INDUCED GLUON RADIATION OF HIGH ENERGY QUARK IN FINITE-SIZE QCD MEDIUM

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Abstract

We study the induced gluon radiation of a high energy quark in a finite-size QCD medium. For a sufficiently energetic quark produced inside a medium we find the radiative energy loss $\Delta E_q \propto L^2$, where L is the distance passed by quark in the medium. In this regime ΔE_q has a weak dependence on the initial quark energy E_q. It is dominated by the gluon radiation with Feynman x close to zero and unity. The L^2 dependence turns to L^1 as the quark energy decreases. Numerical calculations are performed for a cold nuclear matter and a hot quark-gluon plasma. For a quark incident on a nucleus we predict $\Delta E_q \approx 0.1 E_q (L/10\,\mathrm{fm})^\beta$, with β close to unity.

In the recent work[1] we developed a new path integral approach to the Landau-Pomeranchuk-Migdal effect[2,3] in QED and QCD. Here we report on the evaluation within our technique of the radiative energy loss of a fast quark, ΔE_q, propagating through a finite-size QCD medium.[4] We consider both a cold nuclear matter and a hot quark-gluon plasma (QGP). Following previous works[5-7] we model QGP by a system of static scattering centres described by the Debye screened potential $\propto \exp(-r\mu_D)/r$, where μ_D is the color screening mass. For the screening mass we use perturbative formula $\mu_D = (1+n_F/6)^{1/2} g_s T$, where $g_s = \sqrt{4\pi\alpha_s}$ is the QCD coupling constant, T is the temperature of QGP. We assume that a fast quark produced at $z=0$ through a hard mechanism propagates in a medium of extent L along z axis.

Neglecting the multigluon emission the radiative energy loss can be written as

$$\Delta E_q = E_q \int\limits_0^1 dx x \frac{dP}{dx}\,, \tag{1}$$

where E_q is the initial quark energy, x is the Feynman variable for the radiated gluon, and dP/dx is the probability of gluon radiation as function of x. In the approach of Ref. 1 an evaluation of dP/dx is reduced to solving a two-dimensional Schrödinger equation in the impact parameter space. The longitudinal coordinate z plays the role of time. This Schrödinger equation describes evolution of the light-cone wave function of a spurious three-body $q\bar{q}g$ color singlet system. The relative positions of the constituents of the $q\bar{q}g$ system in the impact parameter space are $\boldsymbol{\rho}_q = -\boldsymbol{\rho}x$, $\boldsymbol{\rho}_{\bar{q}} = 0$, $\boldsymbol{\rho}_g = (1-x)\boldsymbol{\rho}$. The corresponding Hamiltonian has the form

$$H = \frac{\mathbf{p}^2}{2\mu(x)} + v(\boldsymbol{\rho}, z)\,, \tag{2}$$

$$v(\boldsymbol{\rho}, z) = -i\frac{n(z)\sigma_3(\rho, x)}{2}\,. \tag{3}$$

Here $\mu(x) = E_q x(1-x)$ is the reduced "Schrödinger mass", $n(z)$ is the medium density, and $\sigma_3(\rho, x)$ is the cross section of interaction of the $q\bar{q}g$ system with a medium constituent (color centre for QGP and nucleon for nuclear matter). In the case of QGP on the rhs of (3) summation over triplet (quark) and octet (gluon) color states is implicit.

In order to simplify the analysis we neglect the $q \to qg$ spin-flip transitions which give a small contribution to the energy loss. Then the radiation rate is given by[1]

$$\frac{dP}{dx} = 2\mathrm{Re}\int\limits_0^\infty d\xi_1 \int\limits_{\xi_1}^\infty d\xi_2 \exp\left[-\frac{i(\xi_2-\xi_1)}{L_f}\right] g(\xi_1,\xi_2,x)\left[K(0,\xi_2|0,\xi_1) - K_v(0,\xi_2|0,\xi_1)\right]\,. \tag{4}$$

Here the generalization of the QED vertex operator of Ref. 1 to QCD reads

$$g(\xi_1,\xi_2,x) = \frac{\alpha_s[4-4x+2x^2]}{3x}\cdot\frac{\mathbf{p}(\xi_2)\cdot\mathbf{p}(\xi_1)}{\mu^2(x)}\,, \tag{5}$$

K is the Green's function for the Hamiltonian (2), K_v is the vacuum Green's function, $L_f = 2E_q x(1-x)/[m_q^2 x^2 + m_g^2(1-x)]$ is the so called gluon formation length (time), m_q is the quark mass and m_g is the mass of radiated gluon. The latter plays the role of an infrared cutoff removing contribution of the long-wave gluon excitations which cannot be treated perturbatively. In contrast to the expression for the bremsstrahlung spectrum for an electron incident on a target of Ref. 1, in which the integration over ξ_1 starts from $-\infty$, in (4) we integrate over ξ_1 from $\xi_1 = 0$, i.e. from the point where a fast quark is produced by hard scattering.

The three-body cross section entering the imaginary potential (3) can be expressed in terms of the dipole cross section for color singlet $q\bar{q}$ pair, $\sigma_2(\rho)$,[8)]

$$\sigma_3(\rho, x) = \frac{9}{8}[\sigma_2(\rho) + \sigma_2((1-x)\rho))] - \frac{1}{8}\sigma_2(x\rho)\,. \tag{6}$$

The radiation rate is dominated by the contribution from $\rho \lesssim 1/m_g$,[1)] where $\sigma_2(\rho) = C_2(\rho)\rho^2$ and $C_2(\rho)$ has a smooth (logarithmic) dependence on ρ.[9,8)] This allows one to estimate the energy loss replacing $C_2(\rho)$ by $C_2(1/m_g)$. Then $\sigma_3(\rho,x) = C_3(x)\rho^2$, with $C_3(x) = \{9[1 + (1-x)^2] - x^2\}C_2(1/m_g)/8$, and the Hamiltonian (1) takes the oscillator form with the frequency

$$\Omega = \frac{(1-i)}{\sqrt{2}}\left(\frac{n(z)C_3(x)}{\mu(x)}\right)^{1/2} = \frac{(1-i)}{\sqrt{2}}\left(\frac{n(z)C_3(x)}{E_q x(1-x)}\right)^{1/2}.$$

Making use of the oscillator Green's function after some algebra one can represent the bremsstrahlung rate (4) in the form

$$\frac{dP}{dx} = Ln\frac{d\sigma^{BH}}{dx}S(\eta, l)\,, \tag{7}$$

where

$$\frac{d\sigma^{BH}}{dx} = \frac{4\alpha_s C_3(x)(4 - 4x + 2x^2)}{9\pi x[m_q^2 x^2 + m_g^2(1-x)]}\,, \tag{8}$$

is the Bethe-Heitler cross section. The suppression factor $S(\eta, l)$, depending on the dimensionless variables

$$\eta = L_f|\Omega| = \frac{[4nC_3(x)E_q x(1-x)]^{1/2}}{m_q^2 x^2 + m_g^2(1-x)}\,, \tag{9}$$

$$l = L/L_f = \frac{L[m_q^2 x^2 + m_g^2(1-x)]}{2E_q x(1-x)}\,, \tag{10}$$

is given by

$$S(\eta, l) = S^{(1)}(\eta, l) + S^{(2)}(\eta, l)\,, \tag{11}$$

$$S^{(1)}(\eta, l) = \frac{3}{l\eta^2}\mathrm{Re}\int\limits_0^{l\eta} dy_1 \int\limits_0^{y_1} dy_2 \exp\left(-\frac{iy_2}{\eta}\right)\left\{\frac{1}{y_2^2} - \left[\frac{\phi}{\sin(\phi y_2)}\right]^2\right\}\,, \tag{12}$$

$$S^{(2)}(\eta, l) = \frac{3}{l\eta^2}\mathrm{Re}\int\limits_0^{l\eta} dy_1 \int\limits_0^{\infty} dy_2 \exp\left[-\frac{i(y_1 + y_2)}{\eta}\right]$$
$$\times\left\{\frac{1}{(y_1 + y_2)^2} - \left[\frac{\phi}{\cos(\phi y_1)\,(\tan(\phi y_1) + \phi y_2)}\right]^2\right\}\,, \tag{13}$$

with $\phi = \Omega/|\Omega| = \exp(-i\pi/4)$. The two terms on the rhs of (11) correspond in (4) to the contributions from the integration regions $\xi_1 < \xi_2 < L$ and $\xi_1 < L < \xi_2$, respectively. The variables in (12), (13) in terms of those in (4) are $y_1 = (L - \xi_1)|\Omega|$, $y_2 = (\xi_2 - \xi_1)|\Omega|$ (in (12)) and $y_2 = (\xi_2 - L)|\Omega|$ (in (13)). In arriving at (13) we have used representation of the first Green's function in the square brackets in (4) through convolution of the oscillator Green's function (for the interval (ξ_1, L)) and the vacuum one (for the interval (L, ξ_2)).

In a medium it is either L_f or $1/|\Omega|$ which sets the effective medium-modified formation length $L_f' = \min(L_f, 1/|\Omega|)$, which is the typical value of $\xi_2 - \xi_1$ in (4) for $L \gg L_f'$. The finite-size effects come into play only at $L \lesssim L_f'$, i.e. $l \lesssim l_0 = \min(1, 1/\eta)$. From (11)-(13) we find $S(\eta, l) \approx -l^2 \log l$ as $l \to 0$. The source of this suppression of radiation at small L is obvious: the energetic quark produced through a hard mechanism loses soft component of its gluon cloud and radiation at distances shorter than the time required for regeneration of the quark gluon field turns out to be suppressed. For $l \gg l_0$ $S(\eta, l)$ is reduced to that for the infinite medium. At large and small η it can be approximated as $S(\eta, l = \infty) \approx 3/\eta\sqrt{2}$ ($\eta \gg 1$) and $S(\eta, l = \infty) \approx 1 - 16\eta^4/21$ ($\eta \ll 1$).[1] Notice, that according to (9), (10) $\eta \to 0$ and $l \to \infty$ as $x \to 0, 1$ and the Bethe-Heitler regime takes place in these limits.

Before presenting the numerical result, let us consider the energy loss at a qualitative level. We begin with the case of a sufficiently large E_q such that the maximum value of L_f', $L_f'(max)$, is much bigger than L. Taking into account the finite-size suppression of radiation at $L_f' \gtrsim L$, we find that ΔE_q is dominated by the contribution from two narrow regions of x: $x \lesssim \delta_g \approx Lm_g^2/2l_0E_q$ and $1 - x \lesssim \delta_q \approx Lm_q^2/2l_0E_q$. In both the regions the finite-size effects are marginal and the energy loss can be estimated using the infinite medium suppression factor. For instance,

$$\Delta E_q(x \lesssim \delta_g) \sim \frac{16\alpha_s C_3(0) E_q L n}{9\pi m_g^2} \int\limits_0^{\delta_g} dx S(\eta(x), l = \infty)\,. \tag{14}$$

Using (9) one can show that $\eta(x \lesssim \delta_g) \lesssim 1$ at $L \lesssim m_g^2/2nC_3(0)$. In this region of L in (14) we can put $S(\eta(x), l = \infty) \approx 1$ and find $\Delta E_q \sim 0.25\alpha_s C_3(0) n L^2$, which does not depend on the quark energy. At $L \gg m_g^2/2nC_3(0)$ the typical values of η in (14) are much bigger than unity, and using the asymptotic formula for the suppression factor we obtain $\Delta E_q \sim \alpha_s C_3(0) n L^2$. Similar analysis for x close to unity gives the contribution to ΔE_q suppressed by the factor $\sim 1/4$ as compared to that for small x. Notice that in this L^2 regime, despite the $1/m_{g,q}^2$ infrared divergence of the Bethe-Heitler cross section, ΔE_q has only a smooth m_g-dependence originating from the factor C_3. We emphasize that the above analysis of the origin of the leading contributions makes it evident that L^2 dependence of ΔE_q cannot be regarded as a consequence of the Landau-Pomeranchuk-Migdal suppression of the radiation rate due to small angle multiple scatterings.

The finite-size effects can be neglected and ΔE_q becomes proportional to L if $L_f'(max) \ll L$. If in addition the typical values of η are much bigger than unity, from (1), (7), (8) along with the asymptotic form of $S(\eta, l = \infty)$ at $\eta \gg 1$ one can obtain the following infrared stable result $\Delta E_q \approx 1.1\alpha_s L\sqrt{nC_3(0)E_q}$.

In numerical calculations we take $m_g = 0.75$ GeV. This value of m_g was obtained from the analysis of HERA data on structure function F_2 within the dipole approach to the BFKL equation.[10,11] For scattering of the $q\bar{q}g$ system on a nucleon, we find from the double gluon model[9] $C_2(1/m_g) \sim 1.3-4$ where the lower and upper bounds correspond to the t-channel gluon propagators with mass 0.75 and 0.2 GeV, respectively. The latter choice allows one to reproduce the dipole cross section extracted from the data on vector meson electroproduction.[12] However, there is every indication[10,11] that a considerable part of the dipole cross section obtained in Ref. 12 comes from the nonperturbative effects for which our approach is not justified. For this reason we take $C_2(1/m_g) = 2$ which seems to be plausible estimate for the perturbative component of the dipole cross section.[10] For QGP at $T = 250$ MeV the double gluon formula with the Debye screened gluon exchanges gives $C_2(1/m_g) \approx 0.5$ for triplet centre. For octet centre the result is $C_A/C_F = 9/4$ times larger, here $C_A(C_F)$ is the octet(triplet) second-order Casimir invariant. For quark mass, controlling the transverse size of the $q\bar{q}g$ system at $x \approx 1$, we take $m_q = 0.2$ GeV. Notice that our prediction for ΔE_q is insensitive to the value of m_q.

We calculate ΔE_q for nuclear matter taking $n = 0.15$ fm^{-3} and $\alpha_s = 1/2$. For QGP at $T = 250$ MeV we take $\alpha_s = 1/3$. In the region $L \lesssim 5$ fm the numerical results can be parametrized in the form $\Delta E_q \approx D(L/5\,\mathrm{fm})^\beta$. The D and β as functions of E_q are shown in Fig. 1 (nuclear matter) and Fig. 2 (QGP). To illustrate the m_g-dependence of the predictions, besides the results for $m_g = 0.75$ GeV (solid curve), we also show in Figs. 1, 2 the results for $m_g = 0.375$ GeV (dashed curve). In the region $5 \lesssim L \lesssim 10$ fm β is by 10-20 % smaller than

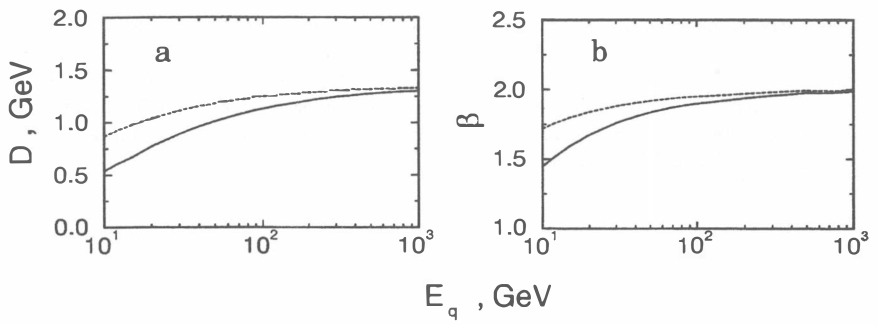

Figure 1: The parameters D (a) and β (b) for the parametrization $\Delta E_q \approx D(L/5\,\mathrm{fm})^\beta$ for nuclear matter. The solid lines correspond to $m_g = 0.75$ GeV, and the dashed ones to $m_g = 0.375$ GeV.

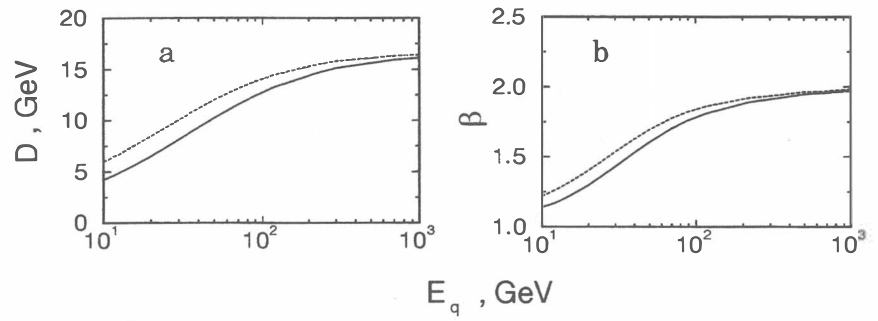

Figure 2: The same as in Fig. 1 but for QGP at $T = 250$ MeV.

for $L \lesssim 5$ fm. Notice that $L_f'(max) \sim 5 - 10$ fm for $E_q \sim 10 - 40$ GeV in nuclear matter, and for $E_q \sim 150 - 600$ GeV in QGP. From Figs. 1, 2 one can conclude that the onset of L^2 regime takes place when $L_f'(max)/L \gtrsim 2$. The closeness of β to unity at $E_q = 10$ GeV for QGP agrees with a small value of $L_f'(max)$ (~ 1 fm). The m_g-dependence of ΔE_q becomes weak at $E_q \gtrsim 50$ GeV. However, it is sizeable for $E_q \sim 10 - 20$ GeV. Our predictions for ΔE_q must be regarded as rough estimates with uncertainties of at least a factor of 2 in either direction. Nonetheless rather large values of ΔE_q obtained for QGP indicate that the jet quenching may be an important potential probe for formation of the deconfinement phase in AA collisions.

We also studied the energy loss of a fast quark incident on a target. In this case the radiation by initial quark is allowed and the lower limit of integration over ξ_1 in (4) must be replaced by $-\infty$. For this situation, as in the case of QED,[13] after expanding the medium Green's function in a series in the potential the spectrum can be represented as a sum of the Bethe-Heitler term and an absorptive correction. For our choice of the gluon mass the absorptive correction is relatively small. This means that $\Delta E_q \propto E_q L n \alpha_s C_3(0)/m_g^2$. For nuclear matter in the region $L \lesssim 10$ fm the numerical calculations give $\Delta E_q \approx 0.1 E_q(L/10\,\mathrm{fm})^\beta$ with $\beta \approx 0.9-1$ for $E_q \lesssim 50$ GeV and $\beta \approx 0.85-0.9$ for $E_q \gtrsim 200$ GeV. This result differs drastically from prediction of Brodsky and Hoyer[14] $\Delta E_q \approx 0.25(L/1\,\mathrm{fm})$ GeV. Our estimate is in a qualitative agreement with the longitudinal energy flow measured in hard pA collisions with dijet final state[15] and the energy loss obtained from the analysis of the inclusive hadron spectra in hA interactions.[16]

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