Theories of Modified Gravity

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I briefly review modified theories of gravity, and in particular discuss massive theories of gravity and current observational bounds placed on the graviton mass.

1 Introduction

1.1 Why modify gravity at large distances?

Why modify gravity at large distances? The principal motivation is cosmological. The main driver for exploring modifications to general relativity are the intertwined dark energy and cosmological constant problems, i.e. the theoretical attempts to reconcile the now compelling picture that the universe is undergoing late time acceleration, together with our understanding of gravity:

- **Old Cosmological Constant Problem**: Why is the Universe not accelerating at a rate determined by the vacuum energy?
- **New Cosmological Constant Problem**: Assuming the above is solved, what gives rise to the remaining vacuum energy or dark energy which leads to the acceleration we observe?
- **Testing GR**: Because it allows us to put better constraints on Einstein Gravity.

Gravity has only been tested over a special range of scales and curvatures [1]. Just as Weinberg constructed his nonlinear version of quantum mechanics, to provide a means to test experimentally the linearity of ordinary quantum mechanics [2], it is important to explore cosmological alternatives to general relativity (and a cosmological constant), to better probe the possible validity of this theory.

1.2 Guiding Principle

The central guiding principle to constructing modifications of general relativity is the theorem that:

General Relativity (with a cosmological constant) is the unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples to matter.

This theorem goes back to the work of Feynman, Weinberg, Deser, Wald and many others [3,5,4,6] and has been approached both from the perspective of the consistency of the equations of motion, the consistency of the non-linear symmetries, and the existence of an S-matrix. Any modification will thus give up one of these properties:

1. Locality,
2. Lorentz Invariance,
3. Massless,
4. Single Spin Two.

A corollary of this theorem is that any theory which preserves Lorentz invariance and locality (1 and 2) leads to new degrees of freedom since we require either new states (4) or the spin 2 field is massive (3), which by Lorentz invariance then has 3 new degrees of freedom. New gravitational degrees of freedom that couple to matter are highly constrained by a variety of tests including: fifth force constraints (e.g. solar system tests), equivalence principle tests, binary pulsar timing, nucleosynthesis, cosmological moduli problems. We thus need some kind of screening mechanism to hide extra degrees of freedom.

1.3 Screening Mechanisms

The different screening mechanisms that arise in theories of dark energy and modified gravity can be understood in relatively simple terms as follows. Let us suppose for simplicity, that the new degrees of freedom that arise are scalars. Let us denote one of these scalar fields by ϕ, and take into account the fact that in a cosmological or astrophysical background it may take on some time/space dependent background value ϕ_b. Accounting for fluctuations we denote $\phi = \phi_b + \delta\phi$. Similarly let us collectively denote the various components of the stress energy tensor by $\rho = \rho_b + \delta\rho$. The generic form of the equations of motion for perturbations is schematically

$$Z(\phi_b,\rho_b)\left[\frac{d^2\delta\phi}{dt^2} - c_s^2(\phi_b,\rho_b)\nabla^2\delta\phi\right] + m^2(\phi_b,\rho_n)\delta\phi = \beta(\phi_b,\rho_b)G_{\rm Newton}\delta\rho\,. \tag{1}$$

The static force between two point masses M_a, M_b will take the schematic form[a]

$$F \approx \frac{M_a M_b G}{r^2}\frac{\beta^2(\phi_b,\rho_b)}{Z(\phi_b,\rho_b)c_s^2(\phi_b,\rho_b)}\exp\left(-m(\phi_b,\rho_b)r\right). \tag{2}$$

Generically these fifth forces may be as strong as the ordinary Newtonian contribution unless one or more of the following conditions are met:

1. The coupling is small, at least in the presence of matter $\beta(\phi_b,\rho_b) \ll 1$. This is realised by quintessence models (where $\beta \approx 0$) or the symmetron mechanism [7] where this occurs in the regions of dense environments.
2. The mass becomes large in the region of dense environments $m(\phi_b,\rho_b) \gg 1/r_{\rm exp}$, as is realised for example by the chameleon mechanism [8].
3. The kinetic term becomes large in dense environments $Z(\phi_b,\rho_b) \gg 1$. This is known as the Vainshtein mechanism [9] and is most famously realised in the context of massive theories of gravity.

[a] Assuming an interaction $\mathcal{L}_{\rm int} \sim \beta(\phi_b,\rho_b)G_{\rm Newton}\delta\phi\delta\rho$

In what follows we shall focus on theories of massive gravity that incorporate the Vainshtein screening mechanism. For a recent review of Vainshtein screening see Ref. [10]. In brief this mechanism works as follows. Associated with the graviton mass scale m, the Planck scale $M_{\rm Pl}$ and mass of a source M, there is a characteristic length scale known as the Vainshtein radius r_V determined by

$$r_V^3 = \frac{1}{M_{\rm Pl} m^2} \frac{M}{M_{\rm Pl}} \,. \tag{3}$$

This is the scale at which the helicity-zero modes of the massive graviton, the would-be progenitors of fifth forces become strongly coupled. At distance $r \ll r_V$ the effective kinetic term for fluctuations around the background takes the form

$$Z \sim \left(\frac{r_V}{r} \right)^A \,, \tag{4}$$

with A a positive power such that $Z \gg 1$. This arises from the nonlinearities of the strong coupling mechanism. This screening of the fifth forces means that the predictions of general relativity are recovered with small corrections. Stated differently, this region is the one for which the spacetime curvature $R \gg m^2$ and so the mass term in massive gravity is negligible relative to the Einstein-Hilbert term. At distances $r \gg r_V$ the mass term comes into play and there are noticeable departures from general relativity, until eventually we reach $r \gg 1/m$ at which point the Yukawa suppression effect kicks in.

2 Massive Gravity

What does it mean to have massive gravity? To understand this we can think of the analogy of how the W and Z bosons become massive in the standard model. There the electroweak symmetry is broken by the vev of the Higgs field via

$$SU(2) \times U(1)_Y \to U(1)_{\rm EM} \,, \tag{5}$$

with the result that the W and Z bosons become massive. The would-be Goldstone mode in the Higgs field becomes the Stückelberg field which gives the boson mass. For instance in the simplified version of the Abelian Higgs model, the single complex Higgs field can be parameterized as

$$\Phi \to (v + \rho) e^{i\pi} \,, \tag{6}$$

where v is the Higgs vev, ρ is the Higgs boson and π is the Stückelberg field (which in the global limit is the Goldstone mode). Under the $U(1)$ symmetry $A_\mu \to A_\mu + \partial_\mu \chi$ and $\pi \to \pi + \chi$.

In massive gravity, the symmetries are the direct product of a local diffeomorphism group and an additional global Poincaré group. The breaking mechanism that gives rise to a mass is the one that leaves behind the diagonal subgroup

$$Diff(M) \times Poincare \to Poincare_{\rm diagonal} \,. \tag{7}$$

Similarly in bigravity models, two copies of the local diffeomorphim group are broken down to a single copy of the diffeomorphism group

$$Diff(M) \times Diff(M) \to Diff(M)_{\rm diagonal} \,. \tag{8}$$

Despite much blood, sweat and tears, an explicit Higgs mechanism for gravity is not known. However, if such a mechanism exists, we do know how to write down the low energy effective theory in the spontaneously broken phase. For an Abelian Higgs this corresponds to integrating out the Higgs boson and working at energy scales lower that the mass of the Higgs boson ρ.

This will then be an effective theory for the Stückelberg field π coupled to the gauge field A_μ.

In massive gravity we follow the same procedure [11]. Diffeomorphism invariance is spontaneously broken, but can be recovered with the introduction of 4 Stuckelberg fields ϕ^a. This is achieved by replacing the Minkowski reference metric for the global Poincare symmetry η_{ab} with the spacetime tensor $f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$. In this way the mass term can be constructed out of scalar combinations of $f_{\mu\nu}$ and $g_{\mu\nu}$ in a way which respects diffeomorphism invariance. The reference metric η_{ab} can be viewed as the vev of a spin-2 Higgs (possibly composite) field $\eta_{ab} = \langle\hat{O}_{ab}\rangle$. The additional 3 degrees of freedom in massive graviton are made manifest by the decomposition

$$\phi^a = x^a + \frac{1}{mM_{\rm Pl}}A^a + \frac{1}{\Lambda_3^3}\partial^a\pi\,, \tag{9}$$

where $\Lambda_3^3 = m^2 M_{\rm Pl}$ and scales introduced are inferred by canonical normalization. A^a caries the 2 helicity-one degrees of freedom of the massive spin 2 field (there is an additional $U(1)$ symmetry $A_\mu \to A_\mu + \partial_\mu\psi$, $\pi \to \pi - m\psi$ which keeps it at 2), and π the helicity-zero. The full form of Lorentz invariant massive gravity with the highest strong coupling scale possible Λ_3 was given in Ref [12]. The precise form is

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_{\rm Pl}^2 R - m^2\sum_{n=0}^{4}\beta_n\mathcal{U}_n\right) + \mathcal{L}_M\,, \tag{10}$$

where the characteristic polynomials $\mathcal{U}_n(K)$ are defined via

$$Det[1+\lambda K] = \sum_{n=0}^{4}\lambda^n\mathcal{U}_n(K)\,, \tag{11}$$

and the matrix K is defined via $K = 1-\sqrt{g^{-1}f}$ which may be equivalently written as $g^{\mu\alpha}f_{\alpha\nu} = g^\mu_\nu - 2K^\mu_\nu + K^\mu_\alpha K^\alpha_\nu$. The square root structure is determined entirely by the requirement that the strong coupling scale is Λ_3. A generic mass term, would give rise to an effective theory whose cutoff is $\Lambda_5 = (m^3 M_{\rm Pl})^{1/5}$.

3 Constraints on the Graviton Mass

Constraints on the graviton mass are summarized in Figure (1) and are discussed at length in Ref. [13 b]. They arise from probing three distinct physical consequences of the graviton having a mass. These are as follows:

- Yukawa Suppression: Due to the mass, static weak field forces will be exponentially suppressed at large distances $V \sim \frac{mMG}{r} \to \frac{mMG}{r}e^{-mr}$.
- Modified Dispersion Relation: Gravitational waves now propagate with a dispersion relation of the form $\omega^2 \approx c^2k^2 + m^2c^2$.
- Fifth forces/New degrees of freedom: Since generically massive gravity theories have additional degrees of freedom which couple to the stress-tensor, then they induces fifth forces which as we have discussed in the previous section must be screened in some sense in dense environments, to avoid immediate conflict with current observations.

[b] We shall review various aspects of this discussion here, and we refer to Ref. [13] for a fairly exhaustive list of references

Figure 1 – A summary of constraints on the graviton mass from de Rham, Deskins, Tolley and Zhou, 2017.

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

3.1 Fifth Forces/New degrees of freedom

Traditionally the strongest constraint on the mass of the graviton comes from lunar laser ranging experiments which probe modifications in the earth-moon system. Despite only testing solar system scales, the accuracy and length of time they can be performed significantly compensates the otherwise negligible physical modification in the orbits due to fifth forces. For models whose effective theory is described by a cubic Galileon, such as the Dvali-Gabadadze-Porratti model then the constraint on the graviton mass implied by a modification of the Newtonian potential $\delta\Phi$ is of order

$$m_g < \delta\Phi \left(\frac{r_S}{a^3}\right)^{1/2} \quad \rightarrow m_g < 10^{-32}\mathrm{eV}\,. \tag{12}$$

where a is the semi-major axis of the lunar orbit and r_S the Schwarzschild radius of the earth. For 'hard-mass' models of massive gravity, or those whose effective theory is described by a quartic Galileon the constraint is slightly weaker

$$m_g < \delta\Phi^{3/4} \left(\frac{r_S}{a^3}\right)^{1/2} \quad \rightarrow m_g < 10^{-30}\mathrm{eV}\,. \tag{13}$$

It should be noted that these are already remarkably strong constraints, and are far stronger than the equivalent constraints on the mass of the photon [21].

Observations of binary pulsars provide a more direct probe of the possible existence of extra degrees of freedom [14], [15] since the extra polarizations of the graviton imply additional modes of gravitational waves. Consequently binary pulsars lose energy faster than in general relativity and so the orbit slows down more rapidly. This gives a constraint of order $m_g < 10^{-27}$eV.

3.2 Yukawa Potential Bounds

In the weak field approximation, a graviton mass leads to a Yukawa potential in place of a Newtonian potential. As such structures cannot be gravitationally stable at distances larger than the Compton wavelength of the graviton. Given that the core of clusters being a typical size of 1-10 Mpc are virialized, then we obtain a constraint $m_g < 10^{-29}$eV [16]. Within the solar system, we can check the force law by the validity of Kepler's law a^3/T^2 =constant, for Earth and Mars. This gives a constraint $m_g < 10^{-23}$eV [17]. In weak lensing, the power spectrum of effective convergence gets corrected by a factor of $k^2/(k^2+m_g^2)$, and so assuming ΛCDM we get $m_g < 10^{-32}$eV [18].

3.3 Direct Detection of Gravitational Waves

Given the recent direct detection of gravitational waves by advanced LIGO [19], associated with the merger of two binary black holes, we have a direct opportunity to put constraints on the mass of the graviton from the modified dispersion relation for gravitational waves. This is through observations of the waveform. During the merger process, the frequency of gravitational waves increases sharply at the end (the so-called 'chirp'). Taking the simplest assumption that $\omega^2 = c^2k^2+m^2c^2$ then the gravitational waveform would be more squeezed in a theory of massive gravity than in GR. This is because the speed of near luminal gravitational waves increases with frequency as

$$\frac{v_g}{c} \approx 1 - \frac{1}{2}\left(\frac{c}{\Lambda_g f}\right)^2 \tag{14}$$

and so the later emitted parts of the waveform would travel faster, causing the overall waveform to bunch up [20]. If Δt_e is the emitted signal duration and Δt_o the observed duration, then accounting for any possible redshifting, the effective time difference due to any possible squeezing of the signal is

$$\Delta t = \Delta t_o - \Delta t_e(1+z)\,. \tag{15}$$

This effect places a constraint on the graviton mass in the form

$$m_g < 4\times 10^{-22}\text{eV}\left(f\Delta t \frac{f}{100\text{Hz}}\frac{200\text{Mpc}}{D}\right)^{1/2}\,, \tag{16}$$

where D is the luminosity distance of the source. The phase distortion $f\Delta t$ can be measured up to $1/\rho$, where ρ is the signal to noise ratio. For the gravitational wave detection $GW150914$ we have $D \sim 400$Mpc, $f \sim 100$Hz and $\rho \sim 23$ implying $m_g < 10^{-22}$eV. For LISA we could in principle have $\rho \sim 10^3$, $D \sim 3$Gpc and $f \sim 10^{-3}$Hz which could put a constraint $m_g < 10^{-26}$eV.

While the above constraint on the graviton mass was discussed already after the first direct detection of gravitational waves [19], this is far from the end of the story of what can be learned about massive gravity from strong gravity physics and gravitational waves. We known in realistic nonlinear theories of massive gravity, such as that discussed above, the graviton mass actually depends on the environment, for instance it can depend on the distance to the black holes, via the background metric. The graviton mass is also likely to vary non-adiabatically during the merger, creating additional non-adiabatic effects in the waveform. The effects of the additional scalar and vector gravitational modes has not been taken into account, and in certain stages of the merger the scalar radiation could feasibly dominate the effect of the tensors. The black hole or neutron star solutions may themselves be modified, which will lead to different quasi-normal modes modifying the final ring-down stages. The Vainshtein suppression may not be active in the merger region and this would need to be dealt with by a proper numerical simulation accounting for the additional degrees of freedom. For instance it is expected that the PN expansion almost certainly doesn't working in the Vainshtein region. Given all these extra effects that have not yet been taken into account, it is not unreasonable to conjecture that the current advanced LIGO constraints on massive gravity theories are already much stronger than those quoted in the literature so far.

4 Summary

Realistic, nonlinear, diffeomorphism invariant effective field theories of modified gravity do exist and can be tested. Many models designed to modify late time cosmology can also be constrained by solar system, astrophysical (e.g. pulsars), strong gravity physics, gravitational waves. The various different screening mechanisms that arise in these theories play a crucial role in their theoretical and observational viability.

Nonlinear theories of massive gravity are examples which will lead to many physical effects different from general relativity, and these theories arise whenever diffeomorphism invariance is spontaneously broken. Constraints on the graviton mass arise from probing essentially three different distinct physical consequences of the existence of a graviton mass, (1) Yukawa suppression, (2) dispersion relation and (3) fifth forces. Looking to the future, strong gravity and gravitational wave physics will place strong constraints on these theories. For example, current constraints on the graviton mass from advanced Ligo need to be improved to better understand the nonlinear dynamics of the helicity-0 mode (Vainshtein effect), and the properties of the black holes and neturon stars in massive gravity (be it Lorentz invariant or Lorentz violating), how the binary merger is modified, and how the quasi-normal modes are modified.

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References

1. D. Psaltis, “Probes and Tests of Strong-Field Gravity with Observations in the Electromagnetic Spectrum,” Living Rev. Rel. **11**, 9 (2008) doi:10.12942/lrr-2008-9 [arXiv:0806.1531 [astro-ph]].
2. S. Weinberg, “Testing Quantum Mechanics,” Annals Phys. **194**, 336 (1989). doi:10.1016/0003-4916(89)90276-5
3. R. P. Feynman, F. B. Morinigo, W. G. Wagner and B. Hatfield, “Feynman lectures on gravitation,” Reading, USA: Addison-Wesley (1995) 232 p. (The advanced book program)
4. S. Weinberg, “Photons and gravitons in perturbation theory: Derivation of Maxwell’s and Einstein’s equations,” Phys. Rev. **138**, B988 (1965). doi:10.1103/PhysRev.138.B988
5. S. Deser, “Selfinteraction and gauge invariance,” Gen. Rel. Grav. **1**, 9 (1970) doi:10.1007/BF00759198 [gr-qc/0411023].
6. R. M. Wald, “Spin-2 Fields and General Covariance,” Phys. Rev. D **33**, 3613 (1986). doi:10.1103/PhysRevD.33.3613
7. K. Hinterbichler and J. Khoury, “Symmetron Fields: Screening Long-Range Forces Through Local Symmetry Restoration,” Phys. Rev. Lett. **104**, 231301 (2010) doi:10.1103/PhysRevLett.104.231301 [arXiv:1001.4525 [hep-th]].
8. J. Khoury, “Chameleon Field Theories,” Class. Quant. Grav. **30**, 214004 (2013) doi:10.1088/0264-9381/30/21/214004 [arXiv:1306.4326 [astro-ph.CO]].
9. A. I. Vainshtein, “To the problem of nonvanishing gravitation mass,” Phys. Lett. **39B**, 393 (1972). doi:10.1016/0370-2693(72)90147-5

10. E. Babichev and C. Deffayet, "An introduction to the Vainshtein mechanism," Class. Quant. Grav. **30**, 184001 (2013) doi:10.1088/0264-9381/30/18/184001 [arXiv:1304.7240 [gr-qc]].
11. N. Arkani-Hamed, H. Georgi and M. D. Schwartz, "Effective field theory for massive gravitons and gravity in theory space," Annals Phys. **305**, 96 (2003) doi:10.1016/S0003-4916(03)00068-X [hep-th/0210184].
12. C. de Rham, G. Gabadadze and A. J. Tolley, "Resummation of Massive Gravity," Phys. Rev. Lett. **106**, 231101 (2011) doi:10.1103/PhysRevLett.106.231101 [arXiv:1011.1232 [hep-th]].
13. C. de Rham, J. T. Deskins, A. J. Tolley and S. Y. Zhou, "Graviton Mass Bounds," Rev. Mod. Phys. **89**, no. 2, 025004 (2017) doi:10.1103/RevModPhys.89.025004 [arXiv:1606.08462 [astro-ph.CO]].
14. C. de Rham, A. J. Tolley and D. H. Wesley, "Vainshtein Mechanism in Binary Pulsars," Phys. Rev. D **87**, no. 4, 044025 (2013) doi:10.1103/PhysRevD.87.044025 [arXiv:1208.0580 [gr-qc]].
15. C. de Rham, A. Matas and A. J. Tolley, Phys. Rev. D **87**, no. 6, 064024 (2013) doi:10.1103/PhysRevD.87.064024 [arXiv:1212.5212 [hep-th]].
16. A. S. Goldhaber and M. M. Nieto, "Mass of the graviton," Phys. Rev. D **9**, 1119 (1974). doi:10.1103/PhysRevD.9.1119
17. C. Talmadge, J. P. Berthias, R. W. Hellings and E. M. Standish, "Model Independent Constraints on Possible Modifications of Newtonian Gravity," Phys. Rev. Lett. **61**, 1159 (1988). doi:10.1103/PhysRevLett.61.1159
18. S. R. Choudhury, G. C. Joshi, S. Mahajan and B. H. J. McKellar, "Probing large distance higher dimensional gravity from lensing data," Astropart. Phys. **21**, 559 (2004) doi:10.1016/j.astropartphys.2004.04.001 [hep-ph/0204161].
19. B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], "GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence," Phys. Rev. Lett. **116**, no. 24, 241103 (2016) doi:10.1103/PhysRevLett.116.241103 [arXiv:1606.04855 [gr-qc]]. B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], "Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. **116**, no. 6, 061102 (2016) doi:10.1103/PhysRevLett.116.061102 [arXiv:1602.03837 [gr-qc]]. B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], "Tests of general relativity with GW150914," Phys. Rev. Lett. **116**, no. 22, 221101 (2016) doi:10.1103/PhysRevLett.116.221101 [arXiv:1602.03841 [gr-qc]].
20. C. M. Will, "Bounding the mass of the graviton using gravitational wave observations of inspiralling compact binaries," Phys. Rev. D **57**, 2061 (1998) doi:10.1103/PhysRevD.57.2061 [gr-qc/9709011].
21. A. S. Goldhaber and M. M. Nieto, "Photon and Graviton Mass Limits," Rev. Mod. Phys. **82**, 939 (2010) doi:10.1103/RevModPhys.82.939 [arXiv:0809.1003 [hep-ph]].