to Vigier (who called inertia an "*unsolved mystery in modern physics* [10]") to Sakharov's elastic spacetime. It is that physicists have not agreed upon a theory that explains the origin of inertia and, inert as it may be, inertia is still in search of an identity.

Physicists do agree, however, that inertia instantaneously opposes acceleration. Newton regarded inertia as an internal property of matter but Mach thought that inertial forces were an external property but collectively linked to all matter in the universe. In general relativity (GR) spacetime curvature is linked to gravitational fields and inertia is accommodated by Einstein's equivalence principle (EEP). GR cannot accommodate a Machian-type inertia and all attempts to explain gravity as the source of inertial forces in any geometrical interpretation fails. Appeals to the equivalence principle leads to circular arguments. Solutions to Einstein's field equations have, surprisingly, been found in rotating Godel universes and empty de Sitter spaces. Vigier's investigations into inertia from a relativistic perspective also revealed other conflicts. Taking Mach's principle into consideration, Vigier reconciled GR and Mach's principle proposing that non-Machian forces operated in "local interactions of a Dirac subquanum aether model stemming from Einstein-de Broglie-Bohm causal stochastic quantum mechanics." Vigier's aether model combines matter and antimatter as well as particles and antiparticles but the aether's density is extremely large and impossible to be detected, observable effects cancelling each other out. Also, the Dirac Sea is a theoretical model of the *vacuum* with negative energy.

Fluctuations in the vacuum can be understood as the production of a weak dipolar fermion / antifermion pair, a harmonic oscillator creation-annihilation type of mechanism. The same fluctuations are also responsible for the Casimir effect and the Van der Waals force corresponding to the potential for a fluctuating dipole-dipole interaction. For additional details on this evolving area of investigation and an argument why we may consider "spacetime forces" responsible for the curvature in matter, rather than the other way around [11]. Rowlands argues that the existence of supersymmetric particle pairs, interacting and modifying spacetime, comes from the duality represented by the fermion and its environment. The isolated fermion represents the action half of Newton's third law, characterized by kinetic energy, continuous variation, and spin in half integral units, while in the case of the fermion interacting with its environment, it is the action and reaction pair, characterized by potential energy, a stable state, and spin in integral units [1, p.434].

The existence of an inert ether refuses to disappear.

5. Zero-point energy (ZPE)

Vacuum, as we know, is not empty space but contains an apparently infinite amount of zero-point energy. Zero-point energy can be interpreted as a classical phenomenon, arising from electromagnetic radiation. It (a) has an energy spectrum of *hw/2* per normal mode of vibration, (b) can be derived from Lorentz invariance, and (c) leads back to classical explanations of the Planck black body spectrum and Bose-Einstein statistics. When an atom or subatomic particle is cooled into its lowest possible energy ground state, zero-point-energy (ZPE), it continues to experience random fluctuations in both position and momentum, according to the Heisenberg uncertainty principle. The more precisely that its position can be defined the more uncertain is its momentum, and vice versa. Heisenberg uncertainty is not only true on the atomic scale but also on the macroscopic scale for objects which contain billions of atoms. Attempts have been made to measure the energy of vacuum fluctuations. It seems impossible to measure ZPE which does not appear to exert a force which can move other objects, i.e. it cannot do any real measurable work. Besides experimental difficulties, a long history of skepticism surrounds this subject as well. Albert Einstein, and colleague Otto Stern, introduced the idea of ZPE in 1913 to explain some contradictory experimental results. After another year of further reflection Einstein declared the idea "*dead as a doornail*." Twenty-three years later, in 1946, Nobel laureate Wolfgang Pauli stated that "*ZPE has no physical reality*." Using more precise instruments, measurements of the atomic spectra of hydrogen have been taken. Trying to see the energy of the vacuum directly is difficult and transferring real energy out of the vacuum fluctuations has failed, until recently [12]. Many skeptics are more convinced that ZPE is real. For a historical summary of attempts to measure vacuum fluctuations see [13].

6. Quaternion mass-charge generators

The Dirac equation, a relativistic wave equation derived in 1928, describes spin-1/2 particles, like electrons and quarks, for which parity is a symmetry. The equation accounts for facts about the hydrogen spectrum and is consistent with both special relativity and quantum mechanics. The Dirac equation implies the existence of a new form of matter, called antimatter, which was experimentally confirmed several years after its derivation. The equation also provides a theoretical justification for the introduction of several component wave functions in Pauli's theory of spin. The wave functions in the Dirac theory are vectors of four complex numbers, called bispinors, two which resemble the Pauli wave function in the non-relativistic limit and in the limit of zero mass, the Dirac equation reduces to the Weyl equation. Rowlands formulates a novel form of the Dirac equation based on duality, zero-totality. Using quaternions, 1 real and 3 unit-imaginary parts, the real part is responsible for mass and three complex vacuum coefficients, **i**, **j**, and **k**, are responsible for discrete, point charges. The charges act as a discrete partitioning of the continuous vacuum responsible for zero-point energy analogous to the "Planck cells" described above. Also, the separate conservation laws for the three charges imply that the three discrete partitions are independent of each other.

A principle of quantum field theory (QFT) requires that charge is conserved locally rather than globally. A particle can be created or annihilated at a point only if the corresponding antiparticle is also created or annihilated simultaneously. Quaternion multiplication is non-commutative and they obey cyclical rules. Reversing the order of multiplication reverses the sign.

$$\mathbf{ij} = \mathbf{-ji}, \quad \mathbf{jk} = \mathbf{-kj}, \ \mathbf{ki} = \mathbf{-ik}$$

$$\mathbf{i^2} = \mathbf{j^2} = \mathbf{k^2} = \mathbf{ijk} = \mathbf{-1}$$

If we regard **ij** as an action of **i** on **j** and then **–ij** is the reaction, equal and opposite. If we write this interaction in the form **ij + ji = 0** it satisfies a zero-totality condition which can be appreciated as a generalization of Newton's third law [14]. Rowlands associates the quaternion operators with the *strong*, *electric*, and **weak** forces considering charge units to be arranged along separate axes. Duality requires that the axes, unlike those of space, should be irrotational, so that one type of charge (say,

strong) can never be converted into another (say, weak or electric). This is the basis of the laws of lepton and baryon conservation and may be the reason why baryon decay has never been detected. He writes,

The connection between the quaternionic operators applied to charge and the (hidden) ones used in the Dirac 4-spinor now gives us a new understanding of the physical meaning of charge as a vacuum generator: **i**, **j**, and **k** are, simultaneously, the respective operators applied to strong, electric and weak charges, and also the creators of the strong, electric and weak vacuum images of a real fermion (which itself may be presumed to be 'generated' by the 'mass' operator, 1). Charge is, in effect, a kind of vacuum state, linked to the quantum field nature of the state vector [1, p.148].
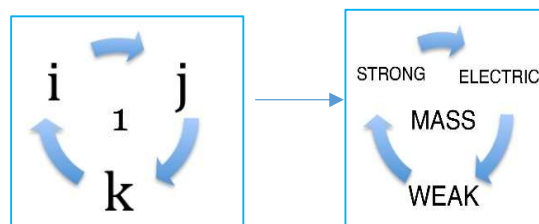
These relationships are shown in Figure 1.

Figure 1. Quaternion operators and associated mass-charge generators.

If we apply the cyclic order rule of quaternion multiplication to the charge generators, we can let **i** = (**s**)trong, **j** = (**e**)lectric and **k** = (**w**)eak, from which it follows that,

$$\textbf{se} = \textbf{- es,}\quad \textbf{ew} = \textbf{- we,}\qquad \textbf{ws} = \textbf{- sw} \tag{1}$$

$$\textbf{s}^2 = \textbf{e}^2 = \textbf{w}^2 = \textbf{sew} = \textbf{- 1} \tag{2}$$

Quaternion multiplication provides a convenient notation for representing geometric orientations and rotations of objects in 3-D, similar to but, simpler than Euler angles. What are other possible interpretations of multiplication and non-commutativity of the three charge operators? There are a couple of ways to understand the non-commutativity found in (1): (a) the charge forces satisfy Newton's third law of motion, (b) the charge forces satisfy Rowlands zero-totality condition. The weak interaction is the mechanism of between subatomic particles that causes radioactive decay. Classically the vacuum behaves as a perfect absorber and re-radiator of radiation. "Radiation reactions are basically just another version of Newton's third law [2, p. 217]."

Regarding the relations in (2), we know that one type of charge (e.g. weak) can never be converted into another (e.g. strong or electric), but weak forces facilitate the decay of neutrons into protons by converting a quark from up to down. This process occurs in a Carbon14 atom, with 6 protons/8 neutrons, when it decays into a Nitrogen14 atom with 7 protons/7 neutrons. β-decay is a type of radioactive decay in which an electron (or positron) and a neutrino are emitted from an atomic nucleus. β-decay in a neutron changes it into a proton, by the emission of an electron. In the reverse process a proton is converted into a neutron by the emission of a positron. Neither the electron or positron, nor the associated neutrino, exist within the nucleus prior to decay, but are created in the decay process.

If we interpret **-1** to mean antimatter then **w**^{2} **= -1** means that *the weak force acting on itself it produces antimatter*. For weak interactions it is known that fermions can exchange three distinct types of force carriers W$^+$, W$^-$, and Z bosons. This suggests another way of writing (2) in terms of β decay so that **(W**$^+$**)**^{2} **= (1W**$^+$**)(1W**$^+$**) = -1** means that mass action, represented by **1**, in the proton and simultaneously (as a creation and annihilation process) in the neutron, produces anti-matter, represented by **-1**. Antimatter provides another physical interpretation for the anti-commutativity found in quaternion multiplication. A down quark of the **w**$^-$ boson decays in the neutron into an up quark, the neutron simultaneously transferring a **w**$^-$ boson to the neutrino which becomes, because of the negative charge,

an electron $\mathbf{e}^-$ and anti-neutrino. See Figure 2 below. The mass operator, **1,** is the real part of the quaternion allowing us to interpret **-1** as representing anti-matter. The three charge forces operating on themselves, or operating collectively, as seen in (2), means that the charge quaternions are responsible for anti-matter production.
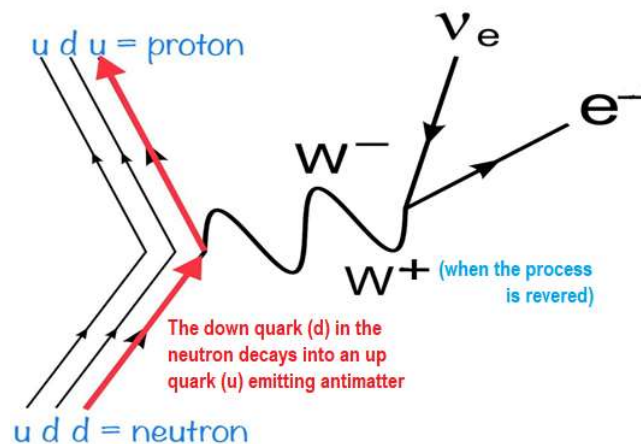

Figure 2. Decay of a neutron into a proton

Observing that $\mathbf{W}^+ + \mathbf{W}^- = \mathbf{0}$ and multiplying both sides by $\mathbf{W}^+$ we get $\mathbf{W}^+\mathbf{W}^+ + \mathbf{W}^+\mathbf{W}^- = \mathbf{0}$. Since $(\mathbf{W}^+)^2 = \mathbf{-1}$ we get $\mathbf{W}^+\mathbf{W}^- = \mathbf{1}$ or that the interaction of the two weak forces produces mass, represented by **1**. If we multiply both sides of the last equation by $(\mathbf{W}^-)^{\mathbf{-1}}$ we get $\mathbf{1W}^+ = \mathbf{1}(\mathbf{W}^-)^{\mathbf{-1}}$ implying that β^- decay is equivalent to inverse β^+ decay reaction, as observed in antiparticle pairs electron-positron. This preliminary study requires a more thorough investigation into the meaning of Rowlands' *mass-charge quaternion interactions* of the nilpotent Dirac equation. As Rowlands emphasizes that many findings become "*immediate consequences of the algebraic structure, including Pauli exclusion, bosons, baryons, CPT transformations, creation and annihilation operators, vacuum states, the structure of the equation itself and the basic symmetries of the Standard Model* [18]."

7. Conclusion

This paper investigated (i) how Newton's third law applies to zero-point fields, (ii) how mass-charge forces can be represented by quaternions, (iii) the relationship between mass-charge quaternions and "Planck cells" in space-time and (iv) the relationship between mass-charge quaternion structure and the Rowlands nilpotent form of the Dirac equation. The novel linkage of the quaternion operators **1**, **i**, **j**, **k** to the three charge operators **s, e**, **w** yields new insights into the mechanics of elementary particles and gauge forces.

Zero-point energy produced by the vacuum charge forces (strong, electric, weak) may reasonably be called *spacetime forces*. Rowlands' assertion that the gauge / gravity duality satisfies a zero-totality condition, implies that spacetime is dynamic, produces discrete inertial forces and creates the curvature properties which are inherited by all matter, as observed in molecular DNA helical structures, which can also be explained by the nilpotent quaternion structure.

As a final point, it is noted in this study that theoretical arguments are supported by experimental research found in the literature supporting quantized inertia over quantized gravity. Real physical connections between an object's inertial mass and the emission of photons are observed in the form of Unruh radiation. Inertial mass is generated by the object's acceleration with respect to surrounding matter. The hypothesis of Sakharov about the effect of vacuum energy on spacetime, viz. its elasticity and curvature, the "quantised inertia" of Rowlands [15] and McCulloch [16] and the inertial effects of the vacuum proposed by Haisch and Rueda [17] all point to a dynamic spacetime that curves matter while simultaneously producing a reverse Machian ether-type cosmological gravitational field.

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