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From Axion—Neutrino Couplings to Axion Thermodynamics: Testing the Axion Mass Hierarchy

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Abstract: The composition and physical state of dark matter remain among the most pressing unresolved questions in modern physics. Addressing these questions is crucial to our understanding of the Universe's structure. In this work, we explore the hypothesis that massive scalar bosons, such as axions, constitute the majority of dark matter. We focus on two key aspects of axion physics: (i) the role of axion–neutrino coupling in generating neutrino mass and (ii) the thermodynamic properties of axion dark matter. We propose that the interaction between neutrinos and axions in the early Universe, prior to hadronic formation, could provide a mechanism for finite neutrino masses. Furthermore, to account for the observed large-scale distribution of dark matter, we extend the Bose–Einstein condensation framework and derive the critical temperature T_c that defines the onset of the condensate phase. Our calculations suggest that this temperature ranges from a few 10^{-3} degrees Kelvin to approximately one Kelvin, depending on the axion scale factor f_a. These findings support the plausibility of axions as viable dark matter candidates and emphasize the importance of future experimental searches for axion–neutrino interactions. Additional astrophysical and laboratory investigations could further refine axion mass constraints and shed light on the role of axion condensates in the evolution of the early Universe.

Keywords: dark matter; axions; neutral pseudoscalars; Bose–Einsten condensate; neutrino-axion couplings

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1. Introduction

Cosmological and astrophysical observations have provided evidence for the existence of dark matter in the Universe [1–4]. A well-established astrophysical observational fact is the behavior of the rotation (or velocity) curves for galaxies, which exhibit a flat trend at large distances from the galactic center [5,6]. This contradicts the prediction of Newton-Kepler's laws [7], in which the velocity should decrease with distance from the galactic center, assuming the presence of a dominant central mass. This contradiction can be rectified if there is a significant amount of dark matter all throughout the galaxy. Dark matter does not emit light, with the mass-to-light ratio of the central bulge being larger than that of visible matter. Furthermore, the possibility that any theory of gravity could explain the behavior of galaxies without considering the presence of dark matter is highly improbable [3] in spite of its assumed distribution.

The evidence supporting the existence of dark matter is purely gravitational and extends to cosmological scales [8–10]. The cosmic microwave background (CMB) radiation is a probe that provides valuable information about the composition of the Universe [11,12].

To explain the CMB and the formation of large-scale structures, dark matter must be cold, meaning its kinetic energy must be small compared to its rest mass, and it must be invisible and stable enough not to decay within the current age of the Universe. Furthermore, the attributes of dark matter, when represented by elementary particles, suggest that it should be electrically neutral, spinless, weakly interacting, and massive [13,14].

Both the temperature anisotropy in the CMB and the clustering of galaxies and structure formation are successfully explained by the Cold Dark Matter Lambda cosmological model (Λ-CDM), i.e., the standard cosmological model in which Λ represents dark energy [13,14]. The Λ-CDM model also explains the abundance of light elements (hydrogen, helium, and lithium) and the accelerated expansion of the Universe, as well as establishing a composition of 71.4% dark energy and 24% dark matter, with baryonic matter representing only 4.6% of the total [14].

Therefore, understanding dark matter requires new physics, possibly in the form of new species of fundamental particles beyond the standard model [15–17]. Among these new particles, the axion is a hypothetical particle that is not predicted by the standard model, the existence of which may answer some of these unsolved questions. This particle has become one of the leading candidates for dark matter; it was introduced by R D Peccei and H R Quinn [18–24].

Axions were conceived to explain the strong charge-parity (CP) symmetry violation within the framework of quantum chromo-dynamics [22]. Further consequences of the introduction of axions in elementary particle physics and their interactions can be found in [22,24].

Axions could also play a role in affecting the propagation of extragalactic neutrinos because their couplings could change the flavor composition of neutrinos [25]. The study of these effects has received considerable attention recently [26,27], as well as the implications of the existence of axions for the extensions of the standard model [28]. In the present work, we focus our attention on the mass mechanism resulting from the axion–neutrino couplings and its consequences in cosmology and in rare decays, the detection of which would demonstrate the need to extend the standard model of electroweak interactions to accommodate finite neutrino masses.

The models of the early Universe seem to suggest the crucial role of gravity in the process of reaching thermal equilibrium [29]. Although the role of particles in thermal equilibrium has been questioned, the possibility of particle-mediated processes leading to equilibrium cannot, in principle, be disregarded. If the associated Lagrangian for such particles contains terms of the form ϕ^4, with ϕ being the field representing the particles, it opens up the possibility of interesting effects. In this work, we review the properties of axions, as they have been formulated in the original work of Peccei and Quinn, and make the connection with neutrino physics by discussing the mechanism of axion–neutrino couplings. We focus on the axion–neutrino couplings as being responsible for a neutrino finite mass. From this, we extracted the limits on both the axion and neutrino masses.

After, we proceed to discuss the propagation of neutrinos in a dark matter background and write the differential cross-section explicitly in terms of the amplitudes derived from the Lagrangian, which describes the current–current interactions between neutrino and dark matter currents. The formalism is written in its covariant form.

Finally, we focus attention on the thermodynamics of finite mass axions, particularly looking at the condensation mechanism anticipated in [29]. Since that proposal [29] did not explicitly estimate the onset of the condensation, we derive—from the formalism of quantum statistical mechanics, applied to neutral scalar bosons like the axions—the corresponding critical temperature.

In other words, in this work, we discuss neutral scalar bosons, that is, axions, as possibly constituting the bulk of dark matter composition, as well as being responsible for finite neutrino masses. In addition, we investigate their thermodynamical properties, such as the Bose–Einstein condensation mechanism.

The paper is organized as follows. In the formalism of Section 2, the theoretical framework of axion–neutrino coupling and its impact on neutrino mass generation is discussed. Next, in Section 3, calculations on axion mass and the critical temperature for Bose–Einstein condensation are presented, analyzing their compatibility with experimental constraints. Finally, Section 4 summarizes the findings and discusses their implications for future theoretical and experimental research.

2. Formalism

To begin with, we shall introduce the properties of the axions, as they were proposed by R Peccei and H Quinn in a series of papers [19–21,23]. The main concept about axions is centered upon the breaking of a U(1) symmetry. Since the model is well known, we shall briefly address some of its basic properties.

2.1. The Peccei-Quinn Model

The existence of the axion was proposed by R D Peccei and H R Quinn [23] back in 1977. It was meant to be a solution to the charge-parity (CP) problem. In their work, the suppression of the neutron electric dipole moment comes from the addition of a U(1) group to the standard model, the generator of which is a neutral massive scalar boson; that is, the axion. The added U(1) symmetry, which is called Peccei-Quinn Symmetry, is a global one, independent of spatial coordinates but which breaks the left-right chiral symmetry. The associated new physical vacuum includes the axion as a field, with its mass resulting from a Higgs-like mechanism acting on the corresponding potential, which consists of quadratic and quartic terms in the axion field (from now on, we shall call it Φ). In a cosmological scenario, after the Big Bang, the temperature started to decrease from its initial value and, according to the axion model, at certain lower values, Peccei-Quinn Symmetry was broken; this gave rise to a phase transition of the first order, which resulted in a non-zero axion mass. However, the mass scale is not determined. This is different from the standard Higgs mechanism. As was described in [23], the coupling of the axion to the neutron suppresses the neutron electric-dipole moment, providing a solution to the strong CP problem. The existence of other channels of interactions between axions, photons, and baryons is proposed in [30].

From the experimental side, many experiments have actually been devoted to the search for axions, like the Axion Dark Matter Experiment (ADMX) [31], the Polarizzazione del Vuoto con LASer (PVLAS) experiment [32], and the Relic Axion Dark-Matter Exploratory Setup (RADES) [33], which is part of the CERN Axion Solar Telescope (CAST) [34]. Up to now, none of these measurements have recorded signals, but the search continues, and several other experiments for dark matter detection are in the development stage.

In addition to their role in cosmology, with reference to dark matter composition [30,35], axions may play a role in neutrino physics [36–38] because the coupling of neutrinos with axions could provide a mechanism to explain non-zero neutrino masses [39].

As was said before, axions are electrically neutral, massive bosons. By making use of the weak identity $m_a f_a \approx m_\pi f_\pi$ for axion mass m_a, we write the expression

$$m_a \approx \frac{6}{f_a/[10^6 \mathrm{GeV}]} \mathrm{eV}. \tag{1}$$

This is the mass of the axion in the simplest U(1) scenario. Taking values of f_a in the domain determined in [19,23], the mass m_a varies within the interval

$$10^{-6}\text{eV} \leq m_a \leq 10^{-3}\text{eV}. \tag{2}$$

2.2. Axion–Neutrino Couplings

In this section, we shall discuss another important consequence of the Peccei-Quinn hypothesis, which, in our opinion, can change the present understanding of neutrino physics substantially. The search for the determination of neutrino properties is, indeed, one of the central problems in physics today. The neutrino should be massive, as was determined from the measurements of neutrino-flavor oscillations. However, the mass scale of the neutrino mass eigenstates is unknown, as their number could relate to sterile neutrinos in addition to active neutrino mass eigenstates. The associated CP phases and the resulting sign of the amplitudes entering the neutrino linear combinations for each flavor are also unknown; only the modulus square of the U-factors and the difference between the squares of the neutrino mass eigenstates are approximately known. The ordering (normal, inverted, or degenerate) of the neutrino mass eigenstates is also unknown. Definitively, the Majorana or Dirac nature of the neutrino remains to be determined. These questions reside in the common area of particle and nuclear physics since, in addition to the search for neutrino properties in oscillation-like devices and in direct reaction with the production of neutrinos, the search for processes beyond the standard model of electroweak interactions is a very active one. In particular, the search for the signals of decays, which are forbidden by the standard model and where neutrinos play a major role, is active. This is the case of the neutrinoless double-beta decay. We shall describe this process later on. However, here, we shall advance our main idea, which consists of the determination of the axion–neutrino couplings as a mass mechanism for neutrinos to make a connection with the axion–neutrino picture [16,40]. Finally, we shall bring together the results from both scenarios to investigate the compatibility between them, meaning the limits for the neutrino mass coming from the Peccei-Quinn mechanism and from the non-observation of the neutrinoless double-beta decay. The Lagrangian for the coupling between neutrinos and axions can be written as [41–44]

$$\mathcal{L} \;=\; \imath \frac{g_a}{m_a} \bar{\psi} \gamma^{\mu} \gamma^5 \psi \partial_{\mu} a \;, \tag{3}$$

where the coupling constant g_a is dimensionless, and ψ and a are the neutrino and axion fields, respectively.

The axion acquires a vacuum expectation value in the presence of a potential [19]

$$V(\Phi) \;=\; -\frac{\mu^2}{2}\left(|\Phi|^2 - \frac{|\Phi|^4}{f_a^2}\right) . \tag{4}$$

Then, the variation of $V(\Phi)$ leads to the breaking of the U(1) symmetry and gives the vacuum expectation value $< \Phi >$

$$a \rightarrow (a(\bar{x}) + < \Phi >) e^{-\imath m_a t} \;, \tag{5}$$

where $< \Phi > = \frac{f_a}{\sqrt{2}}$. This mechanism, which is similar to the Higgs mechanism [45] and gives masses to all other particles but the neutrinos, expresses axion mass in terms of the expectation value of the field Φ that depends on the scale-fixing variable f_a. Then, by adopting its value and replacing it in the time derivative $\partial_0 a$, we obtain the neutrino mass term from Equation (3) after performing the correspondence

$$\begin{aligned} m_\nu^{(0)} &= g_a < \Phi > \\ &= g_a \frac{f_a}{\sqrt{2}}. \end{aligned} \tag{6}$$

In other words, with the Lagrangian introduced above, we obtain the following for the interaction Hamiltonian, written in natural units:

$$\mathcal{H}_{int} \approx g_a \langle \Phi \rangle \psi^\dagger \psi + g_a \vec{\nabla}\Phi(\vec{x}, t).\vec{S}. \tag{7}$$

Here, $\vec{S}$ is the spin operator acting on the neutrino sector.

Therefore, at the lowest order in the coupling g_a, we may introduce the correspondence

$$m_\nu \to g_a \langle \Phi \rangle, \tag{8}$$

as was stated before since $\psi^\dagger \psi$ is neutrino density.

To calculate the contributions to the neutrino mass coming from the spin-dependent term of $\mathcal{H}_{int}$ of Equation (7), or the transition amplitude, we write

$$\mathcal{A}_{i \to f} = -i g_a \int d^4x \, \langle f | \vec{\nabla}\Phi.\mathbf{\vec{S}} | i \rangle, \tag{9}$$

with $|i\rangle$ and $|f\rangle$ being the initial and final states of a free neutrino.

For spin-up and spin-down neutrino states, we obtain

$$\begin{aligned} \langle f | \vec{\nabla}\Phi.\mathbf{\vec{S}} | i \rangle &= i\mathcal{N}_i\mathcal{N}_f \Bigg[\left(1 + \frac{(p'_z p_z - p'_- p_+)}{(E+m)(E'+m)} \right) \frac{\partial \Phi}{\partial z} \\ &+ \frac{(p'_- p_z + p'_z p_+)}{(E+m)(E'+m)} \frac{\partial \Phi}{\partial x} \\ &+ i \frac{(-p'_z p_+ + p'_- p_z)}{(E+m)(E'+m)} \frac{\partial \Phi}{\partial y} \Bigg], \end{aligned} \tag{10}$$

and

$$\begin{aligned} \langle f | \vec{\nabla}\Phi.\mathbf{\vec{S}} | i \rangle &= -i\mathcal{N}_i\mathcal{N}_f \Bigg[\left(1 + \frac{(p'_z p_z - p'_+ p_-)}{(E+m)(E'+m)} \right) \frac{\partial \Phi}{\partial z} \\ &+ \frac{(p'_+ p_z + p'_z p_-)}{(E+m)(E'+m)} \frac{\partial \Phi}{\partial x} \\ &+ i \frac{(p'_+ p_z - p'_z p_-)}{(E+m)(E'+m)} \frac{\partial \Phi}{\partial y} \Bigg], \end{aligned} \tag{11}$$

where $\mathcal{N}_i(\mathcal{N}_f)$ is the norm of the initial (final) state. The field $\Phi(\vec{x}, t)$ depends on position and time. In order to solve these equations, we shall assume a model for the distribution of dark matter. Several DM distributions have been proposed. As an example of spatial distributions of dark matter, we can adopt (for the axions) a directional Gaussian parallel to the neutrino incoming direction (arbitrarily chosen in the z-direction); there will not be a spin-flip term contributing to the amplitude $\langle f | \vec{\nabla}\Phi.\mathbf{\vec{S}} | i \rangle$, and the spin-up contribution will then look like

$$\langle f | \vec{\nabla}\Phi.\mathbf{\vec{S}} | i \rangle = i\mathcal{N}_i\mathcal{N}_f \left(1 + \frac{(p'_z p_z - p'_+ p_-)}{(E+m)(E'+m)} \right) \frac{\partial \Phi}{\partial z}, \tag{12}$$

and for the spin-down contribution, we have

$$\langle f|\vec{\nabla}\Phi.\vec{\mathbf{S}}|i\rangle \quad = \quad -i\mathcal{N}_i\mathcal{N}_f\left(1+\frac{(p'_z p_z - p'_+ p_-)}{(E+m)(E'+m)}\right)\frac{\partial\Phi}{\partial z}. \tag{13}$$

In Equations (10)–(13), $\langle f|\vec{\nabla}\Phi.\vec{\mathbf{S}}|i\rangle$, $\mathcal{N}_i$ and $\mathcal{N}_f$ are the normalization factors of the initial and final neutrino states, and E' and p' are the energy and components of the neutrino momentum in the final state.

Equation (6) expresses the neutrino mass as a result of a sort of conventional Higgs mechanism where the axions play the role of the Higgs bosons, represented by the potential $V(a)$ (see Equation (4)). In other words, Equation (6) gives the neutrino mass as resulting from the coupling with axions in the symmetry-breaking regime. The difference regarding the Higgs mechanism is that the axion mass (see Equation (1)) can be a completely free parameter, depending on the value of f_a.

By taking values of f_a in the domain determined in Equation (2), we obtain values for the coupling constant g_a as a function of m_a and f_a. These values of g_a are constrained by the adopted values for the neutrino mass m_ν. These constraints are determined, among other evidence, by the non-observation of the neutrinoless double-beta decay ($0\nu\beta\beta$) [16]. The $0\nu\beta\beta$ process is forbidden in the context of the standard model. It violates lepton number conservation and requires mediation by massive neutrinos (mass mode) and/or left–right and right–right current couplings. A nucleus with N neutrons and Z protons could decay to a nucleus with N $-$ 2 and Z + 2 protons by emitting two electrons without neutrinos in the final state. From the non-observation of the decay at the level of a half-life lower limit of $T_{1/2}^{0\nu\beta\beta} > 10^{24}$ years, a value for the effective neutrino mass of the order of $m_\nu \approx 0.01$ eV [46] is extracted. Until now, decay has not been observed, and the values extracted from the limits on the half-life are model-dependent. Even if the decay is measured, the mechanism responsible for it could not be determined, but it will show conclusively that the standard model of electroweak interactions in its present version is not the ultimate theory. This means that the standard model should be reformulated in order to include massive neutrinos, both light and heavy, and a second generation of right-handed bosons $(W^\pm, Z_0)_R$.

Going beyond this scheme requires going to the next order in the couplings. The one-loop corrections to the neutrino propagator $S(p)$ is written as

$$\imath S'(p) \quad = \quad \imath S(p) + \imath S(p)(-\imath\Sigma(p))\,\imath S(p)\ , \tag{14}$$

where

$$\Sigma(p) \quad = \quad \int \frac{\mathrm{d}^4k}{(2\pi)^4}\imath g_a\gamma^\alpha \frac{\imath}{p-k-m_\nu}\frac{g_{\alpha\beta}}{k^2-m_a^2}\gamma^\beta \imath g_a\ , \tag{15}$$

p and k are the neutrino and the axion 4-momenta, respectively, m_ν is the neutrino mass, m_a is the axion mass, and $g_{\alpha\beta}$ is the metric. To compute the one-loop correction to the mass (Equation (6)), we shall work in $d = 4+\epsilon$ dimensions and include a parameter ξ that has the dimension of mass. Then, after integration into intermediate momentum, one obtains

$$\Sigma(p) \quad = \quad \frac{g_a^2}{16\pi^2}\Gamma\left(\frac{\epsilon}{2}\right)\int_0^1 \mathrm{d}x[(\epsilon-2)p(1-x)+(4-\epsilon)m] \times\left[\frac{(m^2-m_a^2)x+m_a^2-p^2x(1-x)}{4\pi\xi^2}\right]^{-\epsilon/2}. \tag{16}$$

Taking $\Sigma(p)$ on the shell, the neutrino physical mass is written as

$$m_\nu = m_\nu^0 + \Sigma(p)\Big|_{p^2=m_\nu^2} . \quad (17)$$

So far, we have assumed that the axion field is non-local; that is, the axions as DM components are everywhere. However, for the case of local distributions of DM, one has to also compute the amplitudes given in Equations (10) and (11), which originate in the second term of the Hamiltonian of Equation (7), where the gradient of the axion field is coupled to the spin of the neutrinos. These terms give the second-order contributions to the neutrino mass, but these corrections are, in practice, several orders of magnitude smaller than the leading-order estimate since they are proportional to g_a^2.

2.3. The Search for Neutrino Mass in Rare Nuclear Decays

As we have mentioned, the search for the values of the neutrino mass is one of the hottest topics in theoretical and experimental physics. Among the efforts to determine the value of m_ν, the current search for the observation of neutrinoless double-beta decay is one of the most demanding efforts [16], and it has direct consequences upon the solution to the neutrino mass problem [46]. The process in which a nucleus (with mass A), N neutrons, and Z protons decay into another of the same mass (A) but with N − 2 neutrons and Z + 2 protons is known as nuclear double-beta decay. The decay can eventually use two modes, namely (i) two-neutrino double-beta decay and (ii) the neutrinoless double-beta decay. In the case of the so-called two-neutrino double-beta decay, two neutrons decay into two protons, followed by two electrons and two antineutrinos of the electron flavor. This process is allowed by the standard model of electroweak interactions since it preserves lepton number symmetry. The energy spectrum of the emitted electrons is a continuous one that has an endpoint at the Q-value of the decay. In order to occur, the ground state of the final nucleus should be at a lower energy than the ground state of the initial nucleus. This decay mode, one of the rarest nuclear decays, has been observed in medium- and heavy-mass nuclei, among other candidates like ^{76}Ge, ^{96}Zr, ^{100}Mo, and ^{128,130}Te [7]. The half-life of this decay mode is of the order of 10^{19} years. The situation is much more complex for the case of the other decay mode—the so-called neutrinoless double-beta decay mode—because no neutrinos are emitted, and the two emitted electrons have a single energy channel available, which is just the Q-value of the decay. If it is measured, the signal will be two electrons flying away at the endpoint with each of them sharing half of the Q-value. The questionable aspect is that this decay is forbidden by the rules of the standard model of electroweak interactions because it violates lepton number symmetry in two units. The theoretical consequences of the observation of the neutrinoless double-beta decay are extremely important. It implies that the neutrino is a Majorana neutrino and that right-handed currents and the associated bosons should exist. In other words, it would indicate that the standard model of electroweak interactions is not the ultimate model and that the $SU(2)_{left}$ structure is accepted as valid with the associated $W_\pm$ and Z_0 left triplet of mediators; this should be replaced by a $SU(2)_{left} \times SU(2)_{right}$ structure with the added triplet of $W_\pm$ and Z_0 right-mediating bosons. Moreover, the scheme of neutrino-lepton doublets, which the standard model represents using three left-handed lepton doublets of charged leptons and the corresponding neutrinos, coming in three flavors (electron, tau, and muon flavors), should be enlarged to include right-handed doublets instead of the right-handed singlets of charged leptons without neutrinos. Therefore, in order to accommodate the required degrees of freedom, it would be necessary to introduce light neutrino mass eigenstates and heavy mass neutrino mass eigenstates. The existence of at least three generations of light-mass neutrino mass eigenstates has been demonstrated by the observation of neutrino oscillations. The lower limit for the half-life of the still unobserved neutrinoless double-beta decay mode is of the order of 10^{24} years. The discovery of this nuclear decay mode would, indeed, mark the beginning of a new era in physics. The half-life of the two-neutrino double-beta decay ($2\nu\beta\beta$) is proportional to the inverse of the product of the square of the transition amplitude and the corresponding phase space factor. The decay path is represented by two virtual transitions, one connecting the initial state of the A(N,Z) nucleus with the intermediate A (N − 1,Z + 1) nucleus and another one going from the intermediate to the final A(N − 2,Z + 2) nucleus. The first branch on the decay is stronger than the second branch, and it could reach predominantly Gamow-Teller and Fermi-type transitions, where these states belong to the intermediate nucleus. It can also imply other multipole excitations of the first forbidden type. The transition is represented by the following chain: initial ground state to the set of states of the intermediate nucleus to the ground and excited states of the final nucleus. The associated nuclear matrix elements are strongly quenched by giant resonances, which concentrate practically all of the strength for Gamow-Teller type of transitions. Therefore, the theoretical nuclear models have to deal with large cancellation effects. The use of different nuclear models shows a strong dispersion of values of these calculated nuclear matrix elements. The uncertainties associated with these calculations are related to the control of a variety of parameters, like the single-particle energies, pairing strengths, and the strength of the residual interactions of the proton-neutron type. A major breakthrough concerning theoretical models was the discovery of the effects associated with particle–particle interactions in the proton-neutron channels that compete with particle-hole-type forces, producing a drastic reduction in the values of the nuclear matrix elements for the two-neutrino mode of decay [40,46]. From a theoretical point of view, different techniques have been and are still applied to calculate the matrix elements of this mode, like the shell model, density functional theory, quasi-particle random phase approximation, and even some geometrical models. None of these models could be applied to describe all the transitions in different mass regions. The situation concerning the phase space factors is clearer, although knowledge of the value of axial vector coupling is required, and several attempts have been made to formulate the renormalization effects that affect it, resulting from two-body correlations and meson exchange processes in nuclei. The calculated nuclear matrix elements are of the order of 0.1 or smaller. The main ingredients entering the calculation of neutrinoless double-beta decays ($0\nu\beta\beta$) are different from those of the two-neutrino decay channel. To start with, the virtual transitions to the intermediate nucleus are not restricted to the action of a single multipole of the beta current. Since the intermediate Majorana neutrino is emitted in one vertex and absorbed in the other, this second-order process takes place at a fixed energy, and it does not require the inclusion of energy denominators, which is different from the case of the two-neutrino mode. As with the two-neutrino mode, the expression depends on the fourth power of the axial vector coupling constant, but the nuclear matrix elements are not suppressed. The main difference with respect to the two-neutrino mode is the explicit presence of the square of the average neutrino mass in the formula describing the neutrinoless double-beta decay mode. Therefore, if the mode is experimentally observed, it will set definite limits on the value of the effective neutrino mass, but the mechanism responsible for the decay will not be uniquely determined. Typical values of the nuclear matrix elements calculated at present are of the order of 3–5. Obviously, the search for neutrinoless double-beta decays is not limited to designs based on the accumulation of nuclear masses and the subsequent control of backgrounds. Some of these designs are conducted in large-scale underground labs. Other alternatives are based on the exploration of the structure of states, which might mediate double-beta decay transitions, as is the case for reactions like muon capture via final nuclei participants in double-beta decay chains.

2.4. Propagation of Neutrinos in a Dark Matter Environment

Neutrinos (ν) are produced in practically all astrophysical processes. The interaction of neutrinos from distant sources with other particles yields useful information about these particles. Since dark matter particles are the natural choice for such media, the cross-sections for the ν-dark matter interactions are the proper tool with which to test the composition and properties of dark matter. The evidence for the presence of large amounts of invisible matter has been extracted from the observed rotational curves of galaxies [47] and from the expanding rate of the Universe [48,49]. The composition of the Universe is estimated to be in the intervals of 4.6 to 5.0% for ordinary matter, 24.0 to 25.9% for dark matter (DM), and the remaining 69.1 to 71.4% for dark energy [50]. The exact nature of DM is still unknown; however, there are a number of theoretical models about it [51–57].

The interaction between the CDM and neutrinos may take place through the exchange of mediating neutral bosons [58–60].

Neutrinos produced in astrophysical sources like supernovae (SN), gamma-ray bursts (GRBs), blazars, etc, could interact with dark matter particles on their way to the Earth [4]. From here on, we shall give the expressions needed to calculate the cross-section of neutrino-CDM current–current interactions. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_n + \mathcal{L}_{DM} + \mathcal{L}_{int}, \tag{18}$$

where $\mathcal{L}_n$ is the Lagrangian of a free neutrino of mass m,

$$\mathcal{L}_{n,Dirac} = \bar{\nu}(i\gamma^\mu \partial_\mu - m)\nu, \tag{19}$$

for a Dirac neutrino or

$$\mathcal{L}_{n,Majorana} = \frac{1}{2}\left[\bar{\nu}^c(i\gamma^\mu \partial_\mu)\nu^c + \bar{\nu}(i\gamma^\mu \partial_\mu)\nu - (\bar{\nu} m \nu^c + \bar{\nu}^c m \nu)\right], \tag{20}$$

for a Majorana neutrino. The term $\mathcal{L}_{DM}$ is the Lagrangian for the DM particles of mass M,

$$\mathcal{L}_{DM} = \bar{N}\left(i\gamma^\mu \partial_\mu - M - V(\vec{r})\right)N, \tag{21}$$

where the scalar potential $V(\vec{r})$ determines the assumed DM spatial distribution. The interaction term is given by the expression

$$\mathcal{L}_{int} = -g j^\mu(x) J_\mu(x), \tag{22}$$

where g is a coupling constant, and j^μ and J_μ are the neutrino and DM currents, written in standard notation for spinors. The propagator of the boson mediating the interactions is written as

$$\frac{-g^{\mu\nu} + k^\mu k^\nu / M_0^2}{k^2 - M_0^2}, \tag{23}$$

where k^μ are the 4-momenta components of the boson, and M_0 is its mass. The S-matrix corresponding to this process is given by

$$\begin{aligned} \hat{S} &= e^{-iHt} = I - i\int d^4x\, \hat{\mathcal{H}}_{int}(x) \\ &\quad - \int\int d^4x\, d^4x' \hat{\mathcal{H}}_{int}(x)\hat{\mathcal{H}}_{int}(x'), \end{aligned} \tag{24}$$

up to the second order in the interactions. It yields the scattering amplitude

$$\mathcal{A}^{(1)}_{i\to f} = \langle i|\mathrm{T}\left\{(-i)\int d^4x \hat{\mathcal{H}}_{int}(x)\right\}|f\rangle. \tag{25}$$

where T$\{\cdots\}$ is the time-ordered product, and $|i\rangle$ and $|f\rangle$ are the initial and final states, respectively, with each of them being the product of a neutrino and a DM particle state. The neutrinos and dark matter currents are calculated separately. The cross-section is calculated by applying the well-known S-matrix formalism [45] at the first order in the couplings.

The differential cross-section is written as

$$d\sigma = \frac{1}{|\vec{v}|}\frac{1}{2E_p}\frac{1}{2E_q}\frac{d^3p'}{2E_{p'}(2\pi)^3}\frac{d^3q'}{2E_{q'}(2\pi)^3}\left|\mathcal{A}^{(1)}_{i\to f}\right|^2, \tag{26}$$

where p, q, p', q' denote the momenta of the initial and final states of the neutrino (p, p') and DM (q, q'), respectively.

The velocity distribution of DM has been proposed in different models, as it is discussed by K Freese et al. [4] and in [46]. In terms of these distributions, the differential cross-section becomes

$$\begin{aligned} d\sigma &= \frac{g^2_{eff}}{8\pi^2}\frac{1}{p}\frac{1}{M}\frac{d^3q' f(\frac{\mathbf{q}'}{M})}{E_{q'}}\frac{d^3p'}{p'}\delta^4(p'+q'-p-q) \\ &\times \left[\left(p'E_{q'} - \mathbf{p}'\cdot\mathbf{q}'\right)pM + p'M\left(pE'_q - \mathbf{p}\cdot\mathbf{q}'\right) - M^2(pp' - \mathbf{p}\cdot\mathbf{p}')\right], \end{aligned} \tag{27}$$

We are left with the integration on the outgoing momentum $\mathbf{p}'$ of the neutrino.

The cross-section for the DM distribution with the parameters given in [61] is written as

$$\frac{d\sigma}{d\Omega dp'} = \frac{g^2_{eff}}{8\pi^2}\frac{1}{N}p'^2\left[1 - \frac{M - (|\mathbf{p}| - p')}{E_{q'}}\cos\theta\right]\left[1 - (1-q_0)\frac{|\mathbf{p}-\mathbf{p}'|^2}{M^2v_0^2}\right]^{q_0/(1-q_0)}. \tag{28}$$

This distribution depends on two parameters, q_0 and v_0. We shall take the best-fit values obtained in the high-resolution cosmological N-body simulation performed by Ling et al. in [61], with $q_0 = 0.773$ being the factor entering the power law and $v_0 = 267.2$ km/s being the velocity of DM particles. The dependence on the mass of the DM particles is weaker than the dependence observed for the mass of the mediating boson. Nevertheless, the results may indicate a tendency of the cross-sections to decrease for increasing values of the mass of DM particles. Therefore, the huge difference between the differential cross-section for the proposed distributions may be taken, in principle, as a test of models for DM particles. From the analysis of the expressions presented above, we may conclude by saying that the cross-section for the interaction of neutrinos with DM particles is sensitive to the momentum distribution of DM. We may then expect that the momentum dependence of the proposed DM distributions may be of some use at the time of distinguishing between DM candidates in different models.

In order to complete this revision of physical processes mediated by axions, we shall address the formalism of the Bose–Einstein condensation (BEC) in the next subsection.

2.5. Basic Notions About Bose–Einstein Condensation

The number of bosons, N, in the momentum representation in a finite volume V and at the inverse temperature $\beta = \frac{1}{kT}$ is written [62] as

$$N = \frac{4\pi V}{(2\pi\hbar)^3}\int \frac{dp\ p^2}{e^{\beta(E_p-\mu)} - 1}, \tag{29}$$

where $E_p = \sqrt{p^2c^2 + m^2c^4}$, and μ is the chemical potential that enforces the constraint imposed on the average number of bosons. This integral can be transformed after some straightforward steps into

$$\frac{N}{V} = \frac{1}{4\pi^2}\left(\frac{2mc^2}{\hbar^2c^2}\right)^{\frac{3}{2}} \int dw\, w^{\frac{1}{2}} \frac{e^{-\beta(w-\mu)}}{1-e^{-\beta(w-\mu)}}. \quad (30)$$

Furthermore, by changing the variables, the above integral is further written as

$$\int \rightarrow \frac{1}{\beta^{\frac{3}{2}}} \sum_{n=0} \frac{\lambda^{(n+1)}}{(n+1)^{\frac{3}{2}}} \int dw\, w^{\frac{1}{2}} e^{-w}. \quad (31)$$

Since

$$\int dw\, w^{\frac{1}{2}} e^{-w} = \frac{\sqrt{\pi}}{2}, \quad (32)$$

the density of bosons becomes

$$\frac{N}{V} = \frac{1}{4\pi^2}\left(\frac{2mc^2}{\hbar^2c^2}\right)^{\frac{3}{2}} \frac{\sqrt{\pi}}{2}(kT)^{\frac{3}{2}} \sum_{n=0} \frac{\lambda^{(n+1)}}{(n+1)^{\frac{3}{2}}}. \quad (33)$$

In the above equation $\lambda = e^{\beta\mu}$, which goes to $\lambda \rightarrow 1$ as $\mu \rightarrow 0^-$. Then, the number of particles in the zero momentum state ($p = 0$), N_c, is written as a function of the total number of particles, N_0,

$$N_c = N_0\left[1 - \left(\frac{T}{T_c}\right)^{\frac{3}{2}}\right]. \quad (34)$$

Finally, in this regime, which determines the occupation of the lowest energy state for the bosons, meaning the zero momentum state, the temperature T_c that limits the condensate is cast in terms of the spatial density $\rho = N/V$ as

$$kT_c = 2\pi\left(\frac{\hbar^2c^2}{mc^2}\right)\rho^{\frac{2}{3}}. \quad (35)$$

To obtain this result, we assumed that the axions were massive and non-relativistic. In Section 3, we shall discuss the consequences of the formalism introduced in the previous subsections by performing numerical calculations with appropriate adopted parameters.

3. Numerical Estimates

In this section, we estimate how the effective neutrino mass constrains the axion parameters, and we briefly discuss the conditions under which axions could form a Bose–Einstein condensate (BEC), potentially contributing to the dark matter component of the Universe.

3.1. The Effective Neutrino Mass

In contrast to the estimation $g_a f_a \approx 1$, when imposing the value of the neutrino mass as a constraint, the product $g_a f_a$ differs from unity by several orders of magnitude. For values of $m_\nu \approx 0.1$ eV and $m_a \approx 0.6$ eV, Equations (1), (2), and (6) yield $f_a \approx 10^7$ GeV and $\frac{g_a}{\sqrt{2}} \approx 10^{-17}$. For smaller values of the neutrino mass, the values of the constants f_a and g_a are scaled to small values of the axion mass of the order of 10^{-6} eV, which are still consistent with the limits determined by Equation (2). This result points to the extremely close relationship that exists between the physics of neutrinos and the physics of dark matter. When looking at the information coming from both regimes, we can gain better insights into the fundamental processes involved. Perhaps this is the proper place to stress the link between rare decays, neutrino physics, dark matter physics, and the basic assumptions supporting the theoretical basis of each field.

3.2. BEC Temperature

In order to obtain a value for the temperature T_c, we shall adopt a value for the mass of the axion and a value of its density ρ. The current expression of the mass of the axion can be taken from Equation (1). For density ρ, we chose to parametrize it in terms of the visible matter density, $\rho_{visible}$, as

$$\rho = \alpha\rho_{visible}. \quad (36)$$

The actual value for the density of visible matter and of the other needed parameters are listed in Table 1.

Table 1. Parameters needed to determine the value of the temperature kT_c that limits the condensed phase of axion-like matter.

$\rho_{visible}$ [gr/cm^3]	Fraction α	f_a [2×10^9GeV]
9.9×10^{-30}	5.217	$10^2 \to 10^5$

With them, the value of the critical temperature T_c is readily determined within the domain fixed by the variation of the axion mass, and it is written as

$$T_c = 9.29 \times 10^{-6} f_a[\text{K}] \quad (37)$$

Then, by varying the value of f_a in order to give a mass of the axion in the domain

$$10^{-6}[\text{eV}] \leq mc^2 \leq 10^{-3}[\text{eV}] \quad (38)$$

we obtain, accordingly, the interval

$$10^{-3}[\text{K}] \leq T_c \leq 1.0[\text{K}] \quad (39)$$

for the interval of temperatures, which would limit the condensate.

In performing the calculations, we have assumed that the distribution of matter in the Universe is isotropic and that its visible mass is of the order of 10^{80} m$_p$, with m$_p$ being the mass of the proton. For the radius of the Universe, we have taken the value r$_U$ = 1.3187 $\times$ 10 ^{23} km, calculated from the present-day value of the Hubble expansion rate H$_0$ = 69.96 $\pm$ 1.05 (stat) $\pm$ 1.12 (sys)Km/sec/Mpc.

The results expressed by Equation (39) point to a very interesting prospect: the possibility that the present value of the average absolute temperature of the Universe, which is of the order of a couple of degrees Kelvin, more precisely 2.73 K, might be near the domain of a dark matter component consistent with a Bose–Einstein condensate of neutral scalar axions. At the same time, this result is pretty consistent with the expected values of the axion mass coming from arguments based on the limits of the neutrino mass [16].

4. Conclusions

In this work, we have addressed two fundamental questions in contemporary physics: the origin of the Majorana mass of neutrinos and the nature of dark matter. Through the axion–neutrino coupling mechanism, we have proposed a possible pathway for the generation of neutrino mass and examined Bose–Einstein condensation as a viable scenario for axions being a dominant component of dark matter in the Universe. Our results show that the obtained axion mass values are consistent with the experimental constraints on neutrino mass, particularly those derived from the still-unobserved neutrinoless double-beta decay. Furthermore, we have explored the thermodynamic implications of a finite axion mass, demonstrating that the condensation temperature T_c varies from a few millikelvin to approximately one kelvin, depending on the axion scale factor f_a. These findings reinforce the hypothesis of axions as plausible dark matter candidates and highlight the need for experimental investigations to explore the potential detection of axion–neutrino coupling. Future studies could focus on refining axion mass limits through astrophysical observations and laboratory experiments, as well as analyzing the cosmological consequences of axion condensates in the early Universe.

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