

# ***Chi-Squared* test for constraining free parameters of modified Gravity on Brown Dwarfs**

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**Abstract.** Constraining free parameters of two modified gravity theories such as *beyond* Horndeski and Eddington Inspired Born-Infeld Theories of Gravity have been done precisely with LHS6343C data. However, it remains insufficient to explain the radius and mass relation of Brown Dwarf globally. We still need to identify how well *beyond* Horndeski and Eddington Inspired Born-Infeld Theories of Gravity being compared by the completed observation mass-radius data of Bayliss *et. al.* The provided data gives error-bar that can be used to study more systematically through the standard deviation analysis. We do *Chi-Squared* test to obtain minimum value on the free parameters of both Modified Gravity theories. We compare the two results and constrain the free parameters of both theories on confidence level  $1\sigma$ . We can obtain the more accurate results for determining the limits of free parameters of both theories.

## **1. Introduction**

Constraining free parameters by LHS6343C data remains insufficient to explain the radius and mass relation of Brown Dwarf globally. We provide the completed observation mass-radius data of Bayliss *et. al* [1] to restrict the free parameter on two modified gravity model (MOG). These two models, which are *beyond* Horndeski and Eddington Inspired Born-Infeld Theories of Gravity, need to be considered as test-beds. We choose low mass star like Brown Dwarf as an object that obeys these MOGs. Brown Dwarfs are not practically compact objects due to inadequate massive mass around ( $M < 0.08 M_{\odot}$ ) [2]. Before, Jain finds the strict and independent constraints for *beyond* Horndeski Theories in non-MOG gravity using the mass-radius relation, the Chandrasekhar mass limit and the maximal rotational frequency of white dwarfs [3]. Previously, We have done constraining the free parameters of two MOGs with LHS6343C data and will continue with restricting the constraint by comparing with twelve observational data. We use the same methods as Jain [3] through *Chi-Squared* test to get the constraints.

## **2. Formalism**

### *2.1 Modified Gravity (MOG) and Equation of State (EoS)*

Due to the Vainshtein mechanism which conquers the fifth force effects on small scales, Scalar Tensor Theories of gravity may attend as capable candidate for explaining the present cosmic acceleration [4]. Horndeski Theory is considered as the most general theory as Covariant Galileons form a subclass of that theory. *beyond* Horndeski Theories which acknowledge self-



accelerating solutions thereby presence practical opponents to the  $\Lambda$ CDM model also has its extensions. In these theories, screening mechanism becomes ineffective inside astrophysical objects, specifically, in any other compact objects and other stars [5].

The effects of modifications of gravity are being characterized by dimensionless parameter  $Y$ . This parameter also can yield arbitrary values. For  $Y > 0$ , the gravitational pull inside the star can be weakened by the extra term and vice versa. In a stable stellar configuration, the parameter can only exist for  $-\frac{2}{3} < Y < \infty$  [6].

The Equation of State (EoS) for low mass star like brown dwarf is nearly taken similar to a white dwarf, in which the composition of its interior is slightly the same. We use polytropic relation to relate pressure and energy density in  $P = K \rho^\Gamma$ , where polytropic index  $n=3/2$  for non-relativistic limit,  $\Gamma = 1 + \frac{1}{n}$ . Using mass continuity  $\frac{dm}{dr} = 4 \pi r^2 \rho$ , we can write hydrostatic equation for *beyond* Horndeski model as,

$$\frac{d\rho}{dr} = -\frac{G_N M}{r^2} \frac{[1 + \frac{Y\pi r^3}{M} 2\rho]}{[K \Gamma \rho^{\Gamma-1} + G_N Y \pi r^2]}, \quad (1)$$

where, for  $Y = 0$ , equation (1) reduces to Newtonian gravity.

We also discuss non-relativistic stellar models in EiBI models of gravity. We then will solve the stellar structure equation. We use the conservation of standard mass and  $K$  as constant polytropic with index  $n=3/2$ . This constant is dependent with Temperature in every point in star. The stellar structure equation reads

$$\frac{d\rho}{dr} = -\frac{G_N M}{r^2} \frac{1}{[K \Gamma \rho^{\Gamma-2} + \frac{\kappa}{4}]}, \quad (2)$$

Realistic solar models are reconstructed by this modified stellar structure equation and put constraint on  $\kappa$ [7].

The modified stellar structure equation is originated from modified Poisson equation and has been used to build realistic solar models and put the first observational constraint on  $\kappa$ [7]. On condition, stability for this model is on  $\kappa > 0$ , in contrast, unstable if  $\kappa < 0$ . Regarding value of  $\Gamma$ , positive  $\kappa$  value is in charge to stabilize the models, while negative  $\kappa$  values work in contrast [8].

We model the star as non-relativistic limit. To attain hydrostatic equilibrium, a brown dwarf, which is consist of degenerate electrons, can avoid gravitational collapse by maintaining the Fermi pressure. We integrate the Fermi-Dirac integral using polylogarithm functions  $Li_s(x)$  to obtain more accurate analytical expression for the pressure [9]. We introduce the degeneracy parameter  $\psi$  as temperature effects of equivalency to the fraction of electron Fermi Temperature over electron Fermi Energy. However, the interior of Brown Dwarf is not only composed by electrons but also ionized hydrogen and helium, so the total pressure are the combination of electrons and ions Pressure,  $P = P_F + P_{ion}$ , where the pressure due to ionized gas can be expressed by  $P_{ion} = \frac{k\rho T}{\mu_1 m_H}$ . Total Pressure can be expressed in [9], where polytropic constant that relates to degeneracy parameter  $\psi$  reads,

$$K = \frac{(3\pi^2)^{\frac{2}{3}} \hbar^2}{5 m_e m_H^{\frac{3}{5}} \mu_e^{\frac{3}{5}}} (1 + \gamma + \alpha\psi) \quad (3)$$

where,  $\alpha$  and  $\gamma$  is a constant and  $\psi$  is a degeneracy parameter (for much details, you can see in Ref. [9]).

We adopt the two modified theories in Brown Dwarf to see the narrow limit of free parameter. We also expect that MOG can modify the minimum value of chi-squared not only in Newtonian limit but also closed to Newtonian Limit.

### 2.2 Observational Constraints with Chi-Squared Analysis

Our two models need to be compared by twelve brown dwarfs data accumulated in Bayliss *et al* [1]. The provided data gives error-bar that can be used to study more systematically through the standard deviation analysis. We use fitting parameter on chi-squared test to restrict free parameter and find the minimum value of Chi-squared.

First, we need to choose one point on our theoretical curve  $M_{th}(R)$  from all each point on experimental observation that we will compare. The shortest distance for two points will be best agreements due to its true value. The Deviation from each data is formulated and represented by,

$$\Delta\chi_i^2(R) = \frac{(M_{th}(R)-M_i)^2}{\sigma_{M,i}^2} + \frac{(R-R_i)^2}{\sigma_{R,i}^2} \quad (4)$$

where  $M_i$ ,  $\sigma_{M,i}$ ,  $R_i$  and  $\sigma_{R,i}$  are the mass, mass standard deviation, radius and the radius standard deviation, respectively, of the i-th star. The total chi squared,

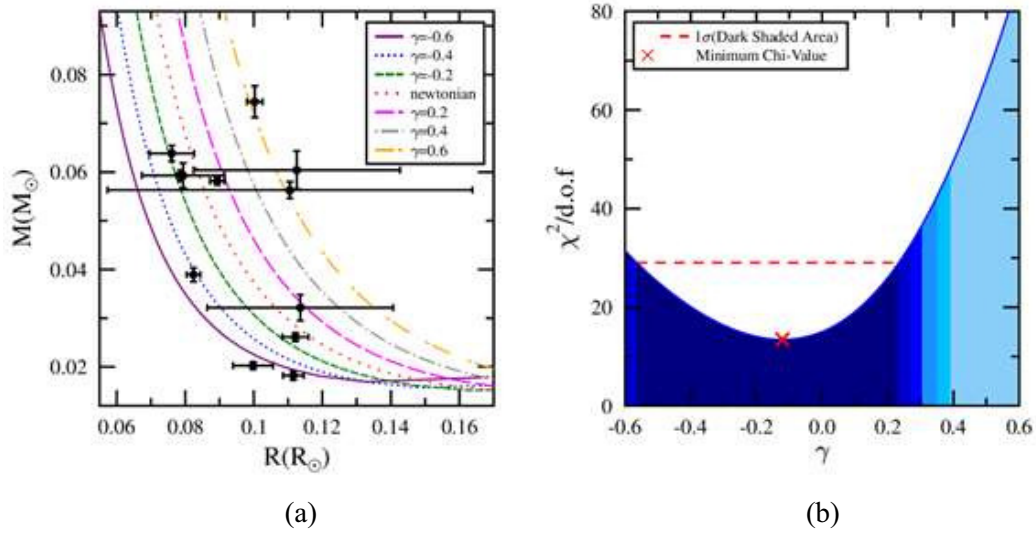
$$\chi^2 = \sum_{i=1}^N \Delta\chi_i^2(R_i) \quad (5)$$

In our case, we choose number of degrees of freedom labeled in,  $d.o.f = 2N - n - 1 = 22$ . 2 is a factor which represents the radius and mass of each stars.  $N=12$  is the number of the stars.  $n=1$  is the number of fitting parameters. We use error-bars on the data listed by Bayliss *et al* [1] as standard deviation. We compare our results to chi-squared distribution for level confidence of  $1\sigma$ . We still cannot find the restricted parameter for  $5\sigma$  which means confidence interval 99.99994%, the exact true value for limiting parameter.

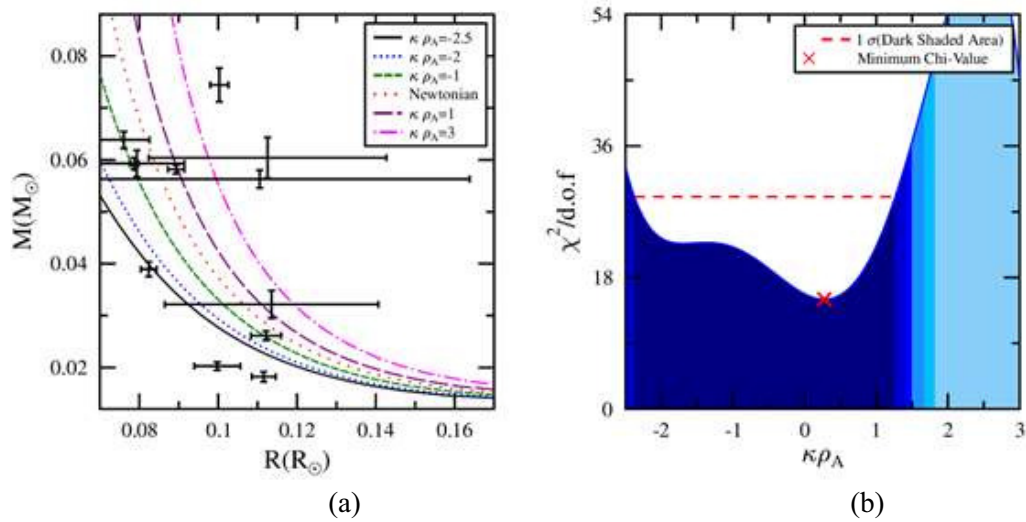
## 3. Results and Discussion

### 3.1 Beyond Horndeski Gravity

On Figure 1, the left figure shows mass and radius relation with twelve brown dwarfs and error-bar data accumulated in Bayliss *et al* [2]. The right figure expresses chi-squared value on different free parameter of *beyond* Horndeski Gravity. On Chi-Squared test, we know that theoretical interval estimation that contains real value can be shown by the confidence interval. Confidence interval can be seen on the Chi-squared distribution, where the distribution has asymmetric form. Chi-squared test can limit the free parameter up to  $-0.565 \leq Y \leq 0.234$  with confidence level of 68.3% ( $1\sigma$ ). The darkest area on the right figure 1 shows minimum value of  $\chi^2/d.o.f$ . Minimum value falls around ( $Y = -0.2$ ), that we can state that MOG can modify the minimum value of chi-squared not only in Newtonian limit but also closed to Newtonian Limit. As we can see the data is spread around  $Y = -0.2$ , that is why the minimum value of chi-squared placed on it.



**Figure 1.** (a) R-M Relation with Observation Data (See Ref. D.Bayliss *et. Al* for detail data) on *beyond* Horndeski parameter and (b)  $\chi^2$  value with d.o.f = 22. The red cross shows the minimum value. Different shaded areas show different confidence levels which the darkest shade refers to  $1\sigma$  confidence level.



**Figure 2.** (a) R-M Relation with Observation Data (See Ref. D.Bayliss *et. Al* for detail data) on *Eddington Inspired Born-Infeld*  $k$  parameter,  $\rho_A = 10^9 \rho_{\odot}$  and (b)  $\chi^2$  value with d.o.f = 22. The red cross shows the minimum value. Different shaded areas show different confidence levels which the darkest shade refers to  $1\sigma$  confidence level.

### 3.2 Eddington Inspired Born-Infeld Gravity

Constraints in EiBI still includes negative value. Chi-squared restricts  $-2.375 \leq \kappa\rho_A \leq 1.266$  ( $\rho_A = \rho_{\odot} \times 10^9$ ). The minimum value falls around  $\kappa\rho_A = 1$ . The same as *beyond* Horndeski Gravity, MOG modifies the value of chi-squared closely to Newtonian limit. The exact constraints in EiBI Theory for dwarfs still needs to be determined and to be compared to the truth restricted constraints for Dwarfs. The minimum scales of compact objects bound together by gravity is determined by the effective Jeans length in EiBI Theory. Estimation of maximum density is delivered by fundamental density inside stable compact stars. The effective Jeans length will obtain conservative constraint. Nevertheless, by considering smaller astrophysical objects, we can obtain stronger constraints on  $\kappa$  (For details, one can see in Ref. [10] for Astrophysical Constraints i.e. Solar System and Neutron Stars).

### 4. Conclusion

We have done Chi-Squared analysis to determine limited free parameter of two modified gravity in Brown Dwarfs. We get the minimum area of chi-squared value that we prove modified gravity can modify the minimum value of chi-squared not only in Newtonian limit but also closed to Newtonian Limit. For *beyond* Horndeski model, it gets minimum around  $Y = -0.2$ , while on EiBI model, we get minimum chi-squared value around  $\kappa\rho_A = 1$ . We also indicate that restricted parameter can be determined by chi-squared distribution based on degrees of freedom.

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