Mass spectra of strangeonium (P-wave) using variational technique in a nonrelativistic formalism.

Shefali Prabhakar[1]*, P. C. Vinodkumar [2], and Tanvi Bhavsar[2]
[1]*Department of physics, Guru Nanak Dev University, Amritsar-143001 , PUNJAB, INDIA*
[2]Deptartment of physics, Sardar Patel University, Vallabh Vidyanagar, Anand 388120,GUJRAT, INDIA.
Electronic addressl: shefalishefali1996@gmail.com

Introduction

Mesons are hadronic subatomic particles composed of one quark and one antiquark. In the present study, theoretical mass spectroscopy of light-light quarkonia (Strangeonium) is shown under the non-relativistic formalism. Although the experimental situation of strangeonium is not well understood and only a few states have been confirmed even then the theoretical analysis of this quarkonia state is significant because of two reasons: Firstly, the mass of the strange quark lies in between the light (up and down) quarks and the heavy (top,charm,bottom) quarks. Secondly, various final states in which the strangeonium is predicted are also the final states in which exotics are claimed.
The quantum mechanical technique, Variational method is adopted here. The interaction potential between strange quark and antistrange quark is taken as coulomb plus power potential expressed with a power exponent. Variational technique acquires the generated trial wavefunction to calculate variational parameter which is then substituted in the derivative of wavefunction at origin to calculate the spin average mass, pseudoscalar mass and vector mass corresponding to each potential index 'ν' at each wavefunction index 'p' describing wavefunction which varies from hydrogenic (p=1) to harmonic type (p=2).

Theoretical methodology of mass spectroscopy

Present study of strangeonium bound state system comprises non-relativistic hamiltonian as given by the following,

$$H = \frac{-1}{2m}\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right) R_{n,l} + V(r)$$

Where,

$$m = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}$$

Here, we have taken the CPP_ν (Coulomb plus power potential) given as,

$$V(r) = \frac{-\alpha c}{r} + A(n)^m r^\nu + V_0$$

A and α_c are the potential parameters. The different values of ν gives different potential forms.
$V_0 = 0.0085$ and $m = 0.38$ are set on purpose to fix the ground state,

$$\alpha_c = \frac{4}{3}\alpha_s;$$

where α_s is the strong running coupling constant expressed as [1]:-

$$\alpha_s = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\left[\frac{m^2 + {m_b}^2}{\Lambda^2}\right]}$$

Λ is the QCD scale factor and m_b is the background mass. n_f is the number of flavours and α_s comes out to be approximately 0.521.
The general form of the generated normalized trial wavefunction is given as,

$$R_{n,l} = \left(\frac{n!p}{\alpha^{\left(\frac{2lp-(2l+3)}{p}\right)}(n+k)!}\right)^{\frac{1}{2}} (\mu r)^p \; e^{-\frac{\mu r^p}{2}} \; L_n^k$$

μ is the variational parameter and L_n^k is the laguerre polynomial. The hyperfine splitting of the strangeonium system are calculated by using following,

$$M_v = M_{sa} + \frac{1}{4}k \quad \text{(for vector meson)}$$

$$M_p = M_{sa} - \frac{3}{4}k \quad \text{(for pseudoscalar meson)}$$

RESULTS AND CONCLUSION

In the present study, we have done the comprehensive study of mass spectra of P-wave of strangeonium through variational approach. The results obtained are in accordance with the experimental values and are concluded in the following tables for hydrogenic and harmonic type wavefunction.

Table I P-wave mass spectra for strangeonium for hydrogenic type wavefunction

nL	State	P	P-Wave (Present Mass Spectra in GeV)			Experimental in GeV [2]	Others in GeV [3]
			ν=0.1	ν=0.5	ν=1.0		
1P	$1\,^3P_2$	1	1.511	1.523	1.573	1.525	1.553
	$1\,^3P_1$	1	1.376	1.404	1.422	1.426	1.409
	$1\,^3P_0$	1	1.306	1.344	1.348	1.500	1.355
	$1\,^1P_1$	1	1.445	1.463	1.498	1.386	1.321
2P	$2\,^3P_2$	1	2.065	2.078	2.445	2.000	1.902
	$2\,^3P_1$	1	1.824	1.969	2.196	1.950	1.833
	$2\,^3P_0$	1	1.704	1.915	2.071	2.000	1.796
	$2\,^1P_1$	1	1.944	2.024	2.321	1.850	1.759
3P	$3\,^3P_2$	1	2.790	2.931	3.375	----	2.137
	$3\,^3P_1$	1	2.303	2.517	2.971	----	2.080
	$3\,^3P_0$	1	2.059	2.309	2.770	----	2.048
	$3\,^1P_1$	1	2.546	2.723	3.173	----	2.023

TableII P-wave mass spectra for strangeonium for harmonic type wavefunction

nL	State	P	P-Wave (Presemt Mass Spectra in GeV)			Experimental in GeV [2]
			ν = 0.1	ν = 0.5	ν = 1.0	
1P	$1\,^3P_2$	2	1.473	1.565	1.592	1.525
	$1\,^3P_1$	2	1.430	1.502	1.491	1.426
	$1\,^3P_0$	2	1.409	1.471	1.440	1.500
	$1\,^1P_1$	2	1.451	1.534	1.541	1.386
2P	$2\,^3P_2$	2	2.188	2.185	2.190	2.000
	$2\,^3P_1$	2	2.108	2.111	2.107	1.950
	$2\,^3P_0$	2	2.069	2.074	2.066	2.000
	$2\,^1P_1$	2	2.148	2.148	2.148	1.850
3P	$3\,^3P_2$	2	2.661	2.788	3.392	----
	$3\,^3P_1$	2	2.574	2.698	3.187	----
	$3\,^3P_0$	2	2.530	2.653	3.084	----
	$3\,^1P_1$	2	2.618	2.743	3.289	----

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