Proc. 8th Int. Conf. Quarks and Nuclear Physics (QNP2018)
JPS Conf. Proc. **26**, 023020 (2019)
https://doi.org/10.7566/JPSCP.26.023020

Three-Baryon Forces in a Quark Model

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(Received February 26, 2019)

We investigate the quark-Pauli effects in three octet-baryons by estimating the eigenvalue of the RGM normalization kernel and the three-body effects generated from the color-magnetic term in the Fermi-Breit interactions in three octet-baryons by estimating the diagonal element of the exchange RGM kernels. The ΛNN system receives minor quark-Pauli effects, whereas the $\Sigma NN(I = 2)$ system regardless of $S = \frac{1}{2}$ or $\frac{3}{2}$, including, e.g., $\Sigma^- nn$ is almost Pauli forbidden. Three-body effects generated from the color-magnetic term depend greatly on the $B_8B_8B_8$ systems and they including hyperon seem not very strong repulsion at the short-range.

KEYWORDS: three-body force, quark-Pauli effect, quark model, octet baryon

1. Introduction

The hyperons appear to be present in the interior of the neutron star with the increasing baryon density. Since their appearance generally leads to the softening of its equation of state, some repulsive mechanism to suppress the role of the hyperons is called for in order to be consistent with the observation that the mass of the neutron star can be twice as heavy as the solar mass. At present the information on two- and three-baryon forces including the hyperons is very much limited, and it is hard to draw clear conclusions on the required repulsion.

The Pauli principle, among the quark-constituents of baryons, often brings important repulsive effects in the two-baryon system. For example, the repulsive Σ single-particle potential in nuclei [1] is considered to originate from the strong Pauli repulsion in the $\Sigma N(I = \frac{3}{2})\,{}^3S_1$ state [2]. It is interesting to study the quark-Pauli effect in the three-baryon systems because it is relevant to the neutron-star structure as well as the hypernuclear structure. The color-magnetic term in the Fermi-Breit interaction generates the short-range repulsion in many B_8B_8 system through the quark-exchange mechanism. It is important to investigate the three-body effect generated from it, in particular, how much it depends on the $B_8B_8B_8$ systems.

2. Quark-Pauli Effects

The octet baryons (B_8) with spin $S = \frac{1}{2}$ contain N,Λ,Σ, and Ξ, all belonging to a member of the flavor $SU(3)$ symmetry $(\lambda\mu) = (11)$ in the Elliott notation [3]. They are classified by the $SU(2) \times U_1$ subgroup label $a = YI$, the hypercharge Y and the isospin I: $N(YI = 1\frac{1}{2})$, $\Lambda(YI = 00)$, $\Sigma(YI = 01)$, $\Xi(YI = -1\frac{1}{2})$. Assuming that B_8 is a three quark cluster, we describe its orbital part $\phi^{(\mathrm{orb})}(123)$ by the $(0s)^3$ harmonic-oscillator wave function with a common size parameter. Since $\phi^{\mathrm{orb}}(123)$ is completely symmetric and the B_8 color wave function $C(123)$ is completely antisymmetric, its spin-flavor part represented by $W^{[3]}(123)$ must be totally symmetric. By specifying the z-components of spin and isospin by S_z and I_z, respectively, a full quark-model description of B_8 reads

$$B_{S_z\alpha}(123) = \phi^{(\mathrm{orb})}(123)W^{[3]}_{S_z\alpha}(123)C(123), \tag{1}$$

where $\alpha = aI_z$.

Equation (1) gives the normalized B_8 wave function that satisfies the required symmetry at the quark level. By combining two B_8 wave functions, it is possible to express the spin-isospin coupled basis in terms of a combination of the spin-flavor coupled basis [4,5]. Physically allowed two-baryon states have to satisfy the generalized Pauli principle that demands the total wave function to be antisymmetric under exchange of quarks. We extend this to a special three-B_8 state in which all nine quarks occupy the same $0s$ harmonic-oscillator function. The orbital configuration of that state is most compact and such a three-B_8 state is expected to be strongly influenced by the quark-Pauli principle.

The total three-baryon wave function defines, in terms of the flavor SU(3) coupled basis, the three-B_8 state with a given Sa that is antisymmetric under the baryon exchange,

$$\Psi_{Sa}((0s)^9 : B_1B_2B_3) = \Psi^{(\mathrm{orb})}(B_1B_2B_3)\Psi^{(\mathrm{SF})}_{Sa}(B_1B_2B_3)\Psi^{(\mathrm{color})}(B_1B_2B_3), \tag{2}$$

where $\Psi^{(\mathrm{orb})}(B_1B_2B_3)$ denotes the orbital wave function with $(0s)^9$-configuration for 9-quarks, and $\Psi^{(\mathrm{color})}(B_1B_2B_3)$ the color wave function, $\Psi^{(\mathrm{SF})}_{Sa}(B_1B_2B_3)$ the totally antisymmetric spin-flavor three-B_8 state, detailed in Ref. [6].

To estimate the quark-Pauli effect we solve an eigenvalue problem of the antisymmetrizer $\mathcal{A}$ that makes the nine-quark wave function antisymmetric under the exchange of quarks among baryons. Since each B_8 cluster is antisymmetric under the quark exchange and the three B_8 baryons are antisymmetrized with respect to the baryon exchanges, $\mathcal{A}$ is reduced to 55 terms of five basis types [7]:

$$\begin{aligned}\mathcal{A} \;=\; & [1 - 9(P_{36} + P_{39} + P_{69}) + 27(P_{369} + P_{396}) + 54(P_{25}P_{39} + P_{35}P_{69} + P_{38}P_{69})]\left(\sum_{\mathcal{P}=1}^{6}(-1)^{\pi(\mathcal{P})}\mathcal{P}\right)\\ & -216P_{25}P_{38}P_{69},\end{aligned} \tag{3}$$

where P_{ij} exchanges quarks i and j and acts on the full orbital, color, spin, and flavor degrees of freedom. The six $\mathcal{P}$ include those quark exchanges that are equivalent to baryon exchanges. Of the five basic types of terms in $\mathcal{A}$, the first is the direct term, and the second, $(P_{36} + P_{39} + P_{69})\left(\sum_{\mathcal{P}=1}^{6}(-1)^{\pi(\mathcal{P})}\mathcal{P}\right)$, involves only the exchange of quark pairs between one baryon pair. Terms in $\mathcal{A}$ of the third to fifth category involve the exchange of quark pairs between different baryon pairs. We will call them as three-body terms.

The matrix element of $\mathcal{A}$ with the basis functions (2) is evaluated by combining the matrix elements of orbital, spin-flavor, and color parts. The orbital matrix element is unity, namely 1, independent of the quark exchange because of the fully symmetric $(0s)^9$ configuration. The color matrix element is also simple: the fifth term in $\mathcal{A}$ has no color matrix element [7], whereas those of the other terms are $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{9}$, for P_{36}, P_{369}, $P_{25}P_{39}$, respectively.

Assuming the eigenfunction of $\frac{1}{3!}\mathcal{A}$ to be

$$\sum_{B_1B_2B_3} C(Sa; B_1B_2B_3)\,\Psi_{Sa}\left((0s)^9 : B_1B_2B_3\right) \tag{4}$$

for NNN, ΛNN, ΣNN and ΞNN systems, eigenvalues μ_{Sa} of the equation

$$\sum_{B_1'B_2'B_3'} \left\langle \Psi_{Sa}\left((0s)^9 : B_1B_2B_3\right) \middle| \frac{1}{6}\mathcal{A} \middle| \Psi_{Sa}\left((0s)^9 : B_1'B_2'B_3'\right) \right\rangle C\left(Sa; B_1'B_2'B_3'\right) = \mu_{Sa}C(Sa; B_1B_2B_3) \tag{5}$$

Table I. Eigenvalues μ_{Sa} in Eq. (5), given in increasing order, for NNN and YNN systems. The expectation value of $\mathcal{A}'$ calculated for each $B_8B_8B_8$ system is given in $\langle\mathcal{A}'\rangle$ column.

$Y\,I$	S	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}	$Y\,I$	S	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}	$Y\,I$	S	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}
$3\,\frac{1}{2}$	$\frac{1}{2}$	NNN	$\frac{100}{81}$	$\frac{100}{81}$	$2\,2$	$\frac{1}{2}$	ΣNN	$\frac{4}{81}$	$\frac{4}{81}$	$1\,\frac{1}{2}$	$\frac{3}{2}$	ΞNN	$\frac{50}{81}$	$\frac{35}{243}$
$2\,0$	$\frac{1}{2}$	ΛNN	$\frac{25}{27}$	0	$1\,\frac{1}{2}$	$\frac{1}{2}$	$\Xi NN_{\nu=1}$	$\frac{25}{27}$	0			$\Sigma\Sigma N$	$\frac{95}{243}$	$\frac{5}{9}$
		ΣNN	$\frac{25}{81}$	$\frac{100}{81}$			$\Xi NN_{\nu=2}$	$\frac{35}{81}$	0			$\Sigma\Lambda N$	$\frac{50}{81}$	$\frac{25}{27}$
	$\frac{3}{2}$	ΛNN	$\frac{25}{27}$	$\frac{25}{27}$			$\Lambda\Lambda N$	$\frac{5}{6}$	0	$1\,\frac{3}{2}$	$\frac{1}{2}$	ΞNN	$\frac{10}{27}$	0
$2\,1$	$\frac{1}{2}$	ΛNN	$\frac{25}{27}$	0			$\Sigma\Sigma N_{\nu=1}$	$\frac{85}{162}$	0			$\Sigma\Sigma N_{\nu=1}$	$\frac{73}{162}$	0
		$\Sigma NN_{\nu=1}$	$\frac{50}{81}$	$\frac{200}{243}$			$\Sigma\Sigma N_{\nu=2}$	$\frac{35}{243}$	$\frac{200}{243}$			$\Sigma\Sigma N_{\nu=2}$	$\frac{235}{486}$	$\frac{4}{81}$
		$\Sigma NN_{\nu=2}$	$\frac{125}{243}$	$\frac{100}{81}$			$\Sigma\Lambda N_{\nu=1}$	$\frac{5}{9}$	$\frac{100}{81}$			$\Sigma\Lambda N_{\nu=1}$	$\frac{5}{18}$	$\frac{200}{243}$
	$\frac{3}{2}$	ΣNN	$\frac{35}{243}$	$\frac{35}{243}$			$\Sigma\Lambda N_{\nu=2}$	$\frac{20}{81}$	$\frac{130}{81}$			$\Sigma\Lambda N_{\nu=2}$	$\frac{85}{162}$	$\frac{100}{81}$

are given in Table I. See Ref. [6] for other $B_8B_8B_8$ systems.

The eigenvalue obtained by solving Eq.(5) is deviated from 1 by the influence of the antisymmetrizer $\mathcal{A}$ that makes the nine-quark wave function totally antisymmetric under the exchange of quarks among the baryons. In particular those eigenfunctions that correspond to vanishing eigenvalues or considerably small eigenvalues are callued Pauli forbidden or almost Pauli forbidden. Three-B_8 baryons cannot occupy such forbidden configurations, namely, they exhibit quark-Pauli repulsion.

We find out that all ΛNN systems are not almost Pauli-forbidden states, that is, the quark-Pauli-blocking effect is weak in the ΛNN systems. The quark-Pauli repulsion is not a candidate for the repulsive three-body hyperon-nucleon interaction speculated in the neutron star because the Λ-particle may appear as first hyperon with increasing baryon density. On the other hand, we see that almost Pauli-forbidden states appear in many systems including the Σ-particle, starting with $\Sigma NN(I = 2)$, namely $\Sigma^- nn$ state. This fact suggests that the quark-Pauli-blocking effect prevents the appearance of the Σ^--particle as the baryon density in the neutron star increases. The Λ and Σ hyperons behave differently with respect to the quark-Pauli repulsion. The ΛNN system receives minor quark-Pauli effects and are allowed to be present in the interior of the neutron star unless ΛNN three-body force is strongly repulsive, whereas the $\Sigma NN(I = 2)$ system regardless of $S = \frac{1}{2}$ or $\frac{3}{2}$, including, e.g., $\Sigma^- nn$ is almost Pauli-forbidden. These details are written in Ref. [6].

3. Estimation of Interaction Kernel

It will be interesting to study three octet-baryon forces using a quark-model Hamiltonian. The spin and flavor SU(3) symmetry developed here should be useful to the extent to which the underlying Hamiltonian is SU(3)-scalar. In the present report, we adopt the color-magnetic term in the Fermi-Breit interaction in the flavor-SU(3) limit, as follows,

$$V = \sum_{i>j} \frac{1}{4}\alpha_S \hbar c \left(-\frac{\pi\hbar^2}{m^2c^2}\right)\left(\lambda_i^c \cdot \lambda_j^c\right)\left\{1 + \frac{2}{3}\left(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right)\right\}\delta(r_{ij})\,, \tag{6}$$

where λ_i^c is the SU(3) color generator of the i-th quark, m the quark mass. We choose to be $mc^2 = 313\,\text{MeV}$ and the width parameter $b = 0.6\,\text{fm}$ in the present case.

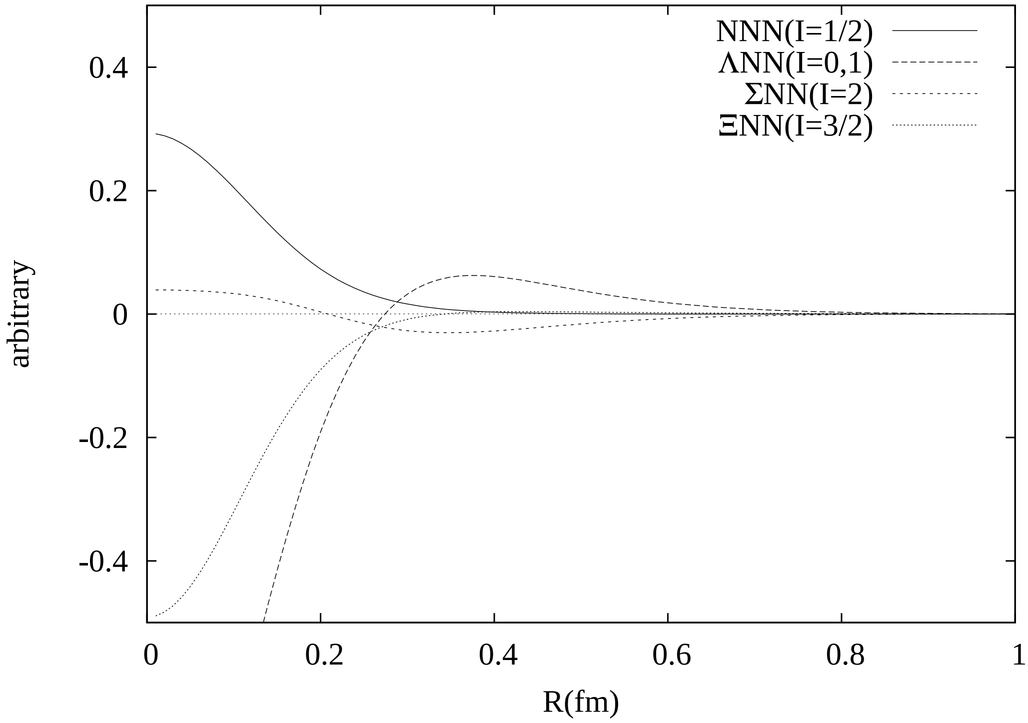

Fig. 1. Diagonal elements of the three-body exchange kernel for equilateral triangular configurations with the relative distance R between three baryons. Solid curve denotes the NNN state with the iso-spin $I = \frac{1}{2}$, long-dashed curve both ΛNN states, short-dashed curve the ΣNN state with $I = 2$, and dotted curve the ΞNN state with $I = \frac{3}{2}$. All are the total spin $S = \frac{1}{2}$.

We study the dependence of the three-baryon force originating from V on the $B_8B_8B_8$ systems with the use of the resonating group method(RGM) formalism for the three-baryon nine-quark system. With the $\mathcal{A}$ of (3) the exchange RGM kernel $K^{(E)}$ for V can be evaluated in terms of the baryon-separation parameters $\vec{R}_a$, $\vec{R}_b$, $\vec{R}'_a$, $\vec{R}'_b$ through

$$K^{(E)}\left(\vec{R}_a, \vec{R}_b; \vec{R}'_a, \vec{R}'_b\right) = \frac{1}{6}\Big\langle \phi(B_1B_2B_3)_{Sa}\delta\left(\vec{R}_{12} - \vec{R}_a\right)\delta\left(\vec{R}_{12-3} - \vec{R}_b\right) \times \left| V \left[\mathcal{A} - \sum_{\mathcal{P}=1}^{6} (-1)^{\pi(\mathcal{P})}\mathcal{P}\right] \right| \phi(B_1B_2B_3)_{Sa}\delta\left(\vec{R}_{12} - \vec{R}'_a\right)\delta\left(\vec{R}_{12-3} - \vec{R}'_b\right)\Big\rangle \quad (7)$$

For details see Refs. [7], [8] and [9].

To determine the importance of the three-body terms generated by the exchange RGM kernel, it is useful to examine first the diagonal element $K^{(E)}\left(\vec{R}_a, \vec{R}_b; \vec{R}_a, \vec{R}_b\right)$. Figure 1 shows this diagonal element for the case where the three baryons form the equilateral triangular configurations. The strength of this diagonal element is chosen to be arbitrary because we aim to investigate the dependence of it on the $B_8B_8B_8$ systems. One can see that the behavior of three-body term depends greatly on the $B_8B_8B_8$ systems, and it looks like that they including hyperon are not necessarily repulsive at the short-range region, as not expected.

This work was supported in part by JSPS KAKENHI Grants, 24654071, 24540261, and 15K05072.

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