THE SPIN RESPONSE OF THE NUCLEON AND ITS IMPLICATION FOR THE GDH SUM RULE AND THE DOUBLE POLARIZATION E ASYMMETRY

Eed M. Darwish[†], Mohamed A. El-Zohry

Physics Department, Faculty of Science
South Valley University, Sohag 82524, Egypt

(Received June 28, 2005)

The contribution of single-pion photoproduction channels to the spin response of the nucleon, *i.e.* the asymmetry of photoabsorption cross sections with respect to parallel and antiparallel spins of photon and nucleon, is calculated over the region of the $\Delta(1232)$-resonance adopting an effective Lagrangian model for the reaction amplitude. Furthermore, the contribution from separate pion photoproduction channels to the Gerasimov–Drell–Hearn integral is explicitly evaluated by integration up to a photon lab-energy of 550 MeV. In addition, the double polarization E asymmetry for the individual pion photoproduction channels is predicted. A quite satisfactory agreement with recent experimental data from the GDH Collaboration is obtained.

PACS numbers: 25.20.Lj, 13.60.Le, 11.55.Hx

1. Introduction

The Gerasimov–Drell–Hearn (GDH) sum rule [1, 2] is one of several dispersive sum rules that connect the Compton scattering amplitudes to the inclusive photoproduction cross sections of the target under investigation. Being based on such universal principles as causality, unitarity and gauge invariance, these sum rules provide a unique testing ground to study the internal degrees of freedom that hold the system together. The GDH sum rule connects the anomalous magnetic moment (a.m.m.) of a particle κ with the energy-weighted integral from π-threshold up to infinity over the spin asymmetry of the total photoabsorption cross section, *i.e.*, the difference of the total photoabsorption cross sections for circularly polarized photons on

[†] Corresponding author. E-mail address: `eeddarwish@yahoo.com`

a longitudinally polarized target with spin parallel $\sigma^{\rm P}$ and antiparallel $\sigma^{\rm A}$ to the photon spin

$$I^{\rm GDH}(\infty) \;=\; \frac{\pi e^2\kappa^2}{M^2}\,S = \int\limits_{\omega_{\rm thr}}^{\infty} \frac{d\omega_\gamma}{\omega_\gamma}\,\left(\sigma^{\rm P}(\omega_\gamma)-\sigma^{\rm A}(\omega_\gamma)\right)\,, \tag{1}$$

where the integral runs over the photon energy in the laboratory frame, ω_γ, from the lowest threshold $\omega_{\rm thr}$ to infinity. The a.m.m. κ can be read off the relation between the total magnetic moment $\vec{\mu}$ and the spin $\vec{S}$ of a particle in its center-of-mass (c.m.) frame, $\vec{\mu} = \frac{e}{M}\,(Q+\kappa)\,\vec{S}$, where eQ is the charge and M is the mass of the particle. The elementary charge e is defined by $e^2/4\pi = 1/137$.

Because the left-hand side (lhs) of (1) is positive, we can draw the qualitative conclusion that the photon prefers to be absorbed with its spin parallel to the target spin. Of course, the sum rule is much more quantitative. The same agent that is responsible for the a.m.m. on the lhs of (1) must also lead to an appropriate energy dependence of the helicity difference of the cross sections, $\sigma^{\rm P}-\sigma^{\rm A}$, such that the sum rule is fulfilled. However, keep in mind one caveat: A basic assumption in deriving (1) is that $\sigma^{\rm P}-\sigma^{\rm A}$ vanishes sufficiently fast such that the integral converges.

Having followed through to this point, the reader may well ask whether we can ever be sure that the GDH sum rule is fulfilled. In a sense we cannot, because ultimately physical questions can only be answered by experiment, and no experiment will ever decide whether an infinite integral converges or diverges in the mathematical sense. However, we should view such questions from a more physical standpoint. First, the GDH sum rule is an ideal testing ground, because the lhs of (1) is given by ground state properties and, in the case of the nucleon, is known to at least eight decimal places. The result is 204.8 μb for the proton and 233.2 μb for the neutron [3, 4]. Thirty years ago, the first estimation for the right-hand side (rhs) of (1) were 261 μb for the proton and 183 μb for the neutron [5]. Over the following years the predictions moved even further away from the sum rule values despite an improving data basis, simply because these data were not sensitive to the helicity difference of the inclusive cross sections. Many explanations for the apparent violation of the sum rule followed, but in view of the more recent experimental evidence we have no intention to deliberate about these models.

Albeit more than thirty years old today, the GDH sum rule still lacks a direct experimental check, since both beam and target have to be polarized and a wide range of photon energies has to be covered, which presents an enormous challenge. Recently, several experiments to test the GDH sum rule both on the proton and on the neutron are now performed or planned at different laboratories (MAMI, ELSA, and LEGS) [6–9]. This makes the study of the GDH sum rule becomes of great interest in the field of intermediate energy nuclear physics. The verification of the GDH sum rule is an important check of our knowledge of the γN interaction and also an important comparison test for nucleon models. Furthermore, the spin asymmetry of the total photoabsorption cross section, entering into the GDH sum rule, is of particular interest.

The importance of polarization observables cannot be overstated. In the case of photoproduction of a single pseudoscalar meson, four complex amplitudes are required to describe the process. Since one phase will always remain ambiguous, this means that seven 'numbers' are required at each kinematic point. The differential cross section provides information only on the sum of the absolute squares of these amplitudes. Polarization observables allow extraction of more information, including phases.

Therefore, the main goal of this paper is to report on an evaluation of the GDH sum rule for the nucleon by explicit integration of the GDH integral up to a photon energy of 550 MeV using a reliable pion photoproduction operator and including the individual contributions from the different charge states of the pion. Their total sum to the spin asymmetry for pion photoproduction on the nucleon will also be evaluated. This asymmetry is a very important quantity which deserves detailed investigations for the various channels. The reason for this is that this asymmetry contains very interesting physics with respect to the hadronic structure of the system describing its optical activity which reflects an internal screw-like or chiral structure. Besides spin asymmetry and the GDH sum rule, the double polarization asymmetry E will also be discussed. These polarization asymmetries are an essential ingredient in the interpretation of various meson production reactions in terms of the various resonances that contribute to the processes as real or virtual intermediate states.

The rest of this article is organized as follows. In the next section we give the formal expressions for the pion photoproduction amplitude that we use as a main tool in our calculations. In Sec. 3 we present the value of the GDH sum rule for the nucleon (proton or neutron). The main results together with a comparison with recent experimental data will be presented and discussed in Sec. 4. Finally, we conclude our results in Sec. 5.

2. The theoretical model

As a starting point, we will consider first the model which we use for studying pion photoproduction on the nucleon

$$\gamma(k, \vec{\epsilon}) + N(p_1) \rightarrow \pi(q) + N(p_2)\,, \tag{2}$$

where we have defined the notation of the four-momenta for the participating particles. The polarization vector of the photon is denoted by $\vec{\epsilon}$. In this work, the effective Lagrangian model of Schmidt *et al.* [10] will be used. This model contains, beside the standard pseudovector Born terms, the resonance contribution from the $\Delta(1232)$-excitation. The individual tree-level contributions to the scattering amplitude are shown in Fig. 1. In the following, we give the formal expressions for these two terms explicitly. We refer to [10] for further details.

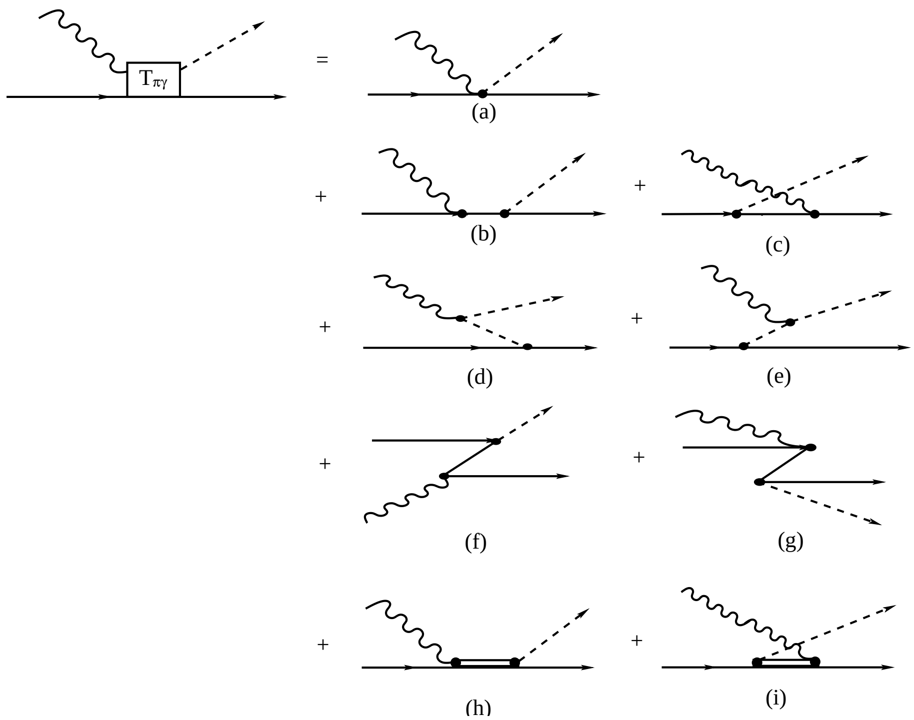

Fig. 1. The tree-level Feynman diagrams for $\gamma N \to \pi N$: (a) the Kroll–Rudermann graph, (b) and (c) the two time-ordered contributions to the direct and crossed nucleon pole graph, (d) and (e) the two time-ordered contributions to the pion pole graph, (f) and (g) the Z-graphs and (h) and (i) the $\Delta(1232)$ resonance graphs. A solid, dashed and wavy line represents a nucleon, pion and photon, respectively.

2.1. The Born terms

First, we consider the nonresonant amplitudes. These are referred to as the Born terms and they are dominant at low energy and for charged pion photoproduction still provide 50% of the cross section in the energy region of the $\Delta(1232)$-resonance. Using graphs (a) to (g) in Fig. 1 the following expression for the matrix element of the Born terms is found [10]

$$
\begin{aligned}
T_{fi}^{\mathrm{B}} \; = \; & (2\pi)^3\delta^3(\vec{p}_2+\vec{q}-\vec{p}_1-\vec{k})\frac{if_{\pi N}}{2m_\pi}\Bigg[2\vec{\sigma}\cdot\vec{\epsilon}\left[\hat{e},\tau_\mu^\dagger\right] \\
& -\left(\frac{\tau_\mu^\dagger\vec{\sigma}\cdot\vec{q}\left(2(\vec{p}_2+\vec{q})\cdot\vec{\epsilon}\,\hat{e}+i\vec{\sigma}\cdot\vec{k}\times\vec{\epsilon}\,(\hat{e}+\hat{\kappa}_N)\right)}{E_{\vec{p}_2+\vec{q}}(\omega_{\vec{q}}+E_{\vec{p}_2}-E_{\vec{p}_2+\vec{q}})}\right. \\
& \left.+\frac{\left(2\vec{p}_2\cdot\vec{\epsilon}\hat{e}+i\vec{\sigma}\cdot\vec{k}\times\vec{\epsilon}(\hat{e}+\hat{\kappa}_N)\right)\tau_\mu^\dagger\vec{\sigma}\cdot\vec{q}}{E_{\vec{p}_2-\vec{k}}(E_{\vec{p}_2}-E_{\vec{p}_2-\vec{k}}-\omega_\gamma)}\right) \\
& +\frac{2\vec{q}\cdot\vec{\epsilon}\vec{\sigma}\cdot(\vec{q}-\vec{k})}{\omega_{\vec{q}-\vec{k}}}\left(\frac{1}{\omega_{\vec{q}}-\omega_{\vec{q}-\vec{k}}-\omega_\gamma}+\frac{1}{\omega_\gamma-\omega_{\vec{q}-\vec{k}}-\omega_{\vec{q}}}\right)\left[\hat{e},\tau_\mu^\dagger\right] \\
& +2M_N\omega_{\vec{q}}\vec{\sigma}\cdot\vec{\epsilon}\left(\frac{\tau_\mu^\dagger\hat{e}}{E_{\vec{p}_2+\vec{q}}(E_{\vec{p}_2+\vec{q}}+E_{\vec{p}_2}+\omega_{\vec{q}})}\right. \\
& \left.+\frac{\hat{e}\tau_\mu^\dagger}{E_{\vec{p}_2-\vec{k}}(E_{\vec{p}_2-\vec{k}}+E_{\vec{p}_2}-\omega_\gamma)}\right)\Bigg] \,, \qquad (3)
\end{aligned}
$$

where $E_{\vec{p}}=\sqrt{M_N^2+\vec{p}^{\,2}}$ and $\omega_{\vec{p}}=\sqrt{m_\pi^2+\vec{p}^{\,2}}$ are the energies of a nucleon and a pion with momentum $\vec{p}$, respectively. M_N and m_π are the nucleon and pion mass, respectively. The isospin projection of the produced pion is given by μ. $\vec{\sigma}$ are the Pauli spin matrices. $\vec{\tau}$ are the isospin matrices. $\hat{e}$ and $\hat{\kappa}_N$ denote nucleon charge and anomalous part of the nucleon magnetic moment, respectively. These are isospin operators of the nucleon and are given by

$$
\begin{aligned}
\hat{e} \; &= \; \frac{e}{2}\left(\mathbb{1}+\tau_0\right) \,, \\
\hat{\kappa}_N \; &= \; \frac{e}{2}\left[\kappa_p\left(\mathbb{1}+\tau_0\right)+\kappa_n\left(\mathbb{1}-\tau_0\right)\right] \,, \qquad (4)
\end{aligned}
$$

where $\kappa_p = \frac{1}{2}(\kappa_s+\kappa_v) = 1.79$ and $\kappa_n = \frac{1}{2}(\kappa_s-\kappa_v) = -1.91$ are the anomalous magnetic moments of the proton and the neutron in units of nuclear magnetons, respectively, and $\kappa_s = -0.12$ and $\kappa_v = 3.70$. Here, we used the πN coupling constant $\frac{f_{\pi N}^2}{4\pi} = 0.0735$ which is given in [11] by fitting the πN scattering data.

2.2. The Δ(1232)-resonance term

The dominant non-Born contribution for photon energies up to 500 MeV is that of the P_{33} pion–nucleon resonance, the $\Delta(1232)$-resonance. Using the nonrelativistic form of the Δ propagator, the various diagrams involving an intermediate $\Delta(1232)$ (see graphs (h) and (i) in Fig. 1) can be calculated. The following expression for the s and u channel contributions in the c.m. frame is obtained [10]

$$\begin{aligned} T_{fi}^{\Delta} &= (2\pi)^3\ \delta^3(\vec{p}_2+\vec{q}-\vec{p}_1-\vec{k}) \\ &\times\Bigg[\frac{F_\Delta\left(q^2\right)}{m_\pi}\,\frac{ef_{\pi N\Delta}G_{\Delta N}^{M1}(W_{\pi N})}{2\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}}}\,\frac{\{\tau_\mu^\dagger,\tau_0\}-\frac{1}{2}\left[\tau_\mu^\dagger,\tau_0\right]}{3} \\ &\times\frac{\vec{\sigma}_{N\Delta}\cdot\vec{q}\vec{\sigma}_{\Delta N}\cdot\vec{k}\times\vec{\epsilon}}{W_{\pi N}-M_\Delta+\frac{i}{2}\Gamma_\Delta(W_{\pi N})} \\ &+\frac{F_\Delta\left(0\right)}{m_\pi}\,\frac{ef_{\pi N\Delta}G_{\Delta N}^{M1}(0)}{2\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}}}\,\frac{\left\{\tau_\mu^\dagger,\tau_0\right\}+\frac{1}{2}\left[\tau_\mu^\dagger,\tau_0\right]}{3} \\ &\times\frac{\vec{\sigma}_{\Delta N}\cdot\vec{k}\times\vec{\epsilon}\vec{\sigma}_{N\Delta}\cdot\vec{q}}{E_{\vec{p}_2}-\omega_\gamma-E^{\Delta}_{\vec{p}_2-\vec{k}}}\Bigg], \end{aligned} \tag{5}$$

where $M_\Delta = 1232$ MeV is the mass of the Δ-resonance. Here $W_{\pi N}$ denotes the invariant mass of the πN-subsystem and it is given by

$$W_{\pi N} \;=\; E_N(q_{\text{c.m.}})+\omega_\pi(q_{\text{c.m.}})\,. \tag{6}$$

The transition spin (isospin) operator $\vec{\sigma}_{N\Delta}=\vec{\sigma}^\dagger_{\Delta N}$ ($\vec{\tau}_{N\Delta}=\vec{\tau}^\dagger_{\Delta N}$) is normalized as

$$\left\langle\tfrac{3}{2}\,||\sigma_{\Delta N}(\tau_{\Delta N})||\,\tfrac{1}{2}\right\rangle \;=\; -\left\langle\tfrac{1}{2}\,||\sigma_{N\Delta}(\tau_{N\Delta})||\,\tfrac{3}{2}\right\rangle = 2\,. \tag{7}$$

The energy dependent and complex coupling $G^{M1}_{\Delta N}(W_{\pi N})$ is given as in [12] by

$$G^{M1}_{\Delta N}(W_{\pi N}) = \begin{cases} \mu^{M1}(W_{\pi N})e^{i\Phi^{M1}(W_{\pi N})} & \text{for}\quad W_{\pi N}>m_\pi+M_N \\ 0 & \text{else} \end{cases}\,, \tag{8}$$

where $\mu^{M1}(W_{\pi N})$ is given by

$$\mu^{M1}(W_{\pi N}) = \mu_0+\mu_2\left(\frac{q_\Delta}{m_\pi}\right)^2+\mu_4\left(\frac{q_\Delta}{m_\pi}\right)^4 \tag{9}$$

and the phase $\Phi^{M1}(W_{\pi N})$ by

$$\Phi^{M1}(W_{\pi N}) = \frac{q_\Delta^3}{a_1 + a_2 q_\Delta^2}\,. \tag{10}$$

q_Δ is the on-shell pion momentum in the πN c. m. frame on the top of the resonance, *i.e.*, when the invariant mass $W_{\pi N}$ of the πN state equals the mass of the Δ-resonance

$$W_{\pi N} = \omega_\pi(q_\Delta) + E_N(q_\Delta) \;=\; M_\Delta\,. \tag{11}$$

It is given by

$$q_\Delta = \sqrt{\frac{\left(W_{\pi N}^2 - m_\pi^2 - M_N^2\right)^2 - 4m_\pi^2 M_N^2}{4W_{\pi N}^2}}\,. \tag{12}$$

The free parameters $\mu_0 = 4.16$, $\mu_2 = 0.542$, $\mu_4 = -0.0757$, $a_1 = 0.185$ fm^{-3} and $a_2 = 4.94$ fm^{-1} are fitted to the experimental data for the $M_{1+}^{3/2}$-multipole of pion photoproduction [11,12].

As in [10] we have introduced a hadronic monopole form factor

$$F_\Delta(q^2) = f_{\pi N\Delta}\frac{\Lambda_\Delta^2 + q_\Delta^2}{\Lambda_\Delta^2 + q^2}\,. \tag{13}$$

The coupling constant $\frac{f_{\pi N\Delta}^2}{4\pi} = 1.393$ and the cutoff $\Lambda_\Delta = 315$ MeV are fixed in [12,13] to fit the πN scattering phase shift in the P_{33} channel and is also used in the calculations of this work.

3. The GDH sum rule for the nucleon

The GDH sum rule for the nucleon, *i.e.*, for proton and neutron, offers a good experimental field of study in nuclear physics. It gives also a possibility of studying problems which might arise in a quite simple system before studying the GDH sum rule for a more complicated systems (deuteron or ^{3}He). In the reaction on the nucleon, all absorptive processes for proton and neutron can be quite simply involved in the value of the GDH integral. This gives the possibility to examine indirectly at least the theoretical results of the GDH sum rule. In addition, one can obtain very fast an indication about how good and reliable the used models are.

In the following, one must first distinguish clearly between the GDH sum rule

$$I_N^{\mathrm{GDH}}(\infty) = \frac{\pi e^2 \kappa_N^2}{2M_N^2} \tag{14}$$

and the value of the GDH integral

$$I_N^{\rm GDH}(\infty) = \int\limits_{\omega_{\rm thr}}^{\infty} \frac{\sigma^{\rm P}(\omega_\gamma) - \sigma^{\rm A}(\omega_\gamma)}{\omega_\gamma} d\omega_\gamma \,. \tag{15}$$

This distinction is very important since the value of the GDH sum rule in (14), which depends only on observables which are already measured, is a model independent value. On the other hand, the used model enters explicitly in the calculation of the GDH integral in (15). Then, the absorption cross sections of polarized photons must be calculated.

Before one calculates the contributions of different absorptive processes to the GDH sum rule, one must first compute the constant in (14). This constant depends only on the static properties of the nucleon. Using these properties, one can compute the values of the GDH sum rule for both proton and neutron. Since proton and neutron have large anomalous magnetic moments, one finds correspondingly large GDH sum rule predictions for them, *i.e.*,

$$I_p^{\rm GDH}(\infty) \;=\; 204.8 \;\; \mu{\rm b} \qquad \text{and} \qquad I_n^{\rm GDH}(\infty) \;=\; 233.2 \;\; \mu{\rm b}\,. \tag{16}$$

Although this sum rule is known for more than 35 years, it has never been evaluated by a direct integration of experimental data on $\sigma^{\rm P} - \sigma^{\rm A}$.

There has been much interest in recent years in testing the GDH sum rule by determining the integral in (15). Direct experimental data on the spin-dependent photoproduction cross section has become available recently [14]. However, many of the published tests in the literature rely on theoretical models for the photoproduction helicity amplitudes which are only partially constrained by unpolarized photoproduction data [11, 15, 16].

4. Results and discussion

In this section we will classify our discussion into four classes. In the first one, we offer the results for spin asymmetry $\sigma^{\rm P} - \sigma^{\rm A}$ for circularly polarized photons on a longitudinally polarized nucleon target with spin parallel $\sigma^{\rm P}$ and spin antiparallel $\sigma^{\rm A}$ to the photon spin for the individual pion photoproduction channels in comparison with experiment. Predictions for the polarized differential cross section difference $(d\sigma/d\Omega)^{\rm P} - (d\sigma/d\Omega)^{\rm A}$ in comparison with the available experimental data will be discussed in the second class. In the third class, we present our results for the double polarization asymmetry E. In the last one, an explicit evaluation of the nucleon GDH integral for the four pion production reactions as a function of the upper limit of integration will be discussed.

In comparison with experimental data we concentrate our discussion on π^0- and π^+-production on the proton since data for π^0- and π^--production on the neutron are not available with respect to the absence of any free neutron targets. In what follows, the solid curves depict the results of the full calculations, *i.e.*, when both the Born terms and the contribution from the $\varDelta$-resonance were taken into account. The dashed curves denote the results when only the contribution from the Born terms was considered. The dotted curves displayed the results when the contribution from the $\varDelta$-resonance was neglected and the anomalous magnetic moment of the nucleon is zero.

4.1. The spin asymmetry

We start the discussion with presenting our results for the spin asymmetry

$$\varDelta\sigma = \sigma^{\rm P} - \sigma^{\rm A} \tag{17}$$

as depicted in Fig. 2 for the individual pion photoproduction channels. The helicity dependent total cross sections give valuable information on the nucleon spin structure and allow the determination of the dominant contribution to the GDH integral.

It is obvious from Fig. 2 that the charged pion production channels (see right panels) have qualitatively a similar behaviour, whereas a totally different behaviour is seen for the neutral channels (see left panels in Fig. 2). The spin asymmetry $\Delta\sigma$ has a peak at energy in the region of the $\varDelta(1232)$-resonance which comes mainly from the contribution of the $\varDelta$-excitation. The dashed curves indicate also that the contribution from Born terms is small, in particular when both the photon and the nucleon have parallel spins. It is obvious from (1) that for $\kappa_N \neq 0$ the particle possesses an internal structure. However, the opposite is not true, in general. A particle having a vanishing or very small κ_N need not be point-like or nearly point-like. When the anomalous magnetic moment of the nucleon goes to zero, one observes that $\sigma^{\rm P}$ is smaller than $\sigma^{\rm A}$ [17] and therefore $\Delta\sigma$ has negative values. On the other hand, one observes that the spin asymmetry $\sigma^{\rm P} - \sigma^{\rm A}$ do vanish for π^0-production on the neutron when $\kappa_N = 0$. It is also clear from the dashed curves that the contribution from Born terms is negligible in this case. We see also that the helicity difference $\Delta\sigma$ starts with negative values in the case of charged pion channels which is not the case for neutral ones. These negative values come mainly from higher values in $\sigma^{\rm A}$ for charged pion photoproduction channels.

In the case of π^+- and π^--production channels, we see that the helicity difference $\Delta\sigma$ fluctuates much more strongly than the total cross sections. The threshold region is dominated by the s-wave pion production,

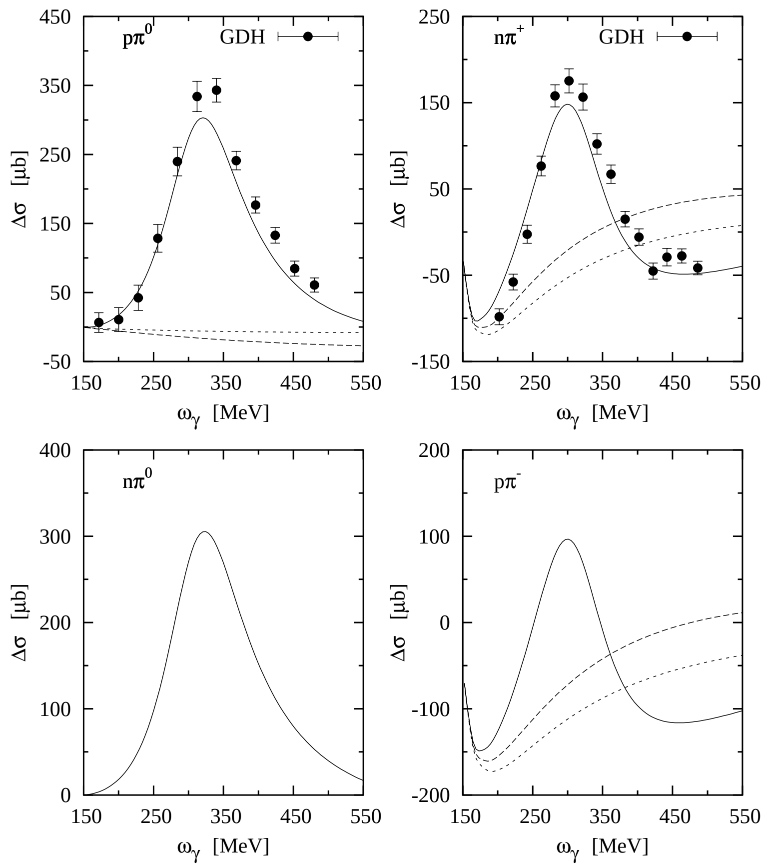

Fig. 2. Difference of helicity dependent total cross sections $\Delta\sigma = \sigma^{\mathrm{P}} - \sigma^{\mathrm{A}}$ as a function of photon lab-energy for different pion production channels. The solid (dashed) curves show the results using the Born terms with (without) the Δ-resonance contribution. The dotted curves show the results of the Born terms when $\kappa_N = 0$. The experimental data are from the GDH Collaboration [14].

i.e., intermediate states with spin $\frac{1}{2}$ that can only contribute to the cross section σ^{A}. In the region of the $\Delta(1232)$-resonance with spin $\frac{3}{2}$, both helicity cross sections contribute, but since the transition is essentially $M1$, we find $\sigma^{\mathrm{P}}/\sigma^{\mathrm{A}} \simeq 3$, and the helicity difference $\Delta\sigma$ becomes large and positive. We observe that the large background of nonresonant photoproduction in the total cross section has almost disappeared in the helicity difference, *i.e.*, the background is "helicity blind".

The dotted curves in Fig. 2 display the results using a very simple model for pion photoproduction on the nucleon. In this model we put a zero value for the anomalous magnetic moment of the nucleon, *i.e.*, $\kappa_N = 0$, and neglect the contribution from the Δ-resonance, *i.e.*, $T^{\Delta}_{fi} = 0$. As a result one sees that, the production of the π^0-meson is suppressed very strongly. In the case of π^0-production on the neutron it disappears even completely. The reason for that stems from the fact that the neutron is a neutral particle and therefore the photon can attack with the production of the π^0-meson only on the anomalous magnetic moment which was switched off in this very simple model. This suppression of the cross section does not arise however with the charged pions, the amount of $\sigma^{\rm P}$, $\sigma^{\rm A}$ and hence $\Delta\sigma$ becomes still larger.

Now, we compare our results for the spin asymmetry $\sigma^{\rm P} - \sigma^{\rm A}$ with experimental data from the GDH collaboration [14]. It is clear from the top two panels in Fig. 2 that the agreement of our results for the spin asymmetry with these experimental data is quite satisfactory. Only at the energies in the Δ-region a small underestimation is found which results from an overestimation in $\sigma^{\rm A}$.

4.2. The helicity difference $(d\sigma/d\Omega)^{\rm P} - (d\sigma/d\Omega)^{\rm A}$

Next we consider the polarized differential cross sections difference

$$\Delta(d\sigma) = \left(\frac{d\sigma}{d\Omega}\right)^{\rm P} - \left(\frac{d\sigma}{d\Omega}\right)^{\rm A} \tag{18}$$

as shown in Fig. 3 for the individual pion production channels as a function of emission pion angle θ_π in the c.m. system at photon lab-energy of $\omega_\gamma = 370$ MeV. In the case of π^0-production on the proton and neutron, one observes small negative values at extreme backward and forward pion angles which stem from small positive values which appear in $(d\sigma/d\Omega)^{\rm P}$ [17]. We have to remember that the contribution of the Born terms in the case of the charged pion photoproduction reactions is not negligible and has a large contribution in the $\Delta(1232)$-region. These terms play an important role in the case of low photon energies. For charged pion production channels (see the right two panels in Fig. 3), one sees that the situation is similar to the case of $(d\sigma/d\Omega)^{\rm P}$. The dashed curves demonstrate the importance of the Born terms. At extreme forward and backward pion angles, it is very obvious that the helicity difference $\Delta(d\sigma)$ has negative values which come mainly from higher positive values in $(d\sigma/d\Omega)^{\rm A}$. As in the case of polarized total cross sections, the dotted curves in Fig. 3 display the results when $\kappa_N = 0$ and $T^{\Delta}_{fi} = 0$. Here, one sees also that the production of the π^0-meson is suppressed very strongly. It disappears even completely in the case of π^0-production on the neutron. This suppression does not arise with the charged pions.

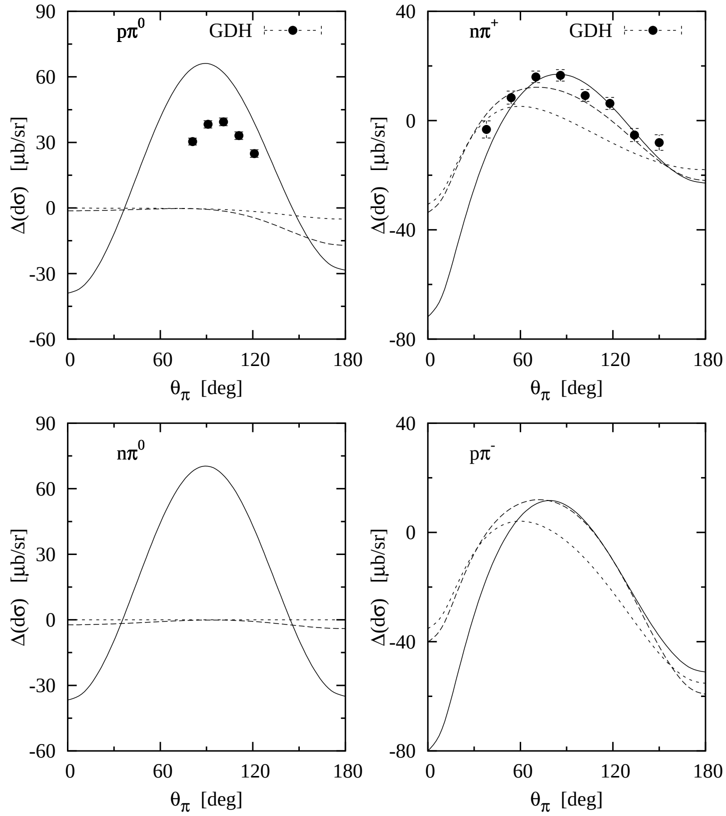

Fig. 3. Difference of helicity dependent differential cross sections $\Delta(d\sigma) = (\frac{d\sigma}{d\Omega})^{\mathrm{P}} - (\frac{d\sigma}{d\Omega})^{\mathrm{A}}$ as a function of pion angle in the c.m. frame for different pion production channels at $\omega_\gamma = 370$ MeV. Lines description as in Fig. 2. The experimental data are from the GDH Collaboration [14].

We close this section by comparing our results for the helicity difference $\Delta(d\sigma)$ with the recent experimental data from the GDH collaboration [14] as shown in the top two panels of Fig. 3. In the case of $\vec{\gamma}\vec{p} \to p\pi^0$ reaction, discrepancies are obtained. An experimental check of these predictions at extreme forward and backward pion angles is needed. The agreement between our prediction for the helicity difference $\Delta(d\sigma)$ and the experimental data from [14] in the case of $\vec{\gamma}\vec{p} \to n\pi^+$ reaction is quite satisfactory.

4.3. The double polarization asymmetry E

The double polarization asymmetry E is defined as

$$E(\theta_\pi) \;=\; \frac{(\frac{d\sigma}{d\Omega})^{\mathrm{A}} - (\frac{d\sigma}{d\Omega})^{\mathrm{P}}}{(\frac{d\sigma}{d\Omega})^{\mathrm{A}} + (\frac{d\sigma}{d\Omega})^{\mathrm{P}}} \;=\; \frac{(\frac{d\sigma}{d\Omega})^{\mathrm{A}} - (\frac{d\sigma}{d\Omega})^{\mathrm{P}}}{2(\frac{d\sigma}{d\Omega})_{\mathrm{unpol}}}\,, \tag{19}$$

where $(\frac{d\sigma}{d\Omega})_{\rm unpol}$ denotes the unpolarized differential cross section. In Fig. 4 the behaviour of the helicity E-asymmetry is plotted as a function of emission pion angle θ_π in the c.m. frame for individual pion photoproduction channels at photon lab-energy of $\omega_\gamma = 370$ MeV. Fig. 5 shows the helicity E-asymmetry as a function of photon energy in the laboratory frame

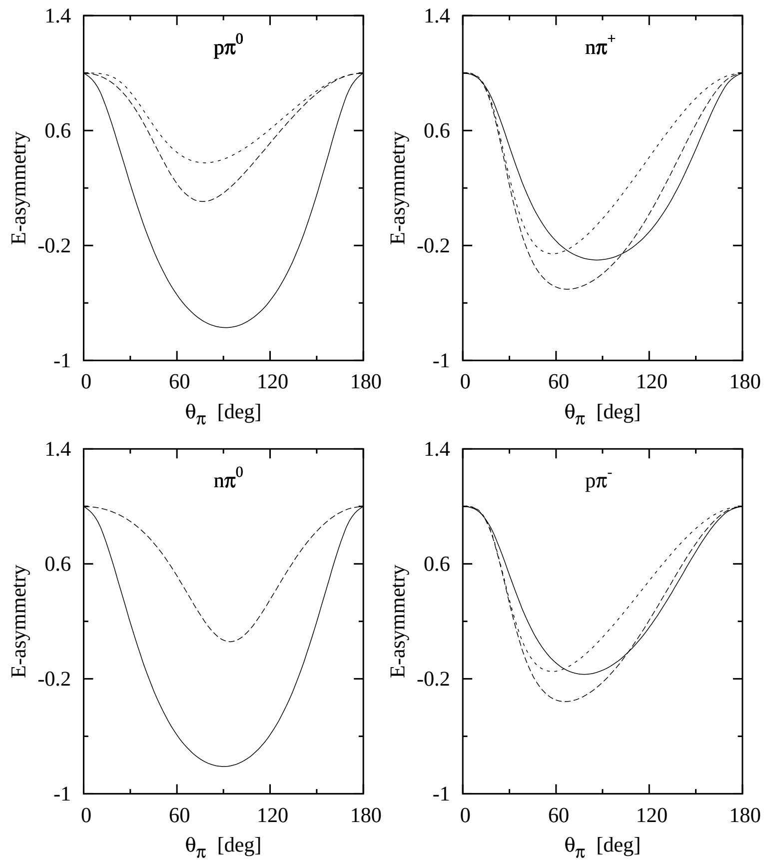

Fig. 4. The helicity E-asymmetry (see (19) for its definition) as a function of pion angle in the c.m. frame for different pion production channels at $\omega_\gamma = 370$ MeV. Lines description as in Fig. 2.

$$E(\omega_\gamma) \;=\; \frac{\sigma^{\rm A} - \sigma^{\rm P}}{\sigma^{\rm A} + \sigma^{\rm P}} \;=\; \frac{\sigma^{\rm A} - \sigma^{\rm P}}{2\sigma_{\rm tot}}\,, \tag{20}$$

where $\sigma_{\rm tot}$ denotes the unpolarized total cross section. In both figures, the full curves represent the results of the full calculations, the dashed curves

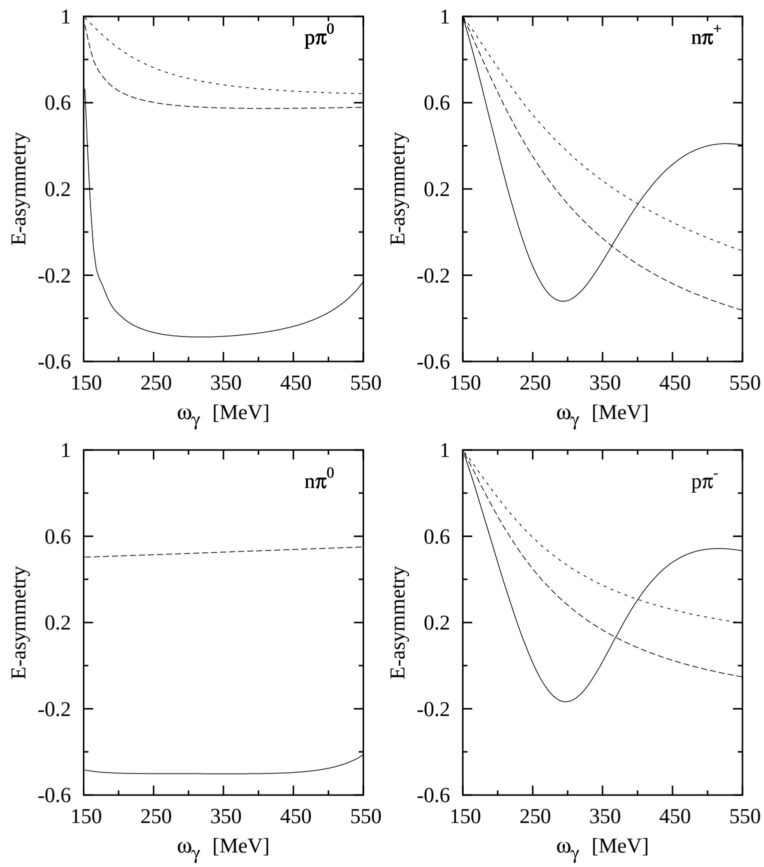

Fig. 5. Energy dependence of the double polarization asymmetry E (see (20) for its definition) for different pion photoproduction channels. Lines description as in Fig. 2.

represent the results in which the contribution from the Δ-resonance was set to zero and the dotted curves represent the results in which both the contribution from the Δ-resonance and the anomalous magnetic moment of the nucleon were set to zero. The difference between the full and dashed curves indicates the sensitivity of this observable to the contribution from the $\Delta(1232)$-resonance. As is clearly seen in these, the influence of the $\Delta(1232)$-resonance is strong, especially in the case of neutral pion photoproduction channels. In contrast to this, one notes for the charged pion channels a much closer behaviour between the full calculations and the ones when the Δ-resonance was set to zero.

One notes also that the helicity E-asymmetry has qualitatively a similar behaviour for all pion photoproduction channels, although the maximum value of E equals unity at $\theta_\pi = 0°$ and $180°$. The curves begin with unity and decrease by increasing the pion angle up to a minimum value at $\theta_\pi \simeq 90°$ and then increase again to unity. The negative values in E come mainly from higher positive contribution in $(d\sigma/d\Omega)^{\mathrm{P}}$. As in the case of polarized total and differential cross sections, the dotted curves in Figs. 4 and 5 display the results when $\kappa_N = 0$ and $T^{\Delta}_{fi} = 0$. One sees also here that the production of the π^0-meson is suppressed very strongly and disappears even completely in the case of π^0-production on the neutron.

4.4. The GDH integral

In this section we present the results for our evaluation of the GDH integral as depicted in Fig. 6 for the individual contributions from the different charge states of the pion for the $\gamma N \to \pi N$ reaction. Their total sum to

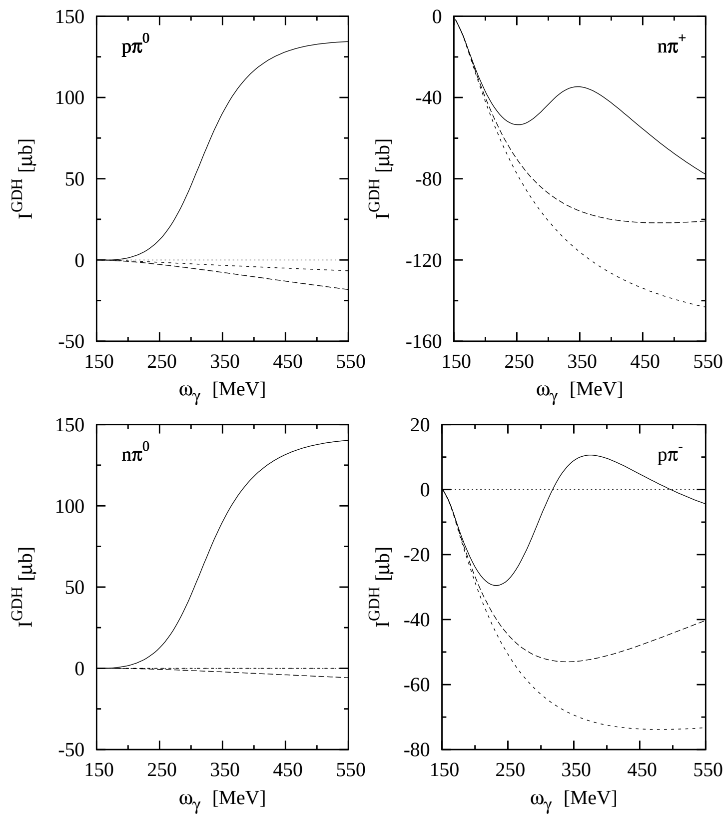

Fig. 6. The GDH integral as a function of the upper limit of integration for different pion production channels. Lines description as in Fig. 2.

the GDH sum rule is shown in Fig. 7. It is obvious that a large positive contribution to the GDH sum rule comes from the π^0-production channels, whereas the charged pion channels give a negative but — in absolute size — smaller contribution to the GDH value.

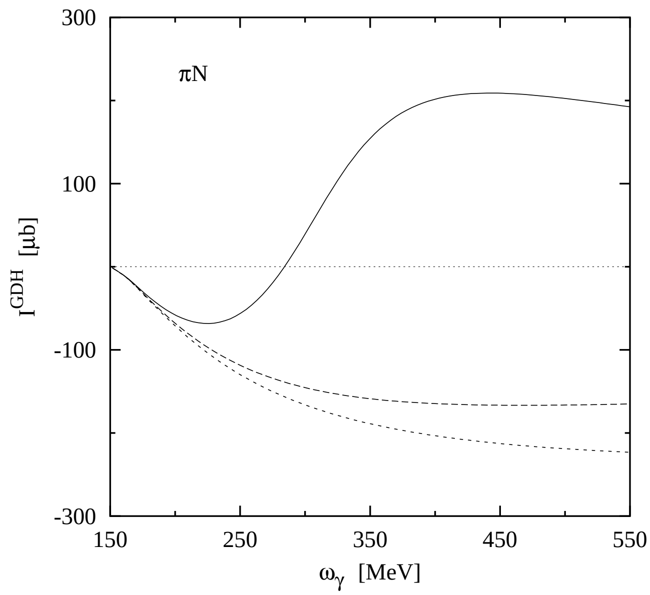

Fig. 7. Summation of contributions of the four pion photoproduction channels to the nucleon GDH sum rule as a function of the upper integration energy. Lines description as in Fig. 2.

Up to an energy of 500 MeV, one finds for the total contribution of the single-pion photoproduction channels a value

$$I_N^{\mathrm{GDH}}(500\,\mathrm{MeV}) = 202.3\,\mu\mathrm{b}\,. \tag{21}$$

The upper integration limit of 500 MeV is chosen, because in this work, we consider only single-pion production. In comparison with the value $I_N^{\mathrm{GDH}}(500\,\mathrm{MeV}) = 243.1\,\mu\mathrm{b}$ of Hanstein *et al.* [16], we found a small deviation. It is worthwhile first to point out that the model in [16] contains not only the Born and the Δ-resonance terms but also contributions from other resonances are considered.

Also, we have evaluated the GDH integral using a very simple model for pion photoproduction on the nucleon. This can be achieved if the anomalous magnetic moment of the nucleon goes to zero in our calculations, *i.e.*, $\kappa_N = 0$, and the contribution from the Δ-resonance vanishes, *i.e.*, $T_{fi}^{\Delta} = 0$. Looking to the left-hand side of (1) one can directly expect that, the value of the GDH integral goes to zero using these substitutions. The value $I_N^{\rm GDH}(500\,{\rm MeV}) = -219.2\,\mu{\rm b}$ is obtained in this case (see also the dotted curve in Fig. 7).

Furthermore, we have also evaluated explicitly the GDH integral for the nucleon by integrating the difference of the two total photoabsorption cross sections with photon and nucleon spins parallel and antiparallel up to a photon energy of 550 MeV. For the GDH value from explicit integration up to 550 MeV, one finds for the total contribution of the single-pion photoproduction channels the value

$$I_N^{\rm GDH}(550\,{\rm MeV}) = 190.4\,\mu{\rm b}\,. \tag{22}$$

A very interesting and important result is the large negative contribution from the charged pion production channels and the large positive contribution comes from the neutral channels to the GDH value. Hopefully, this low energy feature of the GDH sum rule could be checked experimentally in the near future.

5. Conclusions

The main subject of this paper was the investigation of the helicity structure of the partial cross sections and their contributions to the GDH sum rule for the nucleon. Contribution from single pion photoproduction has been explicitly evaluated. For the $\gamma N \to \pi N$ photoproduction amplitude, an effective Lagrangian model is used which contains, beside the standard pseudovector Born terms, the resonance contribution from the $\Delta(1232)$-excitation.

In the case of spin asymmetry $\sigma^{\rm P} - \sigma^{\rm A}$, we obtained qualitatively a similar behaviour for the polarized total cross sections in the case of charged pion channels, but a totally different one is seen in the case of neutral channels. In comparison with experiment, we found that the spin asymmetry $\sigma^{\rm P} - \sigma^{\rm A}$ has a quite satisfactory agreement. Only at energies in the Δ-region a small underestimation is found which stems from an overestimation in $\sigma^{\rm A}$. With respect to the results for the helicity difference $(d\sigma/d\Omega)^{\rm P} - (d\sigma/d\Omega)^{\rm A}$, we observed a peak when $\theta_\pi \simeq 90^\circ$ in the case of charged and neutral pion production channels. In comparison with experiment, discrepancies are obtained in the case of π^0-production on the proton. For π^+-production on the proton, a quite satisfactory agreement with experiment has been found. Regarding the results for the double polarization asymmetry E, we found that E has qualitatively a similar behaviour for all pion photoproduction channels. It is equal to unity at $\theta_\pi = 0^\circ$ and 180°. We found also that the influence of the $\Delta(1232)$-resonance is strong, especially for neutral pion photoproduction channels. When $\kappa_N = 0$ and $T_{fi}^{\Delta} = 0$ we found that the production of the

π^0-meson is suppressed very strongly and disappears even completely in the case of π^0-production on the neutron. Finally, the contributions of separate channels of single-pion photoproduction to the GDH integral have been explicitly evaluated by integration up to photon lab-energy of 550 MeV. The total value of the GDH-integral of $I_N^{\rm GDH}(550~{\rm MeV}) = 190.4~\mu{\rm b}$ has been found. We found that a large positive contribution to the GDH integral cames from the π^0-production channels whereas the charged pion channels gave a negative but — in absolute size — smaller contribution to the GDH value.

The studies we have discussed here will serve as the basis for further investigations including polarization observables in a more complete way. For instance, this work could be continued by further refinement of the pion production operator above the two pion threshold. This may result in a better agreement between experimental data and theoretical predictions.

We wish to thank the members of the GDH Collaboration at Mainz, Germany for providing us with their experimental data. We would also like to thank Prof. M.M. Mustafa for useful discussions.

REFERENCES

[1] S.B. Gerasimov, *Yad. Fiz.* **2**, 598 (1965) (*Sov. J. Nucl. Phys.* **2**, 430 (1966)).

[2] S.D. Drell, A.C. Hearn, *Phys. Rev. Lett.* **16**, 908 (1966).

[3] V. Pascalutsa, B.R. Holstein, M. Vanderhaeghen, *Phys. Lett.* **B600**, 239 (2004).

[4] D. Drechsel, L. Tiator, *Ann. Rev. Nucl. Part. Sci.* **54**, 69 (2004).

[5] I. Karliner, *Phys. Rev.* **D7**, 2717 (1973).

[6] *Proceedings of the 2nd Int. Symposium on the Gerasimov-Drell-Hearn 2002 Sum Rule and the Spin Structure of the Nucleon*, Genova, Italy, 3–6 July, 2002, eds. M. Anghinolfi, M. Battaglieri, R. De Vita, World Scientific, Singapore, 2003.

[7] *Proceedings of the NSTAR2002 Workshop on the Physics of Excited Nucleons*, Pittsburgh, Pennsylvania, USA, 9–12 October, 2002, eds S.A. Dytman, E.S. Swanson, World Scientific, Singapore, 2003.

[8] Proceedings of the NSTAR2004 Workshop on the Physics of Excited Nucleons, Grenoble, France, 24–27 March, 2004.

[9] Proceedings of the 10$^{\rm th}$ International Symposium on Meson-Nucleon Physics and the Structure of the Nucleon (MENU04), Beijing, China, August 29–Sept. 4, 2004.

[10] R. Schmidt, H. Arenhövel, P. Wilhelm, *Z. Phys.* **A355**, 421 (1996).

[11] R.A. Arndt *et al.*, The Scattering Analysis Interactive Dial-In Program (SAID), data available via telnet to VTINTE.PHYS.VT.EDU, Virginia Polytechnic Institute, Blacksburg, Virginia. For further references see, for example, R.A. Arndt, I.I. Strakovsky, R.L. Workman, *Phys. Rev.* **C53**, 430 (1996); R.A. Arndt, R.L. Workman, Z. Li, L.D. Roper, *Phys. Rev.* **C42**, 1853 (1990).

[12] P. Wilhelm, H. Arenhövel, *Nucl. Phys.* **A593**, 435 (1995).

[13] H. Poepping, P.U. Sauer, X.-Z. Zhang, *Nucl. Phys.* **A474**, 557 (1987).

[14] J. Ahrens *et al.*, *Eur. Phys. J.* **A21**, 323 (2004); J. Ahrens *et al.*, *Phys. Rev. Lett.* **84**, 5950 (2000); J. Ahrens *et al.*, *Phys. Rev. Lett.* **88**, 232002 (2002); J. Ahrens *et al.*, *Phys. Lett.* **B551**, 49 (2003); I. Preobrajenski, Dissertation, Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz, 2001.

[15] D. Drechsel, O. Hanstein, S. Kamalov, L. Tiator, *Nucl. Phys.* **A645**, 145 (1999); MAID Program, Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz, Germany, `http://www.kph.uni-mainz.de/de/MAID/`

[16] O. Hanstein, D. Drechsel, L. Tiator, *Nucl. Phys.* **A632**, 561 (1998).

[17] E.M. Darwish, M.A. El-Zohry, submitted for publication.