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A natural and simple UV completion of the QCD axion model

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ABSTRACT

The novel PQ mechanism replaces the strong CP problem with some challenges in a model building. In particular, the challenges arise regarding i) the origin of an anomalous global symmetry called a PQ symmetry, ii) the scale of the PQ symmetry breaking, and iii) the quality of the PQ symmetry. In this letter, we provide a natural and simple UV completed model that addresses these challenges. Extra quarks and anti-quarks are separated by two branes in the Randall-Sundrum $\boldsymbol{R}^4 \times S^1/\boldsymbol{Z}_2$ spacetime while a hidden SU(N_H) gauge field condensates in the bulk. The brane separation is the origin of the PQ symmetry and its breaking scale is given by the dynamical scale of the SU(N_H) gauge interaction. The (generalized) Casimir force of SU(N_H) condensation stabilizes the 5th dimension, which guarantees the quality of the PQ symmetry.

1. Introduction

Small parameters in the Standard Model (SM) are outstanding mysteries of particle physics because they imply severe fine-tunings among bare parameters and/or quantum corrections. Although the very small CP phase in the QCD sector is technically natural, source of hadronic CP-violation typically produces $\mathcal{O}(10^{-4})$ threshold corrections to the CP phase in the QCD sector in the SM. This fine-tuning problem implies physics beyond the SM as a UV theory, such as the QCD axion model that addresses the smallness of the CP phase ($\lesssim 10^{-10}$) by the PQ mechanism [1–4]. However, one should be careful when constructing a UV theory so that it indeed solves the problem without any costs for additional fine-tunings of other parameters. For example, the QCD axion models introduce a very precise global symmetry called a PQ symmetry that is anomalous under the SU(3)$_c$ gauge symmetry. Such a global symmetry is expected to be broken by quantum gravity effects [5–15]. In addition, the energy density of the axion may exceed the observed dark matter (DM) density unless the scale of the PQ symmetry breaking is of the order $10^{11\text{-}12}$ GeV or smaller [16–18]. There is also a lower bound of the order 10^8 GeV from the energy loss in the supernova SN 1987A [19,20]. These introduce a new energy scale that is much smaller than the Planck scale and much larger than the electroweak scale. Thus, the PQ mechanism replaces the strong CP problem with the following challenges in a model building:

- origin of the PQ symmetry,
- quality of the PQ symmetry,
- scale of the PQ symmetry breaking.

Several studies proposed explanations of the quality problem. The PQ symmetry is realized as an accidental symmetry from discrete gauge symmetries [21–26], abelian gauge symmetries [27–31], and non-abelian gauge symmetries [14,32–40]. Models with an extra dimension are also proposed in this context [41–49]. An intermediate scale can be introduced without a fine-tuning by a dynamical symmetry breaking of a gauge symmetry that simultaneously breaks the PQ symmetry [50].

In this letter, we provide a simple UV model that naturally realizes the PQ mechanism, combining the ideas proposed in Refs. [42,51]. We consider a warped $\boldsymbol{R}^4 \times S^1/\boldsymbol{Z}_2$ spacetime, where two branes, called IR and UV branes, are placed at the orbifold fixed points. We separately put extra quarks Q and $\bar{Q}$ into the different branes and introduce a SU(N_H) gauge field in the bulk. Then the chiral symmetry, or the PQ symmetry, is guaranteed by the separation in the five-dimensional space [42,43,45,47,48] and is spontaneously broken by the SU(N_H) condensation. Since the condensation scale of SU(N_H) gauge theory is determined by dimensional transmutation, its energy scale can be naturally as small as $10^{8\text{-}12}$ GeV [50]. The size of the extra dimension (radion) is

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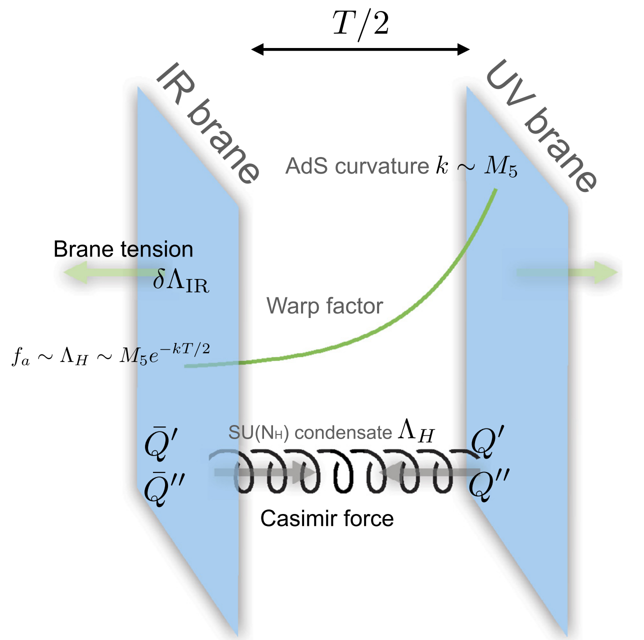

Fig. 1. Schematic illustration of the model in the warped $\boldsymbol{R}^4 \times S^1/\boldsymbol{Z}_2$ spacetime. The horizontal line represents the 5th dimensional space.

stabilized without introducing additional ingredients in the model. The SU(N_H) condensation energy depends on the size of the 5th dimension, which provides a potential for the radion to be stabilized around the PQ scale [51,52]. In addition, the warp factor ameliorates (though not completely addresses) the electroweak hierarchy problem [53] and the cutoff scale is reduced to the PQ symmetry breaking scale rather than the Planck scale. In summary, the answers to the above-mentioned issues in the PQ mechanisms are as follows:

- brane separation of Q and $\bar{Q}$,
- radion stabilization at the PQ scale,
- dynamical scale of SU(N_H).

Compared with the other related works with the 5th dimension [41–46,49], our model automatically stabilizes the size of the 5th dimension. This is a remarkable progress for the model building in the extra-dimensional scenario. In the previous works, the size of the extra-dimension is set by hand and the radion stabilization is assumed. The stabilization mechanism is non-trivial and usually requires a complicated setup; one may add a massive bulk scalar field with different boundary conditions on the branes in the Goldberger-Wise mechanism [54]. In our model, however, the radion stabilization is automatically realized at the PQ phase transition because of the condensation energy of SU(N). This dramatically simplify the UV theory of the model.

2. QCD axion model in $\boldsymbol{R}^4 \times \boldsymbol{S}^1/\boldsymbol{Z}_2$ spacetime

The model proposed in this study is similar to the one proposed in Ref. [42] but features a warped extra dimension [43]. More specifically, we consider an SU(N_H) gauge theory in a warped $\boldsymbol{R}^4 \times S^1/\boldsymbol{Z}_2$ spacetime [53]. We introduce a pair of chiral fermions $Q'(\mathbf{3}, \boldsymbol{N}_H)$ and $\bar{Q}'(\bar{\mathbf{3}}, \bar{\boldsymbol{N}}_H)$ and $N_F - 3$ pairs of chiral fermions $Q''_i(\mathbf{1}, \boldsymbol{N}_H)$ and $\bar{Q}''_i(\mathbf{1}, \bar{\boldsymbol{N}}_H)$, where the arguments represent how the fermions transform under the SU(3)$_c \times$ SU(N_H) gauge group. We collectively denote the fermions as Q ($\supset Q', Q''$) and anti-fermions as $\bar{Q}$ ($\supset \bar{Q}', \bar{Q}''$). Namely, if we explicitly write the flavor index, they are given by

$$Q_i = Q'_a \delta_{ai} \qquad (i = 1, 2, 3), \tag{1}$$

$$Q_i = Q''_{i-3} \qquad (i \geq 4), \tag{2}$$

and similarly for $\bar{Q}$, where a represents the color index. The fields Q and $\bar{Q}$ are localized on UV and IR branes, respectively, while the SU(N_H) gauge field lives in the bulk (see Fig. 1). The standard model (SM) particles are localized on the IR brane. The metric is given by

$$ds^2 = e^{-2kT(x)|y|} g_{\mu\nu} dx^\mu dx^\nu - T^2(x) dy^2, \tag{3}$$

where μ, ν run from 0 to 3, $g_{\mu\nu}$ is the 4D induced metric, $y \in (-1/2, 1/2)$ represents the coordinate for the 5th dimension with $\mathbf{Z}_2$ symmetry $y \leftrightarrow -y$, and k is the AdS curvature. The parameter $T(x)$ represents the size of the extra dimension. We denote T_0 as the size at present and define the radion field by $\mu \equiv k e^{-kT(x)/2}$.

The Lagrangian in the bulk is given by

$$\mathcal{L}_{\rm bulk} = \frac{1}{2} M_5^3 R - V_5 - \frac{1}{4 g_{c5}^2} G_{AB} G^{AB} - \frac{1}{4 g_{h5}^2} F_{AB} F^{AB} + \mathcal{L}_{\rm CS}, \tag{4}$$

where M_5 and R are the 5D Planck mass and the Ricci scalar, V_5 $(= -6 M_5^3 k^2)$ is a bulk cosmological constant, G_{AB} and F_{AB} are the 5D gauge field strengths of SU(3)$_c$ and SU(N_H), g_{c5} and g_{h5} are their gauge coupling constants, and $\mathcal{L}_{\rm CS}$ is a Chern-Simons term that cancels gauge anomalies [42].[1]

The Lagrangians on the IR and UV branes are given by

$$\mathcal{L}_{\rm IR} = \mathcal{L}_{\bar{Q}} - \frac{\tau_{c,\rm IR}}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\tau_{h,\rm IR}}{4} F_{\mu\nu} F^{\mu\nu} - V_{\rm IR} \tag{5}$$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{Q} - \frac{\tau_{c,\rm UV}}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\tau_{h,\rm UV}}{4} F_{\mu\nu} F^{\mu\nu} - V_{\rm UV}, \tag{6}$$

respectively, where $\mathcal{L}_Q$ and $\mathcal{L}_{\bar{Q}}$ are kinetic terms of Q and $\bar{Q}$ and their higher-dimensional terms that will be discussed later. We include localized kinetic terms for the gauge fields into both branes. We omit the SM Lagrangian that may be localized on the IR brane for notational simplicity. The IR and UV brane tensions are rewritten as

$$V_{\rm IR} = -6 M_5^3 k + \delta V_{\rm IR}, \quad V_{\rm UV} = 6 M_5^3 k + \delta V_{\rm UV}, \tag{7}$$

respectively.

Every mass parameter on the IR brane is exponentially suppressed by the warp factor $e^{-kT_0/2}$ when measured with the 4D Einstein metric. The hierarchy problem is then ameliorated for $kT_0 \gg 1$ [53]. However, it is not our primary motivation to consider the warped extra-dimension. The 4D (reduced) Planck scale is given by $M_{\rm Pl}^2 = M_5^3 k^{-1} (1 - e^{-kT_0})$.

3. Radion stabilization

Next, we explain the radion stabilization in our model, following Refs. [51] and [52]. We can consider a four-dimensional effective field theory by the KK decomposition and integrating out heavier particle than μ. We then obtain a 4D effective action for the zero-mode gauge field of SU(N_H) with a gauge coupling of [55,56]

[1] If one considers a grand unified theory (GUT), all SM gauge fields, including U(1)$_Y$, must live in the bulk. This does not affect our discussion.

$$\frac{1}{g_{h4}^2} = -\frac{b_g}{8\pi^2}\log\left(\frac{k}{\mu}\right) - \frac{b_{\rm UV}}{8\pi^2}\log\left(\frac{k}{E}\right) - \frac{b_{\rm IR}}{8\pi^2}\log\left(\frac{\mu}{E}\right) + \tau_{h,{\rm UV}} + \tau_{h,{\rm IR}} \tag{8}$$

at the energy scale of E ($\lesssim \mu$). Here $b_g \equiv -8\pi^2/(kg_{h5}^2)$, $b_{\rm UV} = 11N_H/3 - N_F/3$, and $b_{\rm IR} = -N_F/3$. When $b_{\rm UV} + b_{\rm IR} > 0$, the SU($N_H$) gauge interaction is asymptotically free and is confined at the energy scale of

$$\Lambda_H(\mu) = \left(k^{b_{\rm UV}}\mu^{b_{\rm IR}}e^{-8\pi^2(\tau_{\rm IR}+\tau_{\rm UV})}\left(\frac{\mu}{k}\right)^{-b_g}\right)^{1/(b_{\rm UV}+b_{\rm IR})} \equiv \Lambda_{H,0}\left(\frac{\mu}{\mu_0}\right)^n, \tag{9}$$

for $\Lambda_H(\mu) \lesssim \mu$, where $\mu_0 \equiv ke^{-kT_0/2}$ and $n \equiv (b_{\rm IR} - b_g)/(b_{\rm UV} + b_{\rm IR})$. Here we explicitly express the μ dependence from the dynamical scale.

The vacuum energy of the condensation can be obtained from the dimensional analysis and is given by

$$\frac{1}{4}\langle T^\mu_\mu\rangle = -\frac{1}{4}\frac{b_{\rm UV}+b_{\rm IR}}{32\pi^2}\left\langle F^{(0)}_{\mu\nu}F^{(0)\mu\nu}\right\rangle \tag{10}$$

$$\sim -\frac{1}{4}\frac{b_{\rm UV}+b_{\rm IR}}{32\pi^2}(4\pi)^2\Lambda_H^4(\mu). \tag{11}$$

Note that this estimation is supported by the lattice simulation for the SM QCD [52]. The result depends on μ via the Beta function. The effective action of the radion field μ is given by

$$S_{\rm radion} = \int d^4x\left[3\left(\frac{M_5}{k}\right)^3(\partial\mu(x))^2 - V(\mu)\right], \tag{12}$$

$$V(\mu) = \delta V_{\rm UV} + \frac{\delta V_{\rm IR}}{k^4}\mu^4 - \frac{b_{\rm UV}+b_{\rm IR}}{8}\Lambda_{H,0}^4\left(\frac{\mu}{\mu_0}\right)^{4n}. \tag{13}$$

We consider a large M_5/k so that quantum gravity effects are negligible. According to the naive dimensional analysis [57], it requires [52]

$$2\pi(M_5/k)^{3/2} \gtrsim 4\cdot 5^{3/4}/\sqrt{3\pi} \quad\leftrightarrow\quad M_5 \gtrsim 0.6\,k. \tag{14}$$

The cosmological constant at the minimum can be made vanishingly small by choosing $\delta V_{\rm UV}$ appropriately, which is the only fine-tuning we require in our model.

The branes repel with each other due to the IR brane tension. At the same time, they are attracted by the condensation energy of SU(N_H). The radion VEV is determined by the balance between these forces. The VEV and the mass of the radion are given by [51]

$$\mu_0 = \left(\frac{n(b_{\rm UV}+b_{\rm IR})k^4}{8\delta V_{\rm IR}}\right)^{1/4}\Lambda_{H,0}, \tag{15}$$

$$m_{\rm radion}^2 = (1-n)\left(\frac{32\pi^2}{3N^2}\right)\left(\frac{\delta V_{\rm IR}}{k^4}\right)\mu_0^2, \tag{16}$$

for $n < 1$, where we implicitly assume that $\mu_0 \gtrsim \Lambda_{H,0}$ since otherwise Eq. (9) cannot be used. From Eq. (7), we expect $\delta V_{\rm UV} \sim M_5^3 k$ or a little bit smaller to avoid a fine-tuning in the IR brane tension. We also expect that M_5/k satisfies Eq. (14) but is not significantly larger than unity. Then the expression in the parentheses in Eq. (15) is of order (but is larger than) unity. Hence we obtain $e^{-kT_0/2} = \mu_0/k \sim \Lambda_{H,0}/k$. As we will see shortly, we consider

$$\frac{\Lambda_{H,0}}{N_H} \sim f_a \quad \left(= \mathcal{O}(10^{8\text{-}12})\ {\rm GeV}\right), \tag{17}$$

so that the warp factor is estimated as

$$e^{-kT_0/2} \sim 10^{-(6\text{-}10)} \quad\leftrightarrow\quad kT_0 \simeq 28\text{-}46. \tag{18}$$

This is not small enough to completely address the hierarchy problem between the electroweak and the Planck scales but ameliorates it by a factor of the order $N_H f_a/k$. This is another advantage of our model. It does not only address the issues in the QCD axion model but also ameliorates the hierarchy problem.

4. Origin of PQ symmetry

Now we shall explain how the PQ mechanism is realized at an intermediate scale and how the quality of the symmetry is guaranteed in the model. In this explanation, we follow Ref. [42]. For a moment, let us consider SU(N_H) gauge interaction and omit the SU(3)$_c$ gauge interaction. Then there are N_F pairs of chiral fermions, Q_i and $\bar{Q}_i$, in the SU(N_H) gauge theory. Since Q_i and $\bar{Q}_i$ are separately placed on different branes, the operators involving both Q_i and $\bar{Q}_i$ are exponentially suppressed and the model possesses an approximate $U(N_F)_V \times U(N_F)_A$ flavor symmetry, where $U(N_F)_V$ and $U(N_F)_A$ represent the vector and axial transformations, respectively. In particular, the vector mass term $M_{Q_i}Q_i\bar{Q}_i$ is suppressed as

$$M_{Q_i} \propto e^{-cM_5T_0}, \tag{19}$$

with $c = \mathcal{O}(1)$. However, because of the chiral anomaly of the SU(N_H) gauge theory, $U(N_F)_A$ is broken to $SU(N_F)_A$. In addition, one may write the following higher-dimensional operators on each brane that explicitly break the flavor symmetry:

$$\mathcal{L}_Q \supset \frac{y_Q}{M_5^{3N_H-4}}(Q)^{2N_H} + {\rm H.c.}, \qquad \mathcal{L}_{\bar{Q}} \supset \frac{y_{\bar{Q}}}{M_5^{3N_H-4}}(\bar{Q})^{2N_H} + {\rm H.c.}, \tag{20}$$

for an odd N_H, where y_Q and $y_{\bar{Q}}$ are coupling constants and we omit the flavor indices for notational simplicity. For an even N_H, we obtain similar terms with a replacement of $N_H \to N_H/2$. One can forbid these terms by making Q and $\bar{Q}$ charged under U(1)$_Y$ and/or U(1)$_{B-L}$. For a moment, we neglect these symmetry-breaking operators and come back to this issue later.

As we discussed, the SU(N_H) gauge interaction confines at the energy scale of $\Lambda_{H,0}$. Then the chiral condensate develops such as

$$\left\langle Q_i\tilde{\bar{Q}}_j\right\rangle \sim \Lambda_{H,0}^3\delta_{ij} \tag{21}$$

where $\tilde{\bar{Q}}_i$ ($\equiv e^{-3kT_0/4}\bar{Q}_i$) is a rescaled field of $\bar{Q}_i$ to canonicalize the kinetic term in the four-dimensional effective field theory. As a result, the $U(N_F)_V \times SU(N_F)_A$ flavor symmetry is spontaneously broken to the $U(N_F)_V$ symmetry. The number of composite NG bosons would then be $N_F^2 - 1$, where a factor of -1 comes from a massive pseudo-NG boson due to the chiral anomaly of the SU(N_H) gauge theory.

Now let us add the SU(3)$_c$ gauge interaction, where the SU(3) ($\in$ SU(N_F)$_V$) flavor symmetry is promoted to the SU(3)$_c$ gauge symmetry. The $U(N_F)_V \times SU(N_F)_A$ flavor symmetry is then explicitly broken by $SU(3)_c$ gauge interactions down to $U(1)_{\rm PQ} \times U(N_F-3)_V \times SU(N_F-3)_A$, where $U(N_F-3)_V$ and $SU(N_F-3)_A$ are the vector and axial transformation on $Q''(\mathbf{1}, \boldsymbol{N}_H)$ and $\bar{Q}''(\mathbf{1}, \boldsymbol{N}_H)$, respectively. The U(1)$_{\rm PQ}$ symmetry is defined by

$$\begin{cases} Q'(\bar{Q}') \to e^{i\alpha/3}Q'(\bar{Q}'), \\ Q''(\bar{Q}'') \to e^{-i\alpha/(N_F-3)}Q''(\bar{Q}''). \end{cases} \tag{22}$$

This is anomalous under the $SU(3)_c$ gauge interaction. Accordingly, the associated pseudo-NG boson is identified as the axion. In summary, $U(1)_{PQ} \times U(N_F-3)_V \times SU(N_F-3)_A$ is spontaneously broken to $U(N_F-3)_V$ by the condensation of $SU(N_H)$ (see Eq. (21)). Hence, there are $(N_F-3)^2-1$ NG bosons as well as the axion in the effective theory below the condensation scale. The PQ symmetry breaking scale is therefore identified as the condensation scale $\Lambda_{H,0}$. Since the condensation scale is determined by the dynamical scale of $SU(N_H)$, the smallness of its energy scale (compared to the fundamental scales, e.g., the Planck scale) is explained by the dimensional transmutation [50].

Now we go back to the symmetry-breaking operator, which particularly breaks the PQ symmetry. The vector mass $M_{Q_i} Q_i \bar{Q}_i$ with Eq. (19) induces a shift in the strong CP phase of the order $M_{Q_i} e^{-kT_0/2} f_a / m_a^2$, where m_a is the axion mass at the low energy and f_a ($\simeq \Lambda_{H,0}/N_H$) is the axion decay constant. Requiring $\Delta a / f_a \lesssim 10^{-10}$ to solve the strong CP problem, we find a sufficient condition of

$$cM_5 T_0 \gtrsim 144 + 4\ln(f_a/10^{12}\ \text{GeV}). \tag{23}$$

Since $M_5 \gtrsim k$ from Eq. (14) and $kT_0 \sim 28\text{-}46$ from Eq. (18), the strong CP phase is sufficiently small for $c \gtrsim 3\text{-}5$. For example, one may take $k = 2 \times 10^{18}$ GeV, $M_5 = 2.3 \times 10^{18}$ GeV, $T_0^{-1} = 4 \times 10^{16}$ GeV and $c = 3$, which satisfies all constraints with $f_a \simeq 10^{12}$ GeV. Thus the quality of the PQ symmetry is explained by the brane separation in our model [42]. Since the same vector mass explicitly breaks the flavor symmetry, the $(N_F-3)^2-1$ singlet NG bosons have very tiny masses. The resulting masses are less than of order $10^{-5} m_a \simeq 6 \times 10^{-11}\ \text{eV}(f_a/10^{12}\ \text{GeV})^{-1}$ for $c \gtrsim 3\text{-}5$.

One can forbid terms in Eq. (20) by imposing $U(1)_Y$ and/or $U(1)_{B-L}$ charges on Q and $\bar{Q}$, motivated by grand unified theories. If one wrote those terms, they would also give masses to the axion and may induce a shift in the axion VEV. For example, they lead to

$$\frac{\Delta a}{f_a} \sim 10^{-12} y_Q y_{\bar{Q}} \left(\frac{f_a}{10^8\ \text{GeV}} \right)^9 \times \left(\frac{M_5}{M_{\text{Pl}}} \right)^{-5} \left(\frac{e^{-kT_0/2} M_5}{N_H f_a} \right)^{-5}, \tag{24}$$

for the case of $N_H = 3$, where $e^{-kT_0/2}$ is the warp factor that enhances the coefficient of the operator of Q^{2N_H}. The result is sufficiently small to explain the smallness of the strong CP phase for the case of $N_H = 3$ with $f_a \sim 10^8$ GeV. The axion decay constant f_a can be 10^{12} GeV or larger for the case of $N_H \geq 5$ for an odd N_H and $N_H \geq 10$ for an even N_H. Thus the quality of the PQ symmetry is actually explained by the brane separation in our model [42,43].

5. Cosmological scenario

Finally, we explain the cosmological scenario of our model. Since $Q'(\mathbf{3}, \boldsymbol{N}_H)$ and $\bar{Q}'(\bar{\mathbf{3}}, \bar{\boldsymbol{N}}_H)$ contribute to the $U(1)_{PQ} \times SU(3)_c \times SU(3)_c$ chiral anomaly, the domain wall number of the axion is equal to N_H and hence $f_a \sim \Lambda_{H,0}/N_H$. To avoid the inhomogeneous Universe due to the production of stable domain walls, we consider the pre-inflationary PQ symmetry breaking scenario. Then the energy density of the coherent oscillation of the axion is determined by the misalignment mechanism such as [58,59]

$$\Omega_a h^2 \simeq 0.12\, \theta_{\text{ini}}^2 \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^{1.165}, \tag{25}$$

where h is the Hubble parameter in units of 100 km/s/Mpc and θ_{ini} is the initial misalignment angle. The axion decay constant f_a should be of order 10^{12} GeV to explain all DM unless θ_{ini} is small. Such a "small" energy scale of f_a is naturally realized in our model due to the dimensional transmutation.

We predict the $(N_F-3)^2-1$ light singlet NG bosons in addition to the axion for the case of $N_F \geq 5$. They are fuzzy DM and their total abundance is given by

$$\Omega_f h^2 \simeq \sum_i 3 \times 10^{-11}\, \theta_{\text{ini,f},i}^2 \left(\frac{m_{f,i}}{10^{-20}\ \text{eV}} \right)^{1/2} \times \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^2, \tag{26}$$

where the summation is taken for $(N_F-3)^2-1$ NG bosons, $m_{f,i}$ is the mass of NG boson i, and $\theta_{\text{ini,f},i}$ is the initial misalignment angle for the NG boson i. Note that $m_{f,i} \sim 10^{-20}$ eV can be realized for $f_a = 10^{12}$ GeV if c is larger by a factor of 1.3 (i.e., $c = 4\text{-}6.5$ for $kT_0 \sim 28\text{-}46$) than its minimal value to solve the strong CP problem. If the flavor symmetry has a $U(1)_{EW}$ anomaly, which is the case, e.g., Q and $\bar{Q}$ are charged under $U(1)_Y$, the NG bosons with masses around 10^{-20} eV can be observed by polarimetric imaging of supermassive black holes [60].

The axion acquires quantum fluctuations during inflation, whose amplitude is proportional to the energy scale of inflation. Those modes result in isocurvature density perturbations [61–63]. The constraint by the Planck collaboration implies that the energy scale of inflation has to be smaller than of order 10^7 GeV [64] if the axion is all DM. In fact, a small energy scale of inflation may be a natural consequence of the anthropic landscape [65], where inflations occur at infinitely many vacuum states with different energy scales. It is a possible move to a different vacuum state by a quantum tunneling process. Since the rate of upward tunneling (namely Hawking-Moss transition) is strongly suppressed compared with that of downward tunneling, it would take a journey to a lower energy scale. Eventually, there is an option to go through a slow-roll region and reach a habitable vacuum with a vanishingly small vacuum energy. Accordingly, we expect that the last slow-roll inflation is a small-scale one, which is consistent with the isocurvature constraint.

Since we consider the pre-inflationary PQ symmetry breaking scenario, the reheating temperature (and the maximal temperature) should be lower than the condensation scale of $SU(N_H)$, namely $N_H f_a$. The colored would-be NG bosons acquire effective masses of the dynamical scale order due to the radiative correction from the $SU(3)_c$ gauge interaction and are not produced after inflation. If one considered a scenario where they are produced, $\bar{Q}'$ would have be made charged under $U(1)_Y$ so that the triplet NG bosons have the same SM charges with the SM down quarks and can decay into SM particles. If the reheating temperature after inflation is as high as of the order $f_a/10$, the $(N_F-3)^2-1$ massless singlet NG bosons as well as the axion are thermalized [66,67] and contribute to the energy density of the Universe as dark radiation [68–70]. The resulting abundance is conveniently expressed by the effective neutrino number, which is given by [67,71]

$$N_{\text{eff}} \simeq N_{\text{eff}}^{(\text{SM})} + 0.027 \times (N_F-3)^2, \tag{27}$$

where $N_{\text{eff}}^{(\text{SM})}$ ($\simeq 3.046$) is the SM prediction. Even if $N_F = 4$, the deviation from the SM prediction would be measured by CMB-S4 in the future [72] (see also Ref. [73]).

6. Summary and discussion

Summarizing the conditions on our model parameters, we require $M_5^3/k = M_{\text{Pl}}^2$ ($\simeq 2.4 \times 10^{18}$ GeV), $M_5/k \gtrsim 0.6$, $T_0^{-1} =$

$k/(28\text{-}46)$, and $144/(M_5 T_0) = \mathcal{O}(1)$. These are satisfied, e.g., $k = 2 \times 10^{18}$ GeV, $M_5 = 2.3 \times 10^{18}$ GeV, $T_0^{-1} = 4\text{-}7 \times 10^{16}$ GeV. There are no small parameters nor fine tunings in our model, except for the notorious fine tuning on the vanishing 4-dimensional cosmological constant. The intermediate PQ scale, $10^{8\text{-}12}$ GeV, is realized by the dimensional transmutation. It is automatically close to the IR-brane-cutoff scale, namely the radion VEV. This implies that if one wants to consider a GUT, the PQ scale should be as large as the GUT scale.

One can consider the case with $f_a \lesssim 10^{11}$ GeV, where the axion cannot explain all DM. The constraint on the energy scale of inflation implied to avoid isocurvature density perturbations is not applicable here. In fact, such a small f_a is favored in our scenario in light of the hierarchy problem because the warp factor decreases with f_a. Although the hierarchy problem is not completely solved, it is ameliorated by many orders of magnitude due to the Randall-Sundrum mechanism. Helioscopes can search for solar axions with a relatively small f_a. The sensitivity of the IAXO experiment is expected to reach $N_H f_a \sim 10^9$ GeV in the future [74].

Our stabilization mechanism is based on the fact that the condensation energy of SU(N_H) gauge field depends on the size of the extra dimension via its beta function. This may be regarded as a generalized Casimir energy as it is the vacuum energy in the SU(N_H) gauge theory and depends on the distance between the branes (boundaries). In the same spirit, the cosmological constant in the bulk as well as the brane tensions may also be regarded as a generalized Casimir energy. It was known that Casimir forces are attractive if the boundary conditions on the two boundaries respect interchange symmetry [75–77]. Recently, Jiang and Wilczek found a loophole of this theorem by inserting an intermediate chiral material in the bulk [78]. Their result implies that Casimir forces can be repulsive if the medium in the bulk does not respect the interchange symmetry. A similar logic may apply to the generalized Casimir energy, namely the cosmological constant, brane tensions, and condensation energy. The gauge field has a symmetric configuration, which results in an attractive force from the condensation energy. On the other hand, the cosmological constant in the bulk implies that the brane tensions should not be symmetric to satisfy the Einstein equation. As a result, the interchange symmetry is broken in the presence of the cosmological constant as well as the brane tensions and thus we can generate either attractive or repulsive forces. Combining these results, the size of the extra dimension is stabilized by the balance between the attractive and repulsive Casimir forces of SU(N_H) and the cosmological constant.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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