# Nonlinear supersymmetric general relativity and origin of mass

Kazunari Shima[1] and Motomu Tsuda[2]

*Laboratory of Physics, Saitama Institute of Technology*
*Fukaya, Saitama 369-0293, Japan*

## Abstract

We explain the relation between the large scale structure (the cosmology) and the low energy particle physics, e.g. the observed mysterious relation between the (dark) energy density of the universe and the neutrino mass, which gives a new insight into the origin of mass, based upon nonlinear supersymmetric general relativity towards the unity of nature beyond (behind) the standard model.

*Nonlinear supersymmetric general relativity (NLSUSY GR)* [1], which is based upon the general relativity (GR) principle and the nonlinear (NL) representation [2] of supersymmetry (SUSY) [3, 4], proposes a new paradigm called the SGM (*superon-graviton model*) scenario [1, 5, 6] for the unified description of space-time and matter beyond (behind) the standard model.

The NLSUSY [2] is known as a symmetry which represents a priori *spontaneous SUSY breaking (SSB)* and the basic NLSUSY action [2] is described in terms of only spin-1/2 Nambu-Goldstone (NG) massless fermions. Also, the NLSUSY model is recasted (related) rigorously to various linear (L) SUSY theories with the SSB (*NL/L SUSY relation*), which has been shown by many authors in the various cases [7]-[12].

In NLSUSY GR, a new (generalized) space-time, *SGM space-time* [1], is introduced, where the tangent space-time has the NLSUSY structure, i.e. it is specified not only by the $SO(3,1)$ Minkowski coodinates $x_a$ but also by $SL(2,C)$ Grassmann coordinates $\psi_\alpha^i$ $(i = 1, 2, \cdots, N)$ for NLSUSY. The new Grassmann coordinates in the new (SGM) space-time means the coset parameters of $\frac{superGL(4R)}{GL(4R)}$ which can be interpreted as the NG fermions (*superons*) associated with the spontaneous breaking of super-$GL(4R)$ down to $GL(4R)$. The fundamental action in NLSUSY GR is given in the Einstein-Hilbert (EH) form in SGM space-time by extending the geometrical arguments of GR in Riemann space-time, which has a priori promising large symmetries isomorphic to $SO(10)$ $(SO(N))$ super-Poincaré (SP) group [5].

The SSB in NLSUSY GR due to the NLSUSY structure is interpreted as the phase transition of SGM space-time to Riemann space-time with massless superon (fermionic matter), i.e. *Big Decay* [6, 11] which subsequently ignites the Big Bang and the inflation of the present universe. In the SGM scenario all (observed) particles are assigned uniquely into a single irreducible representation of $SO(N)$ $(SO(10))$ SP group as an on-shell supermultiplet of $N$ LSUSY. And they are considered to be realized as (massless) eigenstates of $SO(N)$ SP composed of $N$ NG fermion-superons through the NL/L SUSY relation after Big Decay.

Since the cosmological term in NLSUSY GR gives the NLSUSY model [2] in *asymptotic* Riemann-flat (an ordinary vierbein $e^a{}_\mu \to \delta^a{}_\mu$) space-time, the scale of the SSB in NLSUSY GR induces (naturally) a fundamental mass scale depending on the cosmological constant and through the NL/L SUSY relation it gives a simple explanation of the mysterious (observed) numerical relation between the (four dimensional) dark energy density of the universe and the neutrino mass [6] in the vacuum of the $N = 2$ SUSY QED theory (in two-dimensional space-time $(d = 2)$ for simplicity) [13].

In order to explain the above low energy physics in NLSUSY GR, i.e. *the relation between the large scale structure and the low energy particle physics*, let us begin with the fundamental EH-type action of NLSUSY GR in SGM space-time given by [1]

$$L_{\rm NLSUSYGR}(w) = \frac{c^4}{16\pi G}|w|\{\Omega(w) - \Lambda\}, \tag{1}$$

[1] E-mail:shima@sit.ac.jp
[2] E-mail:tsuda@sit.ac.jp

where $G$ is the Newton gravitational constant, $\Lambda$ is a (*small*) cosmological constant, $\Omega(w)$ is the the unified scalar curvature in terms of the unified vierbein $w^a{}_\mu(x)$ (and the inverse $w_a{}^\mu$) defined by

$$w^a{}_\mu = e^a{}_\mu + t^a{}_\mu(\psi), \quad t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^i\gamma^a\partial_\mu\psi^i - \partial_\mu\bar{\psi}^i\gamma^a\psi^i), \tag{2}$$

and $|w| = \det w^a{}_\mu$. In Eq.(2), $e^a{}_\mu$ is the ordinary vierbein of GR for the local $SO(3,1)$, $t^a{}_\mu(\psi)$ is the stress-energy-momentum tensor (i.e. the mimic vierbein) of the NG fermion $\psi^i(x)$ for the local $SL(2,C)$ and $\kappa$ is an arbitrary constant of NLSUSY with the dimemsion $(\text{mass})^{-2}$. Note that $e^a{}_\mu$ and $t^a{}_\mu(\psi)$ contribute equally to the curvature of space-time, which may be regarded as the Mach's principle in ultimate space-time.

The NLSUSY GR action (1) possesses promissing large symmetries isomorphic to $SO(N)$ ($SO(10)$) SP group [5]; namely, $L_{\rm NLSUSYGR}(w)$ is invariant under

$$[\text{new NLSUSY}] \otimes [\text{local GL(4,R)}] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation}] \\ \otimes [\text{global SO(N)}] \otimes [\text{local U(1)}^N].$$

Note that the no-go theorem is overcome (circumvented) in a sense that the nontivial $N$-extended SUSY gravity theory with $N > 8$ has been constructed in the NLSUSY invariant way.

The SGM (*empty*) space-time for *everything* described by the (*vacuum*) EH-type NLSUSY GR action (1) is unstable due to NLSUSY structure of tangent space-time and decays spontaneously to Riemann space-time with the NG fermion-superons (matter) described by the ordinary EH action with the cosmological term, the NLSUSY action for the $N$ NG fermions and their gravitational interactions, i.e. by the following SGM action;

$$L_{\rm SGM}(e,\psi) = \frac{c^4}{16\pi G} e|w_{\rm VA}|\{R(e) - \Lambda + T(e,\psi)\}, \tag{3}$$

where $R(e)$ is the scalar curvature of ordinary EH action, $T(e,\psi)$ represents highly nonlinear gravitational interaction terms of $\psi^i$, and $|w_{\rm VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$ is the determinant in the NLSUSY model [2]. The second cosmological term in the action (3) reduces to the NLSUSY action [2], $L_{\rm NLSUSY}(\psi) = -\frac{1}{2\kappa^2}|w_{\rm VA}|$, i.e. the arbitrary constant $\kappa$ of NLSUSY is now fixed to

$$\kappa^{-2} = \frac{c^4\Lambda}{8\pi G} \tag{4}$$

in Riemann-flat $e_a{}^\mu(x) \to \delta^\mu_a$ space-time. Note that the NLSUSY GR action (1) and the SGM one (3) possess different asymptotic flat space-time, i.e. SGM-flat $w_a{}^\mu \to \delta^\mu_a$ space-time and Riemann-flat $e_a{}^\mu \to \delta_a{}^\mu$ space-time, respectively. The scale of the SSB in NLSUSY GR (Big Decay) induces a fundamental mass scale depending on the $\Lambda$ through the relation (4).

It is interesting and important to investigate the low energy physics of NLSUSY GR through the NL/L SUSY relation. In asymptotic Riemann-flat space-time, we focus below on the relation between the NLSUSY model and a LSUSY QED theory for the minimal and realistic $N = 2$ [10] SUSY (in the $d = 2$ case for simplicity) [11, 12]; namely,

$$L_{N=2{\rm SGM}}(e,\psi) \stackrel{e^a{}_\mu \to \delta^a_\mu}{\longrightarrow} L_{N=2{\rm NLSUSY}}(\psi) = L_{N=2{\rm SUSYQED}}(V,\Phi) + [\text{tot. der. terms}]. \tag{5}$$

In the relation (5), the $N = 2$ NLSUSY action $L_{N=2{\rm NLSUSY}}(\psi)$ for the two (Majorana) NG-fermion superons $\psi^i$ $(i = 1,2)$ is written in $d = 2$ as follows;

$$\begin{aligned} L_{N=2{\rm NLSUSY}}(\psi) &= -\frac{1}{2\kappa^2}|w_{\rm VA}| = -\frac{1}{2\kappa^2}\left\{1 + t^a{}_a + \frac{1}{2!}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a)\right\} \\ &= -\frac{1}{2\kappa^2}\left\{1 - i\kappa^2\bar{\psi}^i\partial\!\!\!/\psi^i - \frac{1}{2}\kappa^4(\bar{\psi}^i\partial\!\!\!/\psi^i\bar{\psi}^j\partial\!\!\!/\psi^j - \bar{\psi}^i\gamma^a\partial_b\psi^i\bar{\psi}^j\gamma^b\partial_a\psi^j)\right\}, \end{aligned} \tag{6}$$

where $\kappa$ is a constant with the dimension $(\text{mass})^{-1}$, which satisfies the relation (4) in the $d = 4$ case.

On the other hand, in Eq.(5), the $N = 2$ LSUSY QED action $L_{N=2{\rm SUSYQED}}(V,\Phi)$ is constructed from a $N = 2$ minimal off-shell vector supermultiplet $V$ and a $N = 2$ off-shell scalar one $\Phi$. Indeed, the most general $L_{N=2\mathrm{SUSYQED}}(V,\Phi)$ in $d=2$ with a Fayet-Iliopoulos $D$ term and Yukawa interactions, is given in the explicit component form as follows for the massless case;

$$\begin{aligned} L_{N=2\mathrm{SUSYQED}}(V,\Phi) \ = \ & -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D \\ & + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2 \\ & + f(A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j - A^2 D + \phi^2 D + \epsilon^{ab}A\phi F_{ab}) \\ & + e\Big\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \bar{\lambda}^i \chi B^i + \epsilon^{ij}\bar{\lambda}^i \nu B^j \\ & - \frac{1}{2}D(B^i)^2 + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5\nu \Big\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2. \end{aligned} \qquad (7)$$

where $(v^a, \lambda^i, A, \phi, D)$ $(F_{ab} = \partial_a v_b - \partial_b v_a)$ is the $V$ containing $v^a$ for a $U(1)$ vector field, $\lambda^i$ for doublet (Majorana) fermions and $A$ for a scalar field in addition to $\phi$ for another scalar field and $D$ for an auxiliary scalar field, while $(\chi,\ B^i,\ \nu,\ F^i)$ is the $\Phi$ containing $(\chi, \nu)$ for two (Majorana) are fermions, $B^i$ for doublet scalar fields and $F^i$ for auxiliary scalar fields. Also $\xi$ is an arbitrary dimensionless parameter giving a magnitude of SUSY breaking mass, and $f$ and $e$ are Yukawa and gauge coupling constants with the dimension $(\mathrm{mass})^1$ (in $d=2$), respectively. The $N=2$ LSUSY QED action (7) can be rewritten as the familiar manifestly invariant form under the local $(U(1))$ gauge transformation in the superfield formulation (for further details see Ref.[12]).

In the relation (equivalence) of the two theories (5), the component fields of $(V,\Phi)$ in the $N=2$ LSUSY QED action (7) are expanded as composites of the NG fermions $\psi^i$, i.e. as *SUSY invariant relations*,

$$(V,\Phi) \sim \xi\kappa^{n-1}(\psi^i)^n |w_{\mathrm{VA}}| + \cdots \ \ (n=0,1,2), \qquad (8)$$

where $(\psi^i)^2 = \bar{\psi}^i\psi^j, \epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j, \epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j$, which are very promissing features for SGM scenario. The explicit form [11] of the SUSY invariant relations (8) are obtained *systematically* in the superfield formulation (for example, see Refs.[7, 9, 12]) and the familiar LSUSY transformations on the component fields of the supermultiplet are reproduced in terms of the NLSUSY transformations on the $\psi^i$ contained. Note that *a four NG-fermion self-interaction term (i.e. the condensation of $\psi^i$)* appears only in the auxiliary fields $F^i$ of the scalar supermultiplet $\Phi$ as the origin of the familiar local $U(1)$ gauge symmetry of LSUSY theory [11, 12]. Is the condensation of NG-fermion superons the origin of the local $U(1)$ gauge interaction? The relation (5) are shown explicitly (and systematically) by substituting Eq.(8) into the LSUSY QED action (7) [11, 12].

Now we briefly show the (physical) vacuum structure of $N=2$ LSUSY QED action (7) related (equivalent) to the $N=2$ NLSUSY action (6) [13]. The vacuum is determined by the minimum of the potential $V(A,\phi,B^i,D)$ in the action (7), which is given by using the equation of motion for the auxiliary field $D$ as

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2 - \phi^2 + \frac{e}{2f}(B^i)^2 + \frac{\xi}{f\kappa}\right\}^2 + \frac{1}{2}e^2(A^2+\phi^2)(B^i)^2 \geq 0, \qquad (9)$$

In the potential (9) the configurations of the fields corresponding to vacua in $(A,\phi,B^i)$-space, which are $SO(1,3)$ or $SO(3,1)$ invariant, are classified according to the signatures of the parameters $e, f, \xi, \kappa$. The particle (mass) spectra are obtained by expanding the field $(A,\phi,B^i)$ around the vacua. We have found that two different vacua appear in the $SO(3,1)$ isometry [13], one of which are described by means of the resulting (physical) model with

one charged Dirac fermion $({\psi_D}^c \sim \chi + i\nu)$, one neutral (Dirac) fermion $({\lambda_D}^0 \sim \lambda^1 - i\lambda^2)$,
one massless vector (a photon) $(v_a)$,
one charged scalar $(\phi^c \sim \theta + i\varphi)$, one neutral complex scalar $(\phi^0 \sim \rho + i\omega)$,

which are the composites of NG-fermion superons and the vacuum breaks SUSY spontaneously (the local $U(1)$ is not broken) (for further detailes, e.g. mass spectra, $\cdots$ etc., see [13]).

As for the cosmological significances of $N=2$ SUSY QED in SGM scenario, the (physical) vacuum for the above model simply explains the observed mysterious (numerical) relation between *the (dark) energy density of the universe* $\rho_D$ $(\sim \frac{c^4\Lambda}{8\pi G})$ and *the neutrino mass* $m_\nu$,

$$\rho_D^{\rm obs} \sim (10^{-12}GeV)^4 \sim (m_\nu)^4 \sim \frac{\Lambda}{G} \ (\sim {g_{\rm sv}}^2),$$

provided $-\xi f \sim O(1)$ and $\lambda^i$ is identified with neutrino ($m_{\lambda^i}^2 = \frac{-4\xi f}{\kappa}$), which gives a new insight into the origin of mass [6, 13]. ($g_{\rm sv}$ is the superon-vacuum coupling constant.) Furthermore, the neutral scalar field $\rho$ ($\sim m_\nu$) of the radial mode in the vacuum may be a candidate of *the dark matter*, provided the $N = 2$ LSUSY QED structure is preserved in the realistic large $N$ SUSY GUT model. (Note that $\omega$ in the model is a NG boson and disappears provided the corresponding local gauge symmetry is introduced as in the standard model.)

Recently, we have been shown that *the magnitude of the bare electromagnetic coupling constant e (i.e. the fine structure constant* $\alpha = \frac{e^2}{4\pi}$*) is determined* in the NL/L SUSY relation (i.e. the over-all compositness condition) between the $N = 2$ NLSUSY model and the $N = 2$ LSUSY QED theory (in $d = 2$) from the *general* auxiliary-field structure in the *general* off-shell vector supermultiplet [14].

The similar investigations in $d = 4$ are urgent, and the extension to $N = 5$ is important in SGM scenario and to $N = 4$ is suggestive for the anomaly free nontrivial $d = 4$ field theory. Also NLSUSY GR with extra space-time dimensions equipped with the Big Decay is an interesting problem, which can give the framework for describing all observed particles as elementary *à la* Kaluza-Klein. Linearizing the SGM action (3), $L_{\rm SGM}(e, \psi)$, on curved space-time, which elucidates the topological structure of space-time, is a challenge. The corresponding NL/L SUSY relation will give the supergravity (SUGRA) [15, 16] analogue with the vacuum breaking SUSY spontaneously. The physical and mathematical meanings of the black hole as a singularity of space-time and the role of the equivalence principle are to be studied in detail in NLSUSY GR and SGM scenario. Finally we just mention that NLSUSY GR and the subsequent SGM scenario for spin-3/2 NG fermions [5, 17] is in the same scope.

## References


[1] K. Shima, *European Phys. J.* **C7** (1999) 341; *Phys. Lett.* **B501** (2001) 237.

[2] D.V. Volkov and V.P. Akulov, *JETP Lett.* **16** (1972) 438; *Phys. Lett.* **B46** (1973) 109.

[3] J. Wess and B. Zumino, *Phys. Lett. B* **49** (1974) 52.

[4] Yu. A. Golfand and E.S. Likhtman, *JETP Lett.* **13** (1971) 323.

[5] K. Shima and M. Tsuda, *Phys. Lett.* **B507** (2001) 260; *Class. Quant. Grav.* **19** (2002) 5101.

[6] K. Shima and M. Tsuda, *PoS HEP2005* (2006) 011.

[7] E.A. Ivanov and A.A. Kapustnikov, *J. Phys.* **A11** (1978) 2375; *ibid* **G8** (1982) 167.

[8] M. Roček, *Phys. Rev. Lett.* **41** (1978) 451.

[9] T. Uematsu and C.K. Zachos, *Nucl. Phys.* **B201** (1982) 250.

[10] K. Shima, Y. Tanii and M. Tsuda, *Phys. Lett.* **B525** (2002) 183; *ibid* **B546** (2002) 162.

[11] K. Shima and M. Tsuda, *Mod. Phys. Lett.* **A22** (2007) 1085; *ibid* **A22** (2007) 3027.

[12] K. Shima and M. Tsuda, *Phys. Lett.* **B666** (2008) 410; $N = 2$ SUSY QED in nonlinear/linear SUSY relation, *Mod. Phys. Lett.* **A**, at press, arXiv:0810.2015 [hep-th].

[13] K. Shima and M. Tsuda, *Phys. Lett.* **B645** (2007) 455; K. Shima, M. Tsuda and W. Lang, *ibid* **B659** (2008) 741, Erratum *ibid* **B660** (2008) 612, *ibid*, at press, arXiv:0710.1680 [hep-th].

[14] K. Shima and M. Tsuda, Gauge coupling constant, compositeness and supersymmetry, arXiv:0810.5006 [hep-th].

[15] D. Z. Freedman, P. van Nieuwenhuisen and S. Ferrara, *Phys. Rev. D* **13**, 3214 (1976).

[16] S. Deser and B. Zumino, *Phys. Lett. B* **62**, 335 (1976).

[17] N. S. Baaklini, *Phys. Lett. B* **67**, 335 (1977).