# BEAM TRANSPORT SYSTEM FROM A LASER WAKEFIELD ACCELERATOR TO A TRANSVERSE GRADIENT UNDULATOR*

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## Abstract

The transport and matching of electron beams generated by a laser wakefield accelerator (LWFA) is a major challenge due to their large energy spread and divergence. Strong focusing magnets and a chromatic correction are required.

This contribution discusses the layout of the electron beam transport optics for a diagnostic beamline at the LWFA in Jena, Germany. The aim of this optics is to match the beam's betatron functions and the dispersion to the field of a transverse gradient undulator (TGU) such that bandwidth of the undulator radiation is kept at the natural limit despite the large energy spread.

## INTRODUCTION

Combining laser wakefield accelerators (LWFAs) and short period undulators might allow for the design of compact undulator light sources or even so called table-top free electron lasers (FELs) [1, 2]. The intrinsically short bunches of a LWFA result in high peak currents at electron energies of up to 1 GeV, potentially enabling for the design of a high-gain FEL in the x-ray range. The major drawback in the realization of such a setup is the large relative energy spread of the LWFA of 1 % and more, which deteriorates the characteristic monochromatic spectrum of the undulator radiation.

To overcome this limitation, the concept of a transverse gradient undulator (TGU) was proposed [2, 3]: The electron beam is dispersed in the deflection plane of the undulator, i.e. in the $xz$-plane, and the lateral spread of the electron energy is matched to the magnetic flux density amplitude $B_y(x)$ of an undulator with a transverse field gradient $\frac{dB_y}{dx}$.

A challenging part for the realization of such a setup is the beam transport system from the LWFA to the TGU. Up to now for the focusing of the electrons from the LWFA to a planar undulator basically a set of permanent magnet quadrupoles was used [4]. However, such a transport system will not work for a setup with a TGU, which requires a proper matching of the dispersion at the entrance of the undulator and the control of the transverse beam parameters of all energies in the range of the energy acceptance of the undulator.

The transport system has to ensure that the reference trajectories of each energy, i.e. the trajectories with no initial transverse momentum, enter the undulator at the $x$ position of the matched flux density amplitude and that all reference particles propagate on parallel trajectories along the undulator. Furthermore all beamlets with different energies should enter the undulator with the optimum transverse beam parameters, i.e. the optimum beam size and convergence or, in terms of the Twiss parameters, the according $\alpha$ and $\beta$.

A first experiment with a TGU at the LWFA in Jena is planned. The target bandwidth of the undulator radiation is kept in the range of 1 % with a relative energy spread of the electrons of $\pm 10$ %. In this contribution the possible layout of the beam transport system and its limitations are discussed.

## PARAMETERS OF THE LWFA

The design electron energy for the experiment is 120 MeV with an energy spread of $\pm 10$ %, including also jitter of the central energy. To estimate the initial parameters for the beam transport system the values inside the plasma $\sigma_x = 0.7$ μm and a divergence of 2.5 mrad, which were measured via the betatron radiation emitted by the electrons inside the plasma, are used [5]. The bunch length is taken as $\sigma_z = 2.5$ fs [6]. The maximum bunch charge expected in the experiment is 10 pC. ASTRA [7] calculations assuming a gaussian bunch with these parameters show that space charge effects are negligible already 0.2 mm downstream the source, where a geometrical emittance of 2.5 nm rad is reached. Even assuming a background with 1 nC and the same bunch size the emittance does not increase further.

At the LWFA of the ALPHA-X beam line with similar laser parameters and a similar target [8] an average geometrical emittance of $\varepsilon \sim 8.8$ nm rad and $\sigma_{x'} = 2$-4 mrad were measured and an effective source size of $\sigma_x = 3$ μm determined. So the emittance and the source size are larger than the values estimated with the ASTRA simulations.

As initial parameters for the simulation presented in this paper, a divergence of $\sigma_{x'} = 2.5$ mrad and the source size $\sigma_x = 4$ μm are assumed. To take the variations in the size of the source into account, $\sigma_{x,y}$ is varied in the range of 1 to 8 μm. Furthermore the same initial values in $x$ and $y$ are taken with a beam waist located at the exit of the LWFA.

## DESIGN OF THE TGU

The superconducting undulator consists of two cylindrical coils with 100 full undulator periods and a period length of 10.5 mm. The dispersed electron beam passes the undulator slightly off axis in the region of the maximum transverse field gradient. At the position of the central energy the flux density amplitude $B_0$ is 1.1 T, the undulator parameter $K$ is 1.1 and the transverse gradient $\frac{dK}{dx}$ equals 149 /m.

To match the electron beam to the undulator the most important parameters are the entrance position and angle of each monoenergetic beamlet. In linear approximation this requires fot the dispersion $D = 20$ mm and its derivative $D' = 0$. Considering the focusing effects of the undulator in linear approximation, one expects negligible focusing in the deflection plane $(x, z)$ despite the transverse gradient and the usual focusing due to the longitudinal field components in the $(y, z)$-plane. The smallest beta functions along the undulator are therefore achieved with $\alpha_x = 2.6$, $\beta_x = 1.6$ m and $\alpha_y = 0$, $\beta_y = 0.7$ m at the entrance of the undulator (for details see [10]).

## LINEAR BEAM TRANSPORT SYSTEM

For the beam transport system normal conducting, water-cooled quadrupoles were designed. They can be operated in vacuum to keep the gap as small as possible. The iron yoke is $l_z = 80$ mm long. The quadrupoles achieve a maximum $k$-parameter for $E_0 = 120$ MeV of $k_q \approx 100$ /m$^2$. The chromatic correction is done using combined quadrupole-sextupole magnets keeping the system as compact as possible. The two dipoles with rectangular poles of length $l = 50$ mm have a maximum deflection angle of 60 mrad.

Due to the large divergence of the source it is best to move the first magnet as close as possible to the exit of the LWFA. To avoid a damage of the magnets by the laser radiation a minimum distance of 0.3 m to the LWFA is kept. The minimum possible distance from the last magnet to the entrance of the undulator inside the cryostat is 0.5 m.

The linear beam transport system was simulated with the MAD-X Twiss module [11]. Assuming a beam waist at the exit of the LWFA, the initial Twiss parameters are $\beta_{x,y} = 1.6$ mm, $\alpha_{x,y} = 0$ according to the parameters in the previous section.

As a basic layout the achromat in Fig. 1 was chosen. The achromatic section, which is completely symmetric in the case of an achromat, consists of two dipoles and three quadrupoles to control the dispersion. With one triplet at the beginning and a second triplet before the undulator the transverse beam parameters are matched to the parameters required by the undulator.
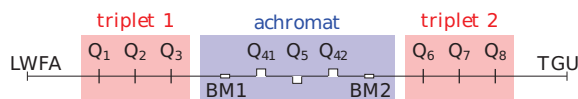


Figure 1: Schematic layout of the transport system with nine focusing magnets and two bending magnets.
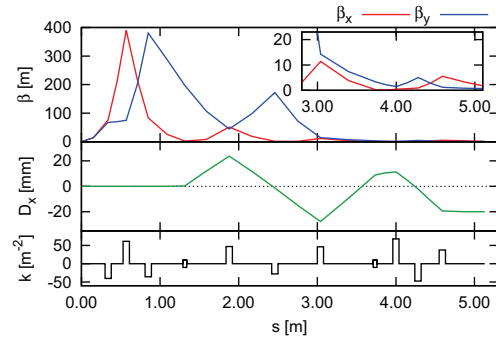
Figure 2: Layout of the dispersive transport system. The inlay shows a zoom on the beta functions.

For the concept of the TGU a dispersed beam with parallel monoenergetic beamlets is required at the entrance of the undulator. To achieve the required parameters of $D$ and $D'$, the achromatic system was modified by changing the quadrupole strength of the achromat and triplet 2 and slight changes of the magnet position in these sections. The beta function and the dispersion of this layout are shown in Fig. 2. The previously achromatic section is not symmetric any more. Note that isochronicity would be a further requirement particularly for an FEL, which was, however, not included in this study.

## TRACKING OF BUNCHES

The beam transport system is supposed to transport electrons in a large energy range. Therefore sextupoles are necessary to correct the chromatic effects and non-linear effects will play a major role. For nonlinear studies the PTC tracking module [12] was used. The beam is considered as a composition of monoenergetic beamlets of different energies, as the transverse phase space distributions of each particle energy must match the required parameters of the undulator. The following criteria are important:

- The entrance angle of each beamlet: In $x$ it should be smaller than 0.1 mrad to ensure a parallel propagation of the beamlets along the undulator. In $y$ it is not that critical, but should be kept small because the coupling between $x$ and $y$ increases with increasing distance to the midplane of the undulator.

- The transversal phase space distribution of each beamlet: To match the required parameters for each energy it should be similar to the phase space distribution of the central energy in case of no sextupole correction. Especially the beam size in $x$ and the position of the beam waist should be conserved in order to prevent a broadening of the undulator radiation spectrum.

- The entrance position: In $x$ it defines the peak wavelength of the undulator radiation. So a mismatch leads to a broadening of the total spectrum. For the current state of these studies the entrance position and the field gradient of the undulator were not yet optimized. This can be done by varying the magnetic flux density $B_0$
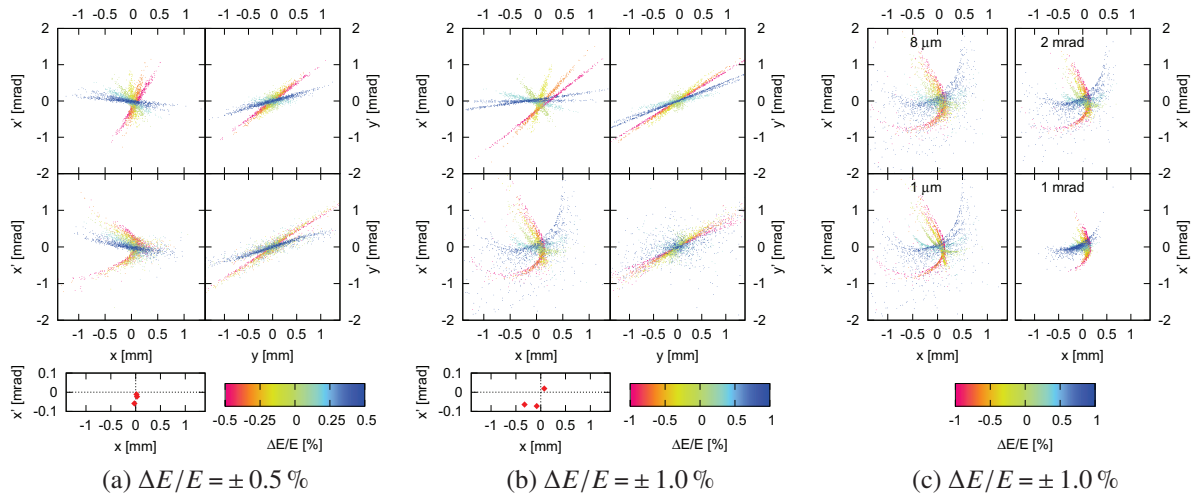
(a) $\Delta E/E = \pm\,0.5\,\%$          (b) $\Delta E/E = \pm\,1.0\,\%$          (c) $\Delta E/E = \pm\,1.0\,\%$

Figure 3: Phase space distributions at the end of the beam transport line for seven monoenergetic beamlets and the average $x$ and $x'$ with $\Delta E/E$ (a) $\pm\,0.5\,\%$ and (b) $\pm\,1.0\,\%$ , at the top is the uncorrected, at the bottom the corrected phase space distribution. (c) Variation of the source size and the divergence for an energy spread of $\pm\,1\,\%$. *left*: The source size is set to 1 μm, and 8 μm with $\sigma_{x'} = 2.5$ mrad. *right*: The divergence is set to 2.0 mrad and 1.0 mrad with $\sigma_x = 4$ μm.
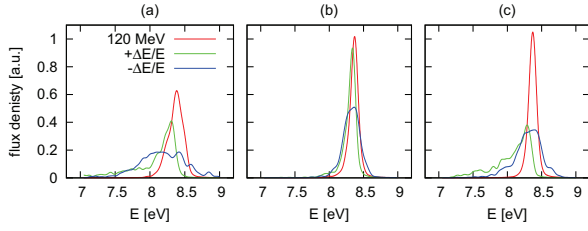


Figure 4: Undulator radiation spectra for monoenergetic beamlets with the different energy spread and initial divergence:  $\Delta E/E = 0.5\,\%$, (a)  $\sigma_{x'} = 2.5$ mrad and (b) $\sigma_{x'} = 1.0$ mrad, respectively; (c) $\Delta E/E = 1.0\,\%$, $\sigma_{x'} = 1.0$ mrad. Displayed is the central, the minumum and the maximum energy. The entrance position in $x$ is not optimized in this case.

of the undulator and the linear dispersion of the beam transport system iteratively.

The chromatic correction will be done with combined quadrupole sextupole magnets. Looking at the beta function and the dispersion there are four quadrupoles that can be used for the correction: $Q_{41}$ and $Q_{42}$, where the dispersion and the beta functions are large, and $Q_6$ and $Q_8$, where the dispersion is large, but the beta function is small. The phase space distribution should not be distorted, as it would be done placing a sextupole at a position with a large beta function, so mainly the two combined function magnets at the location of the small beta function should be used for the correction.

Figure 3a,b show the uncorrected and corrected phase space distributions for the dispersive transport system with seven monoenergetic gaussian bunches of different energies. For an energy spread of $\pm\,0.5\,\%$ it is possible to compensate the chromatic error up to a certain degree. Further increase of the sextupole strength leads to distortion of the distribution

rather than correcting the chromatic effects as shown in Fig. 3b.

For the planned setup a chromatic correction in the range of at least $\pm\,2\,\%$ would be desirable. To achieve this one has to find the reason for the limitation of the chromatic correction.

Figure 3c shows the variation of the source size and the divergence of the source. Increasing the source size leads to smearing out the transverse momenta, but no significant change in the beam size and no improvement on the chromatic correction. Decreasing the initial divergence from 2.5 mrad to 2.0 mrad and 1.0 mrad, respectively, results in a significant reduction of the beam size and, as the stretching of the phase space distribution and the resulting distortion decrease, the chromatic correction works up to 1 %. This also affects the radiation spectra shown in Fig. 4 the three cases mentioned, which are less distorted despite the larger energy spread. Thus, the initial beam divergence can be identified as the limiting factor.

## CONCLUSION

For the initial parameters assumed for the design of the beam transport system it is possible to match the required parameters of the TGU and achieve reasonable undulator radiation spectra up to an energy range of $\pm\,0.5\,\%$. The emittance and in particular the initial divergence of 2.5 mrad are the limiting parameters. By decreasing the divergence, the nonlinear effects of the sextupoles on the phase space distribution decrease and the chromatic correction can be extended to an energy range of $\pm\,1.0\,\%$.

To exploit the whole energy range of $\pm\,10\,\%$ of the undulator in the experiment this range will have to be scanned. The possibility of increasing the usable energy range by employing e.g. higher order multipole corrections will be examined in continuation of the studies presented here.

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