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Field Theory Actions for Ambitwistor Strings

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Abstract

An extensive study on ambitwistors models is presented. We construct the free ambitwistor string field theory action for the bosonic string, heterotic string, and both GSO sectors of the Type II string. These actions contain higher derivative terms, implying non-unitary states. We also re-examine the bosonic chiral string in the sectorized interpretation, computing its spectrum, kinetic action, and 3-point amplitude. As expected, the bosonic ambitwistor string is recovered in the tensionless limit. Finally, we consider an extension of the bosonic model with current algebras. In this case, we compute the effective action and show that it is essentially the same as the action of the mass-deformed $(DF)^2$ theory found by Johansson and Nohle.

Key Words: Super Strings, Twistors, Ambitwistors, String Field theory .

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Resumo

É apresentado um extenso estudo sobre modelos de ambitwistors. Nós construímos a ação livre da teoria de campo de cordas do ambitwistor para a corda bosônica, corda heterótica e ambos setores GSO da corda tipo II. As ações contêm termos de derivadas mais altas, implicando em estados não unitários. Também reexaminamos a corda quiral bosônica na interpretação setorizada, computando seu espectro, ação cinética e amplitude de 3 pontos. Como esperado, a corda bosônica de ambitwistor é recuperada no limite sem tensão. Finalmente, consideramos uma extensão do modelo bosônico com álgebras de correntes. Nesse caso, calculamos a ação efetiva e mostramos que é essencialmente a mesma ação da teoria de massa-deformada $(DF)^2$ encontrada por Johansson e Nohle.

Palavras-chave: Super Cordas, Twistors, Ambitwistors, Teoria de campos de cordas

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To my family

Chapter 1

Introduction

The search for compact formulas of scattering amplitudes can be traced back to Parke and Taylor [1] in 1986 where they conjectured a simple expression for the maximal helicity violating (MHV) scattering amplitude for n number of particles at tree-level. Adopting this approach, the $10,000$ terms required to calculate the 5-gluon tree scattering reduces to a single term. One step towards a better understanding of this formula was achieved in 2004 by Witten [2] and separately by Berkovits [3] with a new type of string theory in $d = 4$ based on twistor worldsheet variables. Both models were shown to be equivalent and provided a simple way to derive the MHV tree-level amplitude for $\mathcal{N} = 4$ super-Yang Mills.

The idea to describe amplitudes using twistor variables drastically changed how people think about scattering amplitudes and the quest for alternative methods to describe and compute amplitudes became an extremely active area of research. In 2013 the field received a lot of attention by a series of remarkable papers [4–7] by Cachazo-He-Yuan (CHY), where a compact formula for massless tree-level S-matrix in any dimension for spin-$0, 1, 2$ was obtained. The $n-$point amplitude is given by an integral over the moduli space of the Riemann sphere with n-punctures:

$$\mathcal{A}_n(k_n, \epsilon_n) = \int \frac{d^n z}{\text{vol}SL(2,\mathbb{C})} \prod_a {}' \delta\Big(k_a \cdot P(z_a)\Big) \times \mathcal{I}_n(\{k_n, \epsilon_n, z_n\}) \tag{1.1}$$

where k_n^μ, ϵ_n^μ are the external momenta and polarization vectors for the particle n.

This elegant formula contains a lot of ingredients. The most important one, is the delta function which has the property to completely localize all integrals by imposing the scattering equations:

$$k_a \cdot P(z_a) = \sum_{b \neq a} \frac{k_a \cdot k_b}{z_a - z_b} = 0 \quad \text{where} \quad P_m(z) = \sum_{b=1}^{n} \frac{k_m^b}{z - z_b} \tag{1.2}$$

These equations relate kinematic invariants $(k_a \cdot k_b)$ of n massless particles to marked points (z_n) in a 2-sphere.

One has to eliminate 3 delta's to remove the redundancy of the scattering equations, since only $n-3$ are independent. So the amplitude result is just a sum over solutions of the scattering equations, which is amazing because the CHY formula (1.1) transforms the problem of computing Feynman diagrams into an algebraic one (finding solutions to the scattering equations). Even though the S-Matrix is just a sum of algebraic equations, it is important to note that it is not trivial to solve analytically for higher number of particles. The total number of solutions for generic kinematics is $(n-3)!$.

The second crucial element is the integrand $\mathcal{I}_n(k, \epsilon, z)$ responsible to accommodate different theories. A systematic procedure to obtain integrands is yet unknown, but the space of possibilities is restricted by the properties that an amplitude must satisfy, such as multilinearity in polarization vectors, $SL(2, \mathbb{C})$ invariance, mass dimension, gauge invariance, and more. With all these constraints Cachazo *et al*, were able to postulate $\mathcal{I}_n(k, \epsilon, z)$ for the Bi-adjoint scalar, Yang-Mills, gravity and others.

Because of the similarities with the string amplitude, a natural question was if these amplitudes (1.1) could be described by a worldsheet model. The answer came in the same year where Mason and Skinner created the so-called the ambitwistor string [8], followed by Berkovits' supersymmetric version using pure spinor formalism [9]. The CHY formulae were later generalized to different theories [10, 11] and, again, different ambitwistor strings were proposed as their underlying worldsheet model [12].

By construction, ambitwistor strings are two-dimensional chiral theories that contain no dimensionful parameter. As we'll see in 2 the bosonic gauge fixed action takes the form :

$$S_B = \frac{1}{2\pi} \int d^2 z \, (P_m \bar{\partial} X^m + b\bar{\partial} c + \tilde{b}\bar{\partial}\tilde{c}), \tag{1.3}$$

where X_m is the target space coordinates, b, c are the reparametrization ghosts, and $\tilde{b}, \tilde{c}$ are the ghosts associated to the null contraint $P^2 = 0$. At first, they were considered as an infinite tension limit of ordinary string theory, a belief motivated in part by the fact that the spectrum and tree-level amplitude of type II $GSO(+)$ sector are identical to that of the corresponding supergravity. However, for the bosonic and heterotic models, this is not true, since the kinetic term contains higher-derivative terms which imply a non-unitary spectrum. Also the tree-level amplitude for the bosonic string has higher momentum dependence. The $n-$particle scattering amplitude prescription given by Mason and Skinner with 3 fixed vertex operators and $n-3$ integrated vertex is

$$\mathcal{A}_n = \langle c_1 \tilde{c}_1 V_1 c_2 \tilde{c}_2 V_2 c_3 \tilde{c}_3 V_3 \prod_{i=4}^{n} \int d^2 z_i \bar{\delta}(k_i \cdot P(z_i)) V_i \rangle \tag{1.4}$$

Since all the fields are holomorphic the delta function $\bar{\delta}(k \cdot P)$ is needed to insure the integrated vertex operator is well defined in the Riemann surface. When the X_m dependence in the vertex operator is only in the exponent $e^{iX\cdot k}$, the delta function imposes the scattering equations. This can be seen by integrating X in the correlation function (1.4) , the zero-modes of X gives the usual conservation of momentum δ-function, the non-zero modes impose

$$\bar{\partial} P_m(z, \bar{z}) = 2\pi i \sum_{j=1}^{n} k_m^j \delta^2(z - z_j) \quad \text{at genus one} \quad P_m(z) = \sum_{b=1}^{n} \frac{k_b}{z - z_b} \tag{1.5}$$

Now the ambitwistor scattering amplitude has a similar form to the CHY expression, the delta function imposes the scattering equations, the c ghosts take care of the $SL(2, \mathbb{C})$ invariance. The ghost $\tilde{c}$ associated to the null constraint $P^2 = 0$ removes the redundancy of the scattering equations and the vertex operators represent different integrants $\mathcal{I}_n$.

However the vertex operator for the bosonic and heterotic models contain ∂X_m in the vertex operator, as was shown by Berkovits and me in [13], so the X path integral becomes hard to compute. Since these theories have higher derivatives, it is expected to be difficult to define and compute scattering amplitudes. It's important to remark that part of the spectrum in the heterotic model gives the correct equations of motion and amplitude for super Yang-Mills. This is similar to the $d = 4$ twistor string whose spectrum includes super Yang-Mills and a higher derivative theory(conformal supergravity)[14].

In hindsight, the higher derivatives should have been expected since the three-point amplitudes in ambitwistor strings (except for the Type II string) were computed [8] to have higher powers of momenta compared to the usual massless theories. And since there are no dimensionful constants like α' in ambitwistor strings, the higher momentum dependence in the cubic term of the string field theory action implies higher momentum dependence in the quadratic kinetic term. These non-unitary states were first found by Berkovits and me in [13], where we have constructed the most general vertex operator. In contrast, the vertex operators in [8] were assumed to contain only P_m dependence and to be independent of ∂X_m, where X_m and P_m are the spacetime variable and its conjugate momentum. Based on the usual definition of BPZ conjugate [15] we found the kinetic term consistent with the non-unitary states. This definition differs from a previous construction [16] where the BPZ conjugate, was chosen unconventionally to give a kinetic term with the standard unitary massless spectrum. In a subsequent paper, [17] the Ramond sector for the heterotic model was investigated. Previously only the NS-sector received the proper attention. With this in mind H.Flores and I constructed the vertex operator for the fermionic spectrum and computed the susy transformation. Also, the kinetic term was built and despite the non-unitary states in the model, the free action is invariant under the susy transformations.

A new interpretation of the ambitwistor was proposed by Siegel [18] called chiral string, where he introduced a new gauge fixing (HSZ gauge - first investigated in [19]) and also a new boundary condition for X_m. In this prescription, he obtained the same results of Mason and Skinner. In [20] the authors found an alternative method to compute amplitudes for the chiral string model. The boundary condition for X_m is the same as in [18], but the amplitudes are computed in the conformal gauge. In this approach, it was noted that the spectrum of these chiral strings contains a finite number of massive states, depending on the amount of spacetime supersymmetry. For the type II case, for instance, the physical spectrum is independent of the string tension; and two spin-2 states with mass-squared $+1, -1$ were found in the Bosonic model.

In this context, the so-called sectorized string model [21] plays an important role. It was introduced as an alternative pure spinor analog of ambitwistor strings [9], motivated by some inconsistencies in its heterotic version and difficulties in coupling it to the $\mathcal{N} = 2$ supergravity background [22]. As such, it was supposed to be a theory for massless particles only. Nevertheless, it was later shown [23] that the the heterotic sectorized model contains the $\mathcal{N} = 1$ supergravity states together with a single massive multiplet with the same quantum numbers as the first massive level of the (conventional) open superstring. This is possible thanks to a dimensionful parameter whose existence had been overlooked since the chiral worldsheet action has no parameters. Moreover, when this parameter is taken to zero, corresponding to a tensionless limit, one recovers the heterotic ambitwistor string.

Following these ideas, together with T.Azevedo, R.Jusinskas, in [24] we analyze the bosonic incarnation of the sectorized model above and show how the theory can be interpreted in terms of two sectors after a particular gauge-fixing is performed. As in the heterotic case, the two sectors emulate the left- and right-moving sectors of the usual string theory, but all worldsheet fields are holomorphic. Using methods similar to the ones used in [13], we found the same physical spectrum as in the alternative Chiral String [20]. Also in [25] one of the massive spin-2 states was determined to be ghost via a 4-point amplitude analysis based on a "twisted" Kawai–Lewellen–Tye formula. This fact is manifest in the quadratic action we constructed from the vertex operator.

And a careful analysis confirmed that in the tensionless limit, these extra massive states become auxiliary fields which then leads to the higher derivative theory found in [13].

Finally, we consider an extension of the bosonic model by including current algebras, which provide a worldsheet derivation of the so-called $(DF)^2$ + YM theory found by Johansson and Nohle [26]. In particular, the scalar field transforming in some real representation of the gauge group, whose inclusion might seem somewhat contrived in the original construction, appears naturally in the sectorized-string formulation. Theories whose Lagrangians include a $(DF)^2$-type kinetic term were first introduced as a way of obtaining conformal (super)gravity amplitudes (R^2 gravity, in general) from color-kinematics duality [27], and were shown to admit CHY/ambitwistor representations in [28]. Like R^2 gravity, such theories contain "ghost" states which render them non-unitary.

This thesis will present the three projects accomplished throughout my Ph.D. in chronological order. First, in chapter 2 we analyze the free ambitwistor string field theory action for the bosonic, heterotic, and both GSO sectors of the type II string. And show that these models — except type II GSO(+) — contain non-unitary states. In chapter 3 we continue our analysis of the heterotic string, by constructing the action for the fermionic states, we demonstrate that this model is invariant under $\mathcal{N} = 1$ susy. Then in chapter 4, we study the bosonic sectorized model, in which the tensionless limit recovers the bosonic ambitwistor model as expected. Next, we show the extension of the model to include current algebras result in the $(DF)^2 + YM$ theory. Finally chapter 5 summarizes the thesis results and discuss some open problems and perspectives.

Chapter 2

Field theory actions for ambitwistor string and superstring

2.1 Outline

In section 2.2 we start with a review of the bosonic ambitwistor model and use the standard BRST method to compute the spectrum. Then we proceed to construct the kinetic action in terms of gauge-invariant objects and show that the model contains higher derivative terms, which implies non-unitary states. This clarifies the unexpected $A_3 \sim (momenta)^6$ behavior in the three-point amplitude found by Mason and Skinner [8]. Since the theory does not contain a dimensionful parameter the higher derivative in the kinetic term solves this inconsistency. And in sections 2.3 and 2.4, we repeat this procedure for the Neveu-Schwarz states in Type II for both GSO sectors and heterotic ambitwistor string field theory actions. The spectrum for the $GSO(+)$ Neveu-Schwarz sector is the expected supergravity states, however, the spectrum for the $GSO(-)$ sector contains unusual non-unitary states. These non-unitary states are also present in the heterotic model.

2.2 Bosonic ambitwistor string

We first describe the bosonic ambitwistor string. Subsection 2.2.1 defines the model and our notation, subsection 2.2.2 computes the spectrum via BRST cohomology, and subsection 2.2.3 constructs the kinetic string field theory action. The same steps will be later described in sections 2.3 and 2.4 for the Type II and heterotic ambitwistor strings.

2.2.1 Review and notation

The gauge-fixed worldsheet action[8] is

$$S_B = \frac{1}{2\pi} \int d^2z \, (P_m \bar{\partial} X^m + b\bar{\partial} c + \tilde{b}\bar{\partial}\tilde{c}), \tag{2.1}$$

where all matter and ghost fields are left-moving bosons and fermions on the worldsheet. (P_m, X^m) are the matter fields of conformal weight $(1,0)$, (b,c) are the Faddeev-Popov ghosts for reparametrization symmetry of conformal weight $(2,-1)$, and $(\tilde{b},\tilde{c})$ are the Faddeev-Popov ghosts for the null geodesic constraint, $P^2 = 0$, and carry conformal weight $(2,-1)$. The action (2.1) is invariant under the BRST transformation generated by

$$Q = \oint \frac{dz}{2\pi i} \left(cT^M + cT_{\tilde{b}\tilde{c}} + bc\partial c + \frac{1}{2}\tilde{c}P^2 \right) \tag{2.2}$$

where

$$T^M = -P_m \partial X^m, \quad T_{\tilde{b}\tilde{c}} = \tilde{c}\partial\tilde{b} - 2\tilde{b}\partial\tilde{c}, \tag{2.3}$$

and one uses the free field OPE's,

$$P_m(z)X^n(w) \sim -\frac{\delta^n_m}{(z-w)}, \quad b(z)c(w) \sim \frac{1}{(z-w)}, \quad \tilde{b}(z)\tilde{c}(w) \sim \frac{1}{(z-w)}. \tag{2.4}$$

Notice that the XX OPE is regular, so $e^{ik\cdot X}$ does not acquire an anomalous dimension. Furthermore, there are no dimensionful parameters such as α' in the theory. So the physical spectrum defined by the BRST cohomology is not expected to contain massive states. This will be confirmed below, however, we will show that the spectrum contains both unitary and non-unitary massless states.

Physical closed string states should have ghost number 2 where the ghost number is defined as

$$N_{gh} = -\oint \frac{dz}{2\pi i}(bc + \tilde{b}\tilde{c}), \tag{2.5}$$

such that $b, \tilde{b}$ have ghost number -1 and $c, \tilde{c}$ have ghost number 1. In order to compute the ghost number 2 cohomology, Mason and Skinner [8] considered only homogeneous polynomials in P so that their expression for the spin-2 unintegrated vertex operator is

$$V(z) = c(z)\tilde{c}(z)P_m(z)P_n(z)g^{mn}e^{ikX(z)}. \tag{2.6}$$

BRST closedness implies

$$k^m g_{mn} = 0 \quad \text{and} \quad k^2 = 0, \tag{2.7}$$

while BRST exactness gives

$$\delta g^{mn} = k^{(m}\lambda^{n)} \quad \text{and} \quad k^m \lambda_m = 0. \tag{2.8}$$

Equations (2.7) and (2.8) are the usual conditions satisfied by the graviton field in linearized gravity where g_{mn} and λ are the target space metric and infinitesimal diffeomorphism generator. So it is tempting to say that the vertex (2.6) describes the graviton. However, this would present a paradox since the three-point scattering amplitude computed using (2.6) is [8]

$$\langle V(z_1)V(z_2)V(z_3)\rangle = \delta^{26}\left(\sum k\right)(g_2^{rs}k_r^1 k_s^1)(g_3^{mn}k_m^2 k_n^2)(g_1^{pq}k_p^3 k_q^3). \tag{2.9}$$

Since (2.9) behaves like k^6 instead of the k^2 behavior of general relativity and since there are no dimensionful parameters in the theory, one would expect the kinetic term for g_{mn} should also behave like k^6. This suggests that the equation of motion for g_{mn} should be something like $\Box^3 g^{mn} = 0$ instead of the $\Box g_{mn} = 0$ equation implied by (2.7).

In this paper, we aim to clarify this issue. Mason and Skinner constructed the vertex operator using only polynomials in P. However, from the string theory perspective, nothing prevents us from considering vertex operators involving ∂X. By considering the most general vertex operator with ghost number two, we will find that the equation of motion for g_{mn} behaves like k^6.

2.2.2 Bosonic spectrum

The most general vertex operator with ghost number two that is annihilated by b_0 and L_0 is[1]

$$\begin{aligned} V(z) =& c\tilde{c}\Phi_2 + c\partial\tilde{c}\Psi_1 + \partial^2 c\tilde{c}S^{(4)} + c\partial^2\tilde{c}S^{(5)} + \partial^2 cc S^{(2)} + \partial\tilde{c}\tilde{c}\Gamma_1 \\ &+ \partial^2\tilde{c}\tilde{c}S^{(3)} + \tilde{b}\tilde{c}c\partial\tilde{c}S^{(6)} + bc\partial\tilde{c}\tilde{c}S^{(1)} , \end{aligned} \tag{2.11}$$

where

$$\begin{aligned} &\Phi_2 = P^m P^n G^{(1)}_{mn} + \partial X^m \partial X^n G^{(2)}_{mn} + \partial X^m P^n H_{mn} + \partial^2 X^m A^{(1)}_m + \partial P^m A^{(2)}_m, \\ &\Psi_1 = P^m A^{(5)}_m + \partial X^m A^{(6)}_m \ , \qquad \Gamma_1 = P^m A^{(3)}_m + \partial X^m A^{(4)}_m, \\ &H_{mn} = G^{(3)}_{mn} + B_{mn}. \end{aligned} \tag{2.12}$$

The symmetric fields with two indices are represented by $G^{(1)}_{mn}, G^{(2)}_{mn}, G^{(3)}_{mn}$; the anti-symmetric 2-form by $B_{mn} = B_{[mn]}$; the 1-forms by $A^{(1)}_m, \ldots, A^{(6)}_m$; and the scalars by $S^{(1)}, \ldots, S^{(6)}$. These fields have arbitrary dependence on X, e.g., $G^{(1)}_{mn} = G^{(1)}_{mn}(X)$.

The target space fields have gauge symmetry $\delta V = Q\Lambda$, where Λ has ghost number one and also satisfies $b_0\Lambda = L_0\Lambda = 0$. The most general gauge parameter Λ takes the form

$$\Lambda =cP^m\Lambda^{(1)}_m + c\partial X^m\Lambda^{(2)}_m + \tilde{c}P^m\Lambda^{(4)}_m + \tilde{c}\partial X^m\Lambda^{(5)}_m + \partial\tilde{c}\Lambda^{(6)} + bc\tilde{c}\Lambda^{(7)} + c\tilde{b}\tilde{c}\Lambda^{(3)}. \tag{2.13}$$

The vertex (2.11) can be simplified by removing fields that are pure gauge. Whenever the gauge transformation of a field does not involve spacetime derivatives of the gauge parameter, we can eliminate this field without producing gauge-fixing ghosts. By a suitable choice of gauge parameters, it is easy to show that the fields $S^{(2)}, S^{(4)}, S^{(6)}, A^{(1)}_m, A^{(2)}_m$ can be eliminated from the vertex operator (2.11).

[1]Since $\bar{L}_0$ is identically zero, the usual constraints that $L_0 - \bar{L}_0$ and $b_0 - \bar{b}_0$ annihilate the off-shell closed string vertex operator are replaced by the constraints that L_0 and b_0 annihilate the off-shell vertex operator. By L_0 and b_0 we mean the zero-modes of the b-ghost and stress-energy tensor:

$$b_0 = \oint \frac{dz}{2\pi i} z\, b(z) \quad \text{and} \quad L_0 = \oint \frac{dz}{2\pi i} z\, T(z). \tag{2.10}$$

Cohomology: Now that we have the most general vertex operator we can calculate the cohomology. The BRST-closedness condition $QV = 0$ gives the following auxiliary equations

$$
\begin{aligned}
&A^{(5)}_n = -\partial^m G^{(1)}_{mn}, \quad A^{(6)}_m = -\frac{1}{2}\partial^n H_{mn}, \quad A^{(3)}_m = A^{(6)}_m, \quad A^{(4)}_m = -\partial_m S^{(1)},\\
&G^{(3)}_{mn} = \frac{1}{2}\Box G^{(1)}_{mn} - \frac{1}{2}\partial_{(n}\partial^r G^{(1)}_{m)r}, \qquad 2G^{(2)}_{mn} = +\frac{1}{2}\Box G^{(3)}_{mn} + \eta_{mn}S^{(1)},\\
&S^{(5)} = +\frac{1}{2}\partial^n\partial^m G^{(1)}_{mn}, \qquad S^{(3)} = -\frac{1}{2}\partial^m A^{(3)}_m + \frac{3}{2}S^{(1)},
\end{aligned}
\tag{2.14}
$$

together with the equations of motion

$$
\begin{aligned}
\Box G^{m(1)}_m + 4\partial^n\partial^m G^{(1)}_{mn} &= 0,\\
\Box B_{nm} + \partial_n\partial^p B_{mp} - \partial_m\partial^p B_{np} &= 0,\\
\Box^3 G^{(1)}_{mn} - \Box^2\partial_{(n}\partial^p G^{(1)}_{m)p} + 4\eta_{mn}S^{(1)} + 16\partial_m\partial_n S^{(1)} &= 0.
\end{aligned}
\tag{2.15}
$$

The gauge transformations given by $\delta V = Q\Lambda$ for the propagating fields are

$$
\begin{aligned}
&\delta G^{(1)}_{(mn)} = \frac{1}{2}\partial_{(n}\Lambda^{(1)}_{m)} - \frac{1}{6}\eta_{mn}(\partial\cdot\Lambda^{(1)}),\\
&\delta B_{[mn]} = \partial_{[m}\Lambda^{(4)}_{n]},\\
&\delta S^{(1)} = \frac{1}{24}\Box^2(\partial\cdot\Lambda^{(1)}).
\end{aligned}
\tag{2.16}
$$

Although the gauge transformation for the field $G^{(1)}_{mn}$ does not correspond to the linear diffeomorphism of the graviton, we will perform in the next subsection a field redefinition to obtain the usual transformation. However, it is unclear how to interpret this vertex operator as a deformation around the background.

2.2.3 Ambitwistor kinetic term

The standard kinetic term $S[\Psi] = \frac{1}{2}\langle\Psi|(c_0 - \bar{c}_0)Q\Psi\rangle$ for the closed bosonic string was introduced in [15] using the string field defined by the state-operator mapping: $|\Psi\rangle = V(0)|0\rangle$ where $|0\rangle$ is the $SL(2,C)$ vacuum and $|\Psi\rangle$ is constrained to satisfy $(L_0 - \bar{L}_0)|\Psi\rangle = (b_0 - \bar{b}_0)|\Psi\rangle = 0$. For the ambitwistor string, we will have a similar kinetic term; however, since all the fields are holomorphic, we discard the antiholomorphic zero-modes $\bar{L}_0$ and $\bar{b}_0$.

Therefore, we propose for the ambitwistor string kinetic term

$$S[\Psi] = \frac{1}{2}\langle\Psi|c_0 Q\Psi\rangle = \frac{1}{2}\langle I \circ V(0)|\partial c Q V(0)\rangle \tag{2.17}$$

where $|\Psi\rangle$ is constrained to satisfy

$$L_0|\Psi\rangle = b_0|\Psi\rangle = 0. \tag{2.18}$$

The bra state of the string field $\langle\Psi|$ is defined by the usual BPZ conjugate, $\langle\Psi| = \langle 0|I \circ V(0)$ where $I(z) = 1/z$. For a primary field of conformal weight h the conformal transformation I acts as

$$I \circ \phi(y) = (\partial_y I)^h \phi(1/y). \tag{2.19}$$

The variation of $S[\Psi]$ implies $c_0 Q|\Psi\rangle = 0$. The condition $b_0|\Psi\rangle = 0$ turns this into the linearized equations of motion $Q|\Psi\rangle = 0$. The action $S[\Psi]$ is invariant under $|\delta\Psi\rangle = Q|\Lambda\rangle$, where Λ has ghost number one and is annihilated by L_0 and b_0. The proof of gauge invariance and the derivation of the field equations follows exactly as in [15], so it will not be reproduced here. A similar string field theory action was previously proposed in [16], but their construction did not allow insertions of ∂X in the vertex operator and they modified the usual definition of the BPZ inner product to get a massless unitary spectrum.

Let us focus on computing the action for the ambitwistor string vertex operator (2.11). The action can be calculated in two different – but equivalent – ways: using creation and annihilation operator algebra or vertex correlation functions. We will work with the latter.

The gauge parameter (2.13) can set $S^{(2)}, S^{(4)}, S^{(6)}, A^{(1)}, A^{(2)}$ to zero without producing ghosts, so the vertex operator (2.11) simplifies to

$$V(z) = c\tilde{c}\Phi_2 + c\partial\tilde{c}\Psi_1 + c\partial^2\tilde{c}S^{(5)} + \partial\tilde{c}\tilde{c}\Gamma_1 + \partial^2\tilde{c}\tilde{c}S^{(3)} + bc\partial\tilde{c}\tilde{c}S^{(1)}\,, \tag{2.20}$$

where

$$\begin{aligned} \Phi_2 &= P^m P^n G^{(1)}_{mn} + \partial X^m \partial X^n G^{(2)}_{mn} + \partial X^m P^n H_{mn}, \\ \Psi_1 &= P^m A^{(5)}_m + \partial X^m A^{(6)}_m, \quad \Gamma_1 = P^m A^{(3)}_m + \partial X^m A^{(4)}_m. \end{aligned} \tag{2.21}$$

One can verify that the auxiliary field equations of (2.14) imply that

$$\begin{aligned}T(z)V(0) \sim &+ z^{-4}[-c\tilde{c}(H^m_m + 6S^{(5)})] \quad + \quad z^{-3}[c\partial\tilde{c}(-\partial^m A^5_m - 2S^{(5)})] \ + \\ &+ z^{-3}[c\tilde{c}(-2P^m(\partial^n G^1_{mn} + A^{(5)}_m) - \partial X^m(\partial^n H_{mn} + 2A^{(6)}_m))] \ + \\ &+ z^{-3}[\tilde{c}\partial\tilde{c}(+\partial^m A^{(3)}_m + 2S^{(3)} - 3S^{(1)})] + z^{-1}\partial V(0) \\ \sim & z^{-4}[-c\tilde{c}(H^m_m + 6S^{(5)})] \ + z^{-1}\partial V(0).\end{aligned} \tag{2.22}$$

So after applying the auxiliary field equations of (2.14), T has no double or cubic poles with V, which implies that $I \circ V(z) = V(I(z))$ and the string action (2.17) becomes the two point function $\langle V(I(0))\partial c Q V(0)\rangle$. We stress that applying the auxiliary field equations before computing the kinetic term is a trick to simplify the computation. One could have done the calculation in full detail and obtained the same answer.

Using the vacuum normalization $\langle \partial^2 c\partial c c\partial^2\tilde{c}\partial\tilde{c}\tilde{c}\rangle = 4$, the string action becomes

$$\begin{aligned}S = -\int d^{26}X \Big[&+\frac{1}{8}G^{mn(1)}\Box^3 G^{(1)}_{mn} + \frac{1}{4}\partial_r G^{mr(1)}\Box^2\partial^p G^{(1)}_{mp} + 4G^{mn(1)}\partial_n\partial_m S^{(1)} + \\ &+ G^{p(1)}_p \Box S^{(1)} - \frac{1}{2}B^{mn}(\Box B_{mn} + \partial_{[m}\partial^p B_{n]p})\Big].\end{aligned} \tag{2.23}$$

The equations of motion agree with (2.15) and the gauge transformations are those given by (2.16). Note that the kinetic action for $G^{(1)}_{mn}$ involves 6 derivatives, so the inconsistency between the momentum dependence of the 3-point amplitude (2.9) and the momentum dependence of the kinetic term is resolved.

To write the kinetic action in terms of gauge invariant objects, it is convenient to perform a field redefinition since the gauge transformation for $G^{(1)}_{mn}$ is not quite the transformation of the graviton. A convenient field redefinition is

$$h_{mn} - \frac{1}{6}\eta_{mn}h^p_p = G^{(1)}_{mn}, \quad t = 4S^{(1)} - \frac{1}{6}\Box^2 h^p_p, \tag{2.24}$$

to obtain the gauge transformations of linearized gravity

$$\delta h_{mn} = \frac{1}{2}\partial_{(n}\lambda_{m)}\ , \quad \delta t = 0. \tag{2.25}$$

The action (2.23) written in terms of gauge invariant objects becomes

$$S = -\int d^{26}X \left[\frac{1}{2}R_{mn}\Box R^{mn} - \frac{1}{4}R\Box R + tR - \frac{1}{3!}H^{mnp}H_{mnp}\right], \tag{2.26}$$

where we have defined the linearized Ricci tensor and 3-form field strength

$$\begin{aligned} 2R_{mn} &= \partial_m\partial^p h_{np} + \partial_n\partial^p h_{mp} - \Box h_{mn} - \partial_m\partial_n h_p^p, \\ H_{mnp} &= \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}. \end{aligned} \tag{2.27}$$

One can simplify further by shifting t to $t + \Box R/4$ so the term $R\Box R$ drops out of the action.

2.3 Type II ambitwistor

In this section we will describe the Type II ambitwistor string for both GSO Neveu-Schwarz sectors. The spectrum for the $GSO(+)$ Neveu-Schwarz sector will be the usual bosonic massless Type II supergravity states, however, the spectrum for the $GSO(-)$ Neveu-Schwarz sector will have some unusual non-unitary states. Although only the $GSO(+)$ sector is supersymmetric, the $GSO(-)$ sector is expected to appear as intermediate states before summing over spin structures using the RNS formalism. So by analyzing the contribution of individual spin structures to the one-loop partition function of the Type II ambitwistor superstring, one should be able to verify this unusual spectrum for the $GSO(-)$ sector.

2.3.1 Review and notation

For the Type II action we add two fermionic holomorphic worldsheet variables ψ_1, ψ_2, both with conformal weight 1/2. We also introduce two pairs of bosonic Faddev-Popov ghosts: (β_1, γ_1) and (β_2, γ_2). The β's have conformal weight 3/2 while the γ's have conformal weight $-1/2$. The action for this system is

$$S_{tII} = \frac{1}{2\pi}\int d^2z\,(P_m\bar{\partial}X^m + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c} + \psi_1\bar{\partial}\psi_1 + \psi_2\bar{\partial}\psi_2 + \beta_1\bar{\partial}\gamma_1 + \beta_2\bar{\partial}\gamma_2). \tag{2.28}$$

The new field variables have the OPE's

$$\psi_i^m(z)\psi_j^n(w) \sim \delta_{ij}\frac{\eta^{mn}}{(z-w)} \ , \quad \beta_i(z)\gamma_j(w) \sim -\frac{\delta_{ij}}{(z-w)} \quad \text{for} \quad i,j=1,2, \tag{2.29}$$

in addition to the ones obtained in (2.4). The action (2.28) also presents BRST symmetry generated by

$$Q = \oint \frac{dz}{2\pi i}(cT^M + cT_{\tilde{b}\tilde{c}} + cT_{\beta_1\gamma_1} + cT_{\beta_2\gamma_2} + bc\partial c + \frac{1}{2}\tilde{c}P^2 + \gamma_1 P\cdot\psi_1 + \gamma_2 P\cdot\psi_2 - \gamma_1^2\tilde{b} - \gamma_2^2\tilde{b}), \tag{2.30}$$

where

$$\begin{aligned} T^M &= -P_m\partial X^m - \frac{1}{2}\psi_1\cdot\partial\psi_1 - \frac{1}{2}\psi_2\cdot\partial\psi_2, \quad T_{\tilde{b}\tilde{c}} = \tilde{c}\partial\tilde{b} - 2\tilde{b}\partial\tilde{c}, \\ T_{\beta_i\gamma_i} &= -\frac{1}{2}\partial\beta_i\gamma_i - \frac{3}{2}\beta_i\partial\gamma_i \ . \end{aligned} \tag{2.31}$$

The nilpotency of the BRST charge imposes the critical spacetime dimension $d = 10$. In order to write the vertex operator in the picture $(-1,-1)$ we bosonize the ghosts (β_i, γ_i) by introducing a set of fermions (η_i, ξ_i) with conformal weight $(1, 0)$ together with a chiral boson ϕ_i. This new system is described by the free field OPE's

$$\phi_i(z)\phi_j(w) \sim -\delta_{ij}\ln(z-w), \quad \eta_i(z)\xi_j(w) \sim \frac{\delta_{ij}}{z-w}, \tag{2.32}$$

and the change of variables is

$$\beta_i = e^{-\phi_i}\partial\xi_i \ , \quad \gamma_i = \eta_i e^{+\phi_i}. \tag{2.33}$$

The BRST charge (2.30) in terms of bosonized variables (η, ξ, ϕ) is written by replacing

$$T_{\beta_i\gamma_i} = -\frac{1}{2}\partial\phi_i\partial\phi_i - \partial^2\phi_i - \eta_i\partial\xi_i \quad \text{and} \quad \gamma_i^2 = \eta_i\partial\eta_i e^{-2\phi_i}, \tag{2.34}$$

for each pair (β_i, γ_i). The ghost number charge (2.57) is modified to accommodate the (β, γ) system as

$$N_{gh} = -\oint \frac{dz}{2\pi i}(bc + \tilde{b}\tilde{c} + \xi_1\eta_1 + \xi_2\eta_2) \tag{2.35}$$

In addition to the ghost number charge we define the picture number:

$$N_{P_i} = \oint \frac{dz}{2\pi i}(\xi_i \eta_i - \partial\phi_i), \tag{2.36}$$

such that β and γ have picture zero and ghost number -1 and 1 respectively.

2.3.2 Type II spectrum

There are two sectors for Neveu-Schwarz states in superstring theory which contain either GSO parity + or GSO parity −. The vertex operator considered by Mason and Skinner [8] is in the $GSO(+)$ sector. The field content found in [8] is a spin-2 G_{mn}, a scalar G^m_m and a 2-form B_{mn} which agrees with the bosonic fields of $d = 10$ N=2 supergravity. However, the ambitwistor superstring also has a $GSO(-)$ sector that has not yet been fully investigated.

In order to distinguish the two sectors, we introduce the operator $(-)^{parity}$ where the parity of ψ_1 and e^{ϕ_1} is defined to be odd, the parity of ψ_2 and e^{ϕ_2} is defined to be even, and the parity of all other variables $(P_m, X^m, b, c, \tilde{b}, \tilde{c}, \xi_i, \eta_i)$ is defined to be even. One can easily verify that $(-)^{parity}$ commutes with the BRST charge of (2.30).

Although the superstring is only spacetime supersymmetric after truncating out the $GSO(-)$ sector, it will be interesting to compute the spectrum for both sectors. The most general Neveu-Schwarz vertex operator in the picture $(-1,-1)$ with ghost number two and which is annihilated by b_0 and L_0 is

$$\begin{aligned} V(z) =& e^{-\phi_1}e^{-\phi_2}(c\tilde{c}\Phi_1 + c\partial\tilde{c}S^{(1)} + \tilde{c}\partial\tilde{c}S^{(6)}) + \partial\phi_1 e^{-\phi_1}e^{-\phi_2}c\tilde{c}S^{(2)} + \\ &+ e^{-\phi_1}\partial\phi_2 e^{-\phi_2}c\tilde{c}S^{(3)} + \partial\xi_1 e^{-2\phi_1}e^{-\phi_2}(c\tilde{c}\partial\tilde{c}\psi_1\cdot A^{(3)} + c\tilde{c}\partial\tilde{c}\psi_2\cdot A^{(4)}) + \\ &+ e^{-\phi_1}\partial\xi_2 e^{-2\phi_2}(c\tilde{c}\partial\tilde{c}\psi_1\cdot A^{(5)} + c\tilde{c}\partial\tilde{c}\psi_2\cdot A^{(6)}) + \eta_1\partial\xi_2 e^{-2\phi_2}c\tilde{c}S^{(4)} + \\ &+ \partial\xi_1 e^{-2\phi_1}\eta_2 c\tilde{c}S^{(5)}, \end{aligned} \tag{2.37}$$

with

$$\begin{aligned} &\Phi_1 = P\cdot A^{(1)} + \partial X\cdot A^{(2)} + B^{(1)}_{mn}\psi^m_1\psi^n_1 + B^{(2)}_{mn}\psi^m_2\psi^n_2 + H_{mn}\psi^m_1\psi^n_2, \\ &H_{mn} = G_{mn} + B_{mn}. \end{aligned} \tag{2.38}$$

where the fields are represented by six scalars S, six 1-forms A_m, one symmetric two-form G_{mn}, and three antisymmetric 2-forms B_{mn}. Note that the vertex operator (2.37) is defined in the small Hilbert space, i.e does not contain the zero mode of ξ_i. Using the definition of the operator $(-)^{parity}$ the fields can be separated into

$$GSO(+) : H_{mn} = G_{mn} + B_{mn}, A_m^{(4)}, A_m^{(5)}, S^{(4)}, S^{(5)}$$
$$GSO(-) : A_m^{(1)}, A_m^{(2)}, A_m^{(3)}, A_m^{(6)}, B_{mn}^{(1)}, B_{mn}^{(2)}, S^{(1)}, S^{(2)}, S^{(3)}, S^{(6)}. \tag{2.39}$$

Cohomology: As in the bosonic case, the fields in (2.37) have gauge transformations $\delta V = Q\Lambda$, where the gauge field Λ is in the small Hilbert space and satisfies $L_0\Lambda = b_0\Lambda = 0$. So the gauge field with ghost number one is

$$\begin{aligned}\Lambda = {} & \partial\xi_1 e^{-2\phi_1} e^{-\phi_2} c\tilde{c}(\psi_1 \cdot \Lambda^{(1)} + \psi_2 \cdot \Lambda^{(2)}) + \partial\xi_2 e^{-2\phi_2} e^{-\phi_1} c\tilde{c}(\psi_1 \cdot \Lambda^{(3)} + \psi_2 \cdot \Lambda^{(4)}) + \\ & + e^{-\phi_1} e^{-\phi_2}(c\Lambda^{(6)} + \tilde{c}\Lambda^{(7)}) + \partial\xi_1 e^{-2\phi_1} \partial\xi_2 e^{-2\phi_2} c\tilde{c}\partial\tilde{c}\Lambda^{(5)} + \\ & + \partial^2\xi_1 \partial\xi_1 e^{-3\phi_1} e^{-\phi_2} c\tilde{c}\partial\tilde{c}\Lambda^{(8)} + \partial^2\xi_2 \partial\xi_2 e^{-3\phi_2} e^{-\phi_1} c\tilde{c}\partial\tilde{c}\Lambda^{(9)},\end{aligned} \tag{2.40}$$

which can be used to gauge away $(A_m^{(1)}, S^{(1)}, S^{(2)}, S^{(5)})$. After using $QV = 0$ to eliminate the auxiliary fields in the vertex operator (2.37) whose equations of motion do not involve derivatives, the remaining equations of motion and gauge transformations for both sectors are

- $GSO(+)$:

Field equations	Gauge transformations
$\Box G_{mn} - \partial_{(m}\partial^p G_{n)p} + \partial_n\partial_m S^{(4)} = 0,$	$\delta G_{mn} = +\frac{1}{2}\partial_{(m}\Lambda^{(2)}_{n)} + \frac{1}{2}\partial_{(m}\Lambda^{(3)}_{n)},$
$\partial^p\partial^m G_{pm} - \Box S^{(4)} = 0,$	$\delta B_{mn} = +\frac{1}{2}\partial_{[m}\Lambda^{(2)}_{n]} - \frac{1}{2}\partial_{[m}\Lambda^{(3)}_{n]},$
$\Box B_{mn} + \partial_{[m}\partial^p B_{n]p} = 0,$	$\delta S^{(4)} = \partial \cdot \Lambda^{(3)} + \partial \cdot \Lambda^{(2)}.$

- $GSO(-)$:

Field equations	**Gauge transformations**
$\partial^p B^+_{pn} = 0,$	$\delta B^+_{mn} = 0,$
$\Box B^+_{mn} + \partial_{[n} A^{(2)}_{m]} = 0,$	$\delta B^-_{mn} = \partial_{[n}\Lambda^{(4)}_{m]},$
$\Box B^-_{mn} - \partial_{[n}\partial^p B^-_{m]p} = 0,$	$\delta A^{(2)}_m = -4\partial_m \Lambda^{(9)} - \partial_m \partial \cdot \Lambda^{(4)},$

where in the $GSO(-)$ sector we defined $B^{\pm}_{mn} \equiv B^{(1)}_{mn} \pm B^{(2)}_{mn}$. The field content in the $GSO(+)$ sector is the expected one from superstring theory and has a graviton G_{mn} coupled to a scalar $S^{(4)}$, and an antisymmetric 2-form B_{mn}. On the other hand, the spectrum in the $GSO(-)$ sector is unusual and includes two antisymmetric 2-forms and a 1-form. One of the antisymmetric 2-forms has the usual gauge transformation but the other one is gauge invariant.

2.3.3 Ambitwistor kinetic term

The construction of the quadratic action for the superstring is similar to the bosonic construction of section 2.2.3. In addition to the constraints $L_0|\Psi\rangle = b_0|\Psi\rangle = 0$, the string field at ghost-number 2 is also constrained to be in the $(-1,-1)$ picture in the small Hilbert space. The string field $|\Psi\rangle$ is given by the vertex operator (2.37) introduced in the previous section. We have

$$S[\Psi] = \frac{1}{2}\langle\Psi|c_0 Q|\Psi\rangle = \frac{1}{2}\langle I \circ V(0)|\partial c Q V(0)\rangle \tag{2.41}$$

where $I \circ V(z)$ is the conformal transformation (2.19). The vertex operator (2.37), after eliminating gauge fields and auxiliary fields, is a primary field with conformal weight zero, i.e,

$$T(z)V(0) \sim z^{-1}\partial V(0),$$

thus the conformal transformation $I \circ V(z) = V(z^{-1})$ acts as (2.19). So the calculation for the action becomes an ordinary two point function with vacuum normalization $\langle c\partial c\partial^2 c\tilde{c}\partial\tilde{c}\partial^2\tilde{c}e^{-2\phi_1}e^{-2\phi_2}\rangle = 4$. After some algebra, the actions for the $GSO(\pm)$ Neveu-Schwarz sectors are

$$S^{+}=-\frac{1}{2}\int d^{10}x\left[G^{mn}(\frac{1}{2}\Box G_{mn}-\frac{1}{2}\partial_{(m}\partial^{p}G_{n)p})+S^{(4)}(\partial^{p}\partial^{m}G_{pm}-\frac{1}{2}\Box S^{(4)})+\right.$$
$$\left.+B^{mn}(\frac{1}{2}\Box B_{mn}+\frac{1}{2}\partial_{[m}\partial^{p}B_{n]p})\right],\tag{2.42}$$

$$S^{-}=-\frac{1}{2}\int d^{10}x\left[B^{mn(1)}(\Box B^{(1)}_{mn}-\partial_{[n}\partial^{p}B^{(1)}_{m]p}+\partial_{[n}A^{(2)}_{m]})+\right.$$
$$\left.+B^{mn(2)}(\Box B^{(2)}_{mn}-\partial_{[n}\partial^{p}B^{(2)}_{m]p}+\partial_{[n}A^{(2)}_{m]})\right].\tag{2.43}$$

The $GSO(+)$ sector has the standard Type II spectrum – graviton, Kalb-Ramond, and dilaton. In order to make the field content more clear, rewrite the action (2.42) in terms of gauge invariant objects by redefining the fields

$$G_{mn}=h_{mn},\quad R=-\Box h^{p}_{p}+\partial^{m}\partial^{n}h_{mn},\quad \phi=S^{(4)}+h^{m}_{m},$$
$$H_{mnp}=\partial_{m}B_{np}+\partial_{n}B_{pm}+\partial_{p}B_{mn},\tag{2.44}$$

such that the gauge transformations are

$$\delta h_{mn}=+\frac{1}{2}\partial_{(m}\lambda_{n)}\quad,\quad \delta B_{mn}=+\frac{1}{2}\partial_{[m}\omega_{n]}\quad,\quad \delta\phi=0,\tag{2.45}$$

with $\lambda_m=\Lambda^{(2)}_m+\Lambda^{(3)}_m$ and $\omega_m=\Lambda^{(2)}_m-\Lambda^{(3)}_m$. The action for the $GSO(+)$ sector written in term of these gauge covariant objects is

$$S^{+}=-\frac{1}{2}\int d^{10}x\left[h^{mn}\frac{1}{2}\Box h_{mn}+(\partial^{p}h_{np})^{2}-\frac{1}{2}h^{r}_{r}\Box h^{p}_{p}+h^{r}_{r}\partial^{p}\partial^{m}h_{pm}+\phi R\right.$$
$$\left.-\frac{1}{2}\phi\Box\phi+\frac{1}{6}H^{mnp}H_{mnp}\right]\tag{2.46}$$

which agrees with the action found by[16].

On the other hand, the action (2.43) for the $GSO(-)$ sector is unusual. In terms of $B^{\pm}_{mn}=B^{(1)}_{mn}\pm B^{(2)}_{mn}$, the action (2.43) is

$$S^{-}=-\frac{1}{2}\int d^{10}x[\frac{1}{3!}H^{-\,mnp}H^{-}_{mnp}+\frac{1}{3!}H^{+\,mnp}H^{+}_{mnp}+B^{+\,mn}F_{mn}]\tag{2.47}$$

where $F_{mn}=\partial_{[m}A^{(2)}_{n]}$ and $H^{\pm}_{mnp}=\partial_{[m}B^{\pm}_{np]}$. So B^{-}_{mn} has the standard kinetic term for an antisymmetric two-form, but B^{+}_{mn} couples to F^{mn} and does not have the usual gauge invariance of an antisymmetric two-form.

2.4 Heterotic ambitwistor string

2.4.1 Review and notation

The worldsheet action for the heterotic model is similar to the Type II, but the two worldsheet fermions (ψ_1, ψ_2) are replaced by one worldsheet fermion ψ together with a new current action S_J

$$S_{het} = \frac{1}{2\pi} \int d^2z \, (P_m \bar{\partial} X^m + b\bar{\partial} c + \tilde{b}\bar{\partial}\tilde{c} + \psi\bar{\partial}\psi + \beta\bar{\partial}\gamma) + S_J. \tag{2.48}$$

The particular form of the current action S_J is irrelevant, except that it should allow the vertex operator to be written using a current algebra J^a which has conformal weight one and satisfies the OPE

$$J^a(z)J^b(w) \sim \frac{\delta^{ab}}{(z-w)^2} + \frac{f^{ab}_c}{z-w} J^c(w), \tag{2.49}$$

where f^{ab}_c are the structure constants of the algebra. The action (3.1) has BRST symmetry generated by

$$Q = \oint dz(cT^M + bc\partial c + cT_{\tilde{b}\tilde{c}} + cT_{\beta\gamma} + cT_J + \frac{1}{2}\tilde{c}P^2 + \gamma P \cdot \psi - \gamma^2 \tilde{b}), \tag{2.50}$$

with

$$T^M = -P \cdot \partial X - \frac{1}{2}\psi \cdot \partial\psi, \quad T_{\tilde{b}\tilde{c}} = \tilde{c}\partial\tilde{b} - 2\tilde{b}\partial\tilde{c}, \quad T_{\beta\gamma} = -\frac{1}{2}\partial\beta\gamma - \frac{3}{2}\beta\partial\gamma,$$

being the stress energy tensor for the matter and ghost fields. The new feature compared to the Type II ambitwistor, after removing the variables $(\psi_2, \gamma_2, \beta_2)$, is the stress energy tensor T_J associated with the current action S_J with

$$T_J(z)T_J(w) \sim \frac{c_J}{2(z-w)^4} + \frac{2T_J(w)}{(z-w)^2} + \frac{\partial T_J(w)}{(z-w)},$$

where c_J is the central charge. Nilpotency of the BRST charge implies $41 - c_J - \frac{5}{2}D = 0$, so the critical spacetime dimension is $D = 10$ for $c_J = 16$.

2.4.2 Heterotic spectrum

Although the Yang-Mills vertex operator of [8] for the heterotic ambitwistor string has the expected behavior for Yang-Mills scattering amplitudes, the graviton vertex operator proposed by Mason and Skinner (2.1) for the heterotic model has similar issues as in the bosonic model. The three-point graviton scattering amplitude behaves like k^4 as opposed to the expected k^2 behavior of general relativity. After allowing ∂X in the construction of the vertex operator, we will find that the equation of motion for the symmetric 2-form h_{mn} is

$$\Box^2 h_{mn} + \cdots = 0,$$

which is consistent with the momentum behavior of the three-point amplitude. Another unexpected feature of the heterotic ambitwistor string is that the spectrum contains a three-form which is not present in the massless sector of the usual heterotic superstring.

The most general vertex operator in picture (-1) in the small Hilbert space that is annihilated by b_0 and L_0 with ghost number 2 is:

$$\begin{aligned} V(z) =& e^{-\phi}(c\tilde{c}\Phi_{3/2} + c\partial\tilde{c}A^{(2)}\cdot\psi + \partial\tilde{c}\tilde{c}A^{(1)}\cdot\psi) + \partial\phi e^{-\phi}(c\tilde{c}A^{(3)}\cdot\psi) + \\ &+ \partial\xi e^{-2\phi}(\partial\tilde{c}\tilde{c}c\Psi_1 + \partial^2\tilde{c}\tilde{c}cS^{(4)}) + \eta(cS^{(1)} + \tilde{c}S^{(3)}) + \partial\xi e^{-2\phi}(\partial^2 cc\tilde{c}S^{(2)}) + \\ &+ \partial\xi\partial\phi e^{-2\phi}\partial\tilde{c}\tilde{c}cS^{(5)} + \partial^2\xi e^{-2\phi}(\partial\tilde{c}\tilde{c}cS^{(6)}), \end{aligned} \tag{2.51}$$

where

$$\begin{aligned} \Phi_{3/2} &= H^{(1)}_{mn}P^m\psi^n + H^{(2)}_{mn}\partial X^m\psi^n + C_{mnp}\psi^m\psi^n\psi^p + J^a\psi\cdot A^a + \partial\psi\cdot A^{(4)}, \\ \Psi_1 &= P\cdot A^{(5)} + \partial X\cdot A^{(6)} + J^aC^a + B^{(3)}_{mn}\psi^m\psi^n, \quad H^{(i)}_{mn} = G^{(i)}_{mn} + B^{(i)}_{mn}. \end{aligned} \tag{2.52}$$

The target space fields are described by six abelian scalars S, one non-abelian scalar C^a, six abelian 1-forms A_m, one non-abelian 1-form A^a_m, two symmetric 2-forms G_{mn}, three antisymmetric 2-forms B_{mn} and a 3-form C_{mnp}.

Cohomology: The gauge invariance $\delta V = Q\Lambda$ can be used to gauge away $S^{(2)}, S^{(1)}, A^{(4)}_m$, $A^{(3)}_m, B^{(1)}_{mn}$ where the gauge parameter in picture (-1) with ghost number 1 is

$$\begin{aligned} \Lambda =& e^{-\phi}(c\Lambda^{(6)}_m\psi^m + \tilde{c}\Lambda^{(7)}_m\psi^m) + \partial\xi e^{-2\phi}(c\tilde{c}\Phi_1 + c\partial\tilde{c}\Lambda^{(2)}) + \partial^2\xi e^{-2\phi}c\tilde{c}\Lambda^{(8)} + \\ &+ \partial^2\xi\partial\xi e^{-3\phi}\partial\tilde{c}\tilde{c}c\Lambda^{(10)}_m\psi^m + \partial\xi\partial\phi e^{-2\phi}c\tilde{c}\Lambda^{(9)}, \end{aligned} \tag{2.53}$$

with $\Phi_1 = P\cdot\Lambda^{(3)} + \partial X\cdot\Lambda^{(4)} + \psi^m\psi^n\Lambda^{(5)}_{mn} + J^a\Lambda^{a(1)}$.

After using $QV = 0$ to fix all auxiliary fields whose equations do not contain derivatives, the remaining dynamical fields are $G^{(1)}_{mn}, G^{r(2)}_r, B^{(2)}_{mn}, A^a_m$ and C_{mnp}. The equations of motions together with its gauge transformations for these remaining fields are

$$-\frac{1}{4}\Box^2 G^{(1)}_{mn} + \Box\frac{1}{4}\partial_{(m}\partial^p G^{(1)}_{n)p} - \frac{1}{10}\eta_{mn}\Box\partial^r\partial^s G^{(1)}_{rs} - \frac{1}{5}\partial_n\partial_m\partial^r\partial^s G^{(1)}_{rs} + \\ -\frac{1}{20}\eta_{mn}\Box G^{r(2)}_r - \frac{1}{10}\partial_n\partial_m G^{r(2)}_r = 0, \tag{2.54}$$

$$\begin{aligned} \Box A^a_m - \partial_m(\partial^p A^a_p) &= 0, \\ -\Box C_{mnp} + \frac{1}{6}\partial_{[p}B^{(2)}_{mn]} &= 0, \\ \partial^p C_{mnp} &= 0, \end{aligned} \tag{2.55}$$

with gauge transformations

$$\begin{aligned} \delta G^{(1)}_{(mn)} &= -\frac{1}{2}\partial_{(n}\Lambda_{m)} + \frac{1}{4}\eta_{mn}\partial\cdot\Lambda, \quad \Lambda_m = \Lambda^{(6)}_m + \Lambda^{(3)}_m, \\ \delta G^{m(2)}_m &= +\frac{1}{4}\Box\partial\cdot\Lambda, \\ \delta B^{(2)}_{mn} &= \partial_{[m}\Lambda^{(4)}_{n]}, \\ \delta C_{mnp} &= 0, \\ \delta A^a_m &= -\partial_m\Lambda^{a(1)}. \end{aligned} \tag{2.56}$$

2.4.3 Ambitwistor kinetic term

The kinetic term follows exactly the Type II construction of section 2.3.3, so we shall not review it here. The vertex operator (3.53) transforms as a primary field with conformal weight zero after using the equation of motion for the auxiliary fields. Finally, the quadratic term takes the form

$$S = \frac{1}{4}\int d^{10}x \Big[-\frac{1}{4}G^{(1)mn}\Box^2 G^{(1)}_{mn} - \frac{1}{2}\Box(\partial_r G^{(1)nr})(\partial^s G^{(1)}_{sn}) - \frac{1}{5}\Box G^{m(1)}_m\partial^m\partial^n G^{(1)}_{mn} + \\ -\frac{2}{5}(\partial^m\partial^n G^{(1)}_{mn})^2 + \frac{1}{10}G^{r(2)}_r(-\Box G^{m(1)}_m - 2\partial^m\partial^n G^{(1)}_{mn}) - 6B^{(2)mn}\partial^p C_{mnp} + \\ +6C^{mnp}(-\frac{1}{2}\Box C_{mnp} + \frac{1}{4}\partial_{[p}\partial^r C_{mn]r}) + 2A^{am}(\Box A^a_m - \partial_m(\partial\cdot A^a))\Big], \tag{2.57}$$

where $\partial_{[p}C_{mn]r} = 2\partial_p C_{mnr} + 2\partial_m C_{npr} + 2\partial_n C_{pmr}$.

To write the action (2.57) in terms of gauge invariant objects, we redefine the fields

$$G^{(1)}_{mn} = h_{mn} - \frac{1}{4}\eta_{mn}h^p_p, \quad G^{r(2)}_r = t - \frac{1}{4}\Box h^p_p \quad \Rightarrow \quad \delta h_{mn} = -\frac{1}{2}\partial_{(m}\Lambda_{n)}, \quad \delta t = 0. \tag{2.58}$$

Using the field strengths for the gauge and 2-form fields together with the linearized Riemann tensor

$$\begin{aligned} R_{abcd} &= \partial_b\partial_c h_{ad} + \partial_a\partial_d h_{bc} - \partial_a\partial_c h_{bd} - \partial_b\partial_d h_{ac}, \\ F^a_{mn} &= \partial_m A^a_n - \partial_n A^a_m, \\ H_{mnp} &= \partial_p B^{(2)}_{mn} + \partial_m B^{(2)}_{np} + \partial_n B^{(2)}_{pm}, \end{aligned} \tag{2.59}$$

the action (2.57) takes the form

$$\begin{aligned} S = -\frac{1}{4}\int d^{10}x \Big[&\frac{6}{10}R_{mn}R^{mn} + \frac{1}{10}R_{mnpq}R^{mnpq} + \frac{1}{5}tR - 2C^{mnp}H_{mnp} + \\ &-3C^{mnp}\left(\Box C_{mnp} - \frac{1}{2}\partial_{[p}\partial^r C_{mn]r}\right) + F^{amn}F^a_{mn}\Big]. \end{aligned} \tag{2.60}$$

Although the heterotic ambitwistor action correctly describes Yang-Mills, it also has a symmetric two-form field h_{mn} whose kinetic action is neither Einstein nor conformal gravity. In addition, it contains an antisymmetric 2-form $B^{(2)}_{mn}$ and antisymmetric 3-form C_{mnp} with unusual couplings. It is interesting to note, however, that the heterotic ambitwistor string was used in [28] to reproduce MHV amplitudes for conformal gravity in $D = 4$.

Chapter 3

On the Spectrum and Spacetime Supersymmetry of Heterotic Ambitwistor String

3.1 Outline

In section 3.2 we decided to review again the ambitwistor model so the chapter is self-contained, it can be skipped on a first reading. We start in section 3.3, where we use the standard BRST method to compute the equations of motion of the Ramond sector for the heterotic system. These represent the fermionic degrees of freedom of the theory, and our analysis shows that they also follow non-unitary equations of motion. We write a gauge-invariant version of the theory in terms of Fronsdal fields [29]. The kinetic term of the fermionic ambitwistor string field theory action is also computed in section 3.4. It is expressed in terms of gauge-invariant objects and resembles Fronsdal's free action despite having more derivatives. Finally, in section 3.5 we write the supersymmetry transformations of the system. Then we prove the invariance of the action under supersymmetry transformations.

3.2 Ambitwistor Action and Ramond Sector.

We first review the ambitwistor model. Its main purpose is to set the basic definitions and notation. The heterotic ambitwistor model is defined by the free action

$$S = \frac{1}{2\pi}\int \mathrm{d}^2 z \left(\mathrm{p}_m \bar{\partial} x^m + \psi_m \bar{\partial}\psi^m + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c} + \beta\bar{\partial}\gamma + S_{\mathrm{j}}\right) \tag{3.1}$$

where p_m is a worldsheet holomorphic one-form and x^m is an holomorphic coordinate function. The b and c fields together with β and γ are the Faddeev-Popov ghosts of superconformal worldsheet symmetry. Particular to the heterotic model, we have the current action S_{j}; its specific form is irrelevant for us, we only require the existence of a current j^a with conformal weight 1 that satisfies the OPE

$$\mathrm{j}^a(z)\mathrm{j}^b(w) \sim \frac{\delta^{ab}}{(z-w)^2} + \frac{f_c^{ab}\mathrm{j}^c(w)}{(z-w)}, \tag{3.2}$$

being f_c^{ab} the structure constants of the Lie algebra in question. The Ambitwistor model differs from the superstring due to the presence of the $\tilde{b}$ and $\tilde{c}$ ghosts related to the gauge symmetries of the light-cone constrain: $\mathrm{p}^2 = 0$. These ghosts have conformal weights 2 and -1 respectively and both are worldsheet fermions.

Our Majorana spinors ψ^m will be rewritten in the complex linear combinations:

$$\psi^{\pm i} = \frac{1}{\sqrt{2}}\left(\psi^{2i-1} \mp i\psi^{2i}\right) \tag{3.3}$$

for $i = 1, \ldots, 5$ that are subsequently bosonized to

$$\psi^{\pm i}(z) = \exp\Big(\pm\phi_i(z)\Big)c_{\pm e_i} \tag{3.4}$$

with ϕ's satisfying

$$\phi_i(z)\phi_j(\mathrm{w}) \sim +\delta_{ij}\ln(z-\mathrm{w}) \tag{3.5}$$

The (β, γ) system is bosonized with extra fermions (ξ, η)[30], both primaries of conformal weight 0 and 1 respectively:

$$\beta = \partial\xi e^{-\phi_6}c_{e_6} \quad \text{and} \quad \gamma = \eta e^{\phi_6}c_{e_6}. \tag{3.6}$$

This choice follows the conventions of [31] and [32] where we have introduced the cocycles c_{e_i} and c_{e_6}. During the computation of cohomology, cocycle factors are important and must be taken into account. The definition of cocycles depends on the way we order the different ϕ_i. For us the chiral bosons corresponding to ψ^m are ordered from 1 to 5 while the boson coming from the $\beta\gamma$ system is labeled as 6. A review of how to operate with cocycles can be found in [31] and a brief explanation is written in appendix A. The sixth boson has OPE:

$$\phi_6(z)\phi_6(\mathrm{w}) \sim -\ln(z-\mathrm{w}) \tag{3.7}$$

while (ξ, η) form a free system:

$$\xi(z)\eta(\mathrm{w}) \sim \frac{1}{(z-\mathrm{w})} \tag{3.8}$$

The symmetries of this action are encoded in the following BRST charge:

$$Q = \oint \frac{dz}{2\pi i}\left[c\Big(\mathrm{T}_{\mathrm{matter}} + T_{\tilde{b}\tilde{c}} + T_{\beta\gamma} + \mathrm{T}_{\mathrm{j}}\Big) + bc\partial c + \frac{1}{2}\tilde{c}\mathrm{p}^2 + \gamma \mathrm{p}^m\psi_m - \gamma^2\tilde{b}\right] \tag{3.9}$$

provided

$$\mathrm{T}_{\mathrm{matter}} = -\mathrm{p}_m\partial x^m + \frac{1}{2}\sum_{i=1}^{5}\partial\phi_i\partial\phi_i, \qquad \mathrm{T}_{\tilde{b}\tilde{c}} = \tilde{c}\partial\tilde{b} - 2\tilde{b}\partial\tilde{c}, \tag{3.10a}$$

$$T_{\beta\gamma} = -\frac{1}{2}\partial\phi_6\partial\phi_6 - \partial^2\phi_6 - \eta\partial\xi, \quad \texttt{and} \quad \gamma^2 = \eta\partial\eta e^{+2\phi_6}. \tag{3.10b}$$

These are all the stress-energy tensors for $(x^m, \mathrm{p}_m, \psi^m)$, (β, γ) and $(\tilde{b}, \tilde{c})$. We only require for the stress tensor of the current sector, T_{j}, that the following OPE is satisfied:

$$\mathrm{T}_{\mathrm{j}}(z)\mathrm{T}_{\mathrm{j}}(w) \sim \frac{c_j}{2(z-w)^4} + \frac{2\mathrm{T}_{\mathrm{j}}(w)}{(z-w)^2} + \frac{\partial\mathrm{T}_{\mathrm{j}}(w)}{(z-w)}. \tag{3.11}$$

Then, provided the central charge of the current system is 16, it is possible to show that $Q^2 = 0$ when the spacetime is 10-dimensional.

3.3 Cohomology.

In this section, we compute the ghost number 2 BRST cohomology of the Ambitwistor string for states in the Ramond sector. The cohomology of the Neveu-Schwarz sector has already been computed in section 2.4 [13].

We start by writing the most general vertex operator and the most general gauge parameter. Once all equations of motion and gauge transformations are obtained, we solve the algebraic gauge conditions to obtain a set of independent field equations.

3.3.1 Vertex operators.

States are defined by picture number $-1/2$ and ghost number 2 BRST cohomology. We define ghost and picture numbers by the expressions:

$$N_{\text{ghost}} = -\oint \frac{\mathrm{d}z}{2\pi i}\left(bc + \tilde{b}\tilde{c} + \xi\eta\right) \quad \text{and} \quad N_{\text{picture}} = \oint \frac{\mathrm{d}z}{2\pi i}\left(\xi\eta - \partial\phi_6\right). \tag{3.12}$$

Vertex Operator. The most general ghost number 2 and picture number $-1/2$ vertex operator that is annihilated by b_0 is given by the sum,

$$V_R = V_+ + V_-, \tag{3.13}$$

where V_+ and V_- are the $GSO(+)$ and $GSO(-)$ combinations. The $GSO(+)$ vertex operator is given by:

$$V_+ = c\eta S^\alpha e^{\phi/2}\mathbf{A}_\alpha + \tilde{c}\eta S^\alpha e^{\phi/2}\mathbf{B}_\alpha + c\tilde{c}S^{\dot\alpha}e^{-\phi/2}\partial x^m\mathbf{C}_{m\dot\alpha} + c\tilde{c}S^{\dot\alpha}e^{-\phi/2}\mathrm{p}_m\mathbf{D}^m_{\dot\alpha} + \tag{3.14}$$

$$+ c\tilde{c}S^{\dot\alpha}e^{-\phi/2}\mathrm{j}^a\mathbf{E}^a_{\dot\alpha} + c\partial\tilde{c}S^{\dot\alpha}e^{-\phi/2}\mathbf{F}_{\dot\alpha} + c\tilde{c}S^{\dot\alpha}\partial e^{-\phi/2}\mathbf{G}_{\dot\alpha} +$$

$$+ c\tilde{c}\psi_m(\not{\psi}S)^\alpha e^{-\phi/2}\mathbf{H}^m_\alpha + c\tilde{c}\partial\tilde{c}\partial\xi S^\alpha e^{-3\phi/2}\mathbf{I}_\alpha + \tilde{c}\partial\tilde{c}S^{\dot\alpha}e^{-\phi/2}\mathbf{J}_{\dot\alpha} \tag{3.15}$$

while V_- is obtained from V_+ by changing the chirality of our spinors. Notice that the vertices $\psi^m\psi^n S^{\dot\alpha}$ and $\partial S^{\dot\alpha}$ have not been written. In bosonized form, these combinations are related to $\psi\not{\psi}S$ via field redefinitions[31]; there is no need to worry about them.

Gauge vertex. As for the gauge transformations, we parametrize them by ghost number 1 and picture number $-1/2$ vertex operators:

$$\Lambda = cS^{\dot\alpha}e^{-\phi_6/2}\lambda_{\dot\alpha} + \tilde{c}S^{\dot\alpha}e^{-\phi_6/2}\omega_{\dot\alpha} + c\tilde{c}\partial\xi S^\alpha e^{-3\phi_6/2}\mu_\alpha. \tag{3.16}$$

Both expressions (3.13) and (3.16) constitute the basic field content of BRST cohomology.

3.3.2 Equations of motion and gauge symmetries.

For clarity we consider only the $GSO(+)$ sector. The $GSO(-)$ is obtained by replacing chiral indices for anti-chiral and vice-versa. We present the equations of motion organized by ghost number as they were obtained from the OPE of Q and V_+. We also write the worldsheet operator that multiplies the resulting equation of motion.

- For $(2c, 1\tilde{c})$ multiplying $(S^{\dot{\alpha}} e^{-\phi_6/2} c\tilde{c}\partial^2 c)$:

$$+\frac{1}{2}\partial_m \mathbf{D}^m_{\dot{\alpha}} + \mathbf{F}_{\dot{\alpha}} - \frac{3}{8}\mathbf{G}_{\dot{\alpha}} - \frac{9}{4}(\Gamma^m)^{\beta}_{\dot{\alpha}} \mathbf{H}_{m\beta} = 0 \tag{3.17}$$

- For $(0c, 1\tilde{c})$ multiplying $(S^{\dot{\alpha}} e^{3\phi_6/2} \tilde{c}\eta\partial\eta)$:

$$+\mathbf{J}_{\dot{\alpha}} - \frac{i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot{\alpha}} \partial_m \mathbf{B}_{\beta} = 0 \tag{3.18}$$

- For $(1c, 0\tilde{c})$ multiplying $(S^{\dot{\alpha}} e^{3\phi_6/2} c\eta\partial\eta)$:

$$-\frac{i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot{\alpha}} \partial_m \mathbf{A}_{\beta} - \mathbf{G}_{\dot{\alpha}} + \mathbf{F}_{\dot{\alpha}} = 0 \tag{3.19}$$

- For $(1c, 2\tilde{c})$
  - multiplying $(S^{\alpha} e^{-\phi_6/2} c\tilde{c}\partial\tilde{c}\mathrm{p}_m)$:

$$-\frac{1}{2}\Box \mathbf{D}^m_{\dot{\alpha}} + \mathbf{C}^m_{\dot{\alpha}} - \partial^m \mathbf{F}_{\dot{\alpha}} - \frac{i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot{\alpha}} \mathbf{I}_{\beta} = 0 \tag{3.20}$$

  - multiplying $(S^{\dot{\alpha}} e^{-\phi_6/2} c\tilde{c}\partial^2\tilde{c})$:

$$-\frac{1}{2}\partial^m \mathbf{C}_{m\dot{\alpha}} - \mathbf{J}_{\dot{\alpha}} = 0 \tag{3.21}$$

  - multiplying $(S^{\dot{\alpha}} e^{-\phi_6/2} c\tilde{c}\partial\tilde{c}\partial x^m)$:

$$-\frac{1}{2}\Box \mathbf{C}_{m\dot{\alpha}} - \partial_m \mathbf{J}_{\dot{\alpha}} = 0 \tag{3.22}$$

  - multiplying $(S^{\dot{\alpha}} e^{-\phi_6/2} c\tilde{c}\partial\tilde{c}\partial\phi_6)$:

$$+\frac{1}{4}\Box \mathbf{G}_{\dot{\alpha}} + \frac{1}{2}\mathbf{J}_{\dot{\alpha}} + \frac{i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot{\alpha}} \partial_m \mathbf{I}_{\beta} = 0 \tag{3.23}$$

  - multiplying $(c\tilde{c}\partial\tilde{c}\psi^m(\psi S)^{\alpha})$:

$$+\frac{1}{2}\Box \mathbf{H}^m_{\alpha} - \frac{i}{4\sqrt{2}}\partial_m \mathbf{I}_{\alpha} + \frac{i}{8\times 9\sqrt{2}}(\Gamma_m)^{\dot{\beta}}_{\alpha}(\partial\!\!\!/ \mathbf{I})_{\dot{\beta}} + \frac{1}{9\times 4}(\Gamma_m)^{\dot{\beta}}_{\alpha} \mathbf{J}_{\dot{\beta}} = 0 \tag{3.24}$$

- For $1c, 1\tilde{c}$
  - multiplying $(S^\alpha e^{\phi_6/2} c\eta\partial\tilde{c})$:

$$-\frac{1}{2}\Box\mathbf{A}_\alpha + \mathbf{B}_\alpha + 2\mathbf{I}_\alpha - \frac{i}{\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\partial_m\mathbf{F}_{\dot\beta} = 0 \tag{3.25}$$

  - multiplying $(S^\alpha e^{\phi_6/2} c\tilde{c}\eta\partial x^m)$:

$$-\partial_m\mathbf{B}_\alpha + \frac{i}{\sqrt{2}}(\Gamma^n)_\alpha^{\dot\beta}\partial_n\mathbf{C}_{m\dot\beta} = 0 \tag{3.26}$$

  - multiplying $(S^\alpha e^{\phi_6/2} c\tilde{c}\eta \mathrm{p}_m)$:

$$-\partial^m\mathbf{A}_\alpha + \frac{i}{\sqrt{2}}(\Gamma^n)_\alpha^{\dot\beta}\partial_n\mathbf{D}^m_{\dot\beta} + \frac{i}{2\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\mathbf{G}_{\dot\beta} - \frac{i8}{\sqrt{2}}\mathbf{H}^m_\alpha - \frac{i}{\sqrt{2}}\mathbf{H}_{\beta n}(\Gamma^n)^\beta_{\dot\alpha}(\Gamma^m)^{\dot\alpha}_\alpha = 0 \tag{3.27}$$

  - multiplying $(S^\alpha e^{\phi_6/2} c\tilde{c}\partial\eta)$:

$$\begin{aligned}-\mathbf{B}_\alpha + 3\mathbf{I}_\alpha + \frac{i}{\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\mathbf{C}_{m\dot\beta} - \frac{i}{2\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\partial_m\mathbf{G}_{\dot\beta} + \frac{8i}{\sqrt{2}}\partial^m\mathbf{H}_{\alpha m} +& \\ + \frac{i}{\sqrt{2}}(\Gamma^n)_\alpha^{\dot\beta}(\Gamma^m)^\tau_{\dot\beta}\partial_n\mathbf{H}_{\tau m} = 0&\end{aligned} \tag{3.28}$$

  - multiplying $(S^\alpha e^{\phi_6/2} c\tilde{c}\eta\partial\phi_6)$:

$$\begin{aligned}\frac{1}{2}\mathbf{B}_\alpha + 4\mathbf{I}_\alpha + \frac{i}{\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\mathbf{C}_{m\dot\beta} - \frac{i}{\sqrt{2}}(\Gamma^m)_\alpha^{\dot\beta}\partial_m\mathbf{G}_{\dot\beta} + \frac{8i}{\sqrt{2}}\partial^m\mathbf{H}_{\alpha m} +& \\ + \frac{i}{\sqrt{2}}(\Gamma^n)_\alpha^{\dot\beta}(\Gamma^m)^\tau_{\dot\beta}\partial_n\mathbf{H}_{\tau m} = 0&\end{aligned} \tag{3.29}$$

  - multiplying $\left(\eta c\tilde{c}\psi^m(\psi S)^{\dot\alpha} e^{\phi_6/2}\right)$:

$$+\frac{i}{2\sqrt{2}}\left[\frac{1}{4}\partial_m\mathbf{G}_{\dot\beta}-\frac{1}{9\times 8}(\Gamma_m)^{\tau}_{\dot\alpha}(\partial\!\!\!/\mathbf{G})_\tau\right]-\frac{i}{\sqrt{2}}\left[\frac{1}{4}\mathbf{C}_{\dot\alpha m}-\frac{1}{9\times 8}(\Gamma_m)^{\tau}_{\dot\alpha}(\mathbf{C}\!\!\!/)_\tau\right]+$$

$$-\frac{i}{\sqrt{2}}\left[-\frac{1}{4}(\Gamma^n)^{\beta}_{\dot\alpha}\partial_m\mathbf{H}_{\beta n}-(\Gamma^n)^{\beta}_{\dot\alpha}\partial_n\mathbf{H}_{\beta m}+\frac{1}{9}(\Gamma_m)^{\beta}_{\dot\alpha}\partial^n\mathbf{H}_{\beta n}\right]+$$

$$+\frac{1}{36}(\Gamma_m)^{\alpha}_{\dot\alpha}\mathbf{B}_\alpha-\frac{i}{\sqrt{2}}\left[\frac{1}{9\times 8}(\Gamma_m)^{\beta}_{\dot\alpha}(\Gamma^l)^{\dot\beta}_{\beta}(\Gamma^p)^{\tau}_{\dot\beta}\partial_l\mathbf{H}_{\tau p}\right]=0 \tag{3.30}$$

These 14 equations of motion are all invariant under the following 10 gauge transformations:

$$\delta\mathbf{A}_\alpha=+\frac{i}{\sqrt{2}}(\Gamma^m)^{\dot\beta}_{\alpha}\partial_m\lambda_{\dot\beta}+2\mu_\alpha \tag{3.31a}$$

$$\delta\mathbf{B}_\alpha=+\frac{i}{\sqrt{2}}(\Gamma^m)^{\dot\beta}_{\alpha}\partial_m\omega_{\dot\beta} \tag{3.31b}$$

$$\delta\mathbf{I}_\alpha=\frac{1}{2}\Box\mu_\alpha \tag{3.31c}$$

$$\delta\mathbf{H}^m_\alpha=\frac{1}{9\times 4}(\Gamma^m)^{\dot\beta}_{\alpha}\omega_{\dot\beta}+\frac{i}{4\sqrt{2}}\partial_m\mu_\alpha-\frac{i}{8\times 9\sqrt{2}}(\Gamma_m)^{\dot\beta}_{\alpha}(\partial\!\!\!/\mu)_{\dot\beta} \tag{3.31d}$$

$$\delta\mathbf{C}_{m\dot\alpha}=\partial_m\omega_{\dot\alpha} \tag{3.31e}$$

$$\delta\mathbf{D}^m_{\dot\alpha}=\partial_m\lambda_{\dot\alpha}-\frac{i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot\alpha}\mu_\beta \tag{3.31f}$$

$$\delta\mathbf{E}^A_{\dot\alpha}=0 \tag{3.31g}$$

$$\delta\mathbf{F}_{\dot\alpha}=-\frac{1}{2}\Box\lambda_{\dot\alpha}+\omega_{\dot\alpha} \tag{3.31h}$$

$$\delta\mathbf{G}_{\dot\alpha}=\omega_{\dot\alpha}-\frac{2i}{\sqrt{2}}(\Gamma^m)^{\beta}_{\dot\alpha}\partial_m\mu_\beta \tag{3.31i}$$

$$\delta\mathbf{J}_{\dot\alpha}=-\frac{1}{2}\Box\omega_{\dot\alpha} \tag{3.31j}$$

We determined the basic content of ghost number 2 BRST cohomology; all equations of motion have been written between (3.17) and (3.30). This set is highly redundant, and the next step is to use (3.31) to stablish the independent field equations.

3.3.3 Gauge-fixing and independent equations of motion.

In order to find the independent set of equations of motion, we begin by fixing algebraic gauge conditions and solving auxiliary field equations. Let us gauge-fix **A** and **F** to zero using the parameters μ and ω, that is, we choose $\mu = -\mathbf{A}$ and $\omega = -\mathbf{F}$ so that the residual gauge parameters μ' and ω' must satisfy:

$$\mu'_{\alpha} + \frac{i}{2\sqrt{2}}(\Gamma^m)^{\dot\beta}_{\alpha}\partial_m\lambda_{\dot\beta} = 0, \tag{3.32}$$

and

$$\omega'_{\dot\alpha} - \frac{1}{2}\Box\lambda_{\dot\alpha} = 0. \tag{3.33}$$

After this gauge fixing, the following auxiliary field conditions can be imposed:

$$\mathbf{G}^m_{\dot\alpha} = 0, \tag{3.34a}$$

$$\mathbf{B}_{\alpha} = -2\mathbf{I}_{\alpha}, \tag{3.34b}$$

$$\mathbf{C}^m_{\dot\alpha} = +\frac{1}{2}\Box\mathbf{D}^m_{\dot\alpha} + \frac{i}{\sqrt{2}}\left(\Gamma^m\mathbf{I}\right)_{\dot\alpha}, \tag{3.34c}$$

$$\mathbf{J}_{\dot\alpha} = -\frac{1}{4}\Box\,\partial_m\mathbf{D}^m_{\dot\alpha} - \frac{i}{2\sqrt{2}}\left(\partial\!\!\!/\mathbf{I}\right)_{\dot\alpha}, \tag{3.34d}$$

$$\mathbf{H}^m_{\alpha} = \frac{1}{8}\partial\!\!\!/^{\dot\beta}_{\alpha}\mathbf{D}^m_{\dot\beta} - \frac{1}{18\times 8}(\Gamma^m\Gamma_n\,\partial\!\!\!/)^{\dot\beta}_{\alpha}\mathbf{D}^n_{\dot\beta}. \tag{3.34e}$$

At this point it is already clear that there only remains two independent fields given by $\mathbf{D}^m_{\dot\alpha}$ and $\mathbf{I}_{\alpha}$. Moreover, the only remaining gauge parameter is λ. We leave the gluino field $\mathbf{E}^a_{\dot\beta}$ out of the discussion since its equation of motion is already the Dirac equation and it has no gauge transformations.

Finally, the following set of 3 equations,

$$\frac{i}{\sqrt{2}}\partial^m\mathbf{I}_{\alpha} = \Box\left(\frac{1}{4}\partial\!\!\!/^{\dot\beta}_{\alpha}\mathbf{D}^m_{\dot\beta} - \frac{1}{12}(\Gamma^m)^{\dot\beta}_{\alpha}\partial_n\mathbf{D}^n_{\dot\beta}\right) \tag{3.35a}$$

$$2\partial_m\mathbf{D}^m_{\dot\alpha} + (\Gamma_n\Gamma_p)^{\dot\beta}_{\dot\alpha}\partial^n\mathbf{D}^p_{\dot\beta} = 0 \tag{3.35b}$$

$$\not{\partial}_{\alpha}^{\dot{\alpha}}\mathbf{E}_{\dot{\alpha}}^{a} = 0 \tag{3.35c}$$

with the corresponding gauge transformations:

$$\delta\mathbf{D}_{\dot{\alpha}}^{m} = \frac{3}{4}\partial^{m}\lambda_{\dot{\alpha}} - \frac{1}{4}(\Gamma^{mn})_{\dot{\alpha}}{}^{\dot{\beta}}\partial_{n}\lambda_{\dot{\beta}} \tag{3.36a}$$

$$\delta\mathbf{I}_{\alpha} = -\frac{i}{4\sqrt{2}}\not{\partial}_{\alpha}^{\dot{\beta}}\Box\lambda_{\dot{\beta}} \tag{3.36b}$$

defines the spectrum of the theory.

Gauge-invariant description. Consider the following field redefinitions:

$$\mathbf{d}_{\dot{\alpha}}^{m} = \mathbf{D}_{\dot{\alpha}}^{m} - \frac{1}{6}(\Gamma^{m})_{\dot{\alpha}}^{\alpha}\not{\mathbf{D}}_{\alpha} \tag{3.37a}$$

$$\mathbf{i}_{\alpha} = +\frac{i4}{\sqrt{2}}\mathbf{I}_{\alpha} + \frac{1}{6}\Box\not{\mathbf{D}}_{\alpha} \tag{3.37b}$$

such that our gauge transformations are mapped to

$$\delta\mathbf{d}_{\dot{\alpha}}^{m} = \partial^{m}\lambda_{\dot{\alpha}} \quad \texttt{and} \quad \delta\mathbf{i}_{\alpha} = 0. \tag{3.38}$$

The gauge-invariant object is then naturally defined as:

$$\mathbf{F}_{mn\dot{\alpha}} = \partial_{m}\mathbf{d}_{n\dot{\alpha}} - \partial_{n}\mathbf{d}_{m\dot{\alpha}} \tag{3.39}$$

which allows us to write the equations of motion in the following form:

$$\partial_{m}\mathbf{i}_{\alpha} = \Box\mathbf{F}_{m\alpha} \tag{3.40a}$$

and

$$(\not{\mathbf{F}})_{\dot{\alpha}} = 0 \tag{3.40b}$$

where

$$\mathbf{F}_{m\alpha} \equiv (\Gamma^n)_\alpha^{\dot\alpha}\mathbf{F}_{mn\dot\alpha} = (\partial\!\!\!/ \mathbf{d}_m - \partial_m \mathbf{d}\!\!\!/)_\alpha. \tag{3.41}$$

In the formulation of free higher-spin theories $\mathbf{F}_m$ is called Fronsdal tensor[29], it is the analog of the Ricci curvature in spin 2 formulation.

This section started with the most general ghost number 2 picture $-1/2$ vertex operator. Then we obtained all equations of motion from the BRST method together with all gauge transformations parametrized by ghost number 1 picture $-1/2$ vertex operators. By fixing some of this gauge freedom, we have found a independent set of equations of motion that can be parametrized by Fronsdal fields. The next natural step is to write the spacetime action that gives the dynamics of this system.

3.4 Action

The kinetic term of the ambitwistor string field theory was defined in [13]:

$$S[V] = \langle I \circ V^{(-3/2)}(0)\, \partial c\, QV^{(-1/2)}(0)\rangle, \tag{3.42}$$

where $V^{-1/2}$ is the vertex operator (3.13) introduced in the previous section, an element of the small Hilbert space that is also constrained to satisfy $L_0 V = b_0 V = 0$. The RNS string has one additional feature: the picture number. It is necessary to saturate the background charge of supermoduli space to -2, and that is why we need a string field with picture $-1/2$, $V^{-1/2}$, together with a string field with picture $-3/2$, $V^{-3/2}$. We define picture raising, Z, and picture lowering, Y, by the following expressions:

$$Z = c\partial\xi + e^{\phi_6}\mathrm{p}_m\psi^m - \partial(e^{2\phi_6}\eta\tilde{b}) - e^{2\phi_6}\partial\eta\tilde{b}, \tag{3.43}$$

$$Y(z) = \tilde{c}\partial\xi e^{-2\phi_6}, \tag{3.44}$$

so that we can obtain $V^{-3/2}$ from $V^{-1/2}$ via

$$V^{-3/2}(z) = \frac{1}{2\pi i}\oint \frac{dw}{(w-z)} Y(w) V^{-1/2}(z). \tag{3.45}$$

Using the auxiliary gauge-fixing conditions imposed on the previous section, we obtain

$$V^{-3/2} = \tag{3.46}$$

$$+\,\tilde{c}\partial\tilde{c}S^{\alpha}e^{-3\phi_6/2}\mathbf{B}_{\alpha} - c\tilde{c}\partial\tilde{c}\partial\xi S^{\dot{\alpha}}e^{-5\phi_6/2}\partial x^m\mathbf{C}_{m\dot{\alpha}} - c\tilde{c}\partial\tilde{c}\partial\xi S^{\dot{\alpha}}e^{-5\phi_6/2}\mathrm{p}_m\mathbf{D}^m_{\dot{\alpha}}$$
$$- c\tilde{c}\partial\tilde{c}\partial\xi S^{\dot{\alpha}}e^{-5\phi_6/2}j^a\mathbf{E}^a_{\dot{\alpha}} - c\tilde{c}\partial\tilde{c}\partial\xi\psi_m(\not{\psi}S)^{\alpha}e^{-5\phi_6/2}\mathbf{H}^m_{\alpha} - \frac{1}{2}c\tilde{c}\partial\tilde{c}\partial\xi\partial^2\tilde{c}\partial^2\xi S^{\alpha}e^{-7\phi_6/2}\mathbf{I}_{\alpha} \tag{3.47}$$

The composition $I\circ V^{-3/2}$ is the BPZ conjugate of the picture $-3/2$ field with $I=-1/z$. We should be careful when computing the conformal transformation $I\circ V^{-3/2}$ because $V^{-1/2}$ is not primary. From the OPE with the stress-energy tensor

$$T(z)V^{-1/2}(0) \sim z^{-3}S^{\dot{\alpha}}e^{-\phi_6/2}c\tilde{c}\left(\frac{1}{2}\partial_m\mathbf{D}^m_{\dot{\alpha}} + \mathbf{F}_{\dot{\alpha}} - \frac{3}{8}\mathbf{G}_{\dot{\alpha}} - \frac{9}{4}\not{\mathbf{H}}_{\dot{\alpha}}\right) + \cdots \tag{3.48}$$

we obtain a cubic pole contribution that changes the finite conformal transformation to

$$I\circ V = \left[V\big(I(z)\big) + \frac{1}{2}\frac{I''(z)}{[I'(z)]^2}\,\#\big(I(z)\big)\right]. \tag{3.49}$$

where # is cubic pole coefficient. Even after the auxiliary conditions are imposed we still have non-primary contributions that must be taken into account.

To calculate the free action, we fix the normalization $\langle c\partial c\partial^2c\tilde{c}\partial\tilde{c}\partial^2\tilde{c}e^{-2\phi_6}\rangle = 4$, then the correlation function (3.42) gives the following gauge-invariant action:

$$S_R = -\int d^{10}x\left[\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left(\mathbf{F}_{m\alpha} - \frac{1}{2}(\gamma_m)_{\alpha\beta}\not{\mathbf{F}}^{\beta}\right) + \frac{1}{2}(\not{\mathbf{F}})^{\alpha}\mathbf{i}_{\alpha} - \frac{i}{2}\mathrm{Tr}\Big(\mathbf{E}\not{\partial}\mathbf{E}\Big)\right]. \tag{3.50}$$

In this expression we used the symmetric gamma matrices $(\gamma^m_{\alpha\beta}, \gamma_m^{\alpha\beta})$ defined in appendix A. When using these symmetric matrices, the charge conjugation is used to eliminate all dotted indices; different chiralities are just represented by upper and lower indices, *i.e.* $(C^{\alpha\dot{\alpha}}\mathbf{d}^m_{\dot{\alpha}} = \mathbf{d}^{m\alpha})$.

We have written a non-unitary action that gives the equations of motion obtained in (3.40). It closely resembles the gauge-invariant formulation of spin 3/2, the difference being the presence of more derivatives. Let us proceed and study the supersymmetry of this non-unitary system.

3.5 Supersymmetry.

Let us define the supersymmetry generator as

$$\mathbf{Q}_\alpha^{-1/2} = \frac{1}{2\pi i}\oint \mathrm{d}z\, S_\alpha e^{-\phi_6/2} \tag{3.51}$$

Notice that it carries picture, which means that supersymmetry algebra only closes on-shell. We need the picture 1/2 supersymmetric charge:

$$\mathbf{Q}_\alpha^{1/2} = \frac{1}{2\pi i}\oint \mathrm{d}z \Big[i\mathrm{p}_m(\gamma^m)_{\alpha\beta}S^\beta e^{\phi_6/2} + \tilde{b}\eta S_\alpha e^{3\phi_6/2}\Big]. \tag{3.52}$$

to obtain $\{Q_\alpha^{-1/2}, Q_\beta^{1/2}\} = 2\gamma^m_{\alpha\beta}\mathrm{p}_m$. In practice, supersymmetry transformations are written up to equations of motion. One also needs to choose a GSO sector to have well-defined supersymmetry transformations, otherwise there will be branch cuts. Given the generator (3.51), we need use the GSO(+) vertex operator.

3.5.1 Supersymmetry transformations of NS and R sectors.

The Neveu-Schwarz vertex operator in picture -1 was written in [13]:

$$\begin{aligned} V_{NS}^{-1} =& \\ & e^{-\phi_6} c\tilde{c}\Big[\Big(G^{(1)}_{(mn)} + B^{(1)}_{[mn]}\Big)\, \mathrm{p}^m\psi^n + \Big(G^{(2)}_{(mn)} + B^{(2)}_{[mn]}\Big)\, \partial x^m\psi^n + C_{mnp}\psi^m\psi^n\psi^p + \mathrm{j}^a\psi^m A^a_m\Big] \\ & + e^{-\phi_6}c\tilde{c}\partial\psi^m A^{(4)}_m + \partial\phi_6 e^{-\phi_6}c\tilde{c}A^{(3)}_m\psi^m + \partial\xi e^{-2\phi_6}\partial^2\tilde{c}\tilde{c}cS^{(4)} + \eta c S^{(1)} + \partial\xi e^{-2\phi_6}\partial^2 cc\tilde{c}S^{(2)} \\ & + \dots \end{aligned} \tag{3.53}$$

where ... depends only on the previous fields. In [13], the fields $(B^{(1)}_{mn}, A^{(3)}_m, A^{(4)}, S^{(1)}, S^{(2)})$ of (3.53) were gauged to zero. If we choose to keep this gauge, we must observe that in general supersymmetry does not preserve a given gauge condition. Therefore when calculating supersymmetry transformations, we have to choose the gauge parameter Λ:

$$\delta_\zeta V_{NS}^{-1} = \Big[\zeta\mathbf{Q}^{-1/2}, V_R^{-1/2}\Big] + \Big[Q_{BRST}, \Lambda^{-1}\Big], \tag{3.54}$$

which is a vertex operator of ghost number 1 and picture -1, to ensure that $\delta_\zeta(B^{(1)}_{mn}, A^{(3)}_m, A^{(4)}, S^{(1)}, S^{(2)})$ all give zero. In the transformations below, the contributions of $\mathbf{H}$ are due to the gauge-fixing of these auxiliary fields:

$$\delta_\zeta G^{(1)}_{mn} = 2(\zeta\gamma_{(m}\mathbf{D}_{n)}) \tag{3.55}$$

$$\delta_\zeta G^{(2)}_{mn} = \frac{2}{5}(\zeta\gamma_{(n}\mathbf{C}_{m)}) - \frac{48}{5}\partial_{(n}\zeta\mathbf{H}_{m)} \tag{3.56}$$

$$\delta_\zeta B^{(2)}_{mn} = -4(\zeta\gamma_{[n}\mathbf{C}_{m]}) - \frac{48}{5}(\zeta\partial_{[m}\mathbf{H}_{n]}) \tag{3.57}$$

$$\delta_\zeta C_{mnp} = \frac{3}{2}\partial_{[p}(\zeta\gamma_m\mathbf{D}_{n]}) - 24(\zeta\gamma_{[np}\mathbf{H}_{m]}) + 6(\zeta\gamma_{mnp}\not{\mathbf{H}}) \tag{3.58}$$

and using the field redefinitions of [13]:

$$h_{mn} = G^{(1)}_{mn} + \frac{1}{4}\eta_{mn}h^r_r, \quad t = \frac{1}{4}\Box h^m_m + G^{m(2)}_m \quad \texttt{and} \quad B^{(2)}_{mn} = B_{mn} \tag{3.59}$$

we arrive at

$$\delta_\zeta h_{mn} = 2\zeta\gamma_{(m}\mathbf{d}_{n)} \tag{3.60}$$

$$\delta_\zeta t = \zeta\mathbf{i} \tag{3.61}$$

$$\delta_\zeta C_{mnp} = -3(\zeta\gamma_{t[mn}\mathbf{F}^t{}_{p]}) - 3(\zeta\gamma_{[m}\mathbf{F}_{np]}) \tag{3.62}$$

$$\delta_\zeta B_{mn} = -2\Box(\zeta\gamma_{[m}\mathbf{d}_{n]}) - (\zeta\gamma_{mn}\mathbf{i}) + \frac{1}{6}(\zeta\gamma_{mn}\partial_p\mathbf{F}^p) \tag{3.63}$$

$$\delta_\zeta A^a_m = \frac{i}{2}(\zeta\gamma_m\mathbf{E}^a). \tag{3.64}$$

The term $(\zeta\gamma_{mn}\partial_p\mathbf{F}^p)$ is zero if we use the equation of motion $\not{\mathbf{F}} = 0$, and so could not have been obtained from the supersymmetry generator (3.51). This term was added by hand in order to make the action invariant under supersymmetry.

For the Ramond sector the same can be done if we use instead the picture $+1/2$ supersymmetry generator (3.52):

$$\delta_\zeta \mathbf{d}_m^\alpha = +(\gamma^{rs}\zeta)^\alpha \partial_s h_{mr} - 2(\gamma^{np}\zeta)^\alpha C_{mnp} + \frac{1}{3}(\gamma_{mnps}\zeta)C^{nps} \tag{3.65}$$

$$\delta_\zeta \mathbf{i}_\alpha = 2(\zeta\not\partial)_\alpha t - (\gamma^{mnp}\zeta)_\alpha H_{mnp} + \frac{1}{3}(\gamma^{mnp}\zeta)_\alpha \Box C_{mnp} \tag{3.66}$$

$$\delta_\zeta \mathbf{E}^{a\beta} = -\frac{1}{4}F_{mn}(\gamma^{mn}\zeta)^\beta \tag{3.67}$$

At this point, we have obtain the supersymmetry transformations of both NS and R system for the independent fields of the theory in equations (3.60) to (3.59). Let us proceed and check that indeed the total $GSO(+)$ action is supersymmetric invariant.

3.5.2 Supersymmetry invariance of the action.

The action that describes the Neveu-Schwarz sector is

$$S_{NS} = -\int d^{10}x \left[\frac{1}{2}h^{mn}\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) - tR + \frac{1}{4}\mathrm{Tr}(F^{mn}F_{mn}) + \right.$$
$$\left. -C^{mnp}H_{mnp} + \frac{1}{2}C^{mnp}\left(\Box C_{mnp} - \frac{1}{2}\partial_{[p}\partial^r C_{mn]r}\right)\right] \tag{3.68}$$

where H_{mnp} is the field strength for B_{mn} and R_{mn} is the Ricci tensor. This expression is equivalent to the action written in equation (4.13) of [13] if we shift t by $t \mapsto t + R^2$. The equations of motion derived from (3.68) are [1]

$$\Box R_{mn} - \partial_m\partial_n t = 0\ , \quad R = 0\ , \quad \Box C_{mnp} - H_{mnp} = 0\ ,$$

$$\partial^m C_{mnp} = 0\ , \quad \texttt{and} \quad \partial_m F^{mn} = 0. \tag{3.69}$$

Now, the Ramond sector is described by equation (3.50):

$$S_R = -\int d^{10}x \left[\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left(\mathbf{F}_{m\alpha} - \frac{1}{2}(\gamma_m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right) + \frac{1}{2}(\not{\mathbf{F}})^\alpha \mathbf{i}_\alpha - \frac{i}{2}\mathrm{Tr}\left(\mathbf{E}\not\partial\mathbf{E}\right)\right]. \tag{3.70}$$

[1] Notice that we use the fact

$$(\delta h_{mn})\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) = h_{mn}\Box\delta\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) + \text{total derivative}$$

from which we obtain the following set of equations of motion – (3.40):

$$\partial_m \mathbf{i}_\alpha = \Box \mathbf{F}_{m\alpha}\ , \quad \not{\mathbf{F}}^\alpha = 0 \quad \texttt{and} \quad i\not{\partial}_{\alpha\beta}\mathbf{E}^{a\,\beta} = 0. \tag{3.71}$$

From now on, we leave the Yang-Mills system out of the discussion because its supersymmetry transformations and action are already standard. For later use, let us write the supersymmetry transformation for all field strengths:

$$\delta_\zeta R_{mn} = (\zeta\partial_{(m}\mathbf{F}_{n)}) + (\zeta\gamma_{(m}\partial^p\mathbf{F}_{n)p}) \tag{3.72a}$$

$$\delta_\zeta H_{mnp} = 3\Box(\zeta\gamma_{[m}\mathbf{F}_{np]}) - 3(\zeta\gamma_{[mn}\partial_{p]}\mathbf{i}) + \frac{1}{2}(\zeta\gamma_{[mn}\partial_{p]}\partial_\ell\mathbf{F}^\ell) \tag{3.72b}$$

$$\delta_\zeta \mathbf{F}^\alpha_{mn} = -2(\gamma^{rs}\zeta)^\alpha R_{mrsn} + 4(\gamma^{rp}\zeta)^\alpha\partial_{[n}C_{m]rp} - \frac{2}{3}(\partial_{[n}\gamma_{m]rps}\zeta)^\alpha C^{rps} \tag{3.72c}$$

$$\begin{aligned}\delta_\zeta \mathbf{F}_{m\alpha} = &+2(\gamma^n\zeta)_\alpha R_{mn} - 2(\gamma^{lnp}\zeta)_\alpha\partial_l C_{mnp} + \frac{1}{3}(\gamma_{lmnps}\zeta)_\alpha\partial^l C^{nps}\\ &+ 4(\gamma^n\zeta)_\alpha\partial^p C_{mnp} - (\gamma_{mps}\zeta)_\alpha\partial_n C^{nps}\end{aligned} \tag{3.72d}$$

$$\delta\not{\mathbf{F}}^\beta = 2\zeta^\beta R - 6(\gamma^{np}\zeta)^\beta\partial^m C_{npm} \tag{3.72e}$$

3.5.3 Supersymmetry for $(h_{mn}, t, \mathbf{i}, \mathbf{d})$

Let us consider the system:

$$\begin{aligned}\mathbf{S} = -\int d^{10}x\bigg(&\frac{1}{2}h^{mn}\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) - tR\\ &+ \frac{1}{2}\mathbf{d}^{m\alpha}\Box\left(\mathbf{F}_{m\alpha} - \frac{1}{2}(\gamma_m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right) + \frac{1}{2}(\not{\mathbf{F}})^\alpha\mathbf{i}_\alpha\bigg)\end{aligned} \tag{3.73}$$

such that the

S_{NS} **variation** is given by

$$\delta_\zeta(-tR) = -\zeta^\alpha\mathbf{i}_\alpha R - 2t\zeta^\alpha\partial_p\mathbf{F}^p_\alpha$$

$$\delta_\zeta\left[\frac{1}{2}h^{mn}\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right)\right] = 2(\zeta\gamma^m\mathbf{d}^n)\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right)$$

and the

S_R **variation** is given by

$$\delta_\zeta\left(\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left(\mathbf{F}_{m\alpha}-\frac{1}{2}(\gamma_m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right)\right)=2\mathbf{d}^{m\alpha}\Box\left((\zeta\gamma^n)_\alpha R_{mn}-\frac{1}{2}(\zeta\gamma_m)_\alpha R\right)$$
$$=-2(\zeta\gamma^n\mathbf{d}^m)\Box\left(R_{mn}-\frac{1}{2}\eta_{mn}R\right)$$

$$\delta_\zeta\left(\frac{1}{2}(\not{\mathbf{F}})^\alpha\mathbf{i}_\alpha\right)=\zeta^\alpha R\mathbf{i}_\alpha+(\not{\mathbf{F}})^\alpha\not{\partial}_{\alpha\beta}\zeta^\beta t$$
$$=\zeta^\alpha\mathbf{i}_\alpha R+2\zeta^\alpha(\partial_p\mathbf{F}^p_\alpha)t+\partial(...)$$

where we have used (3.72) and $(\not{\partial}\not{\mathbf{F}}\zeta)=2(\zeta\partial_p\mathbf{F}^p)$. It is clear that the sum of all terms cancels and invariance of this system is stablished.

3.5.4 Supersymmetry for $(H_{mnp}, C_{mnp}, \mathbf{d}^\alpha_m, \mathbf{i}_\alpha)$

It remains for consideration the following system:

$$\mathbf{S}=-\int d^{10}x\Bigg(-C^{mnp}H_{mnp}+\frac{1}{2}C^{mnp}\left(\Box C_{mnp}-\frac{1}{2}\partial_{[p}\partial^r C_{mn]r}\right)$$
$$+\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left(\mathbf{F}_{m\alpha}-\frac{1}{2}(\gamma_m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right)+\frac{1}{2}(\not{\mathbf{F}})^\alpha\mathbf{i}_\alpha\Bigg) \tag{3.74}$$

In order to check supersymmetric invariance we have to gather all independent combination of gamma matrices $(\gamma^m, \gamma^{mn}, \gamma^{mnp}, \gamma^{mnpp}, \gamma^{mnpqr})$. So consider the

S_{NS} **variation:**

$$\delta_\zeta(-C^{mnp}H_{mnp})=+3\left[(\zeta\gamma_{tmn}\mathbf{F}^t{}_p)+(\zeta\gamma_m\mathbf{F}_{np})\right]H^{mnp}-3(\zeta\gamma_m\mathbf{F}_{np})\Box C^{mnp}$$
$$-3(\zeta\gamma_{mn}\mathbf{i})\partial_p C^{mnp}-\frac{1}{2}(\zeta\gamma_{mn}\partial_p\mathbf{F}^p)\partial_p C^{mnp}+\partial(\dots)$$

$$\delta_\zeta\left[\frac{1}{2}C^{mnp}\left(\Box C_{mnp}-\frac{1}{2}\partial_{[p}\partial^r C_{mn]r}\right)\right] = -3\left[(\zeta\gamma_{tmn}\mathbf{F}^t{}_p)+(\zeta\gamma_m\mathbf{F}_{np})\right]\Box C^{mnp} + \frac{1}{2}\left[(\zeta\gamma_{mn}\partial^p\mathbf{F}_p)+(\zeta\gamma_m\partial_n\not{\mathbf{F}})\right]\partial_r C^{mnr}+\partial(\dots)$$

and the

S_R **variation:**

$$\begin{aligned}\delta_\zeta\left(\frac{1}{2}\not{\mathbf{F}}\mathbf{i}\right) &= -3(\gamma^{np}\zeta)^\beta\partial^m C_{npm}\mathbf{i}_\beta-\frac{1}{2}\not{\mathbf{F}}^\alpha(\gamma^{mnp}\zeta)_\alpha H_{mnp}+\frac{1}{6}\not{\mathbf{F}}^\alpha(\gamma^{mnp}\zeta)_\alpha\Box C_{mnp}\\ &= +3(\zeta\gamma^{nm}\mathbf{i})\partial^p C_{nmp}+\\ &\quad-\left[\frac{1}{6}(\zeta\gamma^{mnpts}\mathbf{F}_{ts})\Box C_{mnp}-(\zeta\gamma^{tmn}\mathbf{F}_t{}^p)\Box C_{mnp}-(\zeta\gamma^m\mathbf{F}^{np})\Box C_{mnp}\right]\\ &\quad+\left[\frac{1}{2}(\zeta\gamma^{mnpts}\mathbf{F}_{ts})H_{mnp}-3(\zeta\gamma^{tmn}\mathbf{F}_t{}^p)H_{mnp}-3(\zeta\gamma^m\mathbf{F}^{np})H_{mnp}\right]\end{aligned}$$

$$\begin{aligned}&\delta_\zeta\left(\frac{1}{2}\mathbf{d}_m^\alpha\Box\left(\mathbf{F}_\alpha^m-\frac{1}{2}(\gamma^m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right)\right)=\\ &\quad=\left(-2(\gamma^{np}\zeta)^\alpha C_{mnp}+\frac{1}{3}(\gamma_{mnps}\zeta)^\alpha C^{nps}\right)\Box\left(\mathbf{F}_\alpha^m-\frac{1}{2}(\gamma^m)_{\alpha\beta}\not{\mathbf{F}}^\beta\right)\\ &\quad=+2(\mathbf{F}_l{}^m\gamma^{lnp}\zeta)\Box C_{mnp}-4(\mathbf{F}^{mn}\gamma^p\zeta)\Box C_{mnp}+\\ &\quad-\frac{1}{3}(\mathbf{F}_{lm}\gamma^{lmnps}\zeta)\Box C_{nps}-(\mathbf{F}_m{}^n\gamma^{psm}\zeta)\Box C_{nps}+\\ &\quad-\left[\frac{1}{6}(\zeta\gamma^{mnpts}\mathbf{F}_{ts})\Box C_{mnp}-(\zeta\gamma^{tmn}\mathbf{F}_t{}^p)\Box C_{mnp}-(\zeta\gamma^m\mathbf{F}^{np})\Box C_{mnp}\right]\end{aligned}$$

Recall that the γ^{mnpqr} is symmetric and γ^{mnp} is antisymmetric under the spinor indices. Gathering all independent terms we confirm the system is supersymmetric.

Chapter 4

Bosonic sectorized strings and the $(DF)^2$ theory

4.1 Outline

This chapter is organized as follows. In section 4.2, we introduce the sectorized description of the bosonic chiral string, having the Polyakov action in first-order form as our starting point. We then investigate the physical spectrum of the model and analyze its tensionless limit. The kinetic part of its effective action and some results on the tree-level three-point amplitudes are also presented. In section 4.3, the bosonic model is extended with the inclusion of current algebras, and the effective field theory inferred from the three-point functions is shown to agree with the $(DF)^2 + \text{YM} + \phi^3$ theory of Johansson and Nohle. The Appendix B includes further details on the CFT of current algebras that are relevant for this work.

4.2 The bosonic sectorized string

In this section we will rederive some known results for chiral bosonic strings using the sectorized description, including its physical spectrum and tensionless limit analysis.

4.2.1 The Polyakov action in first-order form

The Polyakov action is given by

$$S_P = \frac{\mathcal{T}}{2} \int d\tau d\sigma \sqrt{-g}\{g^{ij}\partial_i X^m \partial_j X_m\}, \tag{4.1}$$

where $\mathcal{T} > 0$ is the string tension, g_{ij} is the worldsheet metric (with inverse g^{ij}) and $g = \det(g_{ij})$, with i, j denoting the usual worldsheet coordinates τ and σ. Spacetime indices $m, n, \ldots$ are raised and lowered with the (mostly plus) Minkowski metric η_{mn}.

In the first order formulation, one can define a classically equivalent action, given by

$$\tilde{S}_P = \int d\tau d\sigma \big\{ P_m \partial_\tau X^m - \tfrac{1}{4\mathcal{T}} e_+ (P_m + \mathcal{T}\partial_\sigma X_m)(P^m + \mathcal{T}\partial_\sigma X^m) \\ - \tfrac{1}{4\mathcal{T}} e_- (P_m - \mathcal{T}\partial_\sigma X_m)(P^m - \mathcal{T}\partial_\sigma X^m) \big\}, \quad (4.2)$$

where $e_\pm$ denote the Weyl invariant Lagrange multipliers related to the worldsheet metric as

$$e_\pm \equiv \frac{1}{g^{\tau\tau}\sqrt{-g}} \mp \frac{g^{\tau\sigma}}{g^{\tau\tau}}. \quad (4.3)$$

Although not manifestly, the action $\tilde{S}_P$ is invariant under worldsheet reparametrizations, generated by

$$H_\pm \equiv (P_m \pm \mathcal{T}\partial_\sigma X_m)(P^m \pm \mathcal{T}\partial_\sigma X^m). \quad (4.4)$$

The corresponding gauge transformations are given by

$$\begin{aligned} \delta X^m &= \tfrac{1}{2} c_+ (P^m + \mathcal{T}\partial_\sigma X^m) + \tfrac{1}{2} c_- (P^m - \mathcal{T}\partial_\sigma X^m), & (4.5a) \\ \delta P_m &= \tfrac{\mathcal{T}}{2} \partial_\sigma [c_+ (P^m + \mathcal{T}\partial_\sigma X^m) - c_- (P^m - \mathcal{T}\partial_\sigma X^m)], & \\ \delta e_+ &= \partial_\tau c_+ + c_+ \partial_\sigma e_+ - e_+ \partial_\sigma c_+, & (4.5b) \\ \delta e_- &= \partial_\tau c_- - c_- \partial_\sigma e_- + e_- \partial_\sigma c_-, & (4.5c) \end{aligned}$$

where c_+ and c_- are local parameters.

4.2.2 The sectorized interpretation

The quantization of the action (4.2) is straightforward, and the usual conformal gauge is obtained when we choose $e_\pm = 1$. We want to discuss, instead, a particular case of the one-parameter (β) family of gauges introduced in [18], which can be cast as

$$e_+ = 1, \quad e_- = \frac{(1-\beta)}{(1+\beta)}. \quad (4.6)$$

For $\beta = 0$, the conformal gauge is recovered. We are interested in the singular gauge $\beta \to \infty$, leading to a chiral worldsheet action. In this limit, $e_\pm = \pm 1$. This singular gauge was proposed in the context of doubled-coordinate field theory in [19]. After a

Wick rotation of the worldsheet coordinate τ, the gauge-fixed action can be written as

$$S = \frac{1}{2\pi}\int d^2z\{P_m\bar{\partial}X^m + b_+\bar{\partial}c_+ + b_-\bar{\partial}c_-\}, \tag{4.7}$$

where the gauge parameters $c_\pm$ have been promoted to anticommuting ghosts with corresponding antighosts $b_\pm$. All fields in S are holomorphic and the string tension $\mathcal{T}$ is now hidden.

A few comments about the gauge fixing (4.6) are in order. For any finite β, a redefinition of the worldsheet coordinates can always bring the gauge fixed action to the conformal gauge. This is hardly surprising, since the physical model should be gauge independent. This was noted by Siegel in [18], but his construction of the chiral string involved another crucial ingredient related to a change in the boundary conditions of the action. At any rate, adopting the singular gauge ($\beta \to \infty$) is useful since then the delta functions realizing the scattering equations become explicit.

It was later noticed that the boundary condition leading to Siegel's new propagator for the target space coordinates could in fact be described by the usual string theory in the conformal gauge ($\beta = 0$), albeit with a different choice of vacuum [33]. In the ambitwistor context, this alternative vacuum was investigated in [34] (and further in [35]) and also arises naturally from the quantization of the action (4.7). As it turns out, this seems to be the only consistent vacuum in the singular gauge $\beta \to \infty$. It might look like a contradiction, but the key idea here is precisely that this is a *singular* gauge which effectively leads to a degenerate worldsheet metric. In other words, the action (4.7) is completely oblivious to the usual conformal gauge in string theory because this gauge choice is not invertible (hence, singular).

In spite of being chiral, the model can be interpreted in terms of two sectors, namely the "+" and the "−", which partially emulate the left and right movers of the usual bosonic string. Each sector has its own *characteristic* energy-momentum tensor given by

$$T_+ = -\frac{1}{4\mathcal{T}}P_m^+P_n^+\eta^{mn} - 2b_+\partial c_+ + c_+\partial b_+, \tag{4.8a}$$

$$T_- = \frac{1}{4\mathcal{T}}P_m^-P_n^-\eta^{mn} - 2b_-\partial c_- + c_-\partial b_-, \tag{4.8b}$$

with

$$P_m^\pm \equiv P_m \pm \mathcal{T}\partial X_m. \tag{4.9}$$

The sectorization is manifest in the BRST charge Q:

$$Q = Q^+ + Q^-, \tag{4.10}$$

$$Q^\pm \equiv \oint \{c_\pm T_\pm - b_\pm c_\pm \partial c_\pm\}. \tag{4.11}$$

Nilpotency of Q requires the number of spacetime dimensions to be $d = 26$.

Note that the complete energy-momentum tensor is given by

$$\begin{aligned} T &= T_+ + T_- \\ &= -P_m \partial X^m - b\partial c - \partial(bc) - \tilde{b}\partial\tilde{c} - \partial(\tilde{b}\tilde{c}), \end{aligned} \tag{4.12}$$

and it is BRST exact, since $\{Q, (b_+ + b_-)\} = T$. In fact, if we define

$$\begin{aligned} c &\equiv \tfrac{1}{2}(c_+ + c_-), & \tilde{c} &\equiv \tfrac{1}{2\mathcal{T}}(c_- - c_+), \\ b &\equiv (b_+ + b_-), & \tilde{b} &\equiv \mathcal{T}(b_- - b_+), \end{aligned} \tag{4.13}$$

the action (4.7) becomes

$$S = \frac{1}{2\pi}\int d^2z\{P_m\bar{\partial}X^m + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}\}, \tag{4.14}$$

while the BRST charge is rewritten as

$$Q = \oint \{cT - bc\partial c + \tfrac{1}{2}\tilde{c}P^m P_m + \tfrac{\mathcal{T}^2}{2}\tilde{c}(\partial X^m \partial X_m - 2b\partial\tilde{c})\}, \tag{4.15}$$

and the familiar Virasoro structure emerges. The tensionless limit of Q is now very clear: it is precisely the BRST operator introduced by Mason and Skinner for the bosonic ambitwistor string [8].

We will see, however, that the sectorized description is more advantageous in the cohomology analysis, for it leads to a natural splitting of the vertex operators in the different mass levels.

4.2.3 Physical spectrum

The BRST cohomology at ghost number zero is given by the identity operator. At ghost number one, the cohomology contains only the zero-momentum states mapped to the operators $c_+ P_m^+$ and $c_- P_m^-$.

Physical states will be defined as elements of the BRST cohomology with ghost number two and annihilated by the zero mode of b. The latter follows from the usual off-shell condition $(b_0 - \bar{b}_0) = 0$ on physical states, but adapted to the chiral model. The most general vertex operator with conformal weight zero satisfying these conditions can be written as

$$V = V_0 + V_+ + V_-, \tag{4.16}$$

where

$$\begin{aligned} V_0 &= c_+c_-P^+_m P^-_n G^{mn} + \mathcal{T}(c_+\partial^2 c_+ + c_-\partial^2 c_-)D + \mathcal{T}(c_+\partial^2 c_+ - c_-\partial^2 c_-)E \\ &\quad + c_+P^+_m(\partial c_+ - \partial c_-)A^m_+ + c_-P^-_m(\partial c_+ - \partial c_-)A^m_-, &(4.17)\\ V_+ &= c_+c_-P^+_m P^+_n H^{mn}_+ + c_-P^+_m(\partial c_+ - \partial c_-)B^m_+ + c_+c_-\partial P^+_m C^m_+ \\ &\quad + \mathcal{T}c_-\partial^2 c_+ F^+ + b_+c_+c_-(\partial c_+ - \partial c_-)G^+, &(4.18)\\ V_- &= c_+c_-P^-_m P^-_n H^{mn}_- + c_+P^-_m(\partial c_+ - \partial c_-)B^m_- - c_+c_-\partial P^-_m C^m_- \\ &\quad + \mathcal{T}c_+\partial^2 c_- F^- + b_-c_+c_-(\partial c_+ - \partial c_-)G^-. &(4.19) \end{aligned}$$

Here, G^{mn}, $H^{mn}_\pm$ $A^m_\pm$, $B^m_\pm$, $C^m_\pm$, D, E, $F^\pm$ and $G^\pm$ are the X dependent fields. This splitting of the terms appearing in the vertex operator is motivated by their mass-level, as will become clear shortly.

In order to determine the physical degrees of freedom, we will analyze each of the vertices in (4.16) separately. For V_0, the equations of motion imposed by BRST closedness are given by

$$\begin{aligned} &A^m_+ = \tfrac{1}{2}\partial_n G^{mn} - \tfrac{1}{2}\partial^m(D-E), \qquad \Box D = \partial_m(A^m_+ + A^m_-), \\ &A^m_- = \tfrac{1}{2}\partial_n G^{nm} - \tfrac{1}{2}\partial^m(D+E), \qquad \Box E = \partial_m(A^m_+ - A^m_-), \\ &\Box G^{mn} = 2\partial^m A^n_- + 2\partial^n A^m_+. \end{aligned} \tag{4.20}$$

These equations become more transparent if we rewrite them in terms of the fields

$$g^{mn} \equiv \tfrac{1}{2}(G^{mn} + G^{nm}), \tag{4.21a}$$

$$b^{mn} \equiv \tfrac{1}{2}(G^{mn} - G^{nm}), \tag{4.21b}$$

$$\phi \equiv \tfrac{\mathcal{T}}{2}G^{mn}\eta_{mn} - \mathcal{T}D, \tag{4.21c}$$

$$g^m \equiv A^m_+ + A^m_- - \tfrac{1}{\mathcal{T}}\partial^m D, \tag{4.21d}$$

$$b^m \equiv A^m_+ - A^m_- - \tfrac{1}{\mathcal{T}}\partial^m E, \tag{4.21e}$$

such that g^m and b^m have algebraic solutions, *cf.* (4.20),

$$g^m = \partial_n g^{mn} - \eta_{np}\partial^m g^{np} + \tfrac{2}{\mathcal{T}}\partial^m\phi, \tag{4.22a}$$

$$b^m = \partial_n b^{mn}, \tag{4.22b}$$

and

$$\begin{aligned}\Box g^{mn} - \partial_p\partial^n g^{mp} - \partial_p\partial^m g^{np} + \eta_{pq}\partial^m\partial^n g^{pq} - \tfrac{2}{\mathcal{T}}\partial^m\partial^n\phi &= 0, && (4.23a)\\ \Box\phi &= 0, && (4.23b)\\ \partial_p(\partial^p b^{mn} + \partial^m b^{np} + \partial^n b^{pm}) &= 0. && (4.23c)\end{aligned}$$

The gauge transformations, with parameters λ^m and ω^m, are simply

$$\delta\phi = 0, \qquad \delta g^{mn} = \partial^{(m}\lambda^{n)}, \qquad \delta b^{mn} = \partial^{[m}\omega^{n]}. \tag{4.24}$$

It is now easy to identify the field content of the massless sector described by the vertex V_0: ϕ corresponds to the dilaton, b^{mn} is the Kalb-Ramond 2-form and g^{mn} is the graviton, satisfying the linearized equation of motion (4.23a).

For the vertices V_+ and V_-, the two sets of equations of motion are very similar to each other and can be displayed collectively as

$$\begin{aligned}&B^m_\pm = \partial_n H^{mn}_\pm - C^m_\pm - \tfrac{1}{2}\partial^m F^\pm, && \left(\tfrac{1}{4}\Box \mp \mathcal{T}\right) C^m_\pm = \mathcal{T} B^m_\pm + \tfrac{1}{2}\partial^m G^\pm,\\ &G^\pm = \tfrac{\mathcal{T}}{2} H^{mn}_\pm \eta_{mn} + \tfrac{1}{2}\partial_m C^m_\pm - \tfrac{3\mathcal{T}}{2} F^\pm, && \left(\tfrac{1}{4}\Box \mp \mathcal{T}\right) F^\pm = \tfrac{1}{2}\partial_m B^m_\pm \mp \tfrac{3}{2} G^\pm,\\ &\left(\tfrac{1}{4}\Box \mp \mathcal{T}\right) H^{mn}_\pm = \tfrac{1}{4}\partial^m B^n_\pm + \tfrac{1}{4}\partial^n B^m_\pm \mp \tfrac{1}{4}\eta^{mn} G^\pm, &&\end{aligned} \tag{4.25}$$

Again, these equations become more transparent after the field redefinitions

$$\begin{aligned}h^{mn}_\pm &\equiv H^{mn}_\pm - \tfrac{1}{4\mathcal{T}}(\partial^n C^m_\pm + \partial^m C^n_\pm) \pm \tfrac{1}{20\mathcal{T}}(\partial^m\partial^n \pm \mathcal{T}\eta^{mn})F^\pm\\ &\quad \mp \tfrac{1}{20\mathcal{T}}(\partial^m\partial^n \pm \mathcal{T}\eta^{mn}) H^{pq}_\pm \eta_{pq}, && (4.26a)\\ f_\pm &\equiv F^\pm - H^{mn}_\pm \eta_{mn}, && (4.26b)\\ c^m_\pm &\equiv C^m_\pm \pm \tfrac{1}{10}\partial^m H^{np}_\pm \eta_{np} \mp \tfrac{1}{10}\partial^m F^\pm, && (4.26c)\end{aligned}$$

which imply (using $d = 26$) that

$$\begin{aligned}\left(\tfrac{1}{4}\Box \mp \mathcal{T}\right) h^{mn}_\pm &= 0, && (4.27a)\\ \partial_n h^{mn}_\pm &= 0, && (4.27b)\\ h^{mn}_\pm \eta_{mn} &= 0, && (4.27c)\end{aligned}$$

with gauge transformations

$$\delta h^{mn}_\pm = 0, \qquad \delta f_\pm = \pm 5\Sigma^\pm, \qquad \delta c^m_\pm = \mathcal{T}\Pi^m_\pm. \tag{4.28}$$

The fields $f_\pm$ and $c^m_\pm$ are pure gauge, therefore $h^{mn}_\pm$ contain all the physical degrees of freedom, corresponding to spin 2 fields with $m^2 = \pm 4\mathcal{T}$.

Tensionless limit

Evidently, in the tensionless limit all the physical states are massless. In fact, if we naively take the $\mathcal{T} \to 0$ limit of the vertex (4.16), it may seem that Mason and Skinner's results are recovered [8]. However, the analysis of such limit has to be done more carefully precisely because all the physical states become massless. In other words, the vertices (4.17), (4.18) and (4.19) should mix in the tensionless limit. Therefore, we should find a convenient combination of the fields G^{mn}, $H^{mn}_{\pm}$ $A^{m}_{\pm}$, $B^{m}_{\pm}$, $C^{m}_{\pm}$, D, E, $F^{\pm}$ and $G^{\pm}$ in (4.16) such that the tensionless limit preserves the most general form of the vertex operator. The solution is

$$\begin{aligned} V \;=\;& c\tilde{c}P_m P_n G^{mn}_{(1)} + c\tilde{c}\partial X_m \partial X_n G^{mn}_{(2)} + c\tilde{c}P_m \partial X_n G^{mn}_{(3)} + c\tilde{c}P_m \partial X_n B^{mn} \\ & + c\tilde{c}\partial^2 X_m A^m_{(1)} + c\tilde{c}\partial P_m A^m_{(2)} + \partial\tilde{c}cP_m A^m_{(3)} + \partial\tilde{c}c\partial X_m A^m_{(4)} \\ & + c\partial\tilde{c}P_m A^m_{(5)} + c\partial\tilde{c}\partial X_m A^m_{(6)} + bc\partial\tilde{c}cS_{(1)} + \partial^2 ccS_{(2)} \\ & + \partial^2\tilde{c}\tilde{c}S_{(3)} + \partial^2 c\tilde{c}S_{(4)} + c\partial^2\tilde{c}S_{(5)} + \tilde{b}\tilde{c}c\partial\tilde{c}S_{(6)}, \end{aligned} \tag{4.29}$$

with

$$\begin{aligned} G^{mn}_{(1)} &\equiv 2\mathcal{T}[\tfrac{1}{2}(G^{mn} + G^{nm}) + H^{mn}_{+} + H^{mn}_{-}], & A^m_{(5)} &\equiv -2\mathcal{T}(A^m_{+} + B^m_{+} + A^m_{-} + B^m_{-}), \\ G^{mn}_{(2)} &\equiv 2\mathcal{T}^3[H^{mn}_{+} + H^{mn}_{-} - \tfrac{1}{2}(G^{mn} + G^{nm})], & A^m_{(6)} &\equiv -2\mathcal{T}^2(A^m_{+} + B^m_{+} - A^m_{-} - B^m_{-}), \\ G^{mn}_{(3)} &\equiv 4\mathcal{T}^2(H^{mn}_{+} - H^{mn}_{-}), & S_{(1)} &\equiv 2\mathcal{T}^2(G^{+} + G^{-}), \\ B^{mn} &\equiv -2\mathcal{T}^2(G^{mn} - G^{nm}), & S_{(2)} &\equiv -(2D + F^{+} + F^{-}), \\ A^m_{(1)} &\equiv 2\mathcal{T}^2(C^m_{+} + C^m_{-}), & S_{(3)} &\equiv -\mathcal{T}^2(2D - F^{+} - F^{-}), \\ A^m_{(2)} &\equiv 2\mathcal{T}(C^m_{+} - C^m_{-}), & S_{(4)} &\equiv \mathcal{T}(2E - F^{+} + F^{-}), \\ A^m_{(3)} &\equiv -2\mathcal{T}^2(A^m_{+} - B^m_{+} - A^m_{-} + B^m_{-}), & S_{(5)} &\equiv -\mathcal{T}(2E + F^{+} - F^{-}), \\ A^m_{(4)} &\equiv -2\mathcal{T}^3(A^m_{+} - B^m_{+} + A^m_{-} - B^m_{-}), & S_{(6)} &\equiv 2\mathcal{T}(G^{-} - G^{+}). \end{aligned} \tag{4.30}$$

Here the notation for the fields was chosen so as to agree with the ambitwistor construction of [13], where it was demonstrated that the free field dynamics associated to the fields above involve higher derivative operators. This result follows naturally from our construction above. For example, we can show that the equation of motion for the fields $G^{mn}_{(1)}$, $G^{mn}_{(2)}$ and $G^{mn}_{(3)}$ can be obtained using (4.23a) and (4.27), and are given (in the gauge $c^m_{\pm} = f_{\pm} = 0$) by

$$G^{mn}_{(2)} = \tfrac{1}{4}\Box G^{mn}_{(3)} - \mathcal{T}^2 G^{mn}_{(1)}, \tag{4.31a}$$

$$2G^{mn}_{(3)} = \Box G^{mn}_{(1)} - \partial_p\partial^n G^{mp}_{(1)} - \partial_p\partial^m G^{np}_{(1)} + \eta_{pq}\partial^m\partial^n G^{pq}_{(1)} - \tfrac{2}{\mathcal{T}}\partial^m\partial^n \tag{4.31b}$$

$$(\Box^2 - 16\mathcal{T}^2)G^{mn}_{(3)} = 0, \tag{4.31c}$$

with $G^{mn}_{(3)}\eta_{mn} = \partial_n G^{mn}_{(3)} = 0$.

Note that, by substituting (4.31b) into (4.31c), we get an equation involving $\Box^3 G_{(1)}^{mn}$ which, in the tensionless limit, has the same form as the one found in [13]. Of course, this had to be the case since the vertex operator (4.29) preserves its form as $\mathcal{T} \to 0$, while the BRST operator reduces to the (bosonic) ambitwistor one, as is evident from (4.15). Indeed, all the other equations of motion can be reproduced in a similar way.

4.2.4 Bosonic kinetic action and 3-point amplitudes

As shown above, $G_{(2)}^{mn}$ and $G_{(3)}^{mn}$ can be seen as auxiliary fields[1] which effectively implement a higher derivative equation of motion for $G_{(1)}^{mn}$. This behavior can be better understood from another point of view, namely in terms of the effective action of the model and, in particular, its kinetic part.

Indeed, the kinetic terms associated to g^{mn} and $h_{\pm}^{mn}$ have opposite signs. Physically, this indicates an instability of the model (ghosts), in agreement with the results of [20]. Such ghosts can usually be described in terms of higher derivative theories and this is precisely what happens here.

Bosonic kinetic action

Inspired by Zwiebach's closed string action [15], the kinetic action for ambitwistor strings was built in [13]. We will use the same prescription for the tensionful model and the kinetic action will be defined by

$$S = \frac{1}{2} \langle V | \partial c \, Q | V \rangle \tag{4.32}$$

where $|V\rangle$ is the state associated to the vertex operator (4.16), obtained from the identity state $|0\rangle$ through the state-operator map

$$|V\rangle = \lim_{z \to 0} V(z) \, |0\rangle \, , \tag{4.33}$$

and $\langle V|$ its BPZ conjugate. In order to simplify the calculations, we will fix the gauge $f_{\pm} = c_{\pm}^{m} = 0$ — cf. equations (4.26) and (4.28) — and use the auxiliary equations of motion in (4.21) to write the vertex operator (4.16) in terms of the fields g^{mn}, b^{mn}, ϕ and $h_{\pm}^{mn}$.

[1]Here the word "auxiliary" should not be understood as "not propagating degrees of freedom," but rather that the degrees of freedom represented by these fields can be incorporated in another one which satisfies a higher-derivative equation of motion — cf. equations (4.31) above.

Now, using the usual ghost measure $\langle c_\pm \partial c_\pm \partial^2 c_\pm \rangle = 2$, it is straightforward to show that the free action can be cast as

$$S_{bosonic} = S_0 + S_+ + S_-, \tag{4.34}$$

where

$$S_0 = 2 \int d^{26}x \{ g^{mn} \Box g_{mn} + \partial_p g^{mp} \partial^q g_{mq} + 2(g+\phi)\partial_m \partial_n g^{mn} - (g+\phi)\Box(g+\phi) + b^{mn} \Box b_{mn} - b^{mn} \partial_m \partial^r b_{nr} \}, \tag{4.35}$$

and

$$S_\pm = 4 \int d^{26}x \{ -h_\pm^{mn} (\Box \mp 4\mathcal{T}) h_{\pm mn} + h_\pm^{mn} \partial_n \partial^r h_{\pm mr} - 2h_\pm \partial_m \partial_n h_\pm^{mn} + h_\pm (\Box \mp 4\mathcal{T}) h_\pm \}, \tag{4.36}$$

with $g = g^{mn}\eta_{mn}$ and $h_\pm = h_\pm^{mn}\eta_{mn}$. As expected, the free field equations of motion derived from S_0 and $S_\pm$ precisely reproduce (4.23) and (4.27). The kinetic terms for g^{mn} and $h_\pm^{mn}$ have opposite signs, consistent with the ghost interpretation.

3-point amplitudes

The 3-point tree level scattering amplitudes for the bosonic chiral string were obtained in [20]. However, it is instructive to redo this analysis here since our unintegrated vertex operators have a different structure and, in particular, do not give rise to a Koba–Nielsen factor. For higher point amplitudes, we would need integrated vertex operators but their definition is still unknown.

It will be convenient to gauge fix the vertex operators in (4.16) and work with momentum eigenstates, such that

$$V_0 = c_+ c_- P_m^+ P_n^- G^{mn} e^{ik\cdot X}, \quad V_\pm = c_+ c_- P_m^\pm P_n^\pm H_\pm^{mn} e^{ik\cdot X}, \tag{4.37}$$

where $G^{mn}, H_\pm^{mn}$ are now seen as polarization tensors satisfying $k_m G^{mn} = k_n G^{mn} = k_m H_\pm^{mn} = \eta_{mn} H_\pm^{mn} = 0$.

In order to compute the 3-point amplitudes, we have to evaluate its OPE reduction by contracting all $P_m^\pm$'s with one another and with the momentum exponentials $e^{ik\cdot X}$. We also need the ghost 3-point function, which has the usual form

$$\langle c_\pm(z) c_\pm(y) c_\pm(w) \rangle = (z-y)(y-w)(w-z). \tag{4.38}$$

By virtue of the sectorized description, it is easy to show that the amplitude factorizes into a product of two open string amplitudes (where $\mathcal{T} \longmapsto -\mathcal{T}$ in the minus sector). With all this in mind, we can compute, for example, the 3-point amplitude involving only massless states. The result is

$$\langle V_0(z_1)V_0(z_2)V_0(z_3)\rangle = G_1^{mn}G_2^{pq}G_3^{rs}T_{mpr}\bar{T}_{nqs}\delta^{26}(k^1+k^2+k^3), \tag{4.39}$$

where

$$T_{mnp} \equiv k_m^2 k_n^3 k_p^1 + 2\mathcal{T}(k_m^2\eta_{np} + k_n^3\eta_{mp} + k_p^1\eta_{mn}) \tag{4.40}$$

and $\bar{T}_{nqs}$ is equal to T_{nqs} with the sign of $\mathcal{T}$ flipped. The amplitude does not depend on the positions of the vertex operator insertions and is, therefore, $SL(2,\mathbb{C})$ invariant. This result is to some extent expected, since the vertex structure is completely analogous to the ordinary bosonic string and the Koba–Nielsen factors are just 1 for three massless vertices. However, the $SL(2,\mathbb{C})$ invariance can be shown for *any* 3-point tree level amplitude, even though the Koba–Nielsen factor is *always* 1 in the chiral model (there are no contractions between the momentum exponentials since the XX OPE is trivial). The amplitudes factorize in the plus and minus sectors, and there is a precise cancelation of the poles and zeros in $z_{ij} \equiv z_i - z_j$.

4.3 Extension of the sectorized model with current algebras

In this section we will explore the extension of the bosonic sectorized model in a target space with dimension $d < 26$ and the introduction of current algebras, i.e. a gauge sector. To the action (4.7), we will add two extra pieces, S_C^+ and S_C^-, describing two current algebras. The new BRST charge preserves its form in (4.10) but now with

$$T_+ = -\frac{1}{4\mathcal{T}}P_m^+P_n^+\eta^{mn} - 2b_+\partial c_+ + c_+\partial b_+ + T_C^+, \tag{4.41}$$

$$T_- = \frac{1}{4\mathcal{T}}P_m^-P_n^-\eta^{mn} - 2b_-\partial c_- + c_-\partial b_- + T_C^-, \tag{4.42}$$

where $T_C^\pm$ denotes the energy-momentum tensor associated to different group manifolds with central charge

$$c^{(\pm)} = 26 - d. \tag{4.43}$$

For now we will focus on the "$-$" sector, which contains the tachyonic excitations. The inclusion of the "+" sector, which has an analogous structure, will be discussed in subsection 4.3.4.

Let us consider an affine Lie algebra associated to some group G, with structure constants $f_{ab}{}^{c}$ $(a, b, \ldots = 1$ to $\dim G)$ and level k. The addition of S_C^- to the action allows us to define currents J_a which are primary conformal fields and satisfy the OPE

$$J_a(z)\, J_b(y) \sim \frac{k\delta_{ab}}{(z-y)^2} + i f_{ab}{}^{c} \frac{J_c(y)}{(z-y)}. \tag{4.44}$$

Here the group generators have been orthonormalized such that the metric δ_{ab} corresponds to a Kronecker delta, and we will make no further distinction between upper and lower indices.

The energy-momentum tensor of the algebra can be obtained using the Sugawara construction and is given by

$$T_C^- \equiv \frac{1}{2(k+g)} \left(J_a, J_a\right), \tag{4.45}$$

where g is the dual Coxeter number, defined through

$$f_{acd} f_{bcd} = 2g\delta_{ab}. \tag{4.46}$$

We use the ordering prescription

$$\left(A, B\right)(y) \equiv \tfrac{1}{2\pi i} \oint \frac{dz}{(z-y)} A(z)B(y), \tag{4.47}$$

which can be understood as the product of two operators $A(z)$ and $B(y)$ in the limit $z \to y$, with singular terms removed.

It is then straightforward to compute the central charge of this model, which is given by

$$\begin{aligned} c^{(-)} &= \frac{k\Delta}{(k+g)}, \\ &\overset{!}{=} 26 - d, \end{aligned} \tag{4.48}$$

where

$$\Delta \equiv \delta^{ab}\delta_{ab} = \dim G. \tag{4.49}$$

The second equality in (4.48) comes from imposing the nilpotency of the BRST operator and constrains the group G and the level k of the current algebra. For example, for a target space with $d = 10$ one of the solutions is $G = SO(32)$ and $k = 1$, while for $d = 4$ we can have $G = SU(5)$ and $k = 55$, and so on. Further constraints on the group should arise from the analysis of anomalies but this will not be discussed in this work.

4.3.1 Physical spectrum

The BRST cohomology now includes additional states with corresponding vertex operators containing the currents J_a, expressed as

$$\begin{aligned} V_J &= c_+c_-P_m^+J_aF_a^m + c_-(\partial c_+ - \partial c_-)J_aF^a + c_+c_-\partial J_aS_a \\ &+ c_+c_-P_m^-J_aG_a^m + c_+(\partial c_+ - \partial c_-)J_aG^a + c_+c_-J_\alpha\varphi_\alpha. \end{aligned} \tag{4.50}$$

Here F_a^m, G_a^m, S^a, F_a, G_a and φ_α are target space fields. The index α belongs to a traceless-symmetric bi-adjoint representation of the group G (see appendix), with dimension

$$\Delta(\alpha) = \frac{\Delta(\Delta+1)}{2} - 1. \tag{4.51}$$

J_α is a primary conformal weight 2 operator defined as

$$J_\alpha \equiv \left(C^{-1}\right)_{\alpha ab} J_{(ab)}, \tag{4.52}$$

where $J_{(ab)}$ is given by the traceless-symmetric ordered product of two currents, *i.e.*

$$J_{(ab)} \equiv \tfrac{1}{2}\left(J_a, J_b\right) + \tfrac{1}{2}\left(J_b, J_a\right) - \tfrac{2(k+g)}{\Delta}\delta_{ab}T_C^-, \tag{4.53}$$

and $\left(C^{-1}\right)_{\alpha ab}$ are the inverse of the Clebsch-Gordan coefficients, $C_{\alpha ab}$. The properties of these coefficients will be discussed in the next subsection and in the appendix. Observe that we could have considered also the trace contribution in the vertex, *e.g.* $c_+c_-T_C^-\varphi$. However, the field φ couples only to the vertex V_- in subsection (4.2.3) and does not change the physical content of the model.

The BRST invariance of the vertex V_J implies the following equations of motion

$$(\Box + 4\mathcal{T})\varphi_\alpha = 0, \tag{4.54a}$$
$$F_a = \tfrac{1}{2}\partial_m F_a^m, \tag{4.54b}$$
$$G_a = \tfrac{1}{2}\partial_m G_a^m - S_a, \tag{4.54c}$$
$$\partial_n(\partial^m F_a^n - \partial^n F_a^m) = 0, \tag{4.54d}$$
$$\partial_n(\partial^m G_a^n - \partial^n G_a^m) = 4\mathcal{T}G_a^m + 2\partial^m S_a, \tag{4.54e}$$

and the gauge transformations can be summarized as

$$\delta F_a^m = \partial^m\Lambda_a, \qquad \delta G_a^m = \partial^m\Omega_a, \qquad \delta S_a = -2\mathcal{T}\Omega_a. \tag{4.55}$$

Since S_a is pure gauge, the physical states described by the vertex (4.50) correspond to a massless vector F_a^m and two fields with negative mass-squared $m^2 = -4\mathcal{T}$ namely the scalar φ_α and the vector G_a^m.

In parallel to subsection 4.2.3, we can prepare the vertex V_J for the tensionless limit analysis. Considering the redefinitions of the worldsheet ghosts of (4.13), V_J can be rewritten as

$$\tfrac{1}{2\mathcal{T}}V_J = c\tilde{c}J_\alpha\varphi_\alpha + c\tilde{c}P_mJ_aA_a^m + c\tilde{c}\partial X_mJ_aB_a^m - c\partial\tilde{c}J_aA_a - \tilde{c}\partial\tilde{c}J_aB_a. \tag{4.56}$$

Here, the fields A_a, A_a^m, B_a and B_a^m are defined in terms of F_a^m, G_a^m, F_a and G_a as

$$\begin{aligned} A_a &\equiv F_a + G_a, & B_a &\equiv \mathcal{T}(F_a - G_a), \\ A_a^m &\equiv F_a^m + G_a^m, & B_a^m &\equiv \mathcal{T}(F_a^m - G_a^m), \end{aligned} \tag{4.57}$$

with gauge transformations $\delta A_a^m = \partial^m\Lambda_a$ and $\delta B_a^m = \mathcal{T}\partial^m\Lambda_a$.

Their equations of motion follow from (4.54) and are given by

$$\begin{aligned} A_a &= \tfrac{1}{2}\partial_m A_a^m, & \mathcal{T}A_a^m - \tfrac{1}{2}\partial_n F_a^{mn} &= B_a^m, \\ B_a &= \tfrac{1}{2}\partial_m B_a^m, & (\Box + 4\mathcal{T})\partial_n F_a^{mn} &= 0. \end{aligned} \tag{4.58}$$

Therefore, the physical spectrum can be described in terms of only two fields, φ_α and A_a^m. The vector B_a^m is auxiliary, helping to implement a quartic equation of motion for A_a^m, which carries the degrees of freedom of both the massless and the massive vector fields, F_a^m and G_a^m. Note, in particular, the tensionless limit renders a massless spectrum with equations of motion $\Box\varphi_\alpha = \Box^2 A_a^m = 0$. As in the bosonic model of section 4.2, this behavior can be easily observed when analyzing the effective field theory associated to the model, which will be done in subsection 4.3.3. The first step will be to determine the 3-point amplitudes using the vertex (4.50).

4.3.2 3-point amplitudes

In order to compute the 3-point amplitude

$$\mathcal{A}_3 \equiv \langle V_J(z)V_J(y)V_J(w)\rangle\,, \tag{4.59}$$

we need to provide further details on the current algebra CFT, in particular the OPE's involving the operator J_α defined in (4.52) and the properties of the Clebsch-Gordan coefficients $C_{\alpha ab}$.

The operator J_α satisfies the following OPE's:

$$T_C^-(z)\, J_\alpha(y) \sim \frac{2J_\alpha}{(z-y)^2} + \frac{\partial J_\alpha}{(z-y)}, \tag{4.60a}$$

$$J_a(z)\, J_\alpha(y) \sim C_{\alpha ab}\frac{J_b}{(z-y)^2} - (T_a)_{\alpha\beta}\frac{J_\beta}{(z-y)}, \tag{4.60b}$$

$$\begin{aligned} J_\alpha(z)\, J_\beta(y) \sim\ & \frac{k\delta_{\alpha\beta}}{(z-y)^4} - (T_a)_{\alpha\beta}\left\{\frac{J_a}{(z-y)^3} + \frac{1}{2}\frac{\partial J_a}{(z-y)^2} + \frac{1}{6}\frac{\partial^2 J_a}{(z-y)}\right\} \\ & + d_{\alpha\beta\gamma}\left\{\frac{J_\gamma}{(z-y)^2} + \frac{1}{2}\frac{\partial J_\gamma}{(z-y)}\right\} \\ & + d_{\alpha\beta abc}\frac{J_{(abc)}}{(z-y)} + d_{\alpha\beta[ab]}\frac{J_{[ab]}}{(z-y)} + e_{a\alpha\beta}\frac{(J_a, T_C^-)}{(z-y)}. \end{aligned} \tag{4.60c}$$

The first OPE states that J_α is a primary operator of conformal dimension 2. The second OPE is connected to the definition of the Clebsch-Gordan coefficients (quadratic pole) and the group transformation of J_α (simple pole). $(T_a)_{\alpha\beta}$ denotes the group generators in the traceless bi-adjoint representation of the group G and satisfy

$$[T_a, T_b]_{\alpha\beta} = if_{abc}\,(T_c)_{\alpha\beta}\,, \tag{4.61a}$$

$$(T_a)_{\alpha\beta} \equiv 2if_{abc}C_{\alpha(ce)}\left(C^{-1}\right)_{\beta(be)}\,, \tag{4.61b}$$

$$(T_aT_b)_{\alpha\alpha} = 2g(\Delta+2)\delta_{ab}, \tag{4.61c}$$

$$(T_aT_a)_{\alpha\beta} = 4g\delta_{\alpha\beta} - 2f_{abc}f_{ade}C_{\alpha(ce)}\left(C^{-1}\right)_{\beta(bd)}\,, \tag{4.61d}$$

The OPE (4.60c) can be used to define the 2-point and 3-point functions involving only J_α's. Operators of conformal dimension 3 appear in the last line (with numerical coefficients $d_{\alpha\beta abc}$, $d_{\alpha\beta[ab]}$ and $e_{a\alpha\beta}$) but they do not contribute to $\mathcal{A}_3$. $J_{(abc)}$ is the totally symmetric traceless normal ordered product of J_a, J_b and J_c, and $J_{[ab]}$ is the antisymmetric product $(J_a, J_b) - (J_b, J_a)$.

The Clebsch-Gordan coefficients $C_{\alpha ab}$ are defined in such a way that

$$C_{\alpha ab}\left(C^{-1}\right)_{\beta ab} = \delta_{\alpha\beta}, \tag{4.62a}$$

$$C_{\alpha ab}\left(C^{-1}\right)_{\alpha cd} = \delta_{(ab)(cd)}, \tag{4.62b}$$

$$C_{\alpha ab}C_{\alpha cd} = \Delta_{(ab)(cd)} + 2k\delta_{(ab)(cd)}, \tag{4.62c}$$

$$C_{\alpha ab}C_{\beta ab} = f_{ade}f_{bce}C_{\beta ab}\left(C^{-1}\right)_{\alpha cd} + 2k\delta_{\alpha\beta}, \tag{4.62d}$$

with

$$\delta_{(ab)(cd)} \equiv \tfrac{1}{2}\delta_{ac}\delta_{bd} + \tfrac{1}{2}\delta_{ad}\delta_{bc} - \tfrac{1}{\Delta}\delta_{ab}\delta_{cd}, \tag{4.63a}$$

$$\Delta_{(ab)(cd)} \equiv \tfrac{1}{2}f_{ade}f_{bce} + \tfrac{1}{2}f_{ace}f_{bde} - \tfrac{2g}{\Delta}\delta_{ab}\delta_{cd}. \tag{4.63b}$$

Finally, the coefficient $d_{\alpha\beta\gamma}$ is defined as

$$d_{\alpha\beta\gamma} \equiv \left(C^{-1}\right)_{\beta ab} \left[(T_a T_b)_{\alpha\gamma} + 2C_{\alpha ac}C_{\gamma bc}\right], \tag{4.64}$$

or

$$C_{\beta ab}d_{\alpha\beta\gamma} = \tfrac{1}{2}\,(T_a T_b)_{\alpha\gamma} + C_{\alpha ae}C_{\gamma be} + (a \leftrightarrow b) - \text{trace}. \tag{4.65}$$

Although not manifestly, $d_{\alpha\beta\gamma}$ is traceless, *i.e.* $d_{\alpha\alpha\gamma} = 0$, and completely symmetric in the exchange of any pair of indices.

The 2-point amplitudes involving the gauge currents can be easily determined through the OPE's (4.44), (4.60b) and (4.60c), and are given by

$$\langle J_a(z)J_b(y)\rangle = \frac{k\delta_{ab}}{(z-y)^2}, \tag{4.66a}$$

$$\langle J_a(z)J_\alpha(y)\rangle = 0, \tag{4.66b}$$

$$\langle J_\alpha(z)J_\beta(y)\rangle = \frac{k\delta_{\alpha\beta}}{(z-y)^4}. \tag{4.66c}$$

The 3-point amplitudes are now straightforward to compute. They can be summarized as

$$\langle J_a(z)J_b(y)J_c(w)\rangle = -ikf_{abc}(z-y)^{-1}(y-w)^{-1}(w-z)^{-1}, \tag{4.67a}$$

$$\langle J_\alpha(z)J_a(y)J_b(w)\rangle = kC_{\alpha ab}(z-y)^{-2}(w-z)^{-2}, \tag{4.67b}$$

$$\langle J_\alpha(z)J_\beta(y)J_a(w)\rangle = k\,(T_a)_{\alpha\beta}\,(z-y)^{-3}(y-w)^{-1}(w-z)^{-1}, \tag{4.67c}$$

$$\langle J_\alpha(z)J_\beta(y)J_\gamma(w)\rangle = kd_{\alpha\beta\gamma}(z-y)^{-2}(y-w)^{-2}(w-z)^{-2}. \tag{4.67d}$$

As one last step before evaluating (4.59), it will be convenient to fix the gauge degrees of freedom of V_J. Using the gauge transformations (4.55), we will choose $S_a = 0$. In this gauge, $\partial_m G_a^m = 0$ as a consequence of the equations of motion. We can use the remaining parameter to fix the transversal gauge for the massless vector, such that the vertex is simplified to

$$V_J = c_+c_-P_m^+ J_a F_a^m + c_+c_-P_m^- J_a G_a^m + c_+c_- J_\alpha \varphi_\alpha. \tag{4.68}$$

Using the tree level measure for the ghosts (4.38), the 3-point amplitude (4.59) can be computed to be

$$\begin{aligned}\mathcal{A}_3 \;=\; & kd_{\alpha\beta\gamma}\langle\varphi_\alpha\varphi_\beta\varphi_\gamma\rangle - 3k\,(T_a)_{\alpha\beta}\langle\varphi_\alpha\partial_m\varphi_\beta(F_a^m+G_a^m)\rangle \\ & -3kC_{\alpha ab}\langle\partial_m\partial_n\varphi_\alpha(F_a^m+G_a^m)(F_b^n+G_b^n)\rangle \\ & -ikf_{abc}\langle\partial_p(F_a^m+G_a^m)\partial_m(F_b^n+G_b^n)\partial_n(F_c^p+G_c^p)\rangle \\ & +6k\mathcal{T}C_{\alpha ab}\eta_{mn}\langle\varphi_\alpha(F_a^m-G_a^m)(F_b^n+G_b^n)\rangle \\ & -6ik\mathcal{T}f_{abc}\eta_{mn}\langle(F_a^m-G_a^m)\partial_p(F_b^n+G_b^n)(F_c^p+G_c^p)\rangle\,. \end{aligned} \tag{4.69}$$

Observe that $\mathcal{A}_3$ is at most linear in $\mathcal{T}(F_a^m - G_a^m)$. If we look at the vertex (4.56), this is easy to understand because the 3-point amplitudes with two or three B_a^m's vanish trivially.

In principle, 4-point amplitudes can be computed using the results of Siegel in [18]. Currently, however, there is no clear definition of the integrated vertex operators and higher point amplitudes cannot be *directly* computed from the chiral model. This problem will be dealt with in a separate paper by one of the authors.

In the next subsection we will propose an effective field theory action for the field content of the previous subsection.

4.3.3 Effective field theory: $(DF)^2 + \mathrm{YM}$

As the main result of this paper, we would like to argue that the effective field theory action corresponding to this extension of the bosonic sectorized model is precisely the action of the $(DF)^2 + \mathrm{YM}$ theory constructed in [26]. Indeed, we have already shown the spectrum to be the same. The action can be decomposed as

$$S_{eff} = S_J^0 + S_J^{int}, \tag{4.70}$$

where S_J^0 is the kinetic part of the action and S_J^{int} corresponds to the interactions.

For the kinetic part, we will proceed like in subsection (4.2.4). For the interaction part, we will analyze the possible vertices that give rise to the 3-point amplitudes displayed in (4.69). Next, we will require the non-linear gauge invariance of the resulting model in order to finally propose its effective action.

4.3.3.1 Kinetic action

As stated above, we will define the kinetic action as

$$S_J^0 \equiv \langle V_J | \partial c\, Q | V_J \rangle \,, \tag{4.71}$$

up to normalization.

In order to further simplify the computation, we will consider the algebraic solutions (4.54b) and (4.54c), such that

$$\begin{aligned} \partial c[Q, V_J] \;\; = \;\; & \tfrac{1}{4\mathcal{T}} c_+ c_- \partial c_+ \partial c_- J_a P_m^+ [\partial_n (\partial^n F_a^m - \partial^m F_a^n)] \\ & + \tfrac{1}{4\mathcal{T}} c_+ c_- \partial c_+ \partial c_- J_a P_m^- [\partial_n (\partial^n G_a^m - \partial^m G_a^n) + 4\mathcal{T} G_a^m] \\ & + \tfrac{1}{4\mathcal{T}} c_+ c_- \partial c_+ \partial c_- \partial J_a [2\mathcal{T} \partial_m G_a^m] \\ & + \tfrac{1}{4\mathcal{T}} c_+ c_- \partial c_+ \partial c_- J_\alpha [\Box \varphi_\alpha + 4\mathcal{T} \varphi_\alpha]. \end{aligned} \tag{4.72}$$

It is then straightforward to show that

$$\begin{aligned} S_J^0 = \int d^d x \{ & \varphi_\alpha (\Box \varphi_\alpha + 4\mathcal{T} \varphi_\alpha) - 2\mathcal{T} F_{ma} (\Box F_a^m - \partial^m \partial_n F_a^n) \\ & + 2\mathcal{T} G_{ma} (\Box G_a^m + 4\mathcal{T} G_a^m - \partial^m \partial_n G_a^n) \}. \end{aligned} \tag{4.73}$$

Note that the kinetic terms of the fields F_a^m and G_a^m have opposite sign in S_J^0. Technically, the sign difference can be traced back to the OPE's of P_m^+ and P_m^- with themselves. As discussed previously, this indicates an instability of the model and we can again reinterpret it in terms of a higher derivative theory. In fact, as we will now show, this behavior is more transparent if we rewrite the action in terms of the vectors A_a^m and B_a^m defined in (4.57). The kinetic action can then be cast as

$$S_J^0 = \int d^d X \, \{ \varphi_\alpha (\Box \varphi_\alpha + 4\mathcal{T} \varphi_\alpha) + 2 B_{ma} \partial_n F_a^{mn} + 2 (B_a^m - \mathcal{T} A_a^m)(B_{ma} - \mathcal{T} A_{ma}) \}, \tag{4.74}$$

with

$$F_a^{mn} \equiv \partial^m A_a^n - \partial^n A_a^m. \tag{4.75}$$

Ignoring for now the interaction terms, observe that the equation of motion for B_a^m is algebraic, given by

$$B_a^m = \mathcal{T} A_a^m + \tfrac{1}{2} \partial_n F_a^{nm}. \tag{4.76}$$

If we replace this solution back in the action, we obtain

$$S_J^0|_B = \int d^d X \, \{ \varphi_\alpha (\Box \varphi_\alpha + 4\mathcal{T} \varphi_\alpha) + \mathcal{T} F_a^{mn} F_{mna} - \tfrac{1}{2} \partial_n F_a^{mn} \partial^p F_{mpa} \}. \tag{4.77}$$

This action can be identified with the kinetic part of the $(DF)^2$+YM theory constructed in [26]. Note that the propagator of A_a^m is given in momentum space by

$$G_{ab}^{mn}(p) = \frac{i\eta^{mn}\delta_{ab}}{p^2(p^2 - 4\mathcal{T})}. \tag{4.78}$$

The pole structure of this propagator agrees with the interpretation given after equation (4.58) that A_a^m effectively describes the massless and the massive vector fields, F_a^m and G_a^m.

4.3.3.2 Cubic vertices and the effective action

As it turns out, the procedure of integrating B_a^m out can be partially extended to interactions. We say "partially" because in this paper we consider only unintegrated vertex operators, therefore only 3-point tree level amplitudes. We expect this integration to hold for higher point vertices as well.

By looking at $\mathcal{A}_3$ in (4.69), it is easy to show that the 3-point vertices in terms of the vectors A_a^m and B_a^m can be schematically expressed as

$$\begin{array}{rclrcl} \varphi^3 & \sim & d_{\alpha\beta\gamma}\varphi_\alpha\varphi_\beta\varphi_\gamma, & \varphi A^2 & \sim & C_{\alpha ab}\varphi_\alpha\partial_n A_a^m\partial_m A_b^n, \\ \varphi^2 A & \sim & (T_a)_{\alpha\beta}\,\varphi_\alpha\partial_m\varphi_\beta A_a^m, & A^3 & \sim & if_{abc}\partial_p A_a^m\partial_m A_b^n\partial_n A_c^p, \\ \varphi AB & \sim & C_{\alpha ab}\eta_{mn}\varphi_\alpha B_a^m A_b^n, & A^2 B & \sim & if_{abc}\eta_{mn}B_a^m\partial_p A_b^n A_c^p. \end{array} \tag{4.79}$$

The idea now is to analyze the possible gauge invariant interactions that can generate these vertices after integrating out B_a^m, which is at most linear in the expressions above. The equation of motion for B_a^m in (4.76) gets modified to

$$B_a^m = \mathcal{T}A_a^m + \tfrac{1}{2}\partial_n F_a^{nm} + c_\# C_{\alpha ab}\varphi_\alpha A_{mb} + id_\# f_{abc}\eta_{mn}\partial_p A_b^n A_c^p + \ldots, \tag{4.80}$$

where $c_\#$ and $d_\#$ are numerical constants and the dots contain other terms necessary to generate the correct gauge transformation for B_a^m (remember that the onshell 3-point amplitude $\mathcal{A}_3$ was computed using gauge-fixed vertex operators). Taking this into consideration and replacing B_a^m in the action, we can show that *all* 3-point vertices come from the operators

$$C_{\alpha ab}\varphi_\alpha F_a^{mn}F_{mnb}, \quad (D\varphi)^2, \quad (DF)^2, \quad F^3, \quad F^2, \quad d_{\alpha\beta\gamma}\varphi_\alpha\varphi_\beta\varphi_\gamma,$$

where F_a^{mn} was redefined to be the non-Abelian field strength

$$F_a^{mn} \equiv (\partial^m A_a^n - \partial^n A_a^m) + igf_{abc}A_b^m A_c^n, \tag{4.81}$$

with coupling constant g, and D^m denotes the covariant derivative with respect to the vector A_a^m. The form of the higher point vertices (4, 5 and 6) is severely restricted by the non-linear gauge invariance of the effective action. Some contributions naturally appear after integrating out B_a^m and we expect them to combine with the input coming from higher-point amplitudes, which involve integrated vertex operators.

Finally, we propose the effective field theory action of the model to be

$$\begin{aligned} S_{eff} = \int d^d x \Big\{ & \tfrac{1}{2}(D_n F_a^{mn})^2 - \mathcal{T} F_a^{mn} F_{mna} + \tfrac{1}{2} D_m \varphi_\alpha D^m \varphi_\alpha - 2\mathcal{T}(\varphi^\alpha)^2 \\ & + \tfrac{g}{3} f_{abc} F^m{}_{na} F^n{}_{pb} F^p{}_{mc} + \tfrac{g}{2} C_{\alpha ab} \varphi_\alpha F_a^{mn} F_{mnb} + \tfrac{g}{3!} d_{\alpha\beta\gamma} \varphi_\alpha \varphi_\beta \varphi_\gamma \Big\}, \end{aligned} \tag{4.82}$$

where g is the coupling constant. This action describes the $(DF)^2 + \text{YM}$ theory of [26].

Moreover, if we include the "+" sector mentioned in the beginning of this section, the effective field theory action describes a more general model with a mirrored set of fields. In particular, if we restrict the gauge symmetry of the "+" sector to be instead a global symmetry, the effective action describes the $(DF)^2 + \text{YM} + \phi^3$ theory. This will be shown next.

4.3.4 Including the other gauge sector: $(DF)^2 + \text{YM} + \phi^3$

We will consider for the "+" sector an affine Lie algebra associated to a group $\hat{G}$ (with structure constants $\hat{f}_{AB}{}^C$) and level $\hat{k}$. Apart from the central charge constraint (4.43), $\{\hat{G}, \hat{k}\}$ are independent of $\{G, k\}$, from the "−" sector. The new currents, $\hat{J}_A$, are completely analogous to the ones discussed there, *e.g.* they satisfy the OPE

$$\hat{J}_A(z)\,\hat{J}_B(y) \sim \frac{\hat{k}\delta_{AB}}{(z-y)^2} + i\hat{f}_{AB}{}^C \frac{\hat{J}_C(y)}{(z-y)}, \tag{4.83}$$

when conveniently normalized. Here, δ_{AB} is a Kronecker delta.

In order to analyze the physical spectrum, we can start with the hatted version of (4.50), defined by

$$\begin{aligned} V_{\hat{J}} \;=\; & c_+ c_- P_m^+ \hat{J}_A \hat{G}_A^m + c_-(\partial c_+ - \partial c_-) \hat{J}_A \hat{G}^A + c_+ c_- \partial \hat{J}_A \hat{S}_A \\ & + c_+ c_- P_m^- \hat{J}_A \hat{F}_A^m + c_+(\partial c_+ - \partial c_-) \hat{J}_A \hat{F}^A + c_+ c_- \hat{J}_{\hat{\alpha}} \hat{\varphi}^{\hat{\alpha}}. \end{aligned} \tag{4.84}$$

It is easy to see that the fields appearing in this vertex operator will satisfy essentially the same equations of motion and gauge transformations as their counterparts in the "−" sector, albeit with one important difference: the replacement $\mathcal{T} \to -\mathcal{T}$. By going through the same steps as in subsection 4.3.1, we find that the physical spectrum in this sector contains a "mirror image" of the physical spectrum in the "−" sector, but with opposite mass-squared.

In addition, we can build a new type of vertex operator involving currents from both sectors. It has the form

$$V_\phi = c_+ c_- J_a \hat{J}_A \phi^{aA}, \tag{4.85}$$

where ϕ^{aA} is a bi-adjoint scalar transforming in the adjoint representation of both gauge groups. BRST closedness implies the equation of motion

$$\Box \phi^{aA} = 0, \tag{4.86}$$

whence ϕ^{aA} is a massless field.

Following the same method used in subsection 4.3.3, the kinetic part of the effective action involving the group indices can be cast as

$$S^0 = S^0_J + S^0_{\hat{J}} + S^0_\phi, \tag{4.87}$$

where S^0_J was given in (4.77) and $S^0_{\hat{J}}$ is its hatted analogue, and

$$S^0_\phi = k\hat{k} \int \mathrm{d}^d X \{\phi_{aA} \Box \phi^{aA}\}. \tag{4.88}$$

As for the interacting part, it clearly contains the corresponding part in (4.82) and its hatted version. Moreover, note that cubic vertices mixing the fields in V_J with those in $V_{\hat{J}}$ can only appear through $\langle V_\phi V_J V_{\hat{J}} \rangle$, since the three-point functions involving $\langle J\hat{J}\hat{J} \rangle$ or $\langle JJ\hat{J} \rangle$ vanish. The non-vanishing three-point functions with insertions of V_ϕ are given by:

$$\begin{aligned}
\langle V_\phi(z)V_\phi(y)V_\phi(w)\rangle &= k\hat{k} f^{abc} \hat{f}^{ABC} \left\langle \phi^{aA}\phi^{bB}\phi^{cC} \right\rangle, && (4.89\text{a})\\
\langle V_\phi(z)V_\phi(y)V_J(w)\rangle &= -ik\hat{k} f_{abc} \left\langle \phi^{aA}\partial_m \phi^{bA}(F^m_c + G^m_c) \right\rangle \\
&\quad -k\hat{k} C_{\alpha ab} \left\langle \phi^{aA}\phi^{bA}\varphi^\alpha \right\rangle, && (4.89\text{b})\\
\left\langle V_\phi(z)V_\phi(y)V_{\hat{J}}(w)\right\rangle &= -ik\hat{k} f_{ABC} \left\langle \phi^{aA}\partial_m \phi^{aB}(\hat{F}^m_C + \hat{G}^m_C) \right\rangle \\
&\quad -k\hat{k} \hat{C}_{\hat{\alpha}AB} \left\langle \phi^{aA}\phi^{aB}\hat{\varphi}^{\hat{\alpha}} \right\rangle, && (4.89\text{c})\\
\left\langle V_\phi(z)V_J(y)V_{\hat{J}}(w)\right\rangle &= \tfrac{1}{2}k\hat{k}\eta_{mn} \left\langle (F^m_a + G^m_a)(\hat{F}^n_A + \hat{G}^n_A)\Box\phi^{aA} \right\rangle \\
&\quad -k\hat{k}\eta_{mn} \left\langle \phi^{aA}\partial_p(F^m_a + G^m_a)\partial^p(\hat{F}^n_A + \hat{G}^n_A) \right\rangle \\
&\quad +k\hat{k} \left\langle \phi^{aA}\partial_n(F^m_a + G^m_a)\partial^m(\hat{F}^n_A + \hat{G}^n_A) \right\rangle. && (4.89\text{d})
\end{aligned}$$

Thus, defining

$$\hat{A}^m_A \equiv \hat{F}^m_A + \hat{G}^m_A, \qquad \hat{F}^{mn}_A \equiv \partial^m \hat{A}^n_A - \partial^n \hat{A}^m_A + ig\hat{f}_{ABC}\hat{A}^m_B\hat{A}^n_C, \tag{4.90}$$

and following arguments similar to the ones given in the previous subsection, we can write the effective action as

$$S_{eff} = S[A,\varphi] + S[\hat{A},\hat{\varphi}] + S[A,\hat{A},\phi], \tag{4.91}$$

where $S[A,\varphi]$ is the right-hand side of (4.82), $S[\hat{A},\hat{\varphi}]$ is its hatted version and

$$\begin{aligned} S[A,\hat{A},\phi] \equiv \int \mathrm{d}^d x \Big\{ \frac{\hat{k}}{2}(D_m\phi^{aA})^2 + \frac{g\hat{k}}{3!} f_{abc}\hat{f}_{ABC}\phi^{aA}\phi^{bB}\phi^{cC} + \frac{g}{2}C_{\alpha ab}\varphi^\alpha\phi^{aA}\phi^{bA} \\ + \frac{g}{2}\hat{C}_{\hat{\alpha}AB}\hat{\varphi}^{\hat{\alpha}}\phi^{aA}\phi^{aB} + g\phi^{aA}F^{mn}_a\hat{F}_{mnA}\Big\}, \end{aligned} \tag{4.92}$$

where the covariant derivative of ϕ^{aA} with respect to both gauge fields is given by

$$D^m\phi^{aA} = \partial^m\phi^{aA} - igf_{abc}A^m_b\phi^{cA} - ig\hat{f}_{ABC}\hat{A}^m_B\phi^{aC}. \tag{4.93}$$

Thus we have found the complete effective action in the gauge sector of the model. Now we would like to make contact with the scalar extension of the $(DF)^2 + \mathrm{YM}$ theory which was introduced by Johansson and Nohle [26]. There, the group $\hat{G}$ (with indices $A, B, \ldots$) is viewed instead as a global symmetry group.[2] In the present chiral string formulation, we can turn off the gauge field $\hat{A}^m_A$ and the scalar $\hat{\varphi}^{\hat{\alpha}}$, effectively taking $S[\hat{A},\hat{\varphi}] \to 0$ and turning the group $\hat{G}$ into a global symmetry at tree level. Moreover, we are free to rescale the field ϕ in order to eliminate $\hat{k}$ from its kinetic term. However, a factor of $\lambda \equiv \sqrt{\hat{k}}$ would still be present in the cubic term (with $\lambda > 0$). After performing these modifications, we can finally write the effective Lagrangian in the same form as in [26]:

$$\begin{aligned} \mathcal{L}_{(DF)^2+\mathrm{YM}+\phi^3} \;=\; & \frac{1}{2}(D_nF^{mn}_a)^2 + \frac{1}{2}(D_m\varphi^\alpha)^2 + \frac{1}{2}(D_m\phi_{aA})^2 + \frac{1}{2}m^2(\varphi^\alpha)^2 + \frac{1}{4}m^2(F^{mn}_a)^2 \\ & + \frac{g}{3}F^3 + \frac{g}{2}C_{\alpha ab}\varphi^\alpha F^{mna}F^b_{mn} + \frac{g}{3!}d_{\alpha\beta\gamma}\varphi^\alpha\varphi^\beta\varphi^\gamma + \frac{g}{2}C_{\alpha ab}\varphi^\alpha\phi^{aA}\phi^{bA} \\ & + \frac{g\lambda}{3!}f_{abc}\hat{f}_{ABC}\phi^{aA}\phi^{bB}\phi^{cC}, \end{aligned} \tag{4.94}$$

where $m^2 = -4\mathcal{T}$.

[2] In the context of the double-copy construction found in [36], this would be the heterotic string group.

Chapter 5

Results and Discussion

An extensive study on ambitwistors models was presented. The first chapter (1) was dedicated to correctly compute the spectrum for the main ambitwistor models: bosonic, type II (both GSO sectors), and heterotic. By constructing the most general vertex operator we showed that these models (expected type II GSO(+)) contain higher derivative equations of motion. Even though higher derivative terms indicate non-unitary states, the result is consistent with the unusual momenta dependence of three-level amplitudes for bosonic ($A_3 \sim k^6 \leftrightarrow \Box^3 h_{mn}$) and spin-2 heterotic ($A_3 \sim k^4 \leftrightarrow \Box^2 h_{mn}$), necessary since ambitwistor does not have a dimensionful constant. In the chapter 2, we computed the cohomology for the heterotic system in the Ramond sector and confirmed the higher derivative terms for the fermions are also present. By constructing the free action, we prove the invariance under supersymmetry transformations. These results are interesting for a few reasons:

Loop: An advantage to described the CHY with ambitwistors is that it provides a natural extension to compute loop amplitudes as simply integrals over higher genus curves. In [37, 38] they provided a formula for 1-loop integrands for type II and super Yang-Mills. The resulting worldsheet can be viewed as a Riemann sphere with two points "glued" together where the loop momenta flows. In ten dimensions the loop amplitudes for these theories have UV divergences, only the integrand were computed. This introduces another difficulty since there is not a unique representation for the integrant. For type II the authors found the generalization of the scattering equations for genus one :

$$k_a \cdot P(z_a) = \frac{k_a \cdot \ell}{z_a} + \sum_{b \neq a} \frac{k_a \cdot k_b}{z_a - z_b} = 0, \tag{5.1}$$

where ℓ is interpreted as the *off-shell* momentum circling the loop. The delta function $\bar{\delta}(k_a \cdot P(z_a))$ that enforces the scattering equations localizes all integrals with exception of the loop $\int d\ell$. It is interesting to point out that this formula was soon interpreted as a forward limit of tree-level amplitudes by the same authors of CHY [39, 40]. One problem inherent to this formulation is to finding solutions to the one-loop scattering equations, which were already hard at the tree-level. Some solutions of (5.1) give rise to unphysical poles, however, using BCFW arguments these poles can be discarded [40].

Although the heterotic model correctly describes super Yang-Mills theory, the graviton spectrum contains non-unitary states. At tree-level, it is possible to recover the Yang-Mills amplitudes by extracting the single-trace amplitude, but at loop level, the single trace gluon amplitude receives contributions from internal supergravity states. This feature also appears in the $d = 4$ twistor string theory [14]. Thus the integrand conjectured in [37, 38] for Yang-Mills is given by replacing one of the Pfaffians (in the gravity integrand) with a Parke-Taylor factor. This trick is also used at tree-level and can be viewed as a realization of $gravity = (YM)^2$. Even though the type II is the only model that can compute 1-loop integrands, would be interesting to verify if the partition functions in [37, 38] reproduce the non-unitary states in the massless spectrum. Before performing the sum over spin structures, one should be able to observe in the partition function the contribution of the states in the GSO$(-)$ sector.

Action: As noted in [28], the $d = 10$ heterotic ambitwistor string has some similarities with the $d = 4$ twistor string which describes $\mathcal{N} = 4$ $d = 4$ conformal supergravity coupled to super Yang-Mills [14]. One could try to generalize the quadratic kinetic term computed here to the full string field theory action including interactions and check if describes a generalization of $N = 4$ $d = 4$ conformal supergravity.

In the last project we reexamined the bosonic chiral string, now in the sectorized interpretation, deriving a few novel results. The spectrum found here in the sectorized string, namely a massless level identical to that of the ordinary bosonic string and two traceless-symmetric fields $h_{\pm}^{mn}$ with mass-squared $m^2 = \pm 4\mathcal{T}$, is the same in the chiral string model [20]. Moreover, we showed that the extra (massive) states can be seen as auxiliary fields leading to the higher derivative gravity theory, which in the tensionless limit $(\mathcal{T} \to 0)$ reduces to the bosonic gravity in [13]. In [25] the massive spin-2 states were determined to be ghosts via a 4-point amplitude analysis based on a "twisted" KLT formula. This fact is manifest in the quadratic action we constructed.

Finally, we showed that the current algebra extension of the bosonic model effectively leads to the $(DF)^2 + \text{YM} + \phi^3$ Lagrangian of [26], with all its fields and couplings coming naturally from standard string (field) theory techniques. The emergence of the higher derivative term $(DF)^2$ from two vector fields of the physical spectrum is particularly interesting. In addition, we would like to point out that the group constants $C_{\alpha ab}$ and $d_{\alpha\beta\gamma}$, their relations and properties emerge naturally in our model and are valid for a generic level k of the algebra. In [26], on the other hand, such relations are obtained by demanding that the gluon amplitudes satisfy the Bern–Carrasco–Johansson relations [27] and our results agree when we take $k \to 0$. This limit corresponds to a projection to the single-trace amplitude sector, which is where we expect our results to match. The multitrace sector of the worldsheet model is "contaminated" by the gravity theory described in section 4.2, much like the Berkovits–Witten twistor string necessarily includes conformal gravity [2, 14].

Analytical continuation for the Chiral String: Even though a lot of development has been made, there are still open questions regarding these models. As mentioned before, the new chiral string introduced in [20], has a finite number of states in their spectrum and may contain massive states depending on the amount of supersymmetry. In this approach, the conformal gauge is adopted instead of the singular gauge HSZ and a new boundary condition is used. The fields now have holomorphic and antiholomorphic components and XX has nontrivial OPE (contrary to the ambitwistor models). The new boundary condition effectively changes the sign in the antiholomorphic piece in the propagator $X(z,\bar{z})X(0,0) \sim ln(z) - ln(\bar{z})$, and similar for the other antiholomorphic fields. This modification on the propagator was interpreted as Bogoliubov transformations, where the role of creation and annihilation are interchanged for the modes coming from $\bar{z}$, and also in [34] as a different choice of vacua.

Given the above consideration, this model is quite close to standard string theory, making the chiral model the best place to study its relationship with string theory. During my period at Stony Brook together with Warren Siegel, we began a project to tackle this problem. We introduced a tuning parameter into the propagator which allows us to go back and forth between string theory and chiral string:

$$X^m(z,\bar{z})X^n(y,\bar{y}) \sim -\frac{\alpha}{2}\eta^{mn}[ln(z-y) + f(\theta)ln(\bar{z}-\bar{y})] \tag{5.2}$$

where $f(\theta) = 1$ and -1 gives the ordinary string theory and chiral string theory respectively. Since, $f(\theta)$ can not be zero, one have to use a complex function to go from 1 to -1 without passing through zero. The same parameter $f(\theta)$ is introduced for other anti-holomorphic fields, like $\bar{b}, \bar{c}$ and $\bar{\psi}^m$. Then with this new propagator, in principle, one can compute tree-level amplitudes. The type II spectrum is massless in the chiral string and agrees with the low energy limit of the string theory. The vertex is identical to string theory, but the OPE's are different for the anti-holomorphic fields. For both three-point and four-point amplitudes the only thing that differs from string theory, modulo an overall factor of $f(\theta)$ in front, is the Koba-Nielsen factor. One can see this by realizing that all terms in the correlation function contain the same number of wick contraction, so same powers of $f(\theta)$. Since we are considering massless states, the kinematics $k_i \cdot k_j = 0$ implies that $\langle \prod_{i=1}^{3} e^{ik_i \cdot X_i} \rangle = 1$, so the three-point amplitude is identical to that of ordinary strings. After fixing $(z_1 = 0, z_2 = z, z_3 = 1, z_4 = \infty)$ the 4-point amplitude can be casted as

$$\mathcal{A}_4(s,t,u) = \int d^2 z z^{-s}(1-z)^{-t}\bar{z}^{-sf(\theta)}(1-\bar{z})^{-tf(\theta)} F[z,(1,2,3,4)]\bar{F}[\bar{z},(1,2,3,4)] \quad (5.3)$$

where $F[z,(1,2,3,4)]$ is a function of the kinematics, polarization vectors and integer powers of $z,(1-z)$. This factor is identical to ordinary string theory. We tried to solve this integral by two methods. First was using the KLT decomposition for closed string amplitude and the second was to rewrite (5.3) using a Melling transformation. Both results gave the same answer:

$$\mathcal{A} = K_0 \bar{K}_0 \frac{1}{stu} \frac{\Gamma(-f(\theta)u)\Gamma(-f(\theta)t)}{\Gamma(f(\theta)s)} \frac{\Gamma(-s)}{\Gamma(t)\Gamma(u)} \quad (5.4)$$

Note that this agrees with type II string theory, for $f(\theta) = 1$ you get kinematic function $(K_0\bar{K}_0)$ times the Virasoro-Shapiro-like factor. Also for $f(\theta) = -1$, all the massive poles in the gamma functions cancel, leaving only the massless term $1/stu$, as found in [20]. However, this answer is not *stu* symmetric for arbitrary $f(\theta)$, which is the biggest problem with our result, and unfortunately, we were unable to find a satisfactory *stu* symmetric amplitude. One can force the amplitude to be symmetric, by multiplying it with $\sin(\pi s)/\sin(\pi f(\theta)s)$. Of course, this is not a satisfactory solution, and more development is needed. The difficulty is to find the proper contour region of integration to derive a *stu* symmetric amplitude, and to justify the *anzats* (5.2). This is a rather ambitious project that might help find new interpretations of chiral string type models and understand how the transition between the string theory spectrum and the chiral model occurs.

Appendix A

Ramond sector, cocycles and Gamma matrices

Spinor indices in 10 dimensions can be distinguished between chiral and anti-chiral. We denote chiral indices by undotted greek letters, α, while anti-chiral indices are represented by dotted greek letters, $\dot{\alpha}$. Both run from 1 to 16. Spinor indices are 5-dimensional vector representations of $u(5)$:

$$\dot{\alpha}=\frac{1}{2}\begin{pmatrix}- & - & - & - & -\\ - & - & - & + & +\\ - & + & + & + & +\end{pmatrix} \quad \text{and} \quad \beta=\frac{1}{2}\begin{pmatrix}+ & + & + & + & +\\ + & + & + & - & -\\ + & - & - & - & -\end{pmatrix}. \tag{A.1}$$

where an anti-chiral index, $\dot{\alpha}$, must have an even number of plus signs, and a chiral index, β, must have an odd number of plus signs. Each of these combinations has 16 independent components represented as $\mathbf{16}=\mathbf{1}+\mathbf{10}+\mathbf{5}$.

A.1 The Ramond Sector.

The Ramond sector of the Ambitwistor string is defined by the antiperiodic boundary conditions of ψ^m:

$$\psi^m(e^{2\pi i}z)=-\psi^m(z). \tag{A.2}$$

We follow[30] and implement these boundary conditions via spin fields. That is, we have a conformal primary $S(z)$ that twists a periodic ψ:

$$\psi^m\left(z+(w-z)e^{2\pi i}\right)S(z)=-\psi^m\left(w\right)S(z). \tag{A.3}$$

This implies that a state $|\alpha\rangle$ created from the vacuum $|0\rangle$ via

$$|\alpha\rangle = S^{\alpha}(0)|0\rangle \tag{A.4}$$

should transform as a spacetime spinor. Notice that, due to the presence of S forcing ψ to be in the Ramond sector, this state must belong to an irreducible representation of the zero-mode Clifford algebra of ψ^m: $\{\psi_0^m, \psi_0^n\} = \eta^{mn}$, which implies

$$\psi_0^m|\alpha\rangle = \frac{1}{\sqrt{2}}\Gamma^{m\,\alpha}_{\dot{\beta}}|\dot{\beta}\rangle. \tag{A.5}$$

A.2 Bosonization and cocycles.

Because S^{α} twists the boundary conditions of ψ^m, the system is not free and OPE's are difficult to compute. Bosonization is a technique that allows us to deal with free fields only. Bosonization assigns for a pair of complex fermions one chiral boson, which means that we have to break manifest $so(10)$ invariance down to $u(5)$.

Spin Fields. The bosonization of spin fields is given by

$$S^{\alpha}(z) = \exp\Big(\alpha \cdot \phi(z)\Big)c_{\alpha} \tag{A.6}$$

where α is a chiral spinor index. The same expression is valid for anti-chiral spin fields by just replacing α for $\dot{\alpha}$. The factor c_{α} is a cocycle phase that guarantees the correct anticommutation relations.

Cocycles. The anticommuting fermionic algebra is reproduced in the bosonic system via the Baker-Campbell-Hausdorff formula:

$$e^{\phi(z)}e^{\pm\phi(z')} = e^{\pm\phi(z')}e^{\mp\phi(z')}e^{\phi(z)}e^{\pm\phi(z')} = -e^{\pm\phi(z')}e^{\phi(z)} \tag{A.7}$$

provided for $|z'| = |z|$ we have

$$\Big[\phi(z'), \phi(z)\Big] = \pm i\pi \quad \texttt{which implies} \quad \phi(z)\phi(0) \sim \ln z \tag{A.8}$$

Now, if we are given more than one pair of fermions, they won't naturally anticommute because $[\phi_i, \phi_j] = 0$. This is corrected by the introduction of cocycles[31]:

- Order all bosons of the theory: ϕ_i where $i = 1, \ldots, N$;
- Then multiply each exponential by a factor $(-)^{N_1+\ldots+N_{i-1}}$, where N_i is the fermion number operator:

$$N_i = -\oint \frac{\mathrm{d}z}{2\pi i}\,\overline{\psi}_i\psi_i = \oint \frac{\mathrm{d}z}{2\pi i}\,\partial\phi_i. \tag{A.9}$$

For example, if we consider two pairs of fermions, the bosonization becomes

$$\psi_1 = e^{\phi_1}, \quad \overline{\psi}_1 = e^{-\phi_1} \tag{A.10}$$

with

$$\psi_2 = e^{\phi_2}(-)^{N_1}, \quad \overline{\psi}_2 = e^{-\phi_2}(-)^{N_1} \tag{A.11}$$

where now ψ_1 and ψ_2 anticommute

$$e^{\phi_1}e^{\phi_2}(-)^{N_1} = e^{\phi_2}e^{\phi_1}(-)^{N_1} = e^{\phi_2}(-)^{N_1}(-)^{-N_1}e^{\phi_1}(-)^{N_1} = -e^{\phi_2}(-)^{N_1}e^{\phi_1} \tag{A.12}$$

provided

$$\left[N_i, e^{n\phi_j}\right] = n\delta_{ij}e^{n\phi_j}. \tag{A.13}$$

Thus, for more than one pair of fermions, we need to introduce the cocycle phase factors:

$$c_i = (-)^{N_1+\ldots+N_{i-1}}. \tag{A.14}$$

Consider the vector

$$\partial\phi = (N_1, N_2, \ldots, N_5) \tag{A.15}$$

then the cocycle factor can be written as

$$c_{\pm e_i} = \exp\left[\pm i\pi\langle e_i M \partial\phi\rangle\right] \tag{A.16}$$

where e_i is 1 in the ith component and zero elsewhere, $\langle\ \rangle$ is a matrix inner product and M is a lower triangular matrix with entries ± 1:

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 \end{pmatrix}.$$

The signs of M are arbitrary at this point, but they can be specified studying the charge conjugation matrix[31].

The cocycle factors of spin fields, c_α and $c_{\dot\alpha}$, are given the following expressions:

$$c_\alpha = \exp\left[i\pi\langle \alpha M \partial\phi\rangle\right] \quad \texttt{and} \quad c_{\dot\alpha} = \exp\left[i\pi\langle \dot\alpha M \partial\phi\rangle\right] \tag{A.17}$$

Gamma Matrices. To motivate the construction of gamma matrices and show how cocycles work, let us consider the OPE between ψ^i and S^α. Using expressions (3.4) and (A.6) we have to compute the OPE of $e^{\phi_i(z)}c_i$ with $e^{\alpha\phi(\mathrm{w})}c_\alpha$. Notice that c_i will pass through $e^{\alpha\phi}$ and due to Baker-Campbell-Hausdorff we obtain an extra phase:

$$c_i e^{\alpha\phi} = e^{i\pi\langle e_i M\partial\phi\rangle} e^{\alpha\phi} = e^{i\pi\langle e_i M \alpha\rangle} e^{\alpha\phi} c_i \tag{A.18}$$

so that our OPE becomes

$$e^{\phi_i(z)}c_i \; e^{\alpha\phi(\mathrm{w})}c_\alpha \sim (z-\mathrm{w})^{\alpha\cdot e_i}\, e^{i\pi\langle e_i M\alpha\rangle} e^{(e_i+\alpha)\phi} c_{i+\alpha}. \tag{A.19}$$

Notice that we obtain a branch-cut if $\alpha\cdot e_i = \alpha_i = -1/2$ which in turn implies that the sum $e_i+\alpha$ must be an anti-chiral index $\dot\beta$. Therefore given

$$e^{\phi_i(z)}c_i \; e^{\alpha\phi(\mathrm{w})}c_\alpha \sim (z-\mathrm{w})^{-1/2}\, e^{i\pi\langle e_i M\alpha\rangle} e^{\dot\beta\phi} c_{\dot\beta}, \tag{A.20}$$

we see that it becomes natural to define the gamma matrices as

$$(\Gamma^j)_{\dot\alpha}^{\ \beta} = \sqrt{2}\delta\left(e_j+\beta-\dot\alpha\right) e^{i\pi\langle e_j M\dot\alpha\rangle} \tag{A.21a}$$

and

$$(\Gamma^j)_{\alpha}^{\ \dot\beta} = \sqrt{2}\delta\left(e_j+\dot\beta-\alpha\right) e^{i\pi\langle e_j M\alpha\rangle} \tag{A.21b}$$

giving us the final result:

$$\psi^i(z)S^\alpha(\mathrm{w}) \sim \frac{1}{\sqrt{2}}\frac{\Gamma^{i\,\alpha}_{\ \dot\beta}S^{\dot\beta}(\mathrm{w})}{(z-\mathrm{w})^{1/2}}. \tag{A.22}$$

The explicit representation is written in terms of the Pauli-matrices via

$$\Gamma^{\pm e_j} = (\pm i)^{j-1}\sqrt{2}\left(\sigma^3\otimes\right)^{j-1}\sigma^{\mp}\left(\otimes 1\right)^{5-j} \tag{A.23}$$

and one can convert between $u(5)$ and covariant $so(10)$ using

$$\Gamma^{2j-1} = \frac{1}{\sqrt{2}}\left(\Gamma^{e_j} + \Gamma^{-e_j}\right) \tag{A.24a}$$

and

$$\Gamma^{2j} = \frac{i}{\sqrt{2}}\left(\Gamma^{e_j} - \Gamma^{-e_j}\right) \tag{A.24b}$$

Notice that in our construction, the notation γ^μ is reserved for the symmetric gamma matrices:

$$\gamma^\mu_{\alpha\beta} = \Gamma^{\mu\,\dot\beta}_{\alpha} C_{\dot\beta\beta} \tag{A.25a}$$

$$\gamma^{\mu\,\alpha\beta} = \Gamma^{\mu\,\alpha}_{\dot\beta} C^{\dot\beta\beta} \tag{A.25b}$$

as it is common in the literature. In above equations, C denotes the charge conjugation matrix which is the next topic in our discussion.

Charge Conjugation Matrix. We define C as

$$C^{\beta\dot\beta} = \delta\left(\beta + \dot\beta\right) e^{i\pi\beta M\dot\beta} \tag{A.26a}$$

and

$$C^{\dot\beta\beta} = -\delta\left(\dot\beta + \beta\right) e^{i\pi\dot\beta M\beta} \tag{A.26b}$$

and with these convetions we have $C^{\beta\dot\beta} = C^{\dot\beta\beta}$. These expressions can be motivated by studying the OPE of S^α and $S^{\dot\beta}$.

It is also common to use only undotted indices when describing spinors in 10d. Charge matrices act as metrics on the spinor space and can remove all dotted indices. For us all spinors are defined with upper indices and then anti-chiral ones are written as

$$S_\beta = C_{\beta\dot\beta} S^{\dot\beta}. \tag{A.27}$$

This notation is used together with the symmetric gamma representation.

Appendix B

Current algebra CFT

In this appendix we will discuss some general properties of the CFT of gauge sector of section 4.3.

As mentioned in the text, we are using the ordering prescription (4.47), which can be understood as the product of two operators $A(z)$ and $B(y)$ in the limit $z \to y$ with the removal of singular terms. Note that this prescription is neither commutative nor associative:

$$(A, B) \neq (B, A)\,, \tag{B.1}$$

$$((A, B), C) \neq (A, (B, C))\,. \tag{B.2}$$

The energy-momentum tensor of the algebra can be obtained using the Sugawara construction and it is defined by

$$T \equiv A\,(J_a, J_a)\,, \tag{B.3}$$

where A is a numerical constant to be determined by imposing the OPE

$$J_a(z)\,T(y) \sim \frac{J_a}{(z-y)^2}. \tag{B.4}$$

In order to do that, we can compute first

$$\begin{aligned} J_a(z)\;(J_b, J_c)\,(y) \quad \sim \quad & ik\frac{f_{abd}\delta_{dc}}{(z-y)^3} - f_{abd}f_{dce}\frac{J_e}{(w-y)^2} \\ & + \frac{k\delta_{ac}J_b}{(z-y)^2} + \frac{if_{acd}}{(z-y)}\,(J_b, J_d) \\ & + \frac{k\delta_{ab}J_c}{(z-y)^2} + \frac{if_{abd}}{(z-y)}\,(J_d, J_c)\,. \end{aligned} \tag{B.5}$$

It implies that

$$J_a(z)\,T(y) \sim 2Ak\frac{J_a}{(z-y)^2} + Af_{acd}f_{bcd}\frac{J_b}{(w-y)^2}. \tag{B.6}$$

Now we introduce the dual Coxeter number, g, defined through

$$f_{acd}f_{bcd} = 2g\delta_{ab}. \tag{B.7}$$

Therefore we can fix A to

$$A = \frac{1}{2(k+g)}. \tag{B.8}$$

Now we can compute the central charge of the model through the OPE

$$T(z)T(y) \sim \frac{c/2}{(z-y)^4} + \frac{2T}{(z-y^2)} + \frac{\partial T}{(z-y)}. \tag{B.9}$$

The result is

$$c = \frac{k\Delta}{(k+g)}. \tag{B.10}$$

This is the central charge of the gauge sector.

Building additional primary operators

One of the operators we need for the computation of 3-point amplitudes is related to the ordered product of two currents, (J_a, J_b). Observe, however, that this product is not symmetric. In fact, we can show that

$$(J_a, J_b) - (J_b, J_a) = if_{abc}\partial J_c. \tag{B.11}$$

Therefore, we can define the operator $J_{ab} = J_{ba}$ as

$$J_{ab} \equiv \tfrac{1}{2}(J_a, J_b) + \tfrac{1}{2}(J_b, J_a), \tag{B.12}$$

$$= (J_a, J_b) - \tfrac{i}{2}f_{ab}{}^{c}\partial J_c, \tag{B.13}$$

which can be further decomposed in two irreducible pieces: its trace, proportional to T, and a traceless part.

Observe that any rank two tensor T_{ab} can automatically generate a symmetric traceless tensor $T_{(ab)}$ via a multiplication by the projector

$$\delta_{(ab)(cd)} \equiv \tfrac{1}{2}\delta_{ac}\delta_{bd} + \tfrac{1}{2}\delta_{ad}\delta_{bc} - \tfrac{1}{\Delta}\delta_{ab}\delta_{cd}.$$

It acts as an identity operator for the indices (ab), as

$$\delta_{(ab)(ef)}\delta_{(ef)(cd)} = \delta_{(ab)(cd)}. \tag{B.14}$$

The pair (ab) is an explicit realization of the index α introduced in section (4.3), labeling the field φ_α of the vertex operator (4.50).

As it turns out, the symmetric traceless projection picks only the primary part of the operator (J_a, J_b):

$$T(z)\,\delta_{(ab)(cd)}\,(J_c, J_d)\,(y) \sim \delta_{(ab)(cd)}\frac{2\,(J_c, J_d)}{(z-y)^2} + \delta_{(ab)(cd)}\frac{\partial\,(J_c, J_d)}{(z-y)}. \tag{B.15}$$

This is the only dimension 2 primary operator that can be build out of the currents J_a. In addition, we will define the operator

$$\Delta_{(ab)(cd)} \equiv \tfrac{1}{2} f_{ade} f_{bce} + \tfrac{1}{2} f_{ace} f_{bde} - \tfrac{2g}{\Delta}\delta_{ab}\delta_{cd}, \tag{B.16}$$

which is also symmetric and traceless in the index pairs (ab) and (cd), and the power series

$$C_{(ab)(cd)} = \delta_{(ab)(cd)} - 2\sum_{n=1}^{\infty}(-1)^n\frac{(2n-2)!}{(8k)^n n!}\,(\Delta^n)_{(ab)(cd)}\,, \tag{B.17}$$

satisfying

$$C_{(ab)(ef)}C_{(ef)(cd)} = \delta_{(ab)(cd)} + \tfrac{1}{2k}\Delta_{(ab)(cd)}. \tag{B.18}$$

This is a realization of the Clebsch-Gordan coefficients, $C_{\alpha ab}$, introduced earlier. By construction,

$$\left(C^{-1}\right)_{(ab)(cd)} \equiv \delta_{(ab)(cd)} + \sum_{n=1}^{\infty}(-1)^n\frac{(2n)!}{(8k)^n n!}\,(\Delta^n)_{(ab)(cd)}\,. \tag{B.19}$$

Let us now define the dimension 2 primary operator

$$J_{(ab)} \equiv \left(C^{-1}\right)_{(ab)(cd)}(J_c, J_d)\,, \tag{B.20}$$

which satisfies the OPE

$$J_a(z)\,J_{(bc)}(y) \sim 2kC_{(ad)(bc)}\frac{J_d}{(z-y)^2} - (T_a)_{(bc)(de)}\frac{J_{(de)}}{(z-y)},$$

where

$$(T_a)_{(bc)(de)} \equiv -2i\left(C^{-1}\right)_{(bc)(fg)} f_{afh} C_{(gh)(de)} \tag{B.21}$$

Observe that $(T_a)_{(bc)(de)}$ constitutes a representation of the group generator, as

$$\begin{aligned} [T_a, T_b]_{(de)(fg)} &\equiv (T_a)_{(de)(hi)} (T_b)_{(hi)(fg)} - (T_b)_{(cd)(gh)} (T_a)_{(gh)(ef)} \, . \\ &= i f_{abc} (T_c)_{(de)(fg)} \, . \end{aligned} \tag{B.22}$$

In addition, it satisfies

$$(T_a T_a)_{(bc)(de)} = 2g\delta_{bd}\delta_{ce} + 2g\delta_{be}\delta_{cd} - f_{bdf} f_{cef} - f_{bef} f_{cdf}, \tag{B.23a}$$

$$(T_a T_b)_{(cd)(cd)} = 2g(\Delta + 2)\delta_{ab}. \tag{B.23b}$$

At the next conformal level, there are only two primary operators that can be build out of J_a, defined as

$$J_{[ab]} \equiv \tfrac{1}{2} (J_a, \partial J_b) - \tfrac{1}{2} (J_b, \partial J_a) - \tfrac{i}{3} f_{abc} \partial^2 J_c + iC f_{abc} (J_c, T) \, , \tag{B.24a}$$

$$J_{(abc)} = J_{abc} - C \left[\delta_{bc} (J_a, T) + \delta_{ac} (J_b, T) + \delta_{ab} (J_c, T) \right] , \tag{B.24b}$$

where

$$J_{abc} \equiv \tfrac{1}{3} (J_a, J_{bc}) + \tfrac{1}{3} (J_b, J_{ac}) + \tfrac{1}{3} (J_c, J_{ab}) \, , \tag{B.25a}$$

$$C = \frac{2(k+g)}{k\Delta + 2(k+g)}. \tag{B.25b}$$

They are naturally generated in the OPE algebra. For example,

$$\begin{aligned} J_{(bc)}(z) \, J_{(ad)}(y) \sim \; & 2k^2 \frac{\delta_{(ad)(bc)}}{(z-y)^4} + \tfrac{4k(k+g)}{\Delta} \delta_{(ad)(bc)} \left\{ \frac{2T}{(z-y)^2} + \frac{\partial T}{(z-y)} \right\} \\ & -k \, (T_e)_{(bc)(ad)} \left\{ \frac{2J_e}{(z-y)^3} + \frac{\partial J_e}{(z-y)^2} + \tfrac{1}{3} \frac{\partial^2 J_e}{(z-y)} \right\} \\ & + \tfrac{1}{2} \left(C^{-1} \right)_{(ad)(gh)} (T_g T_h)_{(bc)(ef)} \left\{ \frac{2J_{(ef)}}{(z-y)^2} + \frac{J_{(ef)}}{(z-y)^2} \right\} \\ & + D_{d(bc)(ef)} \frac{J_{(aef)}}{(z-y)} + D_{a(bc)(ef)} \frac{J_{(def)}}{(z-y)} \\ & + D_{(bc)(ad)[ef]} \frac{J_{[ef]}}{(z-y)} + E_{e(bc)(ad)} \frac{(J_e, T)}{(z-y)}, \end{aligned} \tag{B.26}$$

where $D_{a(bc)(de)}$, $D_{(ab)(cd)[ef]}$ and $E_{a(bc)(de)}$ are given in terms of the structure constants of the group, but their precise expression will not be needed here. The OPE above was presented in the main text with the indices α, β in equation (4.60c).

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