Supersymmetry Breaking and Its Mediation in String Theory and Particle Physics

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Abstract

In this thesis, we study spontaneous supersymmetry (SUSY) breaking and its mediation from various perspectives. We begin by motivating SUSY from both phenomenological and more theoretical points of view. After undertaking a brief review of the structure and history of SUSY, we move on to study gauge-mediated SUSY breaking.

We further develop a general formalism for describing the soft parameters generated in theories of gauge mediation. Using this formalism we give a general proof of the finiteness of the soft parameters. Then, specializing to weakly coupled models, we shed new light on the UV sensitivity of the soft masses. Finally, we prove that the parameter space described by our formalism is physical and realizable in calculable, weakly coupled models. This result opens up the possibility of completely new soft spectra not typically associated with gauge mediation.

Next, we make contact with string theory and realize a supersymmetric extension of the Standard Model as the low energy theory of a D3 brane probing a del Pezzo singularity in a gravitational decoupling limit we describe. More importantly, we formulate new topological conditions under which the abelian gauge bosons of the D3-brane theory are rendered massive upon UV completing the singularity into a compact manifold.

Shifting our focus to SUSY breaking in hidden sectors, we analyze how to generate metastable SUSY breaking quantum field theories in string theory. We give a simple set of geometrical conditions for understanding the resulting gauge theory.

In the last part of the thesis, we will introduce a novel string theoretical scheme for mediating SUSY breaking using D-brane instantons.

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Chapter 1

Introduction

Any discussion of supersymmetry (SUSY) and the physical world must necessarily begin with an apology, for not only is SUSY absent at the energies that govern the world as we experience it, but our most energetic particle accelerators—magnifying glasses onto the violent world of high-energy quantum physics—have not detected even a trace of SUSY to date.

Therefore, the study of supersymmetry phenomenology is really the study of supersymmetry breaking. However, before trying to understand how SUSY is broken and how this information is encoded in the observables we hope to probe at future particle colliders, we should mention what SUSY is. In short, supersymmetry entails a radical revision of nature. Indeed, SUSY dictates that the four spacetime dimensions are accompanied by extra quantum dimensions describing the translations of bosons into fermions, with every matter fermion having an accompanying bosonic degree of freedom of equal mass and every bosonic force carrier having a fermionic partner of equal mass. When SUSY is broken, the masses of the fermions and bosons are no longer degenerate. In particular, the supersymmetry partners (or superpartners) of the known matter and gauge field particles can be very massive. Therefore, in order to study supersymmetry in collider experiments, we must have access to energy scales of the order of the masses of these superpartners.

While the status of SUSY is in some sense analogous to the initial status of the electroweak theory of Glashow, Salam, and Weinberg (GSW) before the discovery of the massive W and Z bosons, it is worthwhile to pause and appreciate what a dramatic leap SUSY entails. Perhaps, then, the situation is more analogous to the status of general relativity (GR) before its first experimental confirmation.

Still, there are a host of reasons to believe that SUSY is a symmetry of physics at very high energies and to hope that its low energy avatars will reveal themselves at the upcoming Large Hadron Collider (LHC) experiment deep beneath the French and Swiss border at the European particle physics lab CERN.

One of the main reasons to believe that SUSY exists at high energies is that it solves the so-called "hierarchy problem," namely, the perturbative instability of the weak scale ($\sim 10^3$ GeV) relative to the UV scale, Λ, at which new physics enters. This instability arises in the Higgs sector of the Standard Model (SM). The basic point is that, in the absence of a deeper underlying symmetry, the Higgs particle is unprotected against large quantum corrections to its mass and so

$$m_H^2 \sim -|y_y|^2 \int^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_t^2} \tag{1.0.1}$$

where we have written the leading quantum correction to the Higgs mass, namely the contribution due to the exchange of a virtual top quark pair. If $\Lambda \sim 10^{16}$ GeV, then (1.0.1) is roughly 10^{32} GeV^2. However, for phenomenological reasons, the Higgs should have a mass of order 100 GeV. This fact implies that one needs to introduce a bare mass parameter for the Higgs that cancels the radiative corrections to an extremely high precision. This incredibly unnatural tuning is the hierarchy problem.

Supersymmetry is a symmetry that can tame the divergences in (1.0.1), so long as it is broken at the TeV scale. In particular, since the top quark has a superpartner called the scalar top (or "stop" for short) we must include it in quantum corrections to the Higgs mass. Virtual exchanges of the stop also give quadratic contributions to the Higgs mass, but they enter with opposite sign because they have opposite statistics. As a result, one is left with a less singular logarithmic divergence in the Higgs mass

$$\delta m_H^2 \sim -\widetilde{m}_t^2 \frac{y_t^2}{16\pi^2} \log \frac{\Lambda^2}{\widetilde{m}_t^2} \tag{1.0.2}$$

Finally, in addition to solving the hierarchy problem, SUSY has additional benefits.[1] For example, if we assume SUSY is broken at the TeV scale (the scale that the LHC will probe), then it naturally gives rise to gauge coupling unification at the GUT scale, 10^{16} GeV. Furthermore, if we assume a discrete symmetry called an R-parity under which the visible matter and its superpartners have opposite charge, then SUSY furnishes a dark matter candidate in the form of a stable lightest supersymetric particle (LSP).

The motivations we have given for SUSY so far lie purely in the realm of particle physics, but there are more fundamental, although perhaps more indirect, reasons to believe in SUSY as well. In the deep UV, say of order roughly the Planck scale, we expect gravitational effects to become important. In particular, GR and the SM must coexist as part of a larger quantum theory. Of course, the problems of quantizing gravity are well known. At the perturbative level, the gravitational coupling has inverse mass dimensions (for $d > 2$), with $G_N \sim M^{2-d}$ in d spacetime dimensions and so the theory appears non-renormalizable. Hence, quantized gravity seems to lack a proper perturbative definition. At a more general non-perturbative level, one can also see that gravity is non-renormalizable (see [1] for a more complete development of the argument we are about to give). The basic point is that in the UV any renormalizable quantum field theory (QFT) in d dimensions is a conformal field theory (CFT) and has an entropy that goes as

$$S \sim E^{\frac{d-1}{d}} \tag{1.0.3}$$

where E is the energy. On the other hand, from studying black hole entropy, it is reasonable to estimate that the gravitational entropy should scale as

$$S \sim E^{\frac{d-2}{d-1}} \tag{1.0.4}$$

[1] The 'little hierarchy problem' may still persist, however. This problem refers to the albeit much smaller tuning of the Higgs mass still required for generic values of the parameters in (1.0.2).

These arguments present clear contradictions and mean that gravity must be completed into another structure that goes beyond a QFT—the leading candidate for this UV completion is string theory,[2] and string theory is rather deeply wedded to SUSY.

Therefore, SUSY seems rather well motivated both from the bottom-up and from the top-down. There are, of course, many caveats to this statement. For example, there are other potential solutions to the hierarchy problem, such as the extra-dimensional scenarios of Randall and Sundrum [3] involving the appropriate localization of different SM degrees of freedom in a warped geometry. However, these setups do not naturally include the possibility of gauge coupling unification or a natural dark matter candidate.

In this thesis we will not have much more to say about the possible alternatives to SUSY. This is not out of a desire to ignore other scenarios for physics beyond the Standard Model or to condemn them to the level of implausible speculation. However, because the idea of SUSY is so well-motivated and the intricacies of its structures are so beautiful, we will focus exclusively on supersymmetry and its realizations in string and field theory.

Let us briefly summarize what is to follow in the rest of this thesis. In the remainder of this section we will introduce SUSY more formally, discuss the minimal supersymmetric extension of the Standard Model (MSSM), describe spontaneous and dynamical SUSY breaking, and also sketch how SUSY breaking is transmitted or 'mediated' to the MSSM fields. In the last part of the introduction, we will describe how to embed these ideas into string theory.

In Chapter 2 we will study a particular method for mediating SUSY breaking called "gauge mediation," which relies purely on field theoretical degrees of freedom. We will further develop a general formalism for describing the soft parameters generated by gauge mediation and show that, under certain assumptions, it is possible to cover the resulting parameter space. In particular, this result opens up the possibility of completely new soft spectra not typically associated with gauge mediation. We will then shed new light on

[2] In fact, one of the deepest statements in string theory is the so-called AdS/CFT correspondence first conjectured by Maldacena [2] that relates gravitational string theories in d dimensions to CFTs in $d-1$ dimensions. This correspondance naturally relates the results in (1.0.4) and (1.0.3) upon the substitution $d \to d-1$ in (1.0.3).

various issues relating to the UV sensitivity of our observables and give a general proof of the finiteness of the soft masses in gauge mediation.

In Chapter 3, we will make contact with string theory and realize a supersymmetric extension of the Standard Model as the low energy theory of a D3 brane probing a del Pezzo singularity in a limit where gravitational dynamics decouples. More importantly, we will formulate new topological conditions under which the abelian gauge bosons of the D3-brane theory are rendered massive upon UV completing the singularity into a compact manifold.

In Chapter 4, we shift our focus to SUSY breaking in "hidden" sectors and analyze how to generate metastable SUSY breaking QFTs in string theory. We will give a simple set of geometrical conditions for understanding the resulting gauge theory.

In Chapter 5, we will introduce a novel string theoretical scheme for mediating SUSY breaking from a hidden sector field theory, realized on a set of D-branes, to a toy supersymmetric model of particle physics realized on a geometrically separated set of D-branes. In this setup, the mediators will be D-brane instantons stretching between the two sectors. This mediation scheme has no analog in field theory. In particular, it is an interesting alternative to the scheme presented in Chapter 2.

1.0.1 Supersymmetry

In this subsection, we will give a brief introduction to SUSY and also, where possible, a few details about its historical development. The birth of SUSY spawned an intense and fertile period in physics that began in the early 1970s. It involved an interplay between the development of a two-dimensional string worldsheet SUSY by Ramond, Neveu, Schwartz, Gervais, Sakita and others [4] and the development of a four-dimensional version of SUSY pioneered by Wess and Zumino [5]. However, we will not have much to say about worldsheet SUSY in this section. Instead, we will specialize to the case of four dimensions since this will be of more immediate interest in particle physics.

As we have discussed above, supersymmetry is a symmetry that rotates bosons into fermions and vice versa. Therefore, the generators of this symmetry must be spacetime spinors. The SUSY algebra is given by

$$\begin{aligned} \{Q^a_\alpha,\ Q^b_\beta\} &= 2\sqrt{2}\epsilon_{\alpha\beta}Z^{ab} \\ \{Q^\dagger_{\dot{\alpha}a},\ Q^\dagger_{\dot{\beta}b}\} &= 2\sqrt{2}\epsilon_{\dot{\alpha}\dot{\beta}}\overline{Z}^{ab} \\ \{Q^a_\alpha, Q^\dagger_{\dot{\alpha}b}\} &= 2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu\delta^a_b \end{aligned} \tag{1.0.5}$$

where $Z^{ab} = -Z^{ba}$ is the central charge matrix and α, β are spacetime spinor indices with $\alpha, \beta = 1, 2$ in four dimensions. The indices $a, b = 1, ..., \mathcal{N}$ label the number of $Q, Q^\dagger$ supercharges. Note that for $\mathcal{N} = 1$, the central charge is necessarily vanishing, i.e. $Z = 0$.

The form of the SUSY algebra (1.0.5) is quite interesting since it mixes the Poincaré spacetime symmetries with internal quantum numbers (i.e., the spins) of physical fields. In general, such extensions of the Poincaré symmetries of the S-matrix are tightly constrained by the S-matrix's analyticity properties.

Indeed, Coleman and Mandula [6] proved a no-go theorem showing that, given certain assumptions, any symmetry algebra of the S-matrix that includes the Poincaré group must be of the form of a direct product of the Poincaré symmetry and an internal symmetry group.

However, in their seminal work, Golfand and Likhtman [7] were able to circumvent the Coleman-Mandula theorem by extending the Poincaré algebra and including anti-commutators and spinor generators as well in a graded Lie algebra. In short, they had 'discovered' supersymmetry. A few years later, Haag, Lopuszanski, and Sohnius [8] extended the assumptions of the Coleman-Mandula theorem to include the possibility of graded Lie algebras and showed that SUSY is the *only* possible such extension of the Poincaré algebra.

Given this algebra it is rather straightforward to construct the various allowed representations. Since our ultimate goal is to describe supersymmetric particle physics we will specialize not only to four dimensions but also to $\mathcal{N} = 1$ SUSY. While extended SUSY (i.e., $\mathcal{N} > 1$) is of enormous theoretical interest and will come into play in the later chapters of this thesis when we consider string compactifications and D-branes, the fundamental phenomenological problem with $\mathcal{N} > 1$ theories of particle physics is that the matter representations are necessarily vector-like. Since the Standard Model is chiral, this fact presents an insurmountable obstacle to describing the real world at low energies.

In four dimensions, the $\mathcal{N}=1$ SUSY algebra has a few basic multiplets. At the massless level, the representations necessarily pair helicity λ and helicity $\lambda-1/2$ states—the gauge bosons fit into multiplets with spin 1/2 partner gauginos, the matter fermions fit into multiplets with spin 0 bosonic partners, and the graviton fits into a multiplet with a spin 3/2 partner gravitino. At the massive level, a SUSY representation groups spin j, spin $j-1/2$, and spin $j-1$ particles of equal mass into supermultiplets (if $j=1/2$ there is also a truncated massive multiplet with spin 1/2 and spin 0). Using the superspace idea pioneered by Salam and Strathdee [9], we can package the degrees of freedom relevant to SUSY gauge theories into the following neat set of off-shell superfield representations of the SUSY algebra

$$\begin{aligned}
&\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F \\
&V = \theta\sigma^\mu\overline{\theta}V_\mu + i\theta^2\overline{\theta}\lambda^\dagger - i\overline{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\overline{\theta}^2 D \\
&W_\alpha = -i\lambda_\alpha + \theta_\alpha D - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu} - \theta^2(\sigma^\mu D_\mu\lambda)_\alpha \\
&\mathcal{J} = J + i\theta j - i\overline{\theta}\overline{j} - \theta\sigma^\mu\overline{\theta}j_\mu + \frac{1}{2}\theta^2\overline{\theta}\overline{\sigma}^\mu\partial_\mu j - \frac{1}{2}\overline{\theta}^2\theta\sigma^\mu\partial_\mu\overline{j} - \frac{1}{4}\theta^2\overline{\theta}^2\partial^2 J \\
&\mathcal{J}_\mu = j_\mu + \theta^\alpha(S_{\mu\alpha} + ...) + \overline{\theta}_{\dot{\alpha}}(\overline{S}^{\dot{\alpha}}_\mu + ...) + \theta\sigma^\nu\overline{\theta}\Big(2T_{\nu\mu} - \frac{2}{3}\eta_{\nu\mu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}_R\Big) + ... \\
&X = x + \frac{4}{3}\theta^\mu_{\alpha\dot{\alpha}}\overline{S}^{\dot{\alpha}}_\mu + \theta^2\Big(\frac{2}{3}T + i\partial_\mu j^\mu_R\Big)
\end{aligned} \tag{1.0.6}$$

where θ_α and $\overline{\theta}_{\dot{\alpha}}$ are anticommuting Grassman spinors. The field Φ is a so-called chiral superfield that contains matter fermions, ψ, complex bosonic partner ϕ fields, and an auxiliary superfield F. These superfields are called chiral because their lowest component, ϕ, is annihilated by the antichiral supercharges, $Q^\dagger_{\dot{\alpha}}$. Similarly, the gauge field strength, $F_{\mu\nu}$, is packaged into a chiral superfield, but this time with the lowest component given by the spin 1/2 partner of the vector boson known as the gaugino, λ. The vector bosons are packaged into a real superfield $V=V^\dagger$, along with a partner gaugino and an auxiliary field D. The conserved currents, j^μ, corresponding to the bosonic symmetries of the theory, are packaged into linear superfields, $\mathcal{J}$, while the current related to the supercharge, the spin 3/2 supercurrent $S_{\mu\alpha}$, is packaged into a superfield with the spin 2 stress tensor $T_{\mu\nu}$ and the spin 1 superconformal R-current, $j_{\mu R}$.[3] The R-current is related to a particular R-symmetry. In general, an R-symmetry is defined as an automorphism of the SUSY algebra. In the case of $\mathcal{N} = 1$ SUSY, any R-symmetry is necessarily a $U(1)$ symmetry with

$$[Q_\alpha, R] = Q_\alpha, \quad [Q^\dagger_{\dot{\alpha}}, R] = -Q^\dagger_{\dot{\alpha}} \tag{1.0.7}$$

The final multiplet in (1.0.6) captures, among other things, the trace and superconformal R-anomalies of the theory. In the case that a theory has superconformal symmetry, $X = 0$.

Returning to the main thrust of our section, we note that if a Lagrangian description of a SUSY gauge theory is available, we may write the dynamics of the theory in terms of a real Kähler potential, K, and a complex holomorphic function of the chiral superfields, W, called a superpotential

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger, V) + \Big(\int d^2\theta W(\Phi) + \frac{1}{4}\int d^2\theta W^\alpha W_\alpha + c.c.\Big) \tag{1.0.8}$$

Specializing for simplicity to the case of a $U(1)$ gauge theory, the Lagrangian in (1.0.8) is invariant under the complexified gauge transformations which send $V \to V + i(\Lambda - \Lambda^\dagger)$ and $\Phi \to e^{-iq\Lambda}\Phi$. The moduli space of supersymmetric solutions of (1.0.8) is then given by the stationary solutions of W modulo the complexified gauge transformations. When we discuss D-branes probing extra-dimensional singular manifolds in the later part of this thesis, the resulting moduli spaces will prove to be very important since they furnish a description of the geometry that the D-brane sees.

As a final note, we should stress the importance of the holomorphy of the superpotential in (1.0.8). Indeed, this fact can be used to prove that the superpotential is not renormalized to any order in perturbation theory [10].[4] Perturbatively, the physical fields and couplings

[3] Let us briefly remark as an aside that since the supercurrent and stress tensors are related by SUSY and, equivalently, since the SUSY algebra relates supercharges to the momentum operators, it should be clear that making supersymmetry local necessarily leads to a theory of gravity. Theories of this type are called supergravity theories and are the low-energy approximations to string theory.

[4] This statement is the generalization of the idea we described in the introduction, namely that SUSY protects the Higgs mass from large corrections.

only receive quantum corrections from the Kähler potential. The only allowed quantum corrections to the superpotential itself are non-perturbative in nature. Such non-perturbative corrections will play a starring role in the final chapters of this thesis.

1.0.2 Supersymmetry and particle physics

The sum total of our knowledge of particle physics is contained in the SM. The SM is a gauge theory with gauge group $SU(3) \times SU(2) \times U(1)$. The basic fields of the SM are

$$\begin{aligned}
&Q_i = (\mathbf{3}, \mathbf{2}, \mathbf{1/6}), \quad \overline{u}_i = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{-2/3}), \quad \overline{d}_i = (\mathbf{3}, \mathbf{1}, \mathbf{1/3}) \\
&L_i = (\mathbf{1}, \mathbf{2}, \mathbf{-1/2}), \quad \overline{e}_i = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \\
&H = (\mathbf{1}, \mathbf{2}, \mathbf{1/2}) \\
&G = (\mathbf{8}, \mathbf{1}, \mathbf{0}), \quad W = (\mathbf{1}, \mathbf{3}, \mathbf{0}), \quad B = (\mathbf{1}, \mathbf{1}, \mathbf{0})
\end{aligned} \tag{1.0.9}$$

where $i = 1, 2, 3$ is a generation label, and we have written the explicit representations of the SM gauge group under which the various fields transform on the RHS of (1.0.9). The Q_i, $\overline{u}_i$, and $\overline{d}_i$ are the quarks, while the L_i and $\overline{e}_i$ are the leptons. H is the Higgs and G, W, and B are the gauge bosons associated with the various gauge group factors.

In addition to the various gauge interactions, the Standard Model contains Yukawa couplings between the Higgs and the various matter fields, with a distinctive hierarchy featuring very large couplings of the Higgs to the third generation particles and much smaller couplings to the first and second generations. These couplings give rise to a large mass hierarchy between the third generation and the rest of the SM matter fields.

It is rather straightforward to embed the SM into a supersymmetric theory. The minimal completion is to promote the Q, $\overline{u}$, $\overline{d}$, L, and $\overline{e}$ fields to chiral superfields while promoting the G, W, and B fields to vector superfields. The only small subtlety is in the Higgs sector. Simply promoting the Higgs field to a chiral superfield leads to $U(1)_Y^3$ and $SU(2)^2U(1)_Y$ anomalies because we have introduced a new chiral fermion, the higgsino. The resolution to this problem is to introduce a second Higgs doublet superfield with opposite $U(1)_Y$ quantum numbers. The full Higgs sector of our theory becomes

$$H_u = (\mathbf{1}, \mathbf{2}, \mathbf{1/2}), \quad H_d = (\mathbf{1}, \mathbf{2}, \mathbf{-1/2}) \tag{1.0.10}$$

Finally, we promote the SM Yukawa interactions to superpotential interactions via

$$W = \overline{u}\mathbf{Y_u}QH_u - \overline{d}\mathbf{Y_d}QH_d - \overline{e}\mathbf{Y_e}LH_d + \mu H_u H_d \tag{1.0.11}$$

where the $\mathbf{Y}$ are matrices in generation space that describe the Yukawa interactions. The simple renormalizable theory we have arrived at is the Minimal Supersymmetric Standard Model (MSSM). We will refer to its generalizations as Supersymmetric Standard Models (SSMs). These are the touchstones of supersymmetric phenomenology. In the third chapter of this thesis, we will construct a simple semi-realistic SSM in string theory.

1.0.3 Supersymmetry breaking

From the discussion in the previous section, we have seen that it is rather trivial to construct supersymmetric extensions of the Standard Model. Still, since SUSY is broken at the energies that we have managed to probe in colliders thus far, we need a way of describing SUSY breaking in the context of the MSSM.

Before describing SUSY breaking in particle physics, let us briefly review the basics of spontaneously broken SUSY. From the SUSY algebra in (1.0.5), it is easy to see that

$$Q_\alpha|0\rangle \neq 0 \Rightarrow \langle 0|H|0\rangle \neq 0 \tag{1.0.12}$$

In other words, the vacuum energy is an order parameter for SUSY breaking. By integrating out the auxiliary F and D fields in a theory of the general form described in (1.0.8), we find the following scalar potential

$$V_{\text{scalar}} = \frac{1}{2}\overline{F}^i F_i + \frac{g^2}{2} D^a D^a \tag{1.0.13}$$

where the F^i and D^a are the auxiliary fields. If we find a vacuum in which at least one F^i or one D^a is non-vanishing, then SUSY is broken.

Unfortunately, the SUSY breaking in the MSSM cannot be spontaneous. Indeed, a simple theorem of Georgi and Dimopoulos [11] shows that SUSY cannot be spontaneously broken in any renormalizable extension of the MSSM. In particular, by studying the classical squark mass matrix, they were able to show that in any such renormalizable SSM there would always be a squark with mass less than or equal to the masses of either the up or the down quarks, a clear experimental impossibility.

To circumvent this situation, we can imagine a scenario in which either loop corrections are strong and hence invalidate the classical assumptions of the Georgi-Dimopoulos theorem or the Kähler potential of the MSSM contains various important non-renormalizable terms. Since such contributions are necessarily suppressed by loop factors and powers of the cutoff, Λ, beyond which the MSSM fails as an effective theory, spontaneous SUSY breaking must occur at high scales in a separate, so-called "hidden" sector of the theory, and then be transmitted to the MSSM by "messenger" interactions. In Chapter 2 of this thesis, we will develop a general language to better understand scenarios in which SUSY breaking is mediated by the gauge interactions of the SSM. In Chapter 5 of this thesis, we will describe a setup in which SUSY breaking is mediated by D-brane instantons in a string theory compactification.

The dynamics of the SUSY breaking hidden sector itself are potentially quite complicated. One particularly attractive way for SUSY to be broken is via dynamical effects. In other words, we imagine the hidden sector as having supersymmetric vacua at tree level, with SUSY breaking being introduced by non-perturbative order $e^{-8\pi^2/g^2}$ effects. Such non-perturbative effects arise naturally in hidden sectors endowed with asymptotically free gauge symmetries.

The first concrete example of dynamical SUSY breaking (DSB) was the model of Affleck, Dine, and Seiberg [12] where instanton effects generate a non-perturbative SUSY-breaking correction to a tree-level supersymmetric superpotential. This model, like the other early models of DSB, features a spontaneously broken R-symmetry. Indeed, the connection between R-symmetry breaking and SUSY breaking is well known [13]. For generic calculable theories, it turns out that an exact R-symmetry is a necessary condition for SUSY breaking, and that a spontaneously broken R-symmetry is a sufficient condition. On the other hand, explicitly broken R-symmetry, if parametrically small, can lead to sufficiently long-lived metastable SUSY breaking vacua. Such theories turn out to be quite elegant and simple.

Indeed, this insight was exploited in the construction of the first long-lived SUSY breaking vacua in $\mathcal{N}=1$ supersymmetric QCD (SQCD) with massive flavors [14]. The basic point was that in the low-energy effective theory of [14], the SUSY vacua were invisible, but since the R-symmetry was anomalous, it was broken by non-perturbative effects and SUSY was restored far away in field space. As we will see in Chapter 4 of this thesis, such non-perturbative R-symmetry breaking SUSY-restoring effects occur rather simply in D-brane realizations of SUSY breaking QFTs.

1.0.4 Mediation and soft parameters

We see now that in order to introduce SUSY at high energies, we are forced into a paradigm where the SSM is an effective theory valid below some cutoff, Λ, corresponding to the scale of the new physics that is responsible for spontaneously breaking SUSY. Of course, treating the SSM as an effective theory is not a big sacrifice since it is a theory that does not include gravity and therefore cannot be the correct final description of nature.

The general picture that we have, then, is that SUSY is explicitly broken in the SSM via the mediating interactions with the hidden sector. The resulting explicit SUSY breaking terms are called "soft terms" because they do not reintroduce quadratic divergences into the effective action of the SSM.[5] Taking the concrete case of the MSSM, the most general set of soft terms are collected in the following Lagrangian:

$$\mathcal{L}_{\rm soft}=\sum_{i=Q_i,\overline{u}_i,\ldots} m^2_{\phi_i}|\phi_i|^2+\Big(\sum_{a=1,\ldots,3}M_a\lambda_a\lambda_a-B\mu H_uH_d+\overline{u}\mathbf{A_u}QH_u+\overline{d}\mathbf{A_d}QH_d+\overline{e}\mathbf{A_e}LH_d+c.c.\Big) \tag{1.0.14}$$

where, through an abuse of notation, we let the $Q_i, \overline{u}_i, \ldots$ stand for the lowest components of the corresponding chiral multiplets. The soft terms therefore correspond to scalar masses for the squarks and sleptons, gaugino masses, SUSY-breaking mass terms for the Higgs doublets, and SUSY breaking trilinear terms for the various scalars (so-called, "A-terms").

With the inclusion of the general set of soft parameters for the MSSM, our theory now has on the order of one hundred couplings. In short, by introducing soft SUSY breaking to

[5] We are assuming that the SSM does not contain any singlets.

the MSSM, we have introduced a veritable plethora of new terms to the theory.

It should therefore not come as much of a surprise that most of this parameter space corresponds to phenomenologically unviable theories. In fact, there are two main problems with most of the parameter space. One problem is flavor changing neutral currents (FCNCs) and the other is large CP violation.

One can immediately understand the problem of FCNCs by considering the SUSY contributions to $K^0 - \overline{K}^0$ mixing. In addition to the standard 1-loop diagram containing a box exchange of quarks and W bosons with insertions of the Cabibbo-Kobayashi-Maskawa (CKM) matrix at each vertex, we must also take into account diagrams with exchanges of squarks and winos. The main problem then arises from insertions of the squark mass matrix in the internal squark propagator lines. Since the corresponding mass matrix does not generally commute with the CKM matrix, the usual Glashow-Iliopoulos-Maiani (GIM) mechanism of the Standard Model is inoperative.[6] Finally, generic phases in the various soft terms lead to unacceptably large CP violation in a variety of well studied systems like the electric dipole moment of the neutron.

The particular pattern of soft terms that one generates depends largely on the type of messenger interactions. Indeed, while the hidden sector produces a non-vanishing vacuum energy, it is the mediating interactions that encode how this SUSY breaking is felt by the various visible sector fields. For example, one simple way to address the problem of FCNCs is to allow for the mediating interactions to be gauge interactions, i.e., "gauge mediation." Since these interactions are flavor-blind, they do not contribute additional sources of flavor violation. In Chapter 2 of this thesis, we will pursue this idea much further. Note, however, that gauge mediation does not have much to say about CP violation or about another problem that we have glossed over above, namely the fact that electroweak symmetry breaking (EWSB) requires $B \sim \mu$ (the so-called μ problem).

[6] Unless, for example, we allow for very massive gauginos among other things. Some work in this direction is described in [15].

1.0.5 Supersymmetric particle physics and string theory

As we have discussed above, particle physics in the form of the Standard Model is a low energy approximation to nature. While we can tame the UV sensitivity of the SM and avoid phenomenological pitfalls by including spontaneously broken SUSY and using its relative quantum smoothness to control the behavior of unwieldy divergences in the Higgs sector, it is clear that this framework cannot be the complete description of nature. In the introduction, we briefly motivated the importance of string theory as a means of describing the quantum gravitational effects that emerge as we probe shorter and shorter distances down to the Planck length. To that more abstract motivation we can add this much more concrete one—the spin 2 graviton emerges in the perturbative spectrum of the closed string. Furthermore, the smoothness of the string world sheet means that we don't have to worry about regulating perturbative divergences as we do in QFT.

That said, we would like to focus in this section on how the degrees of freedom of particle physics—the matter fields and gauge particles—can be embedded within this framework as well. This project is an ambitious one since it seeks to unify our full understanding of high energy physics into one edifice. Still, there have been some modest successes and new ideas. We will formulate some of these new ideas in the later chapters of this thesis.

The first problem in confronting string theory with particle physics is that string theory has many extra dimensions. In the supersymmetric flavors of string theory (all related by various dualities), there are ten dimensions. This means that in order to make contact with particle physics, we need to curl up six of those dimensions.

The first string constructions that make contact with particle physics started appearing in the mid 1980's and were based on compactifications of the $E_8 \times E_8$ heterotic string on a six-dimensional compact manifold, $\mathcal{M}$ [16]. In order to find supersymmetric solutions, one demands that the SUSY variations of the fermionic fields of the theory vanish. From the vanishing of the SUSY variation of the gravitino, one finds a solution with a covariantly constant spinor

$$\nabla_m \zeta = 0 \tag{1.0.15}$$

where ∇_m is the covariant derivative on $\mathcal{M}$. The existence of such a spinor implies that $\mathcal{M}$ is a manifold of $SU(3)$ holonomy, i.e., that $\mathcal{M}$ is a so-called 'Calabi-Yau' manifold. The resulting four-dimensional theory has an $\mathcal{N} = 1$ SUSY. Although we will not get into the details here, within the framework of $E_8 \times E_8$ string theory compactified on a CY, it is possible to find models with SM and GUT gauge groups and with the number of quark and lepton chiral superfields controlled by the dimensions of the various non-trivial Dolbeault cohomology classes. One important problem with these constructions is the existence of moduli, i.e., massless scalar fields. Such excitations are clearly ruled out by experiment.

While there has been some continued interest in heterotic string phenomenology in recent years, in particular in heterotic M-theory constructions [17], D-branes have opened up a new perspective on particle physics and have become the preferred object of study in string phenomenology. Discovered by Polchinski in 1995, D-branes are 1/2 BPS solitonic objects of string theory on which open strings can end (this data is summarized by the Chan-Paton (CP) indices of the strings). For our purposes, the important point is that the open string perturbations of the D-branes contain the spin 0, spin 1/2, and spin 1 fields necessary to realize models of particle physics.

To understand this point further, consider, for concreteness, the case of a stack of N D3 branes in ten-dimensional flat space. It turns out that the low energy and small string coupling worldvolume theory on this stack of branes is a 4 dimensional $\mathcal{N} = 4$ $U(N)$ SYM theory (since the branes are 1/2 BPS, they break half the thirty-two supercharges of the ten-dimensional type IIB supergravity). This theory has an $SO(6)$ R-symmetry that corresponds geometrically to rotations of the directions normal to the D-brane. From the $\mathcal{N} = 1$ perspective, the matter content of the effective world-volume theory contains three chiral multiplets transforming in the adjoint in addition to a vector multiplet. The bosonic components of the theory correspond to motion in the six transverse coordinates and therefore transform as a **6** of the $SO(6)_R$.

While this theory is interesting in its own right, in order to make contact with particle physics, we need matter transforming in chiral representations. One possible way to accomplish this task is to consider higher-dimensional D-branes intersecting at appropriate angles [18]. Indeed, by choosing these angles judiciously, one can construct theories with four-dimensional $\mathcal{N} = 1$ SUSY and chiral bifundamental matter localized at the brane intersections. These types of constructions have been repeatedly exploited in the literature to construct simple toy models of particle physics (see [19] and the references therein).

Another interesting possibility is to consider D-branes probing a singular point of the transverse space [20]. Simple examples of these general scenarios include orbifolds of manifolds that are locally $\mathbf{C}^3$ by various discrete groups $\Gamma \subset SU(3)$. The singular points arise from the fact that we choose the orbifold action to have fixed points. The fact that Γ should be a subgroup of $SU(3)$ corresponds to the fact that we would like to preserve an $\mathcal{N} = 1$ SUSY. By appropriately choosing the orbifold group action on the spacetime and on the CP indices, it is possible to engineer chiral theories. The main consistency condition for these orbifolds is that the twisted tadpoles cancel.[7] This condition is equivalent to the absence of gauge anomalies.

In order to engineer SM-like theories with three generations, the most direct route is to consider orbifolds of $\mathbf{C}^3$ by the groups $\Gamma = Z_3$ or $\Gamma = \Delta_{27}$ and simple variations thereof. The case of Z_3 is thoroughly analyzed in [21]. The basic construction we will use in Chapter 3 is conjectured to be related to the Δ_{27} orbifold studied in [21, 22] by a local deformation of the geometry [23].

Now, given the technology of D-branes that we have briefly outlined above, one can ask what we have gained over the heterotic compactifications of the old days. The main point is that we gain the ability to localize the degrees of freedom we are interested in—the massless open string excitations—at specific points on the compactification manifold.

This localization allows us to consider a bottom-up perspective where the particle physics depends only on the description of the local geometry. One computes the superpotential from various disk amplitudes and parameterizes the remaining couplings in terms of the periods of the closed string fields on the cycles of the singularity wrapped by the D-branes.

[7] If we consider a compact setting, Gauss' law implies that the total Ramond-Ramond charge associated with the D-branes and any orientifold planes that are present must also cancel.

From these quantities we have central charges (the phase alignment of the central charges determines whether SUSY is preserved while the norms are proportional to the gauge couplings) and other gauge invariant data. In an appropriate limit in which UV gravitational effects are decoupled into the distant bulk, the closed string dynamics are fixed and one imagines the periods as parameters of the field theory on the D-branes.

Once the appropriate local gauge theories have been engineered, one attacks the harder problem of embedding the theory in a UV compactification.[8] Typically, such compactifications contain many moduli. These moduli are related to the size (Kähler moduli) and shape (complex structure moduli) of the compactification. One must stabilize these moduli with fluxes and D-brane instanton contributions. Such mechanisms have been well-studied [24]. Interestingly, by turning on appropriate fluxes, one can also arrange for soft SUSY breaking [25].

From this brief and heuristic introduction, we see that it is possible to engineer SM-like gauge theories as world-volume theories of D-branes probing singularities. It is of course also possible to use this technology to build additional, hidden sectors where SUSY is spontaneously broken. We will construct some novel SUSY breaking D-brane sectors in Chapter 4 of this thesis. Furthermore, using stringy objects, such as Euclidean D-branes, we will see that it is possible to mediate SUSY breaking to the visible sector in string compactifications.

One aspect of these constructions that we have not elucidated here is how we should understand the dynamics of the various couplings in the engineered QFTs. In particular, coupling constants generally acquire a quantum scale dependence described by a renormalization group (RG) flow. The answer to this question is both complicated and beautiful. It turns out that what we have neglected to describe is the back-reaction of the branes on the geometry itself.

The simplest illustration of this fact is to imagine a stack of D3 branes sitting at a point

[8] It is a hope of this program that the low energy local string theory may have something to say, upon experimental input from the LHC and future collider experiments, about the details of a UV completion. Much work remains to be done in this direction, and its potential for success is unclear.

in flat space. By considering the back-reaction and looking at the geometry in the 'near horizon' limit (roughly speaking, in the neighborhood of the D-brane), we find the metric of $AdS_5 \times S_5$

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + d\mathbf{x}^2 + dz^2) + R^2 d\Omega_5^2 \tag{1.0.16}$$

The important point is that as one runs in the radial coordinate, z, the four dimensional space time is re-scaled. This is the geometric interpretation of the RG scale.

More significantly, the AdS/CFT duality [2] establishes a precise correspondance between the gravitational picture in (1.0.16) and the $\mathcal{N} = 4$ SYM gauge theory on the stack of D-branes. Our discussion above of the gauge invariant couplings being determined by the closed string geometry then fits beautifully into this edifice. By now thousands of papers have been written on AdS/CFT and the idea of gauge-gravity duality. By considering less symmetric spaces, such as the orbifolds we have discussed above, one can get more interesting and potentially more phenomenologically relevant dualities than the one in (1.0.16).

In this introduction we have only been able to give a small flavor of the interesting structures that lie at the intersection of string theory, particle physics, and supersymmetry. This is a complicated subject that is still, after many years of research, mysterious but full of promise.

Chapter 2

General Aspects of Gauge Mediation

In this chapter, we further develop and then reformulate a language (initially discussed in [27]) for describing the soft parameters of theories with gauge-mediated SUSY breaking in terms of hidden-sector contributions to SM gauge group current two-point functions.[1]

Using this reformulation, we give a general proof of the finiteness of the scalar masses in gauge mediation and further study the UV and IR behavior of the contributing current two-point functions.

We also shed further light on the UV sensitivity of the soft masses in the MSSM and help clarify the role of the supertrace in messenger theories of SUSY breaking. In the last part of the chapter, we show that, under certain assumptions, it is possible for calculable messenger theories to cover the full gauge mediation parameter space that we describe. In particular, this fact leads to many novel signatures not typically associated with gauge mediation.

In the appendix, we comment on the sign of the supertrace in effective messenger theories of gauge mediation and prove a general theorem regarding the sign of the supertrace in theories that are UV completed by integrating out charged chiral matter at a high scale.

[1]This chapter is based on the paper, "Exploring General Gauge Mediation," written in collaboration with P. Meade, N. Seiberg, and D. Shih [26].

We also comment on the case of heavy vector matter.

2.1 Introduction

As we discussed in Chapter 1, low-energy supersymmetry, in its minimal incarnation as the MSSM, is probably the most attractive candidate for physics beyond the Standard Model, since it solves the hierarchy problem and predicts gauge coupling unification. However, as we noted, the MSSM has one major drawback, namely, its immense parameter space. Soft SUSY-breaking introduces $\mathcal{O}(100)$ new parameters compared to the SM. These parameters are highly constrained by stringent experimental limits on flavor-changing neutral currents and CP violation. A conservative ansatz for the parameter space which is automatically consistent with flavor and CP is known as "soft SUSY-breaking universality" (see [28] for a nice review). Here there are five flavor-diagonal sfermion masses, three real gaugino masses, three flavor-diagonal A-terms, and three independent real Higgs mass parameters, for a total of 14 real parameters in all. If one accepts the hypothesis of universality, then the theoretical challenge is to construct models of SUSY-breaking and mediation that automatically produce universal patterns of soft parameters without fine tuning.

Gauge mediation [34, 35, 36, 37, 38, 39, 40, 41, 31, 32, 33], or the idea that SUSY-breaking is communicated to the MSSM via the SM gauge interactions, is a promising partial solution to this challenge.[2] Since the gauge interactions are flavor blind, the soft masses obtained through gauge mediation are automatically flavor universal. However, the absence of CP phases is less automatic in gauge mediation. Also, the Higgs μ and B_μ parameters are not generated in pure gauge mediation, so one typically assumes that additional interactions are present to produce these (for a recent discussion of this see [58]).

Recently in [27], gauge mediation was given a general, model-independent definition: *in the limit that the MSSM gauge couplings $\alpha_i \to 0$, the theory decouples into the MSSM and a separate hidden sector that breaks SUSY.* It follows then that the SM gauge group must be

[2] For a review of gauge mediation from both the model building and phenomenological point of view see [42].

part of a weakly-gauged global symmetry G of the hidden sector. By studying a small set of current-current correlators of G, it was shown that all the dependence of the soft masses on the hidden sector could be encapsulated by three real parameters that determine the sfermion masses, and three complex parameters that determine the gaugino masses. This framework was called "General Gauge Mediation" (GGM) in [27]; for more recent work on GGM, see [45, 51, 52, 53, 54, 55]. In this chapter we will further develop several aspects of GGM and explore its properties and its parameter space.

The definition of GGM must be augmented with several phenomenological and consistency requirements, which we will now review. First, the fact that the gaugino masses are complex in general gauge mediation (GGM) implies that GGM does not solve the SUSY CP problem. So additional mechanisms (such as an R-symmetry as in [29], or having the hidden sector be CP invariant) must be invoked to explain why the gaugino masses are real.[3] For the rest of the chapter, wherever it is relevant, we will *assume* that such a mechanism is at work and only consider CP invariant theories, so that the parameter space of GGM spans $\mathbb{R}^6$. With this assumption, the GGM parameter space comprises a much smaller, but still sizeable subspace of the full "universal" soft mass ansatz.

Additionally, as in [27], we will impose a $\mathbb{Z}_2$ symmetry, called "messenger parity," on our hidden sector. In the context of messengers this is typically defined as an interchange symmetry of the messengers combined with $V \to -V$ [38, 44]. More generally, messenger parity can be defined in terms of the gauge current and its supersymmetric partners, without explicit reference to messengers [27]. This symmetry does not have to be imposed, but it is typically a phenomenological necessity: messenger parity prevents dangerous hypercharge D-terms (which could lead to tachyonic sleptons) from being generated in the hidden sector.

Messenger parity has various other consequences, including one on the sum rules of GGM. The fact that the five flavor-diagonal sfermion masses $(m_Q^2, m_U^2, m_D^2, m_L^2, m_E^2)$ are determined in terms of three real numbers implies that they must satisfy two sum rules

[3] Of course, one can have non-zero phases in this framework as long as they are consistent with the experimental bounds. For convenience though, we will only concentrate on CP invariant hidden sectors.

[27]:

$$\begin{aligned} \mathrm{Tr}\, Y m^2 &\propto m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0 \\ \mathrm{Tr}\,(B-L) m^2 &\propto 2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0. \end{aligned} \tag{2.1.1}$$

These sum rules are valid at the characteristic scale M of the gauge mediated model, and they are preserved by the (one-loop) running of the soft masses in the MSSM. There could in principle be violations to these sum rules arising at higher order in the SM gauge couplings, coming from 3-point functions in the hidden sector. We will show in section 2 that in fact these threshold contributions satisfy the sum rules if one imposes messenger parity on the hidden sector. Additionally, the leading log contributions at all higher orders also satisfy the sum rules. Therefore there are no contributions at any relevant order in the hidden sector which would violate the sum rules and they truly are predictions of GGM.

In [27], it was shown that the GGM parameter space is the most general that can be populated by models of gauge mediation. However, this left open the important question of whether models existed that could actually span this space. For instance there may have been additional relations or inequalities satisfied by the parameters that were not manifest from the analysis of the current-current correlators. Or it could have been that for some regions of the GGM parameter space there was simply no field theory that could populate it. Indeed, a quick survey of existing models of gauge mediation (e.g. the original models of "minimal gauge mediation"[31, 32]) would suggest that this could be the case, as these models clearly do not cover the parameter space. These models are based on a set of weakly coupled "messengers," chiral superfields, Φ^i, that transform under a real representation of the SM gauge group and couple to a field that has a SUSY breaking F-component. This can be expressed as having a generic supersymmetric mass term for the messengers

$$W = M_{ij} \Phi^i \Phi^j \tag{2.1.2}$$

and a SUSY-breaking mass term of the form

$$V \supset f_{ij} \phi^i \phi^j + c.c. \tag{2.1.3}$$

In [45] it was shown that in the context of such models, the right number (6) of parameters in GGM could be realized. However, in their models the full space of GGM was not actually spanned.

In this chapter we further explore the model building possibilities in the context of weakly coupled messengers and show that there *are* models that span the GGM parameter space. This is because there can be additional contributions to the MSSM soft masses from gauge mediation in addition to those of the form (2.1.3), namely "diagonal-type"[47, 46] messenger masses of the form

$$V \supset \xi_{ij}\phi^i\phi^{\dagger j} \tag{2.1.4}$$

Such terms typically arise from D-term breaking, but they can also arise from strong hidden sector dynamics (such as in [43]) where the distinction between F-term and D-term breaking is not obvious.

Using both (2.1.3) and (2.1.4), we demonstrate that there exist weakly coupled messenger models which span the space of GGM. Thus there can be no additional relations for the soft SUSY breaking parameters beyond (2.1.1).

The outline of the chapter is as follows. First, in section 2 we present a reformulation of GGM that does not rely upon superspace and that leads to extremely compact formulas for the gaugino and sfermion soft masses. Using this formalism we will demonstrate both the UV and IR finiteness of the soft masses in GGM. We will then discuss in section 3 the dependence on the various mass scales that can enter the correlation functions. We will further elaborate on the issues of UV sensitivity for SUSY breaking parameters, clearing up some confusion in the existing literature regarding the interpretation of a nonzero messenger supertrace. Finally, in section 4 we present a simple explicit model involving weakly-coupled messengers that spans the entire six-dimensional parameter space of GGM. This model should be viewed merely as an "existence proof" that the entire GGM parameter space can be realized and that there are no additional hidden relations between the parameters that are not obvious from the general formulation. In light of this we believe that future phenomenological studies of gauge mediation should not restrict themselves to the parameterization of minimal gauge mediation (for example see [50]), but instead should explore the entire parameter space of GGM. This should in principle open up new avenues for possible experimental/phenomenological studies that have not yet been explored (for recent work in this direction, see [55]). We finish by collecting a few technical results in two appendices. In Appendix A we will review the role of the supertrace in models with messenger fields. We demonstrate that certain classes of models always generate a particular sign for the supertrace in an effective field theory. In appendix B we collect some general results for the correlation functions of models with arbitrary numbers of messengers.

2.2 General Gauge Mediation: A New and Improved Formulation

2.2.1 Review and reformulation

In this section we wish to review the basic features of GGM. Along the way, we will reformulate and streamline various aspects of it. This will lead to various new physical insights, including a direct proof of the finiteness of the sfermion soft masses in GGM.

To begin, let us describe the setup. Consider a renormalizable hidden sector[4] which is characterized by the scale M and where supersymmetry is broken spontaneously. Suppose that this hidden sector has a global symmetry group $G \supset G_{SM} = SU(3)\times SU(2)\times U(1)$ that is weakly gauged. Suppose further that the only coupling to the visible sector occurs through the SM gauge interactions (so the hidden and visible sectors decouple in the $g_{SM} \to 0$ limit). We will refer to this setup as general gauge mediation, and we are interested in the visible-sector soft masses that arise. As shown in [27], all of the information in the soft masses is encoded in two-point functions of the current superfield of the symmetry group G.

To avoid writing all the gauge theory factors, we will assume for simplicity that $G = U(1)$ in this subsection. Recall now the definition of of the current superfield $\mathcal{J}$

$$D^2\mathcal{J} = 0 \tag{2.2.1}$$

[4] We will consider non-UV-complete scenarios in later sections.

which leads in components to

$$\mathcal{J} = J + i\theta j - i\overline{\theta}\overline{j} - \theta\sigma^\mu\overline{\theta}j_\mu + \frac{1}{2}\theta^2\overline{\theta}\overline{\sigma}^\mu\partial_\mu j - \frac{1}{2}\overline{\theta}^2\theta\sigma^\mu\partial_\mu\overline{j} - \frac{1}{4}\theta^2\overline{\theta}^2\mathbf{\Lambda}J \tag{2.2.2}$$

with $\partial^\mu j_\mu = 0$.

The use of superspace is not essential. Without it, we can replace the definition of the current superfield $\mathcal{J}$ (2.2.1) as follows. We study the hermitian operator J which satisfies

$$\{Q_\alpha, [Q_\beta, J]\} = 0 \tag{2.2.3}$$

where Q_α are the supercharges, which satisfy the SUSY algebra

$$\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu. \tag{2.2.4}$$

Then, we can define

$$\begin{aligned} j_\alpha &\equiv -i[Q_\alpha, J] \\ j_{\dot{\alpha}} &\equiv i[\overline{Q}_{\dot{\alpha}}, J] \\ j_\mu &\equiv -\frac{1}{4}\overline{\sigma}^{\dot{\alpha}\alpha}_\mu \left(\{\overline{Q}_{\dot{\alpha}}, [Q_\alpha, J]\} - \{Q_\alpha, [\overline{Q}_{\dot{\alpha}}, J]\}\right), \end{aligned} \tag{2.2.5}$$

and derive the current conservation by applying two supercharges to this definition of j_μ and using the SUSY algebra (2.2.4).

The relation between the original presentation in superspace with (2.2.1) and this one is similar to the relation between the definition of chiral superfields in terms of $\overline{D}\Phi = 0$ and the definition of chiral operators (the first component of Φ) as $[\overline{Q}, \phi] = 0$.[5] As we will now show, (2.2.3) proves to be extremely useful when computing current-current correlation functions.

The correlators of interest are the nonzero current-current two-point functions

$$\begin{aligned} \langle J(x)J(0)\rangle &= \frac{1}{x^4}C_0(x^2M^2) \\ \langle j_\alpha(x)\overline{j}_{\dot{\alpha}}(0)\rangle &= -i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\left(\frac{1}{x^4}C_{1/2}(x^2M^2)\right) \\ \langle j_\mu(x)j_\nu(0)\rangle &= (\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\left(\frac{1}{x^4}C_1(x^2M^2)\right) \\ \langle j_\alpha(x)j_\beta(0)\rangle &= \epsilon_{\alpha\beta}\frac{1}{x^5}B(x^2M^2) \end{aligned} \tag{2.2.6}$$

[5] We will not pursue it here, but it would be interesting to consider correlators of J's defined by (2.2.3) along with any number of supercharges, in the case when SUSY is unbroken. Perhaps there could be an interesting mathematical structure analogous to operators in the chiral ring.

or in momentum space,

$$\begin{aligned}
\langle J(p)J(-p)\rangle &= \widetilde{C}_0(p^2/M^2) \\
\langle j_\alpha(p)\overline{j}_{\dot\alpha}(-p)\rangle &= -\sigma^\mu_{\alpha\dot\alpha}p_\mu \widetilde{C}_{1/2}(p^2/M^2) \\
\langle j_\mu(p)j_\nu(-p)\rangle &= -(p^2\eta_{\mu\nu} - p_\mu p_\nu)\widetilde{C}_1(p^2/M^2) \\
\langle j_\alpha(p)j_\beta(-p)\rangle &= \epsilon_{\alpha\beta}M\widetilde{B}(p^2/M^2)
\end{aligned} \tag{2.2.7}$$

where now a factor of $(2\pi)^4\delta^{(4)}(0)$ is understood.

These two-point functions encode the mediation of SUSY breaking to the MSSM gaugino and sfermion soft-masses at leading order in the gauge coupling g. Specifically, the gaugino masses are given by

$$M_{gaugino} = g^2 M\widetilde{B}(0). \tag{2.2.8}$$

while the sfermion soft mass-squareds are given by

$$m^2_{sfermion} = g^4 Y^2 A \tag{2.2.9}$$

where Y is the $U(1)$ charge of the sfermion and A is the following linear combination of correlators integrated over momentum:

$$\begin{aligned}
A &\equiv -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\Big(3\widetilde{C}_1(p^2/M^2) - 4\widetilde{C}_{1/2}(p^2/M^2) + \widetilde{C}_0(p^2/M^2)\Big) \\
&= -\frac{M^2}{16\pi^2}\int dy\,\Big(3\widetilde{C}_1(y) - 4\widetilde{C}_{1/2}(y) + \widetilde{C}_0(y)\Big)
\end{aligned} \tag{2.2.10}$$

Using (2.2.3) and (2.2.5), one easily finds that formula for the gaugino mass can be rewritten as

$$M_{gaugino} = -\frac{1}{4}g^2\int d^4x\,\langle Q^2(J(x)J(0))\rangle \tag{2.2.11}$$

where we use the notation

$$Q^2(\ldots) = Q^\alpha Q_\alpha(\ldots) \equiv \{Q^\alpha, [Q_\alpha, (\ldots)]\}. \tag{2.2.12}$$

Indeed, according to (2.2.3)(2.2.5), $Q^2(J(x)J(0)) = 2[Q^\alpha, J(x)][Q_\alpha, J(0)] = -2j^\alpha(x)j_\alpha(0)$. Similar reasoning shows that the action of four supercharges on $J(x)J(0)$ yields

$$\langle \overline{Q}^2(Q^2(J(x)J(0)))\rangle = -8\partial^2(C_0(x) - 4C_{1/2}(x) + 3C_1(x)) \tag{2.2.13}$$

and so the formula for the sfermion mass can be rewritten as

$$m^2_{sfermion} = -\frac{1}{128\pi^2} g^4 Y^2 \int d^4x \, \log(x^2M^2) \langle \overline{Q}^2 (Q^2(J(x)J(0))) \rangle \tag{2.2.14}$$

Note that the order of the four supercharges is not essential – a different ordering of Q and $\overline{Q}$ leads to terms that vanish after using the SUSY algebra and momentum conservation. Note also that the scale M appearing in (2.2.14) is arbitrary (i.e. the dependence on M drops out), since according to (2.2.13) the integrand $\langle \overline{Q}^2 (Q^2(J(x)J(0))) \rangle$ is a total derivative. (The short distance behavior of the correlator, to be discussed below, guarantees that there is no surface term.)

Let us make some brief comments on the results (2.2.11), (2.2.14). In [27] it was shown using the SUSY algebra that when SUSY is unbroken, $B = 0$ and $C_0 = C_{1/2} = C_1$. Hence the gaugino and sfermion masses vanish in the SUSY limit, as they must. Writing the gaugino and sfermion masses as multiple commutators, as we have done here, makes this fact obvious.

It is well known that when supersymmetry is broken at a scale F and the dynamics is characterized by the scale $M \gg \sqrt{F}$, we can effectively describe the soft terms in an expansion in $\frac{F}{M^2}$ using spurions. Then the gaugino masses arise as an F-term and the sfermion masses as a D-term. The expressions (2.2.11) and (2.2.14) generalize this result to the more generic situation of $F \sim M^2$. The small $\frac{F}{M^2}$ limit can be obtained by realizing that in (2.2.11) the two Qs lead to one factor of F and in (2.2.14) the four Qs lead to $|F|^2$.

Another interesting feature of the formula (2.2.14) is that all the information at large momentum is contained within the OPE of J with itself. This observation has immediate implications about the convergence of the momentum integral in (2.2.10) and (2.2.14). In [27] an indirect proof of the convergence of these integrals was given using the fact that otherwise there would be no supersymmetric counterterm that could cancel a divergence in this integral. Here we can easily give a direct proof which is intrinsic to the properties of the hidden sector. The most singular term in the OPE $J(x)J(0)$ is associated with the identity operator. Since this is annihilated by the action of the supercharges in (2.2.14), to get a nonzero result we must use an operator with $\Delta > 0$. Its coefficient is $x^{-4+\Delta}$ and therefore the integral (2.2.14) converges at small x.

Finally, let us examine the low momentum behavior of the integral in (2.2.10). We can exclude any zero-momentum divergences in these integrals by invoking messenger parity $\mathcal{J} \to -\mathcal{J}$. On general grounds, any such zero-momentum poles in the current two point functions in (2.2.7) must be due to massless intermediate one-particle states:

$$\langle \mathcal{J}(x)\mathcal{J}(0)\rangle = \langle 0|\mathcal{J}(x)|\lambda\rangle\langle\lambda|\mathcal{J}(0)|0\rangle + ... \tag{2.2.15}$$

Assuming that the only massless particles in the spectrum are due to spontaneously broken symmetries (bosonic or fermionic), and that messenger parity commutes with all the symmetries of the theory, it follows that the one-point functions on the RHS of (2.2.15) must vanish. Therefore massless modes can never contribute zero-momentum poles to the current two point function, and the integral (2.2.10) must always converge at $p = 0$.

2.2.2 Generalization to the MSSM

Finally, let us briefly generalize the discussion from our $G = U(1)$ toy model to the MSSM, where $G = SU(3) \times SU(2) \times U(1)$. We will label the gauge group factors $U(1)$, $SU(2)$ and $SU(3)$ by $k = 1, 2, 3$ respectively. Then are three complex numbers $B_k \equiv \widetilde{B}_k(0)$ and three real numbers A_k which determine the gaugino and sfermion soft masses. They are defined as above, using the current supermultiplet of the respective gauge group. The soft masses are given to leading order in the α by

$$M_k = g_k^2 M B_k, \qquad m_f^2 = \sum_{k=1}^{3} g_k^4 c_2(f,k) A_k \tag{2.2.16}$$

$f = Q, U, D, L, E$ labels the matter representations of the MSSM, and $c_2(f,k)$ is the quadratic Casimir of f with respect to the gauge group k.

Since the five sfermion masses are determined by three real numbers, they must satisfy two sum rules. These take the form [27]:

$$\begin{aligned} &m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0 \\ &2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0. \end{aligned} \tag{2.2.17}$$

From (2.2.16), it is clear that these sum rules are valid at $\mathcal{O}(\alpha^2)$. However, we can further demonstrate that they are valid at $\mathcal{O}(\alpha^3)$ and to leading-log order for any α, meaning that the sum rules must be satisfied to very high accuracy.

First, it was already shown in [27] that the sum rules are preserved by the MSSM RGEs (neglecting contributions from the Higgs sector proportional to the Yukawa interactions). This takes care of the leading-log corrections. Second, we can consider the $\mathcal{O}(\alpha^3)$ corrections coming from the hidden sector. These arise from various current three-point functions in the hidden sector. It is easy to see that gauge invariance allows only five three-point functions: $SU(3)^3$, $SU(2)^3$, $U(1)^3$, $SU(3)^2U(1)$, $SU(2)^2U(1)$. If one imposes messenger parity (which sends $V_Y \to -V_Y$), this eliminates the mixed three-point functions and the $U(1)^3$, leaving us with only the $SU(3)^3$ and $SU(2)^3$ three point functions. These represent additional contributions to the parameters A_2 and A_3. Their presence does not spoil the sum rules, which only rely on the fact that there are three A's and not that they only receive contributions at a given order in α.

2.3 Sensitivity to UV physics

2.3.1 General remarks

In the previous section, we restricted our analysis to renormalizable, UV-complete hidden sectors. However, it is often the case that our understanding of the hidden sector is incomplete, that we have only an effective description of it at low energies. In this section we would like to make some general comments about the dependence of the MSSM soft-breaking terms on unknown UV physics. This will have immediate applications in the next section, when we wish to use incomplete messenger-spurion models of gauge mediation to cover the parameter space of GGM. With our understanding of the (in)sensitivity of gauge mediation to UV physics, we will be sure that the models we study in the next section are indeed calculating correctly the MSSM soft masses.

We will begin with a more abstract discussion of UV sensitivity in a theory with spontaneously broken SUSY. Then in the next subsection we will give an example to illustrate some of our general comments. The reader may find it useful to reread the general discussion after having gone through the example calculation in the next subsection.

Consider a hidden sector consisting of an effective field theory valid below a UV cutoff scale Λ (which could be e.g. the Planck scale, or some UV scale), with SUSY spontaneously broken at a scale $\sqrt{F}$. As long as $\sqrt{F} \ll \Lambda$, all the soft terms are calculable in terms of the effective theory. The reason is that at energies much larger than $\sqrt{F}$ supersymmetry is restored and all the supersymmetry breaking contributions arise at energies of order $\sqrt{F}$ or smaller.

Now suppose the hidden sector is a messenger model of gauge mediation. Such models are weakly coupled truncations of a more complete theory valid above the scale Λ. They are fully specified by the set of messenger quantum numbers and the set of messenger masses given in (2.1.2), (2.1.3), (2.1.4). In this scheme, the soft parameters are calculable in terms of the messenger mass matrices. Let us denote the scale of the messenger sector by M. Clearly, when we study these models, we are implicitly taking the limit $\Lambda \to \infty$ with M fixed.

Typically one considers the messenger scale M and the SUSY-breaking scale $\sqrt{F}$ to be of the same order. In this case there is no problem and the soft terms are indeed unambiguously calculable, insensitive to the physics above the UV cutoff Λ. However, it is often the case that the messengers at the scale M receive supersymmetry breaking mass splittings which are much smaller than $\frac{F}{M}$. Then, we might want to reconsider the $\Lambda \to \infty$ limit in such a way that the messenger mass splittings are kept finite.

For example, imagine that these mass splittings are or order $\frac{F}{\Lambda}$. Then, the proper decoupling limit is $\Lambda, \sqrt{F} \to \infty$ with fixed $\frac{F}{\Lambda}$ and M. In this case the soft-breaking terms may not be calculable. A simple way to see that is to add to the theory additional messengers with mass of order Λ and supersymmetry breaking mass splittings of order $\frac{F}{\Lambda}$. These messengers contribute to gaugino masses and sfermion mass-squareds additional terms of order $\frac{F}{\Lambda}$ and $(\frac{F}{\Lambda})^2$ respectively. We can view these additional contributions as *finite local* *counterterms* for gaugino masses and sfermion masses which are determined by the details of the high energy theory.

From the point of view of the effective theory, such counterterms are ambiguous, controlled by the choice of UV completion above the scale Λ. It is important to note, however, that any such ambiguity must necessarily arise *only at leading order* in the SUSY breaking parameter F, since higher-order contributions from the UV states are necessarily suppressed by additional powers of $\frac{F}{\Lambda^2}$ (which goes to zero as $\Lambda \to \infty$).

The sensitivity to the UV is particularly dramatic when the supertrace of the messenger spectrum is nonzero [47, 48]. In this case the necessary counterterms include a logarithmically divergent sfermion mass. (See Appendix B for an explicit proof of this fact.) We stress that this divergence is a symptom of the problem, but the problem might arise even if the supertrace vanishes.

We conclude by roughly summarizing the foregoing discussion: if the messenger splittings are parametrically smaller than F/M, the soft-breaking terms in the MSSM are not calculable without further UV input.

2.3.2 Example

Let us now illustrate these general points with a simple example. To that end, consider the messenger theory with superpotential

$$W_{\text{eff}} = M\phi_1\widetilde{\phi}_1 \tag{2.3.1}$$

and Kähler potential

$$K_{\text{eff}} = |X|^2 + |\widetilde{\phi}_1|^2 + \Big(1 + \Big|\frac{X}{\Lambda}\Big|^2 + ...\Big)|\phi_1|^2 \tag{2.3.2}$$

where the ellipsis contains higher dimensional operators and X is a SUSY breaking field with

$$\langle X\rangle = M' + \theta^2 F \tag{2.3.3}$$

It will be convenient to introduce the following notation:

$$x \equiv \frac{M'}{\Lambda}, \qquad y = \frac{F}{M\Lambda} \tag{2.3.4}$$

As described above, we consider the limit $\Lambda \to \infty$ with x and y and the low energy mass parameter M held fixed.

By the general arguments above, we expect that the soft parameters computed in this effective theory are sensitive to large corrections from states at the scale Λ where the description of the physics given by (2.3.1) and (2.3.2) breaks down. Moreover, we expect that such corrections only enter in at leading order in the SUSY-breaking parameter F. We will now explicitly show that this is indeed the case.

Using our messenger GGM formalism developed in Appendix B, or equivalently in this case using the explicit formulas from [47], we find the low energy soft parameters to be

$$B_{\text{eff}} = \frac{Mx}{48\pi^2(1+x^2)^2}\Big(6(1+x^2)y + (2+x^2)y^3\Big) + \mathcal{O}(y^5) \tag{2.3.5}$$

and

$$A_{\text{eff}} = \frac{M^2}{64\pi^4(1+x^2)^2}\left(\left(\log\left(\frac{\Lambda^2_{\text{cutoff}}}{M^2}\right) - 2 + x^2 + 2\log(1+x^2)\right)y^2 + \frac{x^2(6+x^2)}{36(1+x^2)}y^4\right) + \mathcal{O}(y^6). \tag{2.3.6}$$

Note that while B_{eff} is finite, A_{eff} is logarithmically divergent with the UV cutoff Λ_{cutoff}. The appearance of this divergence which multiplies the supertrace in the low energy effective theory

$$\text{STr}\mathcal{M}^2_{\text{IR}} = -\frac{2M^2y^2}{(1+x^2)^2} \tag{2.3.7}$$

reminds us that our theory must be UV completed. Note, however, that even though the gaugino mass parameter is finite, it too will be sensitive to the UV physics as we will see below.

We can regulate the divergence in (2.3.6) by embedding the IR theory in a renormalizable UV theory with the following superpotential

$$W = X\phi_1\widetilde{\phi}_2 + M\phi_1\widetilde{\phi}_1 + \Lambda\phi_2\widetilde{\phi}_2 \tag{2.3.8}$$

and a canonical Kähler potential.[6] Integrating out the heavy fields (with mass Λ) ϕ_2, $\widetilde{\phi}_2$,

[6] Some authors (see e.g.[47]) regularize the theory using dimensional reduction with "ϵ-scalars." We prefer to replace the unphysical ϵ-scalars with physical heavy fields as in (2.3.8).

we readily derive the effective low energy Lagrangian (2.3.1), (2.3.2).[7]

The contribution of the messengers in our full theory (2.3.8) to the soft SUSY breaking masses in the MSSM is manifestly finite. Let's compare it to the calculation in the low energy theory (2.3.5), (2.3.6).

Again, using our messenger GGM formulas we find the following soft parameters

$$B_{\text{full}} = \frac{Mx}{48\pi^2(1+x^2)^2}(2+x^2)y^3 + \mathcal{O}(y^5) \tag{2.3.9}$$

and

$$A_{\text{full}} = \frac{M^2}{64\pi^4(1+x^2)^2}\left(\left(\log\left(\frac{\Lambda^2}{M^2}\right) + 2x^2 + 2\log(1+x^2)\right)y^2 + \frac{x^2(6+x^2)}{36(1+x^2)}y^4\right) + \mathcal{O}(y^6) \tag{2.3.10}$$

We see that B_{eff} and B_{full} differ *only* at leading order in y, with the counterterm given by[8]

$$\delta B = \frac{M}{8\pi^2}\Big(\frac{x}{1+x^2}\Big)y \tag{2.3.11}$$

For the particular UV definition we have chosen, we can understand this term as arising from the rescaling anomaly in the recanonicalization of the IR Kähler potential. Notice, however, that if we had added messengers to the UV theory that did not couple to the light messengers, they would have also contributed at order y to the counterterm in (2.3.11). These contributions cannot be captured by the rescaling anomaly.

Similarly, the difference between A_{full} and A_{eff} is also only at leading order in the SUSY breaking. However, here it includes an infinite counterterm:

$$\delta A = \frac{M^2}{64\pi^4(1+x^2)^2}(\log(\Lambda^2/\Lambda^2_{\text{cutoff}}) + x^2 + \log(1+x^2))y^2 \ . \tag{2.3.12}$$

Again, adding messengers in the UV decoupled from the IR has the effect of generating additional corrections at leading order in the SUSY breaking.

---

[7] In this regularization, we see that the negative sign of the supertrace in (2.3.7) corresponds precisely to what we expect from the general results on integrating out massive chiral matter in Appendix A.

[8] One can check that the full expressions for both B and A in the effective and the full theories agree at *all* higher orders in y and not just at the next-to-leading order we have written down in our expressions above.

With a sharp set of criteria for defining calculable gauge mediation models in hand, we will now explore the covering of the GGM parameter space in the next section. In particular, when using messenger models we will specialize to the case of vanishing supertrace and $\frac{F}{\Lambda} \to 0$.

2.4 Covering the General Gauge Mediation Parameter Space

2.4.1 The general setup

In this section we will demonstrate, using a general model with messengers, that the entire parameter space of GGM can be covered by a calculable weakly coupled field theory.

Consider a theory with N chiral messengers Φ^i, $\widetilde{\Phi}^i$, $i = 1, \ldots, N$ transforming in some vector-like representation $\mathbf{R} \oplus \overline{\mathbf{R}}$ of a gauge group G (which will later be identified with the SM gauge group). The messenger spectrum determines the GGM soft masses, so we will focus on that. The most general messenger spectrum is of the form

$$V_{\text{mass terms}} = (\widetilde{\psi}^T \mathcal{M}_F \psi + c.c.) + \begin{pmatrix} \phi \\ \widetilde{\phi}^* \end{pmatrix}^\dagger \mathcal{M}_B^2 \begin{pmatrix} \phi \\ \widetilde{\phi}^* \end{pmatrix} \tag{2.4.1}$$

with

$$\mathcal{M}_B^2 \equiv \begin{pmatrix} \mathcal{M}_F^\dagger \mathcal{M}_F + \xi & F \\ F^\dagger & \mathcal{M}_F \mathcal{M}_F^\dagger + \widetilde{\xi} \end{pmatrix} \tag{2.4.2}$$

Here $\mathcal{M}_F$, ξ, $\widetilde{\xi}$ and F are all $N \times N$ matrices. We take $\mathcal{M}_F$ to be diagonal with real, positive entries without loss of generality. ξ and $\widetilde{\xi}$ are Hermitian; and F is complex. The off-diagonal parameters F can arise from "F-term breaking" e.g. from a superpotential coupling to spurion field. The diagonal parameters ξ can arise from "D-term breaking" e.g. from FI-U(1) terms. More generally, the general spectrum shown in (2.4.1) can arise from complicated non-Abelian dynamics such as in [43].

We will impose the following restrictions on the messenger spectrum, motivated by phenomenology and overall consistency:

- In order to avoid the SUSY CP problem, we require all the mass parameters to be real

$$\xi = \xi^*, \qquad \widetilde{\xi} = \widetilde{\xi}^*, \qquad F = F^* \ . \tag{2.4.3}$$

- In order to guarantee that no dangerous FI-term for hypercharge is generated, we impose invariance under messenger parity [1,2][9]

$$\Phi^i \leftrightarrow \widetilde{\Phi}^i. \tag{2.4.4}$$

  This restricts the parameters to satisfy

$$\xi = \widetilde{\xi}\,, \qquad F = F^T \ . \tag{2.4.5}$$

- Since we want our theory to be calculable and insensitive to UV physics, we require vanishing messenger mass-squared supertrace. This translates to

$$\mathrm{Tr}\,\xi = 0 \tag{2.4.6}$$

- In the case where $G = SU(3) \times SU(2) \times U(1)$, we want the gauge couplings to unify. This restricts the messengers to be in complete $SU(5)$ representations. Furthermore, we limit the number of representations such that the theory remains perturbative.

- The messengers must be non-tachyonic for consistency of the model. So this puts upper limits on the magnitudes of the entries in ξ and F.

Finally, we note that if the messengers are in a reducible representation

$$\mathbf{R} = \bigoplus_R (n_R \times R) \tag{2.4.7}$$

then the messenger mass matrices must be block-diagonal. Each block couples the messengers with the same R. Consequently, all of the statements above hold for each R separately, and the leading-order in α contributions from each R to the soft masses are additive.

[9] Actually, the authors of [2] considered another action for this symmetry which maps chiral superfields to anti-chiral superfields. Such a symmetry does not commute with the Lorentz symmetry. However, if we also impose CP symmetry, our choice is equivalent to theirs.

2.4.2 Covering the GGM parameter space of a toy $U(1)$ visible sector

In this subsection we will consider a simplified theory with only $G = U(1)$ symmetry and messengers with charges ± 1. This example is instructive because the detailed representation theory of the messengers does not play an important role in this case. It will also be useful in the next subsection when we consider the full $G = SU(3) \times SU(2) \times U(1)$ case.

Here there is only one A parameter and only one B parameter and covering the parameter space means finding a theory that covers the range

$$\kappa = \frac{A}{|B|^2} \in (0, \infty). \tag{2.4.8}$$

Notice that $\kappa \to 0$ corresponds to the limit of either a very massive gaugino or vanishing sfermion mass, while $\kappa \to \infty$ corresponds to either a very massive scalar or vanishing gaugino mass.

Let us first ask if we can cover (2.4.8) with a single messenger pair and, at the same time, obey the microscopic constraints on our messenger sector described in the previous subsection. To answer this question, note that the most general single messenger model allowed by messenger parity and vanishing supertrace is of the form

$$\mathcal{M}_F = M, \qquad \mathcal{M}_B^2 = \begin{pmatrix} M^2 & F \\ F & M^2 \end{pmatrix}. \tag{2.4.9}$$

i.e. only minimal gauge mediation is allowed. This model has two parameters, M and F, and spans a two-dimensional subspace of the full A and B parameter space. However, an explicit calculation shows [56] that this subspace is not the full GGM parameter space and that in fact

$$\kappa \in (.37, 1) \tag{2.4.10}$$

where the upper bound for κ is obtained in the limit of small SUSY breaking and the lower bound arises because the messengers cannot be tachyonic.

Next, we try a system with two messengers. Since we are only interested in giving an existence proof of (2.4.8), we will not consider the most general possible two-messenger mass matrix satisfying the conditions above. Instead, we consider the following special mass matrix

$$\mathcal{M}_F = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \tag{2.4.11}$$

and

$$\mathcal{M}_B^2 = \begin{pmatrix} M_1^2 + D & 0 & F_1 & 0 \\ 0 & M_2^2 - D & 0 & F_2 \\ F_1 & 0 & M_1^2 + D & 0 \\ 0 & F_2 & 0 & M_2^2 - D \end{pmatrix} . \tag{2.4.12}$$

This model could arise, e.g. from a simple MGM-like setup with the messengers charged under an additional $U(1)'$ gauge group with a nonzero FI D-term.

With the added assumption

$$F_1, F_2, D \ll M_{1,2}^2 \ . \tag{2.4.13}$$

we can use the techniques of wavefunction renormalization [57, 29] to compute the A and B parameters

$$B = \frac{1}{8\pi^2}\Big(\frac{F_1}{M_1} + \frac{F_2}{M_2}\Big) + \mathcal{O}(F^3, DF) \tag{2.4.14}$$

and

$$\begin{aligned} A &= A_F + A_\xi \\ A_F &= \frac{1}{64\pi^4}\Big(\frac{F_1^2}{M_1^2} + \frac{F_2^2}{M_2^2}\Big) + \mathcal{O}(F^4, DF^2) \\ A_\xi &= \frac{D}{32\pi^4}\log(M_1^2/M_2^2) + \mathcal{O}(DF^2). \end{aligned} \tag{2.4.15}$$

From these expressions, it is straightforward to see that this example in fact covers the range

$$\kappa \in (-\infty, \infty). \tag{2.4.16}$$

First, for $D = 0$ we can set $\frac{F_1}{M_1} \approx -\frac{F_2}{M_2}$ such that B is very small while A is finite. This leads to arbitrarily large $|\kappa|$. However, setting $D = 0$ prevents us from making $|\kappa|$ arbitrarily small. For that, we use nonzero D to set

$$A_\xi < 0 \tag{2.4.17}$$

such that $A = A_F + A_\xi$ is arbitrarily small with fixed B.

We conclude that this example covers the full parameter space of GGM for a $U(1)$ visible sector.

2.4.3 Covering the MSSM GGM parameter space

Let us now generalize the discussion of the previous section to the physically relevant case of $G = SU(3) \times SU(2) \times U(1)$. We will see that, when properly analyzed, this case reduces to the $U(1)$ case considered in the previous subsection.

We would like to find weakly-coupled messenger theories that cover the full GGM parameter space of the MSSM, namely the six parameters A_k, $B_k \in \mathbb{R}^+$, where $k = 1, 2, 3$ labels $U(1)$, $SU(2)$ and $SU(3)$, respectively. A first analysis of this subject was presented by Carpenter, Dine, Festuccia and Mason in [45]. We will extend their analysis, by demanding not only the right number of parameters, but that the entire parameter space can be covered.

As noted above around equation (2.4.7), the messenger mass matrices are block diagonal with respect to different irreps R, and the contribution from messengers of different irreps are additive. It follows then that

$$A_k = \sum_R N_{k,R} A_R \ , \qquad B_k = \sum_R N_{k,R} B_R \tag{2.4.18}$$

where the sum is over the different messenger irreps, and $N_{k,R}$ are the total Dynkin indices of the irrep R with respect to the gauge group k. Notice how the dependence on the gauge group is trivial and factors out completely. The functions A_R and B_R are universal in the sense that they depend only on the mass parameters of the messengers with representation R. In fact, they are identical to what one would compute for n_R $U(1)$ messengers with charges ± 1.

Since we are interested in models that are compatible with unification, we should consider messengers in complete representations of $SU(5)$. The smallest $SU(5)$ representations **5** and **10** can be decomposed under the usual matter representations of the MSSM as

$$\overline{\mathbf{5}} = D \oplus L \ , \qquad \mathbf{10} = Q \oplus U \oplus E. \tag{2.4.19}$$

So we will restrict our attention to $R = Q, U, D, L, E$. Just for reference, the Dynkin indices for these representations are

$$
\begin{aligned}
&N_{1,Q} = \frac{1}{10},\ N_{1,U} = \frac{4}{5},\ N_{1,E} = \frac{3}{5},\ N_{1,D} = \frac{1}{5},\ N_{1,L} = \frac{3}{10} \\
&N_{2,Q} = \frac{3}{2},\ N_{2,L} = \frac{1}{2} \\
&N_{3,Q} = 1,\ N_{3,U} = \frac{1}{2},\ N_{3,D} = \frac{1}{2}
\end{aligned}
\tag{2.4.20}
$$

where in the first line we have used the standard GUT normalization for the $U(1)_Y$ charge.

The expressions (2.4.18) immediately lead to a necessary condition on the messenger content, in order for the model to cover the full parameter space: we need messengers transforming in at least three different irreps. Otherwise, we do not have three linearly independent functions A_R and three linearly independent functions B_R.

This means that any number of messengers in $\mathbf{5} \oplus \overline{\mathbf{5}}$ cannot cover the parameter space (they have only two values of $R = D, L$). Next we can attempt to use messengers in a single copy of $\mathbf{10} \oplus \overline{\mathbf{10}}$. Here we have three values of $R = Q, U, E$ and therefore three linearly independent constants. However, the result (2.4.10) in the $U(1)$ toy example discussion shows that these constants are bounded, $.37 < \kappa_R \equiv \frac{A_R}{|B_R|^2} < 1$. In particular, we cannot make the gauginos arbitrarily heavy compared to the scalars.

As in the $U(1)$ example, we can avoid this difficulty by having at least two copies of the representations and then using D-type supersymmetry breaking. We are therefore led to the following simplest possible models

$$
2 \times (\mathbf{10} \oplus \overline{\mathbf{10}}) \qquad \text{or} \qquad 2 \times (\mathbf{5} \oplus \overline{\mathbf{5}}) \oplus \mathbf{10} \oplus \overline{\mathbf{10}}\ . \tag{2.4.21}
$$

The latter is more "minimal" since it has slightly smaller total Dynkin index (and thus contributes slightly less to the MSSM gauge coupling beta functions). However, the former is easier to analyze, since we can now build a theory that is three copies of the two-messenger models discussed in the previous section, one for each irrep in the $\mathbf{10}$. The small SUSY breaking result (2.4.16) is then enough to show that we can in fact cover the parameter range. This is true even if we take universal fermion mass for each $\mathbf{10} \oplus \overline{\mathbf{10}}$ factor, so we can cover the parameter space without introducing supersymmetric GUT-breaking splittings in the messenger sector. This shows that covering the parameter space is compatible with unification, up to possible threshold corrections coming from the SUSY-splittings.

The analysis of a theory with messenger content $2\times(\mathbf{5}\oplus\overline{\mathbf{5}})\oplus\mathbf{10}\oplus\overline{\mathbf{10}}$ is slightly different since the $\mathbf{10}\oplus\overline{\mathbf{10}}$ representations must have pure F-type breaking. In particular, the Q, U, and E type messengers must satisfy (2.4.10) and so

$$0.37 < \kappa_R < 1 \quad \text{for } R = Q, U, E \tag{2.4.22}$$

Substituting (2.4.22) into (2.4.18), we find six equations for seven non-compact variables ($A(D)$, $A(L)$, $B(D)$, $B(L)$, $B(Q)$, $B(U)$, and $B(E)$) and three compact variables ($\kappa_{Q,U,E}$). However, it is not completely obvious that a real solution exists, because the substitution is quadratic in $B(Q)$, $B(U)$ and $B(E)$. One can check that this is always possible if we take $\kappa_Q > \kappa_E, \kappa_U$. Note that this takes us outside the small SUSY-breaking limit (where $\kappa = 1$) for the E and the U messengers.

These results show that there cannot be any additional field theoretic restrictions on the GGM parameter space. Another consequence of this result is the following. Assume that all the soft terms are measured someday, and our two sum rules (2.1.1) are satisfied. Then, we can derive the six numbers A_k, B_k and try to match them with a more microscopic theory. Our result here shows that whatever these numbers are, we'll be able to obtain them from weakly coupled messengers. In fact, we'll be able to do it in more than one way. This implies that the gaugino and sfermion masses alone will not be enough to distinguish between different gauge mediation scenarios. More input, such as the messenger scale or the SUSY-breaking scale (equivalently, the gravitino mass), will be needed in order to break this degeneracy.

2.5 General results on the effective supertrace

In this section we analyze the effect of integrating out massive modes at tree-level in a renormalizable theory. In particular, we will be interested in the supertrace over the spectrum of the low-energy effective theory. We will assume that the low-energy theory is described by a non-linear sigma model without gauge interactions. Then the supertrace over the light modes is given by the following general formula [30, 49]:

$$\mathrm{STr}\mathcal{M}^2 = 2R_{c\bar{k}}g^{\bar{k}a}g^{\bar{b}c}W_aW^*_{\bar{b}} \tag{2.5.1}$$

where the indices run over the chiral superfields Φ^a comprising the low-energy effective theory; $g^{\bar{a}b}$ is the inverse Kähler metric; $R_{a\bar{b}}$ is the Ricci tensor associated with the Kähler metric, and W is the effective superpotential.

We will show that integrating out massive chiral matter results in a negative semi-definite Ricci tensor, so $\mathrm{STr}\mathcal{M}^2 \leq 0$ in this case. We then show that integrating out massive vector fields results in an indefinite Ricci tensor and correspondingly a supertrace of indefinite sign.

2.5.1 Integrating out massive chiral matter

Consider the most general renormalizable theory of heavy chiral superfields H^A coupled to light chiral superfields ℓ^a. This must have the form (we take the Kähler potential to be canonical)

$$W = \frac{1}{2}\lambda_{Abc}H^A\ell^b\ell^c + \frac{1}{2}M_{AB}H^AH^B + \frac{1}{2}m_{ab}\ell^a\ell^b + ... \tag{2.5.2}$$

where the ellipsis contains unimportant marginal and higher dimensional couplings, and $m \ll M$. Integrating out the heavy fields yields the following equation of motion

$$H^A = -\frac{1}{2}(M^{-1})^{AB}\ell^T\lambda_B\ell + ... \tag{2.5.3}$$

Substituting this into (2.5.2) we obtain the effective superpotential

$$W_{\text{eff}} = \frac{1}{2}m_{ab}\ell^a\ell^b + \mathcal{O}(\ell^4) \tag{2.5.4}$$

We also find the following effective Kähler potential

$$K_{\text{eff}} = \ell^\dagger\ell + \frac{1}{4}\sum_A \left|(M^{-1})^{AB}\ell^T\lambda_B\ell\right|^2 ... \tag{2.5.5}$$

It follows that the Ricci tensor of the effective Kähler metric

$$R_{a\bar{b}} = -\partial_a(g^{\bar{c}d}g_{d\bar{b},\bar{c}}) \tag{2.5.6}$$

is at $\ell = 0$

$$R_{a\bar{b}} = -\delta^{\bar{c}d} g_{a\bar{b},\bar{c}d} = -\sum_A \left((M^{-1}\lambda)^A (M^{-1}\lambda)^{\dagger\overline{A}}\right)_{a\bar{b}} \tag{2.5.7}$$

This is a sum over negative semi-definite matrices, so it is also negative semi-definite. It then follows from (2.5.1) that the effective supertrace over the light fields is non-positive. One application of this result is to gauge mediation models of the type discussed in section 3, where the H^A fields are heavy messengers and the ℓ^a are light messengers and SUSY breaking fields.

2.5.2 Integrating out massive vector superfields

Next we consider what happens when one classically integrates out massive vector superfields. Here it turns out that the Ricci tensor of the effective Kähler metric is indefinite and therefore the supertrace over the light spectrum is also of indefinite sign.

The setup is as in [43]; we will review it here. Consider a gauge theory with matter chiral superfields Φ^a transforming under gauge group G (not necessarily simple), where $a = 1, \ldots, N$ denotes the collective set of gauge and flavor indices. Suppose that the Φ^a acquire supersymmetric vevs ϕ_0 which Higgs the entire gauge group. These vevs must lie along the D-flat moduli space $\mathcal{M}$ defined by the equations:

$$\phi_0^\dagger T^I \phi_0 = 0 \tag{2.5.8}$$

where T^I are the generators of G. Now consider the fluctuations around this point in moduli space:

$$\Phi = \phi_0 + \delta\Phi \tag{2.5.9}$$

We are interested in the effective Kähler potential for these fluctuations induced by integrating out the massive vector supermultiplets of G. In what follows we will work in the unitary gauge discussed in [43]

$$\phi_0^\dagger T^I \delta\Phi = 0 \tag{2.5.10}$$

which guarantees that the fluctuations lie within $\mathcal{M}$. It will be convenient to perform a unitary transformation so that $\delta\Phi^{a=1,\ldots,N-\mathrm{dim}G}$ satisfy (2.5.10) and the other elements of

$\delta\Phi$ are in the orthogonal subspace.

Now according to [43], the effective Kähler potential is given by

$$K_{\text{eff}} = \delta\Phi^\dagger\delta\Phi - \frac{1}{2}(\delta\Phi^\dagger T^I\delta\Phi)h^{-1}_{IJ}(\delta\Phi^\dagger T^J\delta\Phi) + \mathcal{O}(\delta\Phi^6) \tag{2.5.11}$$

where h^{IJ} is the matrix

$$h^{IJ} = \frac{1}{2}\Phi^\dagger\{T^I, T^J\}\Phi \tag{2.5.12}$$

(Note the analogy with the previous subsection: h^{-1}_{IJ} is analogous to $M^{-1\dagger}M^{-1}$ and $T^I_{\bar{b}a}$ is analogous to λ_{Abc}. The only difference is in the type of the indices, which dictates how they are contracted.) As in the previous subsection, we can compute the Ricci tensor at leading order in the fluctuations. However, we must be careful not to differentiate with respect to all the fluctuations $\delta\Phi^a$, but only those which satisfy the gauge condition (2.5.10). In our convenient basis, these are simply the $a = 1, \ldots, N - \dim G$ entries of $\delta\Phi^a$. So the metric is simply

$$g_{a\bar{b}} = \delta_{a\bar{b}} - (\delta\Phi^\dagger T^I)_a h^{-1}_{IJ}(T^J\delta\Phi)_{\bar{b}} - (T^I)_{\bar{b}a}h^{-1}_{IJ}(\delta\Phi^\dagger T^J\delta\Phi) + \mathcal{O}(\delta\Phi^4) \tag{2.5.13}$$

with $a, b = 1, \ldots, N - \dim G$. Therefore, the Ricci tensor at $\delta\Phi = 0$ is:

$$R_{a\bar{b}} = -\delta^{c\bar{d}}g_{a\bar{b},c\bar{d}} = (T^I)^c{}_a h^{-1}_{IJ}(T^J)_{\bar{b}c} + (T^I)_{\bar{b}a}h^{-1}_{IJ}\mathrm{Tr}'T^J \tag{2.5.14}$$

Here the sum is only over indices in the range $1, \ldots, N - \dim G$, and Tr' refers to the restricted trace over the subspace of fluctuations satisfying (2.5.10). Even though the full trace of T^J must vanish due to the anomaly condition, the restricted trace need not vanish since the gauge symmetry is spontaneously broken. This is important, because while the first term in (2.5.14) enjoys definiteness properties, the second term obviously does not. Thus there is no reason to expect the Ricci tensor to have any definiteness property. Indeed, it is straightforward to construct simple examples where $R_{a\bar{b}}$ has both positive and negative eigenvalues.[10] Therefore we conclude in this case that the effective supertrace can have either sign.

[10] For instance, consider a $U(1)$ gauge theory with fields $\Phi^{1,2,3,4}$ having charges $q_1 = +1$, $q_2 = -1$, $q_3 = +q$ and $q_4 = -q$ with $q \neq \pm 1$. The D-flat moduli space is characterized by $\phi_0 = (\Phi^1, \Phi^2, \Phi^3, \Phi^4)$ with

2.6 General results on multiple messenger models

In this final section we write down the GGM correlation functions for a general messenger theory. We then explicitly show that a messenger sector with non-vanishing supertrace generates contributions to the scalar mass-squareds that are logarithmically divergent and proportional to the supertrace.

As in section 4, let us restrict ourselves to the case that the messengers are charged under a $U(1)$ gauge group with mass terms

$$V \supset \xi_{ij}\phi_i\phi_j^* + \widetilde{\xi}_{ij}\widetilde{\phi}_i\widetilde{\phi}_j^* + |M_i|^2(\phi_i\phi_i^* + \widetilde{\phi}_i\widetilde{\phi}_i^*) + f_{ij}\phi_i\widetilde{\phi}_j + f_{ij}^*\phi_i^*\widetilde{\phi}_j^* + M_i\psi_i\widetilde{\psi}_i + M_i^*\overline{\psi}_i\overline{\widetilde{\psi}}_i \quad (2.6.1)$$

and $i = 1, ..., N$. Again, taking the ϕ_i and $\widetilde{\phi}_i$ to have $U(1)$ charge $+1$ and -1 respectively, we find

$$\begin{aligned}
J(x) &= \phi_i^*\phi_i - \widetilde{\phi}_i^*\widetilde{\phi}_i \\
j_\alpha(x) &= -\sqrt{2}i(\phi_i^*\psi_{i\alpha} - \widetilde{\phi}_i^*\widetilde{\psi}_{i\alpha}) \\
\overline{j}_{\dot\alpha}(x) &= \sqrt{2}i(\phi_i\overline{\psi}_{i\dot\alpha} - \widetilde{\phi}_i\overline{\widetilde{\psi}}_{i\dot\alpha}) \\
j_\mu(x) &= i(\phi_i\partial_\mu\phi_i^* - \phi_i^*\partial_\mu\phi_i - \widetilde{\phi}_i\partial_\mu\widetilde{\phi}_i^* + \widetilde{\phi}_i^*\partial_\mu\widetilde{\phi}_i) + \psi_i\sigma_\mu\psi_i - \widetilde{\psi}_i\sigma_\mu\overline{\widetilde{\psi}}_i
\end{aligned} \quad (2.6.2)$$

where we have implicitly summed over i.

Let us now write the various current two-point functions. To perform the calculation, it will be convenient to change basis from the gauge eigenstates appearing in (2.6.2) to the mass eigenstates via the following expressions

$$\phi_i = R_{ia}\cdot\varphi_a, \qquad \widetilde{\phi}_i^* = R_{(i+N)a}\cdot\varphi_a \quad (2.6.3)$$

where $i = 1, ..., N$, $a = 1, ..., 2N$, and R is a $2N \times 2N$ unitary matrix. Let us also denote the bosonic (fermionic) mass eigenvalues by μ_a (M_i). Inserting (2.6.3) into (2.6.2), and

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Φ^i satisfying the equation

$$|\Phi^1|^2 - |\Phi^2|^2 + q(|\Phi^3|^2 - |\Phi^4|^2) = 0 \quad (2.5.15)$$

Going to a point on this moduli space, we can impose the gauge fixing condition (2.5.10) by solving for $\delta\Phi^4$. Substituting back into the Kähler potential (2.5.11) gives the effective Kähler potential for $\delta\Phi^{1,2,3}$. From this one can compute the Ricci tensor at $\delta\Phi = 0$ using $R_{a\bar{b}} = -\delta^{c\bar{d}}g_{a\bar{b},c\bar{d}}$. Then by varying ϕ_0 and q it is easy to find places where $R_{a\bar{b}}$ has both positive and negative eigenvalues.

performing the contractions to evaluate the correlators, we find

$$\begin{aligned}
C_0(p) &= \Big(\overline{R}_{ia}R_{ib} - \overline{R}_{(j+N)a}R_{(j+N)b}\Big)\Big(\overline{R}_{kb}R_{ka} - \overline{R}_{(l+N)b}R_{(l+N)a}\Big)I(p,\mu_a,\mu_b)\\
C_{1/2}(p) &= \frac{p^2+\mu_a^2-M_i^2}{p^2}\Big(R_{ia}\overline{R}_{ia} + R_{(i+N)a}\overline{R}_{(i+N)a}\Big)I(p,\mu_a,M_i)\\
C_1(p) &= \frac{1}{3p^2}\Big((p^2+4\mu_a^2)I(p,\mu_a,\mu_a) + (2p^2-8M_i^2)I(p,M_i,M_i) + 4J(\mu_a) - 8J(M_i)\Big)\\
B &= -4M_iR_{ia}\overline{R}_{(i+N)a}I(0,M_i,\mu_a)
\end{aligned} \tag{2.6.4}$$

where all indices are summed, and we define

$$\begin{aligned}
I(p,m_1,m_2) &= \int \frac{d^4q}{(2\pi)^4}\frac{1}{((p+q)^2+m_1^2)(q^2+m_2^2)}\\
&= \frac{1}{16\pi^2}\Big(\log\frac{\Lambda_q^2}{p^2}+1\Big) + \frac{1}{16\pi^2p^2}\Big(m_1^2\log\frac{m_1^2}{p^2} + m_2^2\log\frac{m_2^2}{p^2} - m_1^2 - m_2^2\Big)\\
&\quad + \mathcal{O}\Big(\frac{1}{p^4},\frac{\log p^2}{p^4}\Big)\\
J(m) &= \int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m^2} = \frac{\Lambda_q^2}{16\pi^2} + \frac{m^2}{16\pi^2}\log\frac{m^2}{\Lambda_q^2}
\end{aligned} \tag{2.6.5}$$

where Λ_q is a momentum cutoff for the q integral.

Let us now show that a non-vanishing messenger supertrace necessarily generates a logarithmically divergent scalar counterterm. Recall first the expression (2.2.10) for the A parameter

$$A \equiv -\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\Big(3C_1(p) - 4C_{1/2}(p) + C_0(p)\Big) \tag{2.6.6}$$

Using (2.6.5), (2.6.4) and focussing on the $\mathcal{O}(1/p^2)$ terms (one can check that the $\mathcal{O}(p^0,\log p)$ terms, and hence the dependence on Λ_q, always vanish in (2.6.6)), we find

$$\delta A = -\frac{1}{64\pi^4}\Big(\mathrm{Tr}\mu^2 - 2\mathrm{Tr}M^2\Big)\log\Lambda^2 = -\frac{1}{128\pi^4}\mathrm{Str}\,\mathcal{M}^2\cdot\log\Lambda^2 \tag{2.6.7}$$

where Λ is the cutoff of the p integral in (2.6.6).

In this example we took the gauge group to be $U(1)$ and took all the messengers to have charge ± 1. More generally, one obtains a charge-weighted supertrace, or to be precise

$$\delta A = -\frac{1}{128\pi^4}\sum_R \mathrm{Str}\,N_R\mathcal{M}_R^2\cdot\log\Lambda^2 \tag{2.6.8}$$

where the supertrace is taken over the subset of messengers transforming in irrep R and N_R is the Dynkin index of irrep R.

Chapter 3

Towards the SSM in String Theory

In this chapter, we move on to string theory embeddings of particle physics and consider a D3 brane probing a del Pezzo 8 (dP_8) singularity.[1] We study the gauge theory on the corresponding exceptional collection of fractional branes in the formal decoupling limit in which the Planck and string scales are taken to be large. We discuss the parameter space of the resulting gauge-invariant couplings.

Then, by studying the relationship between the local homology of the dP_8 singularity and the homology of an imagined UV compactification, we propose a topological condition on the compact space which ensures that all abelian vector bosons, except the one corresponding to hypercharge, become massive. In so doing, we end up with a collection of branes that realizes the symmetries and matter content of the MSSM with only a few extra Higgs doublets. We also comment on the possible non-perturbative generation of μ terms for the Higgs doublets in this setup.

We should note that our topological criteterion for generating massive $U(1)$s from UV compactifications is of general interest because theories of D-branes at singularities rather generically come with many surplus $U(1)$ gauge factors.

[1] This chapter is based on "D-branes at Singularities, Compactification, and Hypercharge," written in collaboration with D. Malyshev, D. R. Morrison, H. Verlinde, and M. Wijnholt [59].

3.1 Introduction

As we have hinted at in the introduction, D-branes near Calabi–Yau singularities provide open string realizations of an increasingly rich class of gauge theories [20, 60, 61]. Given the hierarchy between the Planck and TeV scale, it is natural to make use of this technology and pursue a bottom-up approach to string phenomenology, that aims to find Standard Model-like theories on D-branes near CY singularities. In this setting, the D-brane world-volume theory can be isolated from the closed string physics in the bulk via a formal decoupling limit, in which the string and 4-d Planck scale are taken to infinity, or very large. The clear advantage of this bottom-up strategy is that it separates the task of finding local realizations of SM-like models from the more difficult challenge of finding fully consistent, realistic string compactifications.[2]

In scanning the space of CY singularities for candidates that lead to realistic gauge theories, one is aided by the fact that all gauge invariant couplings of the world-volume theory are controlled by the local geometry; in particular, symmetry breaking patterns can be enforced by appropriately dialing the volumes of compact cycles of the singularity. Several other properties of the gauge theory, however, such as the spectrum of light $U(1)$ vector bosons and the number of freely tunable couplings, depend on how the local singularity is embedded inside the full compact Calabi–Yau geometry.

In this chapter we work out some concrete aspects of this program. We begin with a brief review of the general set of ingredients that can be used to build semi-realistic gauge theories from branes at singularities. Typically these local constructions lead to models with extra $U(1)$ gauge symmetries beyond hypercharge. As our first new result, we identify the general topological conditions on the embedding of a CY singularity inside a compact CY threefold, that determines which $U(1)$-symmetry factors survive as massless gauge symmetries. The other $U(1)$ bosons acquire a mass of order of the string scale. The left-over global symmetries are broken by D-brane instantons.

[2] The general challenge of extending local brane constructions near CY singularities to full-fledged string compactifications represents a geometric component of the "swampland program" of [62], that aims to determine the full class of quantum field theories that admit consistent UV completions with gravity.

In the second half of the chapter, we apply this insight to the concrete construction of an SM-like theory given in [23], based on a single D3-brane near a suitably chosen del Pezzo 8 singularity. We specify a simple topological condition on the compact embedding of the dP_8 singularity, such that only hypercharge survives as the massless gauge symmetry. To state this condition, recall that the 2-homology of a dP_8 surface is spanned by the canonical class K and eight 2-cycles α_i with intersection form $\alpha_i \cdot \alpha_j = -A_{ij}$ with A_{ij} the Cartan matrix of E_8. With this notation, our geometric proposal is summarized as follows:

The world-volume gauge theory on a single D3-brane near a del Pezzo 8 singularity embedded in a compact Calabi-Yau threefold with the following geometrical properties:

(i) the two 2-cycles α_1 and α_2 are degenerate and form a curve of A_2 singularities

(iii) all 2-cycles except α_4 are non-trivial within the full Calabi-Yau three-fold

has, for a suitable choice of Kähler moduli, the gauge group and matter content of the SSM shown in Table 1, except for an extended Higgs sector (with 2 pairs per generation):

	Q_i	u_i^c	d_i^c	ℓ_i	e_i^c	ν_i^c	H_i^u	H_i^d
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$1/6$	$-2/3$	$1/3$	$-1/2$	1	0	$1/2$	$-1/2$

Table 3.1: The matter content of our D-brane model. i counts the 3 generations

More details of this proposal are given in section 4. In section 5, we present a concrete geometric recipe for obtaining a compact CY manifold with all the required properties.

3.2 General Strategy

We begin with a summary our general approach to string phenomenology. In subsection 2.1, we give a quick recap of some relevant properties of D-branes at singularities. The reader familiar with this technology may wish to skip to subsection 2.2.

3.2.1 D-branes at a CY singularity

D-branes near Calabi–Yau singularities typically split up into so-called fractional branes. Fractional branes can be thought of as particular bound state combinations of D-branes, that wrap cycles of the local geometry. In terms of the world-sheet CFT, they are in one-to-one correspondence with allowed conformally invariant open string boundary conditions. Alternatively, by extrapolating to a large volume perspective, fractional branes may be represented geometrically as particular well-chosen collections of sheaves, supported on corresponding submanifolds within the local Calabi–Yau singularity. For most of our discussion, however, we will not need this abstract mathematical description; the basic properties that we will use have relatively simple topological specifications.

There are many types of CY-singularities, and some are, in principle, good candidates for finding realistic D-brane gauge theories. For concreteness, however, we specialize to the subclass of singularities which are asymptotic to a complex cone over a del Pezzo surface X. D-brane theories on del Pezzo singularities have been studied in [23, 63, 64].

A del Pezzo surface is a manifold of complex dimension 2, with a positive first Chern class. Each del Pezzo surface other than $\mathbf{P}^1 \times \mathbf{P}^1$ can be represented as $\mathbf{P}^2$ blown up at $n \leq 8$ generic points; such a surface is denoted by dP_n and sometimes called "the n-th del Pezzo surface".[3] By placing an appropriate complex line bundle (the "anti-canonical bundle") over $X = dP_n$, one obtains a smooth non-compact Calabi–Yau threefold. If we then shrink the zero section of the line bundle to a point, we get a cone over X, which we will call the conical del Pezzo n singularity and denote by Y_0. (More general del Pezzo singularities are asymptotic to Y_0 near the singular point.) To specify the geometry of Y_0, let $ds^2_X = h_{a\bar{b}} dz^a dz^{\bar{b}}$ be a Kähler-Einstein metric over the base X with $R_{a\bar{b}} = 6h_{a\bar{b}}$ and first Chern class $\omega_{a\bar{b}} = 6iR_{a\bar{b}}$. Introduce the one-form $\eta = \frac{1}{3}d\psi + \sigma$ where σ is defined by $d\sigma = 2\omega$ and $0 < \psi < 2\pi$ is the angular coordinate for a circle bundle over the del Pezzo

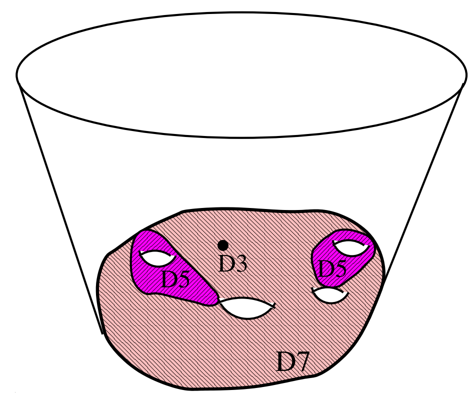
[3] This terminology is unfortunately at odds with the fact that, for $n \geq 5$, dP_n is not unique but actually has $2n - 8$ complex moduli represented by the location of the points.

surface. The Calabi–Yau metric can then be written as follows

$$ds_Y^2 = dr^2 + r^2\eta^2 + r^2 ds_X^2 \tag{3.2.1}$$

For the non-compact cone, the r-coordinate has infinite range. Alternatively, we can think of the del Pezzo singularity as a localized region within a compact CY manifold, with r being the local radial coordinate distance from the singularity. We will consider both cases.

The del Pezzo surface X forms a four-cycle within the full three-manifold Y, and itself supports several non-trivial two-cycles. Now, if we consider IIB string theory on a del Pezzo singularity, we should expect to find a basis of fractional branes that spans the complete homology of X: the del Pezzo 4-cycle itself may be wrapped by any number of D7-branes, any 2-cycle within X may be wrapped by one or more D5 branes, and the point-like D3-branes occupy the 0-cycle within X. The allowed fractional branes, however, typically do not correspond to single branes wrapped on some given cycle, but rather



to particular bound states F_s, each characterized by a charge vector of the form

$$\mathrm{ch}(\mathsf{F}_s) = (\, r_s,\, p_s^{\mathrm{A}},\, q_s\,) \tag{3.2.2}$$

Here $r_s = \mathrm{rk}(\mathsf{F}_s)$ is the rank of the fractional brane F_s, and is equal to the D7-brane wrapping number around X. The number $q_s = \mathrm{ch}_2(\mathsf{F}_s)$ is the 2nd Chern character of F_s and counts the D3-brane charge. Finally, the integers p_s^{A} are extracted from the first Chern class of F_s via

$$p_s^{\mathrm{A}} = \int_{\alpha_{\mathrm{A}}} c_1(\mathsf{F}_s) \tag{3.2.3}$$

where α_{A} denotes an integral basis of $H_2(X)$. Geometrically, p_s^{A} counts the number of times the D5-brane component of F_s wraps the 2-cycle α_{A}.

For a given geometric singularity, it is a non-trivial problem to find consistent bases of fractional branes that satisfy all geometric stability conditions. For del Pezzo singularities, a special class of consistent bases are known, in the form of so-called exceptional collections [76, 63, 64]. These satisfy special properties, that in particular ensures the absence of adjoint matter in the world-volume gauge theory, besides the gauge multiplet. The formula for the intersection product between two fractional branes F_i and F_j of an exceptional collection reads

$$\#(\mathsf{F}_i, \mathsf{F}_j) = \mathrm{rk}(\mathsf{F}_i)\deg(\mathsf{F}_j) - \mathrm{rk}(\mathsf{F}_j)\deg(\mathsf{F}_i) \equiv \chi_{ij} \tag{3.2.4}$$

Here the degree of F_i is given by $\deg(\mathsf{F}_i) = -c_1(\mathsf{F}_i)\cdot K$ with K the canonical class on X. It equals the intersection number between the D5 component of F_i and the del Pezzo surface. The intersection number χ_{ij} governs the number of massless states of open strings that stretch between the two fractional branes F_i and F_j.

The world-volume theory on D-branes near a CY singularity takes the form of a quiver gauge theory. For exceptional collections, the rules for drawing the quiver diagram are: [4] (i) draw a single node for every basis element F_i of the collection, (ii) connect every pair of nodes with $\chi_{ij} > 0$ by an oriented line with multiplicity χ_{ij}. Upon assigning a multiplicity n_i to each fractional brane F_i, one associates to the quiver diagram a quiver gauge theory. The gauge theory has a $U(|n_i|)$ gauge group factor for every node F_i, as well as χ_{ij} chiral multiplets in the bi-fundamental representation $(n_i, \overline{n}_j)$. The multiplicities n_i can be freely adjusted, provided the resulting world volume theory is a consistent $\mathcal{N} = 1$ gauge theory, free of any non-abelian gauge anomalies.

Absence of non-abelian gauge anomalies is ensured if at any given node, the total number of incoming and outgoing lines (each weighted by the rank of the gauge group at the other end of the line) are equal:

$$\sum_j \chi_{ij}\, n_j = 0\,. \tag{3.2.5}$$

This condition is automatically satisfied if the configuration of fractional branes constitute

[4] These rules can be generalized by including orientifold planes that intersect the CY singularity. We will elaborate on this possibility in the concluding section.

a single D3-brane, in which case the multiplicities n_i are such that $\sum_i n_i \,\mathrm{ch}(\mathsf{F}_i) = (\,0, 0, 1\,)$. In general, however, one could allow for more general configurations, for which the charge vectors add up to some non-trivial fractional brane charge.

For a given type of singularity, the choice of exceptional collection is not unique.[5] Different choices are related via simple basis transformations, known as mutations [76]. However, only a subset of all exceptional collections, that can be reached via mutations, lead to consistent world-volume gauge theories. The special mutations that act within the subset of physically relevant collections all take the form of Seiberg dualities [63, 64]. Which of the Seiberg dual descriptions is appropriate is determined by the value of the geometric moduli that determine the gauge theory couplings.

3.2.2 Symmetry breaking towards the SSM

To find string realizations of SM-like theories we now proceed in two steps. First we look for CY singularities and brane configurations, such that the quiver gauge theory is just rich enough to contain the SM gauge group and matter content. Then we look for a well-chosen symmetry breaking process that reduces the gauge group and matter content to that of the Standard Model, or at least realistically close to it. When the CY singularity is not isolated, the moduli space of vacua for the D-brane theory has several components [61], and the symmetry breaking we need is found on a component in which some of the fractional branes move off of the primary singular point along a curve of singularities (and other branes are replaced by appropriate bound states). This geometric insight into the symmetry breaking allows us to identify an appropriate CY singularity, such that the corresponding D-brane theory looks like the SSM.

The above procedure was used in [23] to construct a semi-realistic theory from a single D3-brane on a partially resolved del Pezzo 8 singularity (see also section 4). The final model of [23], however, still has several extra $U(1)$ factors besides the hypercharge symmetry. Such

[5] Each collection corresponds to a particular set of stability conditions on branes, and determines a region in Kähler moduli space where it is valid.

extra $U(1)$'s are characteristic of D-brane constructions: typically, one obtains one such factor for every fractional brane. As will be explained in what follows, whether or not these extra $U(1)$'s actually survive as massless gauge symmetries depends on the topology of how the singularity is embedded inside of a compact CY geometry.

In a string compactification, $U(1)$ gauge bosons may acquire a non-zero mass via coupling to closed string RR-form fields. We will describe this mechanism in some detail in the next section, where we will show that the $U(1)$ bosons that remain massless are in one-to-one correspondence with 2-cycles, that are *non-trivial* within the local CY singularity but are *trivial* within the full CY threefold. This insight in principle makes it possible to ensure – via the topology of the CY compactification – that, among all $U(1)$ factors of the D-brane gauge theory, only the hypercharge survives as a massless gauge symmetry.

The interrelation between the 2-cohomology of the del Pezzo base of the singularity, and the full CY threefold has other relevant consequences. Locally, all gauge invariant couplings of the D-brane theory can be varied via corresponding deformations of the local geometry. This local tunability is one of the central motivations for the bottom-up approach to string phenomenology. The embedding into a full string compactification, however, typically introduces a topological obstruction against varying all local couplings: only those couplings that descend from moduli of the full CY survive. Their value will need to be fixed via a dynamical moduli stabilisation mechanism.

3.2.3 Summary

Let us summarize our general strategy in terms of a systematized set of steps:

(i) Choose a non-compact CY singularity, Y_0, and find a suitable basis of fractional branes F_i on it. Assign multiplicities n_i to each F_i and enumerate the resulting quiver gauge theories.

(ii) Look for quiver theories that, after symmetry breaking, produce an SM-like theory. Use the geometric dictionary to identify the corresponding (non-isolated) CY singularity.

(iii) Identify the topological condition that isolates hypercharge as the only massless $U(1)$.

Look for a compact CY threefold, with the right topological properties, that contains Y_0.

In principle, it should be possible to automatize all three of these steps and thus set up a computer-aided search of SM-like gauge theories based on D-branes at CY singularities.

3.3 $U(1)$ Masses via RR-couplings

The quiver theory of a D-brane near a CY singularity typically contains several $U(1)$-factors, one for each fractional brane. Some of these $U(1)$ vector bosons remain massless, all others either acquire a Stückelberg mass via the coupling to the RR-form fields or get a mass through the Higgs mechanism [20, 74, 75, 69]. We will now discuss the Stückelberg mechanism in some detail.

3.3.1 The $U(1)$ hypermultiplet

To set notation, we first consider the $U(1)$ gauge sector on a single fractional brane. Let us introduce the two complex variables

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}\,, \qquad\qquad S = \rho + i\zeta\,. \tag{3.3.6}$$

Here τ is the usual $SL(2,\mathbf{Z})$ covariant complex coupling, that governs the kinetic terms of the $U(1)$ gauge boson via (omitting fermionic terms)

$$\mathrm{Im}\int d^2\theta\,\frac{\tau}{8\pi}\,W_\alpha W^\alpha = -\frac{1}{4g^2}\,F\wedge *F + \frac{\theta}{32\pi^2}\,F\wedge F \tag{3.3.7}$$

The field S in (3.3.6) combines a Stückelberg field ρ and a Faillet-Iliopoulos parameter ζ. After promoting S to a chiral superfield, we can write a supersymmetric gauge invariant mass term for the gauge field via [20]

$$\int d^4\theta\,\frac{1}{4}(\mathrm{Im}(S-\overline{S}-2V))^2 = \frac{1}{2}(A-d\rho)\wedge *(A-d\rho)\; - \zeta D. \tag{3.3.8}$$

Here D denotes the auxiliary field of the vector multiplet V. Together with the mass term, we observe a Faillet-Iliopoulos term proportional to ζ.

The complex parametrization (3.3.6) of the D-brane couplings naturally follows from its embedding in type IIB string theory. Without D-branes, IIB supergravity on a Calabi–Yau threefold preserves $\mathcal{N}=2$ supersymmetry. Closed string fields thus organize in $\mathcal{N}=2$ multiplets [66, 67]. The four real variables in (3.3.6) all fit together as the scalar components of a single hypermultiplet that appears after dualizing two components of the so called double tensor multiplet [68]. Since adding a D-brane breaks half the supersymmetry, the hypermultiplet splits into two complex $\mathcal{N}=1$ superfields with scalar components τ and S. The hypermultiplet of a single D3-brane derives directly from the 10-d fields, via

$$\begin{aligned} \tau &= C_0 + ie^{-\phi}, \\ dS &= *d(C_2 + \tau B_2) \end{aligned} \tag{3.3.9}$$

A dP_n singularity Y_0 supports a total of $n+3$ independent fractional branes, and a typical D-brane theory on Y_0 thus contains $n+3$ separate $U(1)$ gauge factors. In our geometric dictionary, we need to account for a corresponding number of closed string hypermultiplets.

In spite their common descent from the hypermultiplet, from the world volume perspective τ and S appear to stand on somewhat different footing: τ can be chosen as a non-dynamical coupling, whereas S must enter as a dynamical field. In a decoupling limit, one would expect that all closed string dynamics strictly separates from the open string dynamics on the brane, and thus that all closed string fields freeze into fixed, non-dynamical couplings. This decoupling can indeed be arranged, provided the $U(1)$ symmetry is non-anomalous and one starts from a D-brane on a *non-compact* CY singularity. In this setting, τ becomes a fixed constant as expected, while S completely decouples, simply because the $U(1)$ gauge boson stays massless.

3.3.2 Some notation

As before, let Y_0 be a non-compact CY singularity given by a complex cone over a base X. A complete basis of IIB fractional branes on Y_0 spans the space of compact, even-dimensional homology cycles within Y_0, which coincides with the even-dimensional homology of X. The

2-homology of the n-th del Pezzo surface dP_n is generated by the canonical class $\alpha_0 = k$, plus n orthogonal 2-cycles α_i. Using the intersection pairing within the threefold Y_0, we introduce the dual 4-cycles β^{B} satisfying

$$\alpha_{\mathrm{A}} \cdot \beta^{\mathrm{B}} = \delta_{\mathrm{A}}^{\mathrm{B}} \qquad \mathrm{A}, \mathrm{B} = 0, \ldots, n \tag{3.3.10}$$

The cycle β^0, dual to the canonical class α_0, describes the class of the del Pezzo surface X itself, and forms the only compact 4-cycle within Y_0. The remaining β's are all non-compact and extend in the radial direction of the cone. The degree zero two-cycles α_i, that satsify $\alpha_0 \cdot \alpha_i = 0$, have the intersection form

$$\alpha_i \cdot \alpha_j = -A_{ij} \tag{3.3.11}$$

where A_{ij} equals minus the Cartan matrix of E_n. The canonical class has self-intersection $9 - n$. In the following we will use the intersection matrix

$$\eta_{\mathrm{AB}} = \begin{pmatrix} 9-n & 0 \\ 0 & -A_{ij} \end{pmatrix} \tag{3.3.12}$$

and its inverse η^{AB} to raise and lower A-indices.

3.3.3 Brane action

The 10-d IIB low energy field theory contains the following bosonic fields: the dilaton ϕ, the NS 2-form B, the Kähler 2-form J and the RR p-form potentials C_p, with p even from 0 to 8. (Note that the latter are an overcomplete set, since $dC_p = *_{10} dC_{8-p}$.) From each of these fields, we can extract a 4-d scalar fields via integration over a corresponding compact cycles within Y_0. These scalar fields parametrize the gauge invariant couplings of the D-brane theory. Near a CY singularity, however, α' corrections may be substantial, and this gauge theory/geometric dictionary is only partially under control. We will not attempt to solve this hard problem and will instead adopt a large volume perspective, in which the local curvature is assumed to be small compared to the string scale. All expressions below are extracted from the leading order DBI action. Moreover, we drop all curvature contributions, as they do not affect the main conclusions. To keep the formulas transparant, we omit factors of order 1 and work in $\ell_s = 1$ units. For a more precise treatment, we refer to [69].

The D-brane world-volume theory lives on a collection of fractional branes F_s, with properties as summarized in section 2. Since the fractional branes all carry a non-zero D7 charge r_s, we can think of them as D7-branes, wrapping the base X of the CY singularity r_s times. We can thus identify the closed string couplings of F_s via its world volume action, given by the sum of a Born-Infeld and Chern-Simons term via

$$\mathcal{S} = \int e^{-i_s^* \phi} \sqrt{\det(i_s^*\,(G+B) - F_s)} + \int \sum_p i_s^* C_p \; e^{F_s - i_s^* B} \,. \tag{3.3.13}$$

Here i_s^* denotes the pull-back of the various fields to the world-volume of F_s; it in particular encodes the information of the D7 brane wrapping number r_s. In case the D7 brane wrapping number r_s is larger than one, we need to replace the abelian field strength F_s to a non-abelian field-strength and take a trace where appropriate. [6]

The D5 charges of F_s are represented by fluxes of the field strength F_s through the various 2-cycles within X.

$$p_s^{\mathrm{A}} = \int_{\alpha_{\mathrm{A}}} \mathrm{Tr}(F_s) \tag{3.3.14}$$

Analogously, the D3-brane charge is identified with the instanton number charge.

$$q_s = \int_X \frac{1}{2} \mathrm{Tr}(F_s \wedge F_s) \tag{3.3.15}$$

The D5 charges $p_{s\mathrm{A}}$ are integers, whereas the D3 charge q_s may take half-integer values.

The D-brane world volume action, since it depends on the field strength F_s via the combination

$$\mathcal{F}_s = F_s - i_s^* B \,,$$

[6] In general, there are curvature corrections to the DBI and Chern-Simons terms that would need to be taken into account. They have the effect of replacing the Chern character by [70] $\mathrm{Tr}(e^F) \to \mathrm{Tr}(e^F)\sqrt{\frac{\widehat{A}(T)}{\widehat{A}(N)}}$. We will ignore these geometric contributions here, since they do not affect the main line of argument.

is invariant under gauge transformations $B_2 \to B_2 + d\Lambda$, $A_s \to A_s + \Lambda$, with Λ any one-form. If Λ is single valued, then the fluxes p_s^{A} of F_s remain unchanged. But the only restriction is that $d\Lambda$ belongs to an integral cohomology class on Y. The gauge transformations thus have an integral version, that shifts the integral periods of B into fluxes of F_s, and vice versa. This integral gauge invariance naturally turns the periods of B_2 into angular variables. The relevant B-periods for us are those along the 2-cycles of the del Pezzo surface X

$$b^{\mathrm{A}} \;=\; \int_{\alpha_{\mathrm{A}}} B. \tag{3.3.16}$$

The integral gauge transformations act on these periods and the D-brane charges via

$$\begin{aligned} b^{\mathrm{A}} &\;\to\; b^{\mathrm{A}} + n^{\mathrm{A}} \\ p_s^{\mathrm{A}} &\;\to\; p_s^{\mathrm{A}} + n^{\mathrm{A}}\, r_s \\ q_s &\;\to\; q_s - n_{\mathrm{A}}\, p_s^{\mathrm{A}} - \tfrac{1}{2} r_s n_{\mathrm{A}} n^{\mathrm{A}} \end{aligned} \tag{3.3.17}$$

with n^{A} an a priori arbitrary set of integers. These transformations can be used to restrict the b^{A} to the interval between 0 and 1.

Physical observables should be invariant under (3.3.17). This condition provides a useful check on calculations, whenever done in a non-manifestly invariant notation. A convenient way to preserve the invariance, is to introduce a new type of charge vector for the fractional branes, obtained by replacing in the definitions (3.3.14) and (3.3.15) the field strength F_s by $\mathcal{F}_s$:

$$\mathsf{Q}(\mathsf{F}_s) = (\mathbf{r}_s, \mathsf{p}_{s\mathrm{A}}, \mathsf{q}_s) \tag{3.3.18}$$

where $\mathbf{r}_s = r_s$ and

$$\begin{aligned} \mathsf{p}_{s\mathrm{A}} &= \; p_{s\mathrm{A}} - r_s b_{\mathrm{A}} \\ \mathsf{q}_s &= \; q_s + p_s^{\mathrm{A}}\, b_{\mathrm{A}} - \tfrac{1}{2}\, r_s\, b^{\mathrm{A}} b_{\mathrm{A}} \end{aligned} \tag{3.3.19}$$

The new charges can take any real value, and are both invariant under (3.3.17).

The charge vector is naturally combined into the central charge $\mathsf{Z}(\mathsf{F}_s)$ of the fractional brane F_s. The central charge is an exact quantum property of the fractional brane, that can be defined at the level of the worldsheet CFT as the complex number that tells us which linear combination of right- and left-moving supercharges the boundary state of the brane preserves. It depends linearly on the charge vector:

$$\mathsf{Z}(\mathsf{F}_s) = \mathsf{\Pi} \cdot \mathsf{Q}(\mathsf{F}_s), \tag{3.3.20}$$

with $\mathsf{\Pi}$ some vector that depends on the geometry of the CY singularity.

In the large volume regime, one can show that the central charge is given by the following expression: [71, 82, 72]

$$\mathsf{Z}(\mathsf{F}_s) = \int_X e^{-i_s^*(B+iJ)}\,\mathrm{Tr}(e^{F_s}) \tag{3.3.21}$$

where J denotes the Kähler class on Y. Evaluating the integral gives

$$\mathsf{Z}(\mathsf{F}_s) \;=\; \mathsf{q}_s \,-\, \tfrac{1}{2}\,\mathbf{r}_s\,\zeta^{\mathrm{A}}\zeta_{\mathrm{A}} \,-\, i\,\mathsf{p}_{s\mathrm{A}}\,\zeta^{\mathrm{A}}\,, \tag{3.3.22}$$

with

$$\zeta^{\mathrm{A}} \;=\; \int_{\alpha_{\mathrm{A}}} J\,. \tag{3.3.23}$$

With this preparation, let us write the geometric expression for the couplings of the fractional brane F_s. From the central charge $Z(\mathsf{F}_s)$, we can extract the effective gauge coupling via

$$\frac{4\pi}{g_s^2} = e^{-\phi}|\mathsf{Z}(\mathsf{F}_s)|\,, \tag{3.3.24}$$

which equals the brane tension of F_s. In the large volume limit, this relation directly follows from the BI-form of the D7 world volume action. The phase of the central charge

$$\zeta_s = \frac{1}{\pi}\mathrm{Im}\log\mathsf{Z}(\mathsf{F}_s) \tag{3.3.25}$$

gives rise to the FI parameter of the 4-d gauge theory [82]. Two fractional branes are mutually supersymmetric if the phases of their central charges are equal. Deviations of the relative phase generically gives rise to D-term SUSY breaking, and such a deviation is therefore naturally interpreted as an FI-term.

The couplings of the gauge fields to the RR-fields follows from expanding the CS-term of the action. The θ-angle reads

$$\theta_s \;=\; \mathbf{r}_s \theta_X + \mathsf{p}_{s\mathrm{A}} \theta^{\mathrm{A}} + \mathsf{q}_s C_0 \,, \tag{3.3.26}$$

with

$$\theta^{\mathrm{A}} \;=\; \int_{\alpha_{\mathrm{A}}} C_2 \,, \qquad\qquad \theta_X = \int_X C_4 \,. \tag{3.3.27}$$

In addition, each fractional brane may support a Stückelberg field, which arises by dualizing the RR 2-form potential $\mathcal{C}_s$ that couples linearly to the gauge field strength via

$$\mathcal{C}_s \wedge F_s \tag{3.3.28}$$

From the CS-term we read off that

$$\mathcal{C}_s \;=\; \mathbf{r}_s \, c_X \;+\; \mathsf{p}_{s\mathrm{A}} c^{\mathrm{A}} \;+\; \mathsf{q}_s \, C_2 \tag{3.3.29}$$

$$c^{\mathrm{A}} \;=\; \int_{\alpha_{\mathrm{A}}} C_4 \,. \qquad\qquad c_X \;=\; \int_X C_6 \tag{3.3.30}$$

Note that all above formulas for the closed string couplings all respect the integral gauge symmetry (3.3.17).

3.3.4 Some local and global considerations

On dP_n there are $n+3$ different fractional branes, with a priori as many independent gauge couplings and FI parameters. However, the expressions (3.3.22) for the central charges $\mathsf{Z}(\mathsf{F}_s)$ contain only $2n+4$ independent continuous parameters: the dilaton, the (dualized) B-field, and a pair of periods $(b_{\mathrm{A}}, \zeta^{\mathrm{A}})$ for every of the $n+1$ 2-cycles in dP_n. We conclude that there must be two relations restricting the couplings. The gauge theory interpretation of these relations is that the dP_n quiver gauge theory always contains two anomalous $U(1)$ factors. As emphasized for instance in [73], the FI-parameters associated with anomalous $U(1)$'s are not freely tunable, but dynamically adjusted so that the associated D-term equations are automatically satisfied. This adjustment relates the anomalous FI variables and gauge couplings.

The non-compact cone Y_0 supports two compact cycles for which the dual cycle is also compact, namely, the canonical class and the del Pezzo surface X. Correspondingly, we expect to find a normalizable 2-form and 4-form on Y_0. [7] Their presence implies that two closed string modes survive as dynamical 4-d fields with normalizable kinetic terms; these are the two axions θ^0 and θ_X associated with the two anomalous $U(1)$ factors. The two $U(1)$'s are dual to each other: a $U(1)$ gauge rotation of one generates an additive shift in the θ-angle of the other. This naturally identifies the respective θ-angles and Stückelberg fields via

$$\theta^0 = \rho^X \,, \qquad\qquad \rho_0 = \theta_X \,. \tag{3.3.31}$$

The geometric origin of these identifications is that the corresponding branes wrap dual intersecting cycles[8] .

We obtain non-normalizable harmonic forms on the non-compact cone Y_0 by extending the other harmonic 2-forms ω_i on X to r-independent forms over Y_0. The corresponding 4-d RR-modes are non-dynamical fields: any space-time variation of c^{A} with $\mathrm{A} \neq 0$ would carry infinite kinetic energy. This obstructs the introduction of the dual scalar field, the would-be Stückelberg variabel ρ_{A}, which would have a vanishing kinetic term. We thus conclude that for the non-compact cone Y_0, all non-anomalous $U(1)$ factors remain massless. This is in accord with the expectation that in the non-compact limit, all closed string dynamics decouples.[9]

[7]Using the form of the metric of the CY singularity as given in eqn (1), the normalizable 2-form can be found to be $\omega_X = \frac{1}{r^4}\left[\omega - 2\,\frac{dr}{r}\wedge\eta\right]$. The normalizable 4-form is its Hodge dual.

[8]All other D5-brane components, that wrap the degree zero cycles α_i, do not intersect any other branes within Y (see formula 3.2.4). This correlates with the absence of any other mixed $U(1)$ gauge anomalies.

[9]There is a slight subtlety, however. Whereas the non-abelian gauge dynamics of a D-brane on a del Pezzo singularity flows to a conformal fixed point in the IR, the $U(1)$ factors become infrared free, while towards the UV, their couplings develop a Landau pole. Via the holographic dictionary, this suggests that the D-brane theory with non-zero $U(1)$ couplings needs to be defined on a finite cone Y_0, with r cut-off at some finite value Λ. This subtlety will not affect the discussion of the compactied setting, provided the location of all Landau poles is sufficiently larger than the compactification scale.

As we will show in the remainder of this section, the story changes for the compactified setting, for D-branes at a del Pezzo singularity inside of a compact CY threefold Y. In this case, a subclass of all harmonic forms on the cone Y_0 may extend to normalizable harmonic forms on Y, and all corresponding closed string modes are dynamical 4-d fields.

3.3.5 Bulk action

The most general class of string compactifications, that may include the type of D-branes at singularities discussed here, is F-theory. For concreteness, however, we will consider the subclass of F-theory compactification that can be described by IIB string theory compactified on an orientifold CY threefold $Y = \widehat{Y}/\mathcal{O}$. The orientifold map $\mathcal{O}$ acts via

$$\mathcal{O} = (-1)^{F_L}\Omega_p\sigma$$

where F_L is left fermion number, Ω_p is world-sheet parity, and σ is the involution acting on Y. It acts via its pullback σ^* on the various forms present. The fixed loci of σ are orientifold planes. We will assume that the orientifold planes do not intersect the base X of the del Pezzo singularity.

The orientifold projection eliminates one half of the fields that were initially present on the full Calabi–Yau space. Which fields survive the projection is determined by the dimensions of the corresponding even and odd cohomology space $H_+^{(i,j)}$ and $H_-^{(i,j)}$ on Calabi–Yau manifold $\widehat{Y}$. Note that the orientifold projection in particular eliminates the constant zero-mode components of C_2, C_6 and B_2, since the operator $(-1)^{F_L}\Omega_p$ inverts the sign of all these fields.

The RR sector fields give rise to 4-d fields via their decomposition into harmonic forms on Y, which we may identify as elements of the $\overline{\partial}$ cohomology spaces $H^{(p,q)}$. On the orientifold, we need to decompose this space as $H_+^{(p,q)} \oplus H_-^{(p,q)}$, where $\pm$ denotes the eigenvalue under the action of σ^*

$$\omega_\alpha \in H_+^{(1,1)}(\widehat{Y}, \mathbf{Z})\,, \qquad\qquad \widetilde{\omega}_a \in H_-^{(1,1)}(\widehat{Y}, \mathbf{Z})\,,$$

$$\omega^\alpha \in H_+^{(2,2)}(\widehat{Y},\mathbf{Z})\,, \qquad\qquad \widetilde{\omega}^a \in H_-^{(2,2)}(\widehat{Y},\mathbf{Z})\,.$$

The relevant RR fields, invariant under $\mathcal{O} = (-1)^{F_L}\Omega_p\sigma$, decompose as:

$$\begin{aligned} C_2 &= \theta^a(x)\,\widetilde{\omega}_a \\ C_4 &= c^\alpha(x)\,\omega_\alpha + \rho_\alpha(x)\,\omega^\alpha\,, \qquad (3.3.32) \\ C_6 &= c_a(x)\,\widetilde{\omega}^a\,. \qquad (3.3.33) \end{aligned}$$

Here c^α and c_a are two-form fields and ρ_α and θ^a are scalar fields. Similarly, we can expand the Kähler form J and NS B-field as

$$\begin{aligned} J &= \zeta^\alpha(x)\,\omega_\alpha\,, \\ B_2 &= b^a(x)\,\widetilde{\omega}_a\,. \qquad (3.3.34) \end{aligned}$$

We can choose the cohomology bases such that

$$\int_Y \omega_\alpha\wedge\omega^\beta \;=\; \delta_\alpha^\beta, \qquad\qquad \int_Y \widetilde{\omega}_a\wedge\widetilde{\omega}^b \;=\; \delta_a^b\,. \qquad (3.3.35)$$

In what follows, ω_X and $\widetilde{\omega}_X$ will denote the Poincaré dual 2-forms to the symmetric and anti-symmetric lift of X, respectively.

The IIB supergravity action in string frame contains the following kinetic terms for the RR p-form fields

$$\mathcal{S} \;=\; \int[\,G^{ab}\,dc_a\wedge *dc_b \;+\; G_{\alpha\beta}\,dc^\alpha\wedge *dc^\beta\,] \qquad (3.3.36)$$

where $G_{\alpha\beta}$ and G^{ab} denote the natural metrics on the space of harmonic 2-forms on Y

$$G_{\alpha\beta} \;=\; \int_Y \omega_\alpha\wedge *\omega_\beta\,, \qquad\qquad G^{ab} \;=\; \int_Y \widetilde{\omega}^a\wedge *\widetilde{\omega}^b\,. \qquad (3.3.37)$$

The scalar RR fields θ^b and ρ_α are related to the above 2-form fields via the duality relations:

$$*\,d\theta^b \;=\; -G^{ab}dc_a\,, \qquad\qquad *\,d\rho_\alpha \;=\; G_{\alpha\beta}dc^\beta\,. \qquad (3.3.38)$$

The 4-d fields in (3.3.32) and (3.3.34) are all period integals of 10-d fields expanded in harmonic forms. Each of the 10-d fields may also support a non-zero field strength with some quantized flux. These fluxes play an important role in stabilizing the various geometric moduli of the compactification. In the following we will assume that a similar type of mechanism will generate a stabilizing potential for all the above fields, that fixes their expectation values and renders them massive at some high scale. The Stückelberg and axion fields ρ^α and θ_0 still play an important role in deriving the low energy effective field theory, however.

3.3.6 Coupling brane and bulk

Let us now discuss the coupling between the brane and bulk degrees of freedom. A first observation, that will be important in what follows, is that the harmonic forms on the compact CY manifold Y, when restricted to base X of the singularity, in general do not span the full cohomology of X. For instance, the 2-cohomology of Y may have fewer generators than that of X, in which case there must be one or more 2-cycles that are non-trivial within X but trivial within Y. Conversely, Y may have non-trivial cohomology elements that restrict to trivial elements on X. The overlap matrices

$$\Pi_\alpha^{\rm A} = \int_{\alpha_{\rm A}} \omega_\alpha \,, \qquad\qquad \Pi_a^{\rm A} = \int_{\alpha_{\rm A}} \widetilde{\omega}_a \,, \tag{3.3.39}$$

when viewed as linear maps between cohomology spaces $H^{(1,1)}(X, \mathbf{Z})$ and $H_\pm^{(1,1)}(Y, \mathbf{Z})$, thus typically have both a non-zero kernel and cokernel.

As a geometric clarification, we note that the above linear map Π between the 2-cohomologies of Y and X naturally leads to an exact sequence

$$... \to H^2(Y) \to H^2(X) \to H^3(Y/X) \to H^3(Y) \to H^3(X) \to ... \tag{3.3.40}$$

where $X \subset Y$ is the 4-cycle wrapped by the del Pezzo in the CY 3-fold, Y. The cohomology space $H^k(Y/X)$ is referred to as the 'relative' k-cohomology class. The map from $H^2(Y)$ to $H^2(X)$ in the exact sequence is given by our projection matrix Π. Since in our case $H^1(Y) \cong 0$ and $H^1(X) \cong 0$, we have from (3.3.40):

$$\ker[\Pi] \cong H^2(Y/X) \tag{3.3.41}$$

or, in words, the kernel of our projection matrix is just the relative 2-cohomology. Similarly, using the fact that $H^3(X) \cong 0$, we deduce that

$$H^3(Y/X) \cong H^3(Y) \oplus \mathrm{coker}[\Pi] \tag{3.3.42}$$

In other words, the relative 3-cohomology $H^3(Y/X)$ is dual to the space of all 3-cycles in Y plus all 3-chains Γ for which $\partial\Gamma \subset X$.

This incomplete overlap between the two cohomologies has immediate repercussions for the D-brane gauge theory, since it implies that the compact embedding typically reduces the space of gauge invariant couplings. The couplings are all period integrals of certain harmonic forms, and any reduction of the associated cohomology spaces reduces the number of allowed deformations of the gauge theory. This truncation is independent from the issue of moduli stabilization, which is a *dynamical* mechanism for fixing the couplings, whereas the mismatch of cohomologies amounts to a *topological* obstruction.

By using the period matrices (3.3.39), we can expand the topologically available local couplings in terms of the global periods, defined in (3.3.32) and (3.3.34), as

$$b^{\mathrm{A}} = \Pi^{\mathrm{A}}_{\ a}\, b^a\,, \qquad\qquad c^{\mathrm{A}} = \Pi^{\mathrm{A}}_{\ \alpha}\, c^\alpha$$

$$\theta^{\mathrm{A}} = \Pi^{\mathrm{A}}_{\ a}\, \theta^a\,, \qquad\qquad \zeta^{\mathrm{A}} = \Pi^{\mathrm{A}}_{\ \alpha}\, \zeta^\alpha$$

By construction, the left hand-side are all elements of the subspace of $H^{(1,1)}$ that is common to both Y and X. The number of independent closed string couplings of each type thus coincides with the rank of the corresponding overlap matrix.

As a special consequence, it may be possible to form linear combinations of gauge fields A_s, for which the linear RR-coupling (3.3.28) identically vanishes. These correspond to linear combinations of $U(1)$ generators

$$Q = \sum_s k_s\, Q_s$$

such that

$$\sum_s k_s\, \mathbf{r}_s \;=\; 0\,, \qquad\qquad \sum_s k_s\, \mathsf{p}_{s\mathrm{A}}\, \Pi^{\mathrm{A}}_{\ \alpha} \;=\; 0\,. \tag{3.3.43}$$

The charge vector of the linear combination of fractional branes $\sum_s k_s \,\mathsf{F}_s$ adds up to that of a D5-brane wrapping a 2-cycle within X that is trivial within the total space Y. As a result, the corresponding $U(1)$ vector boson $A = \sum_s k_s \, A_s$ decouples from the normalizable RR-modes, and remains massless. This lesson will be applied in the next section.

Let us compute the non-zero masses. Upon dualizing, or equivalently, integrating out the 2-form potentials, we obtain the Stückelberg mass term for the vector bosons A_s

$$G_{XX} \, \nabla\rho^X \wedge *\nabla\rho^X \, + \, G^{\alpha\beta} \, \nabla\rho_\alpha \wedge *\nabla\rho_\beta \tag{3.3.44}$$

with

$$\begin{aligned} \nabla\rho^X \; &= \; d\rho^X - \sum_s \mathbf{r}_s \, A_s \, , \\ \nabla\rho_\alpha \; &= \; d\rho_\alpha - \sum_s \, \mathsf{p}_{s\mathrm{A}} \, \Pi^{\mathrm{A}}_{\alpha} \, A_s \, . \end{aligned} \tag{3.3.45}$$

The vector boson mass matrix reads

$$m^2_{ss'} = G_{XX} \, \mathbf{r}_s \mathbf{r}_{s'} \, + \, G^{\alpha\beta} \, \Pi_\alpha^{\;\mathrm{A}} \Pi_\beta^{\;\mathrm{B}} \, \mathsf{p}_{s\mathrm{A}} \mathsf{p}_{s'\mathrm{B}} \tag{3.3.46}$$

and is of the order of the string scale (for string size compactifications). It lifts all $U(1)$ vector bosons from the low energy spectrum, except for the ones that correspond to fractional branes that wrap 2-cycles that are trivial within Y. This is the central result of this section.

Besides via Stückelberg mass terms, vector bosons can also acquire a mass from vacuum expectation values of charged scalar fields, triggered by turning on FI-parameters. It is worth noting that for the same $U(1)$ factors for which the above mass term (3.3.44) vanishes, the FI parameter cancels

$$\sum_s k_s \, \mathsf{p}_{s\mathrm{A}} \Pi^{\mathrm{A}}_{\alpha} \zeta^\alpha = 0$$

These $U(1)$ bosons thus remain massless, as long as supersymmetry remains unbroken.

3.4 SM-like Gauge Theory from a dP_8 Singularity

We now apply the lessons of the previous section to the string construction of a Standard Model-like theory of [23], using the world volume theory of a D3-brane on a del Pezzo 8 singularity. Let us summarize the set up – more details are found in [23].

3.4.1 A Standard Model D3-brane

A del Pezzo 8 surface can be represented as $\mathbf{P}^2$ blown up at 8 generic points. It supports nine independent 2-cycles: the hyperplane class H in $\mathbf{P}^2$ plus eight exceptional curves E_i with intersection numbers

$$H \cdot H \,=\, 1, \qquad E_i \cdot E_j = -\delta_{ij}, \qquad H \cdot E_i = 0\,.$$

The canonical class is identified as

$$K = -3H + \textstyle\sum_{i=1}^{8} E_i.$$

The degree zero sub-lattice of $H_2(X, \mathbf{Z})$, the elements with zero intersection with $c_1 = -K$, is isomorphic to the root lattice of E_8. The 8 simple roots, all with self-intersection -2, can be chosen as

$$\alpha_i = E_i - E_{i+1}, \qquad {\scriptstyle i=1,\ldots,7} \qquad\qquad \alpha_8 = h - E_1 - E_2 - E_3\,. \tag{3.4.47}$$

A del Pezzo 8 singularity thus accommodates 11 types of fractional branes F_i, which each are characterized by charge vectors $\mathrm{ch}(\mathsf{F}_i)$ that indicate their (D7, D5, D3) wrapping numbers.

Exceptional collections (=bases of fractional branes) on a del Pezzo 8 singularity have been constructed in [76]. For a given collection, a D-brane configuration assigns multiplicity n_i to each fractional brane F_i, consistent with local tadpole conditions. The construction of [23] starts from a single D3-brane; the multiplicities n_i are such that the charge vectors add up to $(0, 0, 1)$. For the favorable basis of fractional branes described in [23] (presumably corresponding to a specific stability region in Kähler moduli space), this leads to an $\mathcal{N}=1$ quiver gauge theory with the gauge group $\mathcal{G}_0 = U(6) \times U(3) \times U(1)^9$. [10]

As shown in [23], this D3-brane quiver theory allows a SUSY preserving symmetry breaking process to a semi-realistic gauge theory with the gauge group

$$\mathcal{G} = U(3) \times U(2) \times U(1)^7.$$

The quiver diagram is drawn in fig 1. Each line represents three generations of bi-fundamental fields. The D-brane model thus has the same non-abelian gauge symmetries, and the same quark and lepton content as the Standard Model. It has an excess of Higgs fields – two pairs per generation – and several extra $U(1)$-factors. We would like to apply the new insights obtained in the previous section to move the model one step closer to reality, by eliminating all the extra $U(1)$ gauge symmetries except hypercharge from the low energy theory.

To effectuate the symmetry breaking to $\mathcal{G}$, while preserving $\mathcal{N}=1$ supersymmetry, it is necessary turn on a suitable set of FI parameters and tune the superpotential W.[11] The D-term and F-term equations can then both be solved, while dictating expectation values that result in the desired symmetry breaking pattern. As first discussed in [61][12] (in the context of $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds), when a Calabi–Yau singularity is not isolated, the moduli space of D-branes on that Calabi–Yau has more than one branch. From a non-isolated singularity, several curves Γ_i of singularities will emanate, each having a generic singularity type R_i, one of the ADE singularities.

In this non-isolated case, on one of the branches of the moduli space, the branes move freely on the Calabi–Yau or its (partial) resolution, and the FI parameters are identified with "blowup modes" which specify how much blowing up is done. But there are additional branches of the moduli space associated with each Γ_i: on such a branch, the FI parameters which would normally be used to blow up the ADE singularity R_i are frozen to zero, and

[10] This particular quiver theory is related via a single Seiberg duality to the world volume theory of a D3-brane near a $\mathbf{C}^3/\Delta_{27}$ orbifold singularity – the model considered earlier in [22] [21] as a possible starting point for a string realization of a SM-like gauge theory.

[11] The superpotential W contains Yukawa couplings for every closed oriented triangle in the quiver diagram, can be tuned via the complex structure moduli, in combination with suitable non-commutative deformations [77] of the del Pezzo surface.

[12] See also [78, 79, 80, 81].

new parameters arise which correspond to positions of R_i-fractional branes along Γ_i. That is, on this new branch, some of the R_i fractional branes have moved out along the curve Γ_i and their positions give new parameters.

The strategy for producing the gauge theory of fig 1, essentially following [23], is this: by appropriately tuning the superpotential (i.e., varying the complex structure) we can find a Calabi–Yau with a non-isolated singularity—a curve Γ of A_2 singular points—such that the classes α_1 and α_2 have been blown down to an A_2 singularity on the (generalized) del Pezzo surface where it meets the singular locus.[13] Our symmetry-breaking involves moving onto the Γ branch in the moduli space, where the α_1 and α_2 fractional brane classes are free to move along the curve Γ of A_2 singularities. In particular, these branes can be taken to be very far from the primary singular point of interest, and become part of the bulk theory: any effect which they have on the physics will occur at very high energy like the rest of the bulk theory.

Making this choice removes the branes supported on α_1 and α_2 from the original brane spectrum, and replaces other branes in the spectrum by bound states which are independent of α_1 and α_2. The remaining bound state basis of the fractional branes obtained in [23] is specified by the following set of charge vectors

$$\begin{aligned}
\mathrm{ch}(\mathsf{F}_1) &= (3, -2K + \sum_{i=5}^{8} E_i - E_4, \tfrac{1}{2}) & \mathrm{ch}(\mathsf{F}_4) &= (1, H - E_4, 0) \\
\mathrm{ch}(\mathsf{F}_2) &= (3, \sum_{i=5}^{8} E_i, -2) & \mathrm{ch}(\mathsf{F}_i) &= (1, -K + E_i, 1) \qquad i = 5, .., 8 \\
\mathrm{ch}(\mathsf{F}_3) &= (3, 3H - \sum_{i=1}^{4} E_i, -\tfrac{1}{2}) & \mathrm{ch}(\mathsf{F}_9) &= (1, 2H - \sum_{i=1}^{4} E_i, 0)
\end{aligned} \tag{3.4.48}$$

Here the first and third entry indicate the D7 and D3 charge; the second entry gives the 2-cycle around wrapped by the D5-brane component of F_i. As shown in [23], the above collection of fractional branes is rigid, in the sense that the branes have the minimum number of self-intersections and the corresponding gauge theory is free of adjoint matter besides the gauge multiplet. From the collection of charge vectors, one easily obtains the

[13] We will give an explicit description of a del Pezzo 8 surface with the required A_2 singularity in the next section.

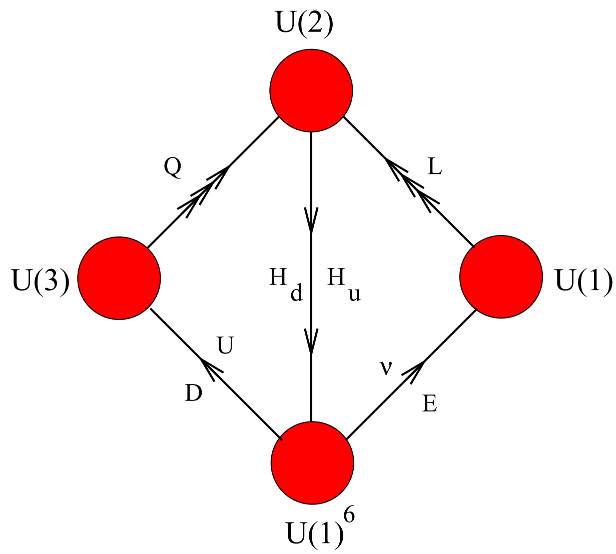


Figure 3.1: The MSSM-like quiver gauge theory obtained in [23]. Each line represents three generations of bi-fundamentals. In the text below we will identify the geometric condition that isolates the $U(1)_Y$ hypercharge as the only surviving massless $U(1)$ gauge symmetry.

matrix of intersection products via the fomula (3.2.4). One finds

$$\#(\mathsf{F}_i, \mathsf{F}_j) = \begin{pmatrix} 0 & -3 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 0 & 3 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & -3 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (3.4.49)$$

which gives the quiver diagram drawn fig 1. The rank of each gauge group corresponds to the (absolute value of the) multiplicity of the corresponding fractional brane, and has been chosen such that weighted sum of charge vectors adds up to the charge of a single D3-brane. In other words, the gauge theory of fig 1 arises from a single D3-brane placed at the del Pezzo 8 singularity.

Note that, as expected, all fractional branes in the basis (3.4.48) have vanishing D5 wrapping numbers around the two 2-cycles corresponding to the first two roots α_1 and α_2

of E_8, since we have converted the FI parameters which were blowup modes for those cycles into positions for A_2-fractional branes.

After eliminating the two 2-cycles α_1 and α_2, the remaining 2-cohomology of the del Pezzo singularity is spanned by the roots α_i with $i = 3, .., 8$ and the canoncial class K. Note that the total cohomology of the generalized del Pezzo surface with an A_2 singularity is 9 dimensional, and that the fractional branes (3.4.48) thus form a complete basis.

3.4.2 Identification of hypercharge

Let us turn to discuss the $U(1)$ factors in the quiver of fig 1, and identify the linear combination that defines hypercharge. We denote the node on the right by $U(1)_1$, and the overall $U(1)$-factors of the $U(2)$ and $U(3)$ nodes by $U(1)_2$ and $U(1)_3$, resp. The $U(1)^6$ node at the bottom divides into two nodes $U(1)^3_u$ and $U(1)^3_d$, where each $U(1)_u$ and $U(1)_d$ acts on the matter fields of the corresponding generation only. We denote the nine $U(1)$ generators by $\{Q_1, Q_2, Q_3, Q^i_u, Q^i_d, \}$. The total charge

$$Q_{tot} = \sum_s Q_s$$

decouples: none of the bi-fundamental fields is charged under Q_{tot}. Of the remaining eight generators, two have mixed $U(1)$ anomalies. As discussed in section 3, these are associated to fractional branes that intersect compact cycles within the del Pezzo singularity. In other words, any linear combination of charges such that the corresponding fractional brane has zero rank and zero degree is free of anomalies.

Hypercharge is identified with the non-anomalous combination

$$Q_Y = \frac{1}{2}Q_1 - \frac{1}{6}Q_3 - \frac{1}{2}\Big(\sum_{i=1}^{3} Q^i_d - \sum_{i=1}^{3} Q^i_u\Big) \tag{3.4.50}$$

The other non-anomalous $U(1)$ charges are

$$\frac{1}{3}Q_3 - \frac{1}{2}Q_1 \;=\; B - L, \tag{3.4.51}$$

together with four independent abelian flavor symmetries of the form

$$Q^{ij}_{u,d} = Q^i_u - Q^j_u, \qquad Q^{ij}_b = Q^i_b - Q^j_b. \tag{3.4.52}$$

We would like to ensure that, among all these charges, only the hypercharge survives as a low energy gauge symmetry. From our study of the stringy Stückelberg mechanism, we now know that this can be achieved if we find a CY embedding of the dP_8 geometry such that only the particular 2-cycle associated with Q_Y represents a trivial homology class within the full CY threefold. We will compute this 2-cycle momentarily.

Let us take a short look at the physical relevance of the extra $U(1)$ factors in the quiver of fig 2. If unbroken, they forbid in particular all μ-terms, the supersymmetric mass terms for the extra Higgs scalars. In the concluding section 6, we return to discuss possible string mechanisms for breaking the extra $U(1)$'s. First we discuss how to make them all massive.

The linear sum (3.4.50) of $U(1)$ charges that defines Q_Y, selects a corresponding linear sum of fractional branes, which we may choose as follows[14]

$$\mathsf{F}_{\rm Y} = \frac{1}{2}\Big(\mathsf{F}_3 - \mathsf{F}_1 - \sum_{i=4,5,9} \mathsf{F}_i + \sum_{i=6,7,8} \mathsf{F}_i\Big) \tag{3.4.53}$$

A simple calculation gives that, at the level of the charge vectors

$$\mathrm{ch}(\mathsf{F}_{\rm Y}) = (\,0\,,\, -\alpha_4,\, \tfrac{1}{2}\,) \qquad \alpha_4 = e_5 - e_4 \tag{3.4.54}$$

We read off that the 2-cycle associated with the hypercharge generator Q_Y is the one represented by the simple root α_4.

We consider this an encouragingly simple result. Namely, when added to the insights obtained in the previous section, we arrive at the following attractive geometrical conclusion: we can ensure that all extra $U(1)$ factors except hypercharge acquire a Stückelberg mass, provided we can find compact CY manifolds with a del Pezzo 8 singularity, such that only α_4 represents a trivial homology class. Requiring non-triviality of all other 2-cycles except α_4 not only helps with eliminating the extra $U(1)$'s, but also keeps a maximal

[14]With this equation we do not suggest any bound state formation of fractional branes. Instead, we simply use it as an intermediate step in determining the cohomology class of the linear combination of branes, whose $U(1)$ generators add up to $U(1)_Y$.

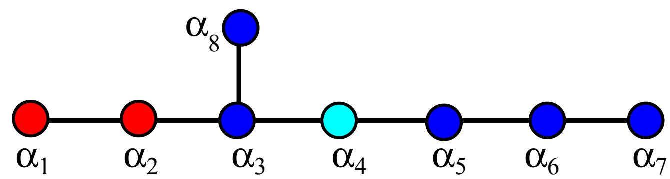


Figure 3.2: Our proposed D3-brane realization of the MSSM involves a dP_8 singularity embedded inside a CY manifold, such that two of its 2-cycles, α_1 and α_2, develop an A_2 singularity which forms part of a curve of A_2 singularities on the CY, and all remaining 2-cycles except α_4 are non-trivial within the full CY.

number of gauge invariant couplings in play as dynamically tunable moduli of the compact geometry. In particular, to accommodate the construction of the SM quiver theory of fig 1, the complex structure moduli of the compact CY threefold must allow for the formation of an A_2 singularity within the del Pezzo 8 geometry[15] , with α_4 representing a trivial cycle and all other cycles being nontrivial. In the next section, we will present a general geometric prescription for constructing a compact CY embedding of the dP_8 singularity with all the desired topological properties.

3.5 Constructing the Calabi–Yau Threefold

It is not difficult to find examples of compact CY threefolds that contain a dP_8 singularity. Since a dP_8 surface can be constructed as a hypersurface of degree six in the weighted projective space $W\mathbf{P}_{(1,1,2,3)}$, one natural route is to look among realizations of CY threefolds as hypersurfaces in weighted projective space, and identify coordinate regions where the CY equation degenerates into that of a cone over dP_8. Examples of this type are the CY threefolds obtained by resolving singularities of degree 18 hypersurfaces $W\mathbf{P}_{(1,1,1,6,9)}$, considered in [83]. This class of CY manifolds, however, has only two Kähler classes, and

[15]This can be done without any fine-tuning, as follows. The complex structure of Y is fixed via the GVW superpotential, which for given integer 3-form fluxes takes the form $W = (n_J + \tau m_J)\,\Omega^J$ where Ω^J denote the periods of the 3-form Ω. Now choose the integer fluxes to be invariant under the diffeomorphisms that act like Weyl reflections in α_1 and α_2. W then has an extremum for Ω^J invariant both Weyl reflections, which is the locus where dP_8 has the required A_2 singularity.

therefore can not satisfy our topological requirement that all 2-cycles of dP_8 except α_4 lift to non-trivial cycles within Y. On the other hand, this example does illustrate the basic phenomenon of interest: since the 2-cohomology of Y has only two generators, most 2-cycles within the dP_8 surface must in fact be trivial within Y.

A potentially more useful class of examples was recently considered in [84], where it was shown how to construct a CY orientifold Y as a T^2-fibration over any del Pezzo surface. The T^2 is represented in hyperelliptic form, that is, as a two sheeted cover of a $\mathbf{P}^1$. The $\mathbf{P}^1$ fibration takes the form $\mathbf{P}(\mathcal{O}_X \oplus K_X)$ with X the del Pezzo surface. The covering space $\widehat{Y}$ of Y has a holomorphic involution σ, which exchanges these two sheets, and the IIB orientifold on this CY surface is obtained by implementing the projection $\mathcal{O} = (-1)^{F_L}\Omega_p\,\sigma$. The $\mathbf{P}^1$-fibration over the del Pezzo has two special sections, X_0 and X_∞, one of which can be contracted to a del Pezzo singularity [84]. The total space of the fibration is the orientifold geometry Y. This set-up looks somewhat more promising for our purpose, since all 2-cycles within X are manifestly preserved as 2-cycles within the orientifold space Y. So a suitable modification the construction, so that only α_4 is eliminated as a generator of $H^2(Y)$ while all other 2-cycles are kept, would yield a concrete example of a CY orientifold with the desired global topology. [16]

Rather than following this route (of trying to find a specific compact CY manifold) we will instead give a general local prescription for how to obtain a suitable compact embedding of the dP_8 singularity, based only on the geometry of the neighborhood of the singularity. This local perspective does not rely on detailed assumptions about the specific UV completion of the dP_8 model, and thus combines well with our general bottom-up philosophy.

[16] A concrete proposal is as follows. Tune the complex structure so that the dP_8 has an automorphism which maps $\alpha_4 \to -\alpha_4$, i.e. which acts as the Weyl reflection $w(\alpha_4)$ on the homology lattice. One way to get such an automorphism is to let X develop an A_1 singularity with α_4 as the (-2) curve. The Weyl reflection then acts trivially on the Calabi-Yau Y, but acts non-trivially on the cohomology and the string theory spectrum on Y. We may then define a new holomorphic involution $\rho = w(\alpha_4) \circ \sigma$ and consider the orientifold $\mathcal{O}' = (-1)^{F_L}\Omega_p\,\rho$. The $O7$-planes are at the same locus as before, but the monodromy is slightly different. The harmonic 2-form associated to α_4 on X still lifts to the cover space $\widehat{Y}$, but as a generator of odd homology $H^{1,1}_-(\widehat{Y})$ instead of $H^{1,1}_+(\widehat{Y})$. Therefore, the FI-parameter and Stückelberg field associated to α_4 are projected out, leading to a massless $U(1)_Y$.

3.5.1 Local Picard group of a CY singularity

To begin, we discuss the local Picard group of a Calabi–Yau singularity, and the effect it has on things such as deformations.

If X is a (local or global) algebraic variety or complex analytic space of complex dimension d, the *Weil divisors on* X, denoted $Z_{d-1}(X)$, are the $\mathbf{Z}$-linear combinations of subvarieties of dimension $d-1$; the *Cartier divisors on* X, denoted $\mathrm{Div}(X)$, are divisors which are locally defined by a single equation $\{f = 0\}$. On a nonsingular variety, $Z_{d-1}(X) = \mathrm{Div}(X)$, so the quotient group

$$Z_{d-1}(X)/\mathrm{Div}(X)$$

is one measurement of how singular the variety X is.

The *principal Cartier divisors* on X, denoted $\mathrm{Div}^0(X)$, are the divisors which can be written as the difference of zeros and poles of a meromorphic function defined on all of X, and the *Picard group of* X is the quotient

$$\mathrm{Pic}(X) = \mathrm{Div}(X)/\mathrm{Div}^0(X).$$

If X is sufficiently small, this is trivial, and one introduces a local version of the group called the *local Picard group*: for a point $P \in X$,

$$\mathrm{Pic}(X, P) = \lim_{\leftarrow} Z_{d-1}(U)/\mathrm{Div}(U),$$

where the limit is taken over smaller and smaller open neighborhoods U of P in X.

Local Picard groups of Calabi–Yau singularities in complex dimesion 3 were studied in detail by Kawamata [85], who showed that $\mathrm{Pic}(X, P)$ is finitely generated. In our context, we are mainly interested in the case where X is a neighborhood of a singular point $P \in X$ which is obtained by contracting a (generalized) del Pezzo surface S in a Calabi–Yau space $\widetilde{X}$ to a point via a map $\pi : \widetilde{X} \to X$.[17] In this case, we can identify $\mathrm{Pic}(X, P)$ with the image of the natural map $\mathrm{Pic}(\widetilde{X}) \to \mathrm{Pic}(S)$. The rank of this image is always at least one: it follows from the adjunction formula that there is always a divisor D_0 on $\widetilde{X}$ such that

[17] We allow $\widetilde{X}$ to have a curve of rational double point singularities, meeting S in a rational double point, which is why S is called "generalized", following the terminology of the mathematics literature.

$D_0 + S$ is the divisor of a meromorphic function on X, and the image of D_0 in $\mathrm{Pic}(S)$ is the anticanonical divisor $-K_S$.

To take a simple, yet important example, suppose that $S = \mathbf{CP}^1 \times \mathbf{CP}^1 \subset \widetilde{X}$ contracts to a Calabi–Yau singular point $P \in X$. There are two possibilities for $\mathrm{Pic}(X, P)$: it may happen that the two homology classes $[\mathbf{CP}^1 \times \{\text{point}\}]$ and $[\{\text{point}\} \times \mathbf{CP}^1]$ are the same in $H_2(\widetilde{X})$, in which case $\mathrm{Pic}(X, P) \cong \mathbf{Z}$ (with the generator corresponding to $-K_S$), or it may happen that those two homology classes are distinct, in which case $\mathrm{Pic}(X, P) \cong \mathbf{Z}^2$. Note that if X is simply a cone over S, the classes will be distinct; on the other hand, the case $\mathrm{Pic}(X, P) \cong \mathbf{Z}$ is closely related to one of the key examples from Mori's original pathbreaking paper [86] which started the modern classification theory of algebraic threefolds.[18]

The calculation of the local Picard group near a singular point depends sensitively on the equation of the point. Mori's example was in fact a form of the familiar conifold singularity. It is common in the study of Calabi–Yau spaces to consider only the "small" blowups of such a singularity (which replace it by a $\mathbf{CP}^1$; however, we could also choose to simply blow up the singular point in the standard way, which would yield $\mathbf{CP}^1 \times \mathbf{CP}^1$ with normal bundle $\mathcal{O}_{\mathbf{CP}^1 \times \mathbf{CP}^1}(-1, -1)$. The "small" blowups exist exactly when the two homology classes $[\mathbf{CP}^1 \times \{\text{point}\}]$ and $[\{\text{point}\} \times \mathbf{CP}^1]$ are distinct; when they are the same, we are in Mori's situation where small blowups do not exist. How do we determine this from the equation? If we can write the equation of the conifold singularity in the form

$$xy - zt = 0 \tag{3.5.55}$$

then the two small blowups are obtained by blowing up the Weil divisors $x = z = 0$ or $x = t = 0$, respectively. However, if there are higher order terms in the equation, the nicely factored form 3.5.55 may be destroyed:[19] this is Mori's case. The case of a del Pezzo contraction, with normal bundle $\mathcal{O}_{\mathbf{CP}^1 \times \mathbf{CP}^1}(-2, -2)$, is similar.[20]

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[18] In Mori's case, the normal bundle of S in $\widetilde{X}$ was $\mathcal{O}_S(-1, -1)$; on our case, the normal bundle is $\mathcal{O}_S(-2, -2)$.

[19] The factored form can always be restored by a local complex analytic change of coordinates, but that change of coordinates may fail to extend over the entire Calabi–Yau.

[20] In that case, the small contractions would yield a curve of A_1 singularities, as was crucial for the analysis of [87].

If the neighborhood of the $\mathbf{CP}^1 \times \mathbf{CP}^1$ is sufficiently large, the difference between the two cases can be detected by the topology of the neighborhood. When the two homology classes $[\mathbf{CP}^1 \times \{\text{point}\}]$ and $[\{\text{point}\} \times \mathbf{CP}^1]$ are the same, there is a 3-chain Γ whose boundary is the difference between the two. Such a 3-chain cannot exist if the two homology classes are distinct, so an analytic change of coordinates which affects the factorizability of 3.5.55 will have the topological effect of creating or destroying such a 3-chain Γ.[21]

3.5.2 Construction of the CY threefold

From this simple example, we can easily obtain more complicated ones, including examples of the type we are interested in. Let S' be a generalized del Pezzo surface obtained from $\mathbf{CP}^2$ by (1) blowing up 5 distinct points $P_4, \ldots, P_8$ to curves $E_4, \ldots, E_8$, (2) blowing up a point P_1 and two points P_2 and P_3 infinitely near to P_1, (3) blowing down two out of the last three exceptional divisors to an A_2 singularity. Note that the line ℓ_{45} through P_4 and P_5 lifts an an exceptional curve E_{45}, and that the same del Pezzo surface S' could be obtained starting from $\mathbf{CP}^1 \times \mathbf{CP}^1$: in that case, one would blow up a point P_{45} to the curve E_{45} observing that the two original $\mathbf{CP}^1$'s which pass through P_{45} lift to exceptional curves E_4 and E_5, and then blowing up P_6, P_7, P_8, P_1, P_2, P_3 as before.

We give an embedding into a Calabi–Yau in the following way. Start with $S_1 := \mathbf{CP}^1 \times \mathbf{CP}^1$ embedded in a Calabi–Yau neighborhood such that the two rulings are homologically equivalent in the Calabi–Yau. We attach rational curves C_{45}, C_6, C_7, C_8 to the del Pezzo surface S_1, meeting transversally at P_{45}, P_6, P_7 and P_8, and consider local divisors D_i meeting C_i transversally at another point, for $i = 45, 6, 7, 8$. We also attach a rational curve C_1 at P_1 which transversally meets the first of a pair of ruled surfaces D_1 and D_2 which together can be contracted to a curve of A_2 singularities. We label the fiber of D_1's ruling which passes through $C_1 \cap D_1$ by C_2, and we label the fiber of D_2's ruling which pass

[21] More details about the topology of this situation can be found in [88].

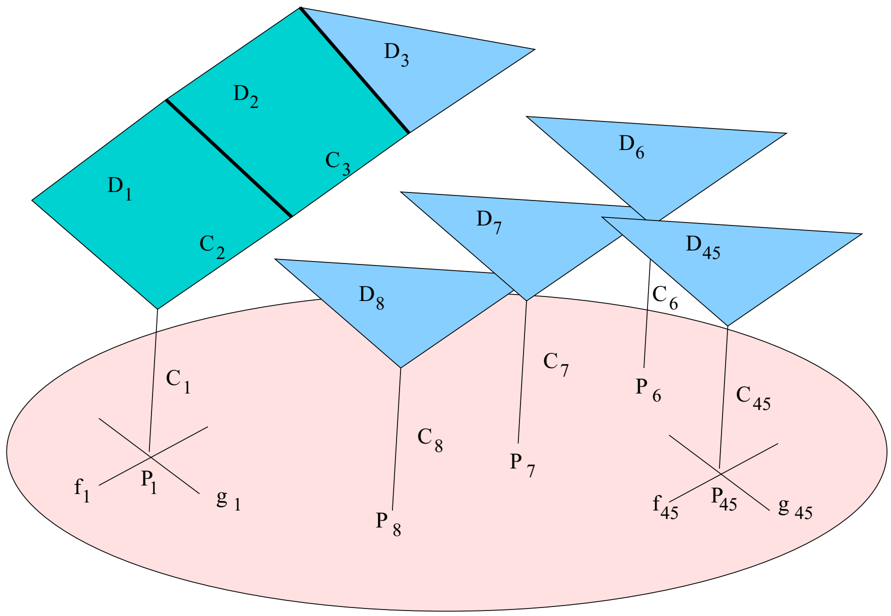

Figure 3.3: Starting point of our construction of a CY threefold with the desired topology. The curves f_i and g_i are fibers in the two rulings on S_1.

through $C_2 \cap D_2$ by C_3. We also consider a local divisor D_3 meeting C_3 transversally away from its intersection with C_2. This is all illustrated in figure 3.3.

Each of the curves and surfaces we have used in this construction can be embedded in a Calabi–Yau neighborhood, and those neighborhoods can be glued together to form a Calabi–Yau neighborhood of the entire structure illustrated in figure 3.3.

We now pass from this structure to the one we want by a sequence of flops. First, we flop the curves C_{45}, C_6, C_7, and C_8, which has the effect of blowing up S_1 at the four points P_{45}, P_6, P_7 and P_8 yielding a del Pezzo surface S_5. The transformed surfaces D_{45}, D_6, D_7, D_8 now meet S_5 in the flopped curves, as indicated in figure 3.4.

Next, we flop the curve C_1, yielding a del Pezzo S_6 on which the point P_1 has been blown up, as indicated in figure 3.5. The transformed surface D_1 meets S_6 in the flopped curve, and the transformed curve C_2 meets S_6 in a point P_2 ("infinitely near" to the first point P_1). When C_2 is now flopped, S_6 is blown up at P_2 to yield S_7, as indicated in figure 3.6. The transformed surface D_2 meets S_7 in the most recently flopped curve.

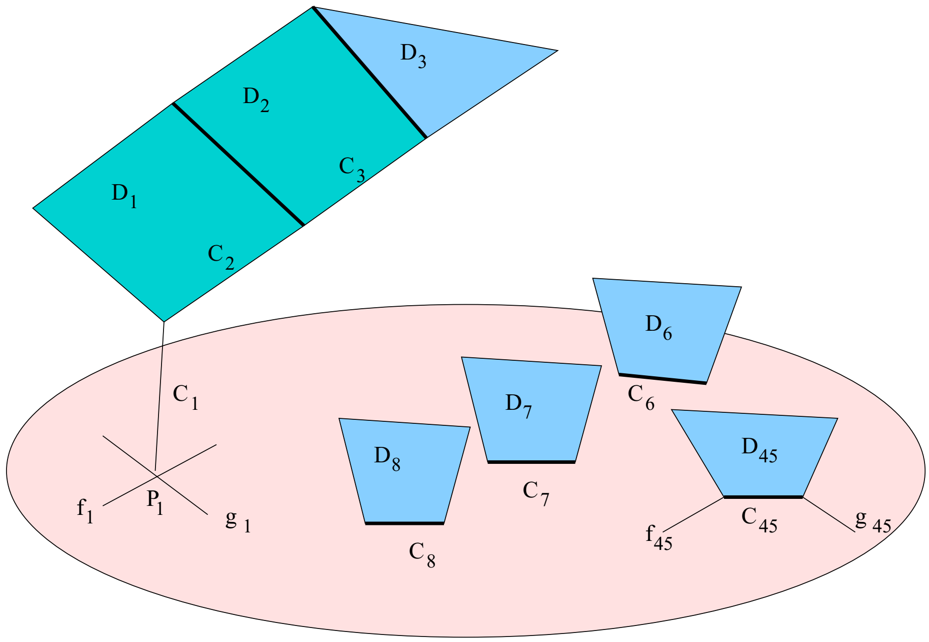

Figure 3.4: The CY threefold after flopping the curves C_{45}, C_6, C_7 and C_8. The curves f_{45}, g_{45}, C_{45}, C_6, C_7, and C_8 are all (-1)-curves on S_5.

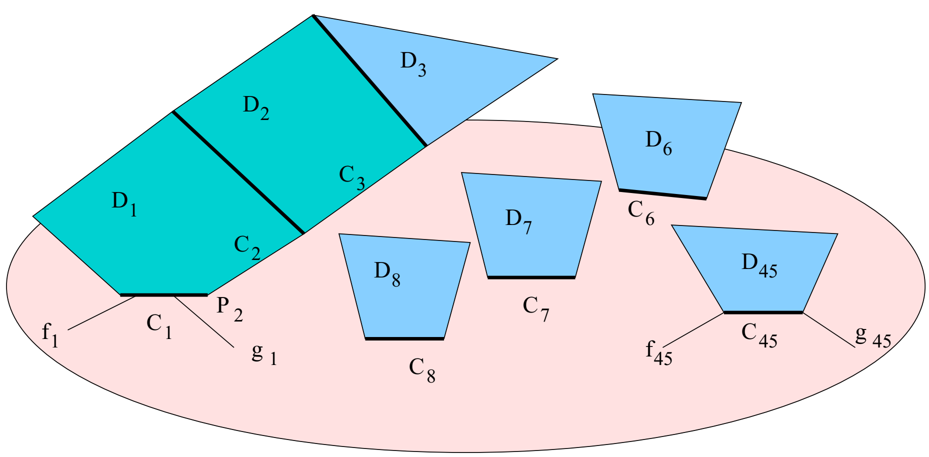

Figure 3.5: The CY threefold after flopping C_1. The curves f_1, g_1, and C_1 are additional (-1)-curves on S_6.

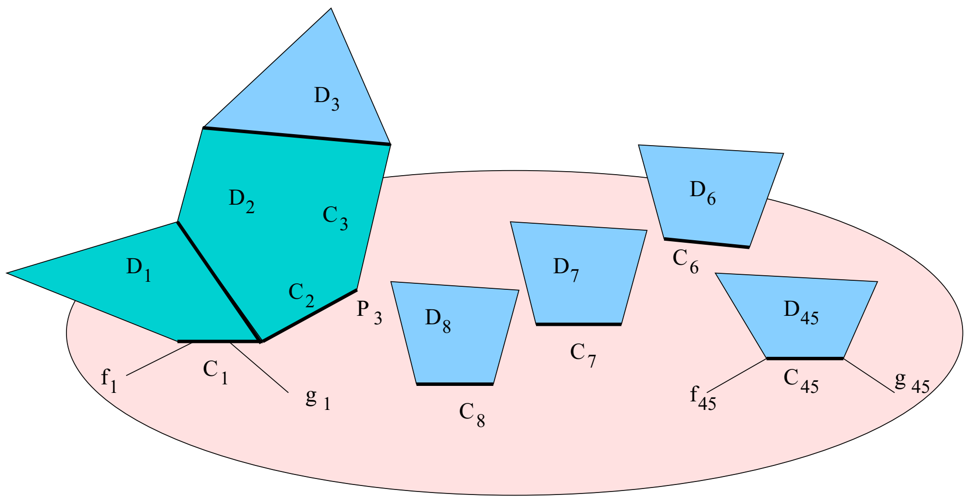

Figure 3.6: The CY threefold after flopping C_2. The curve C_1 has become a (-2)-curve, and C_2 is a (-1)-curve on S_7.

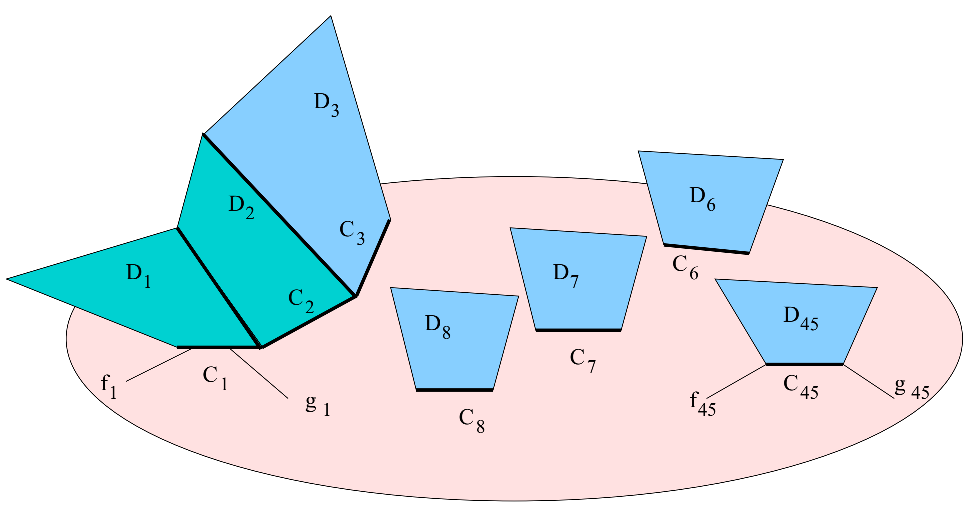

Figure 3.7: The CY threefold after flopping C_3. The curves C_2 has become a (-2)-curve, C_1 remains a (-2)-curve, and C_3 is a (-1)-curve on S_8.

The transformed curve C_3 meets S_7 in a point P_3 ("infinitely near" to P_2), and when C_3 is now flopped, S_7 is blown up at P_3 to yield S_8, as indicated in figure 3.7. (The transformed surface D_3 meets S_8 in the most recently flopped curve.) To match the curves on S_8 to the standard basis for cohomology of a del Pezzo, we set $e_1 = C_1 + C_2 + C_3$, $e_2 = C_2 + C_3$, $e_3 = C_3$, $e_4 = f_{45}$, $e_5 = g_{45}$, and $e_j = C_j$ for $j = 6, 7, 8$ so that $\alpha_1 = C_1$ and $\alpha_2 = C_2$. We can now contract the transforms of D_1 and D_2 to a curve of A_2 singularities, yielding the configuration illustrated in figure 3.8.

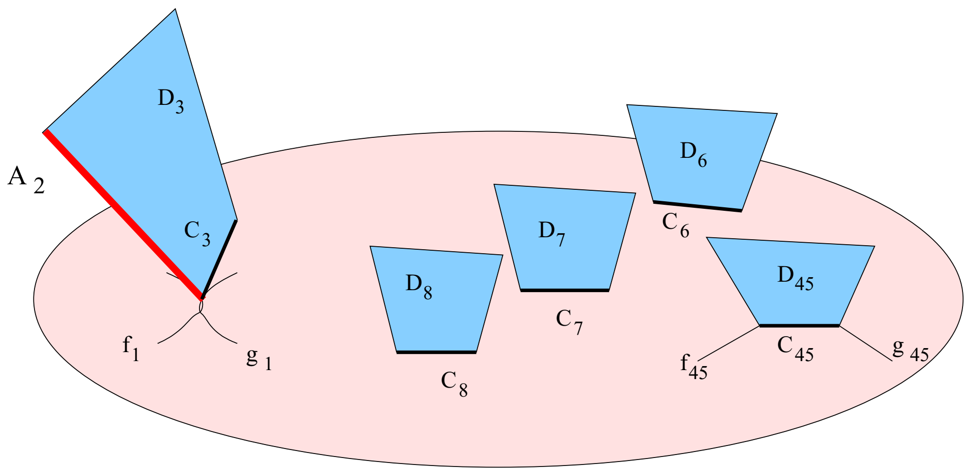

Figure 3.8: The final configuration: a del Pezzo 8 surface with an A_2 singularity, embedded into a Calabi–Yau such that two of its exceptional curves are homologous.

To achieve our final desired singular point, we contract the del Pezzo surface $S' = S_8$ to a point. Let us analyze the properties of this singular point.

First, it is not isolated: there is a curve of A_2 singularities which eminates from our singular point. This is one of the features we needed, because it allows the fractional branes where were supported on α_1 and α_2 to move off of the singular point we are interested in, into the bulk of the Calabi–Yau manifold.

Second, the local Picard group of this singular point has rank 6: the anticanonical divisor $D_0 \equiv -K_S$ and the transformed divisors D_3, D_{45}, D_6, D_7, D_8 generate a subgroup of the local Picard group of rank six; if there were a seventh generator, the map $\mathrm{Pic}(X) \to \mathrm{Pic}(S)$ would be surjective and the original surface S_1 would have had the same property, so that its local Picard group would have had rank 2. But by construction, the surface S_1 had a local Picard group of rank 1. We thus have demonstrated the presence of a 3-chain Γ, with boundary equal to the difference of two exceptional divisors. We can identify this difference with $\alpha_4 = E_5 - E_4$, which therefore does not exist as a homology class in the full Calabi–Yau.

Thus, via the above geometric procedure, we have succeeded in constructing a compact CY threefold with the properties we need for our D-brane construction. The outlined strategy furthermore preserves the main characteristics of our bottom-up perspective, since it only refers to the local Calabi-Yau neighborhood of the singularity and does not rely on unnecessary assumptions about the full string compactification.

An important physical assumption is that the compact embedding preserves the existence of all constituent fractional branes listed in eqn (3.4.48). This is not entirely obvious, since, in particular, the D5 charge around the trivial cycle α_4 is no longer a conserved quantum number: one could imagine a tunneling process, in which the linear combination (3.4.53) of fractional branes combines into a single D5 wrapping α_4, which subsequently self-annihilates by unwrapping along the 3-chain Γ. The tunneling process, however, is suppressed because it is non-supersymmetric and the probability can be made exponentially small by ensuring that the 3-volume (measured in units of D-brane tension) of the 3-chain Γ is large enough.

3.6 Conclusion and Outlook

In this chapter, we further developed the program advocated in [23], aimed at constructing realistic gauge theories on the world-volume of D-branes at a Calabi–Yau singularity. We have seen that several aspects of the world-volume gauge theory, such as the spectrum of light $U(1)$ vector bosons and the number of freely tunable of couplings, depend on the compact Calabi–Yau embedding of the singularity. In section 3, we have worked out the stringy mechanism by which $U(1)$ gauge symmetries get lifted. As a direct application of this result, we have shown how to construct a supersymmetric Standard Model, however with some extra Higgs fields, on a single D3-brane on a suitably chosen Calabi–Yau threefold with a del Pezzo 8 singularity. The final result for the quiver gauge theory is given in fig 2, where in addition all extra $U(1)$ factors besides hypercharge are massive.

3.6.1 $U(1)$ Breaking via D-instantons

At low energies, the extra $U(1)$'s are approximate global symmetries, which, if unbroken, would in particular forbid μ-terms. Fortunately, the geometry supports a plethora of D-instantons, that generically will break the $U(1)$ symmetries. Here we make some basic comments on the generic form of the D-instanton contributions.

The simplest type of D-instantons are the euclidean D-branes that wrap compact cycles within the base X of the CY singularity. The 'basic' D-instantons of this type are in 1-1 correspondence with the space-time filling fractional branes: they are localized in $\mathbf{R}^4$, but otherwise have the same Calabi-Yau boundary state and preserve the same supersymmetries as the fractional branes F_s. Apart from the exponential factor $e^{-8\pi^2/g_s^2 + i\,\theta_s}$, their contribution is independent of Kähler moduli and can thus be understood at large volume. In this limit, the analysis has essentially already been done in [90, 89, 91].[22] The result agrees with the expected field theory answer [92], and sensitively depends on $N_c - N_f$, the number of colors minus the number of flavors for the corresponding node. In our gauge theory we have $N_f > N_c$, in which case the one-instanton contribution to the superpotential is of the schematic form

$$\delta W = \Omega(\Phi)\, e^{-8\pi^2/g_s^2 + i\,\theta_s} \tag{3.6.56}$$

where $\Omega(\Phi)$ is a chiral multi-fermion operator [92]. The theta angle θ_s in general contains an axion field, that is shifted by the anomalous $U(1)$ gauge rotations. Instanton contributions to the effective action thus generally violate the anomalous $U(1)$ symmetries.

[22] For more recent discussions, see [93, 94, 95].

The story for the non-anomalous global $U(1)$ symmetries is analogous. The relevant D-instanton contributions are generated by euclidean D3-branes wrapping the dual 4-cycles Σ_α within Y. The classical D-instanton action reads $S = \mu_3 \mathrm{Vol}(\Sigma_\alpha) - i \int_{\Sigma_\alpha} C_4$ with μ_3 the D3-brane tension. Since $\int_{\Sigma_\alpha} C_4 = \rho_\alpha$ is the Stückelberg field, we observe that the D3-instanton contribution to the superpotential takes the form

$$\delta W = \mathcal{A}(\Phi)\, e^{-\mu_3 \mathrm{Vol}(\Sigma_\alpha) + i\rho_\alpha} \tag{3.6.57}$$

Here $\mathcal{A}(\Phi)$ denotes the perturbative pre-factor, the string analogue of the fluctuation determinant, of the D-instanton.[23] Since the phase factor $e^{i\rho_\alpha}$ transforms non-trivially under the corresponding $U(1)$ rotation, the pre-factor must be oppositely charged. After gauge fixing, the value of ρ_α will get fixed at by minimizing the potential, and ρ_α gets lifted from the low energy spectrum. What remains is a superpotential term that, from the low energy perspective, breaks the global $U(1)$ symmetry.

While we have not yet done the full analysis of these D-instanton effects, it seems reasonable to assume that the desired μ-terms can be generated via this mechanism.[24] Since the D-instanton contribution decreases exponentially with the volume of the 4-cycles Σ_α, it would naturally explain why (some of) the μ-terms are small compared to the string scale.

3.6.2 Eliminating extra Higgses

From a phenomenological perspective, the specific model based on the dP_8 singularity still has several issues that need to be addressed, before it can become fully realistic. Most

[23]Note that, unlike all classical couplings, the D-instanton contributions (3.6.57) are not governed by the local geometry of the singularity, but depend on the size of dual cycles Σ_α that probe the full CY. In fact, eqn (3.6.57) is a direct generalization of the famous KKLT contribution to the superpotential, that helps stabilize all geometric moduli of the compact Calabi–Yau manifold.

[24]In [96] it was argued that μ-terms can not arise in oriented quiver realizations of the SSM, like ours, because they seem forbidden by chirality at the $SU(2)$ node, in case one would consider more than one single brane (so that $SU(2)$ becomes $SU(2N)$). It is important to note, however, that the form of the D-instanton contributions sensitively depends on the rank of the gauge group, and thus may contain terms that at first sight would not be allowed in a large N limit of the quiver gauge theory. The μ terms, in particular, can be viewed as baryon-type operators for $SU(2)$, and thus one can easily imagine that they get generated via D-instantons.

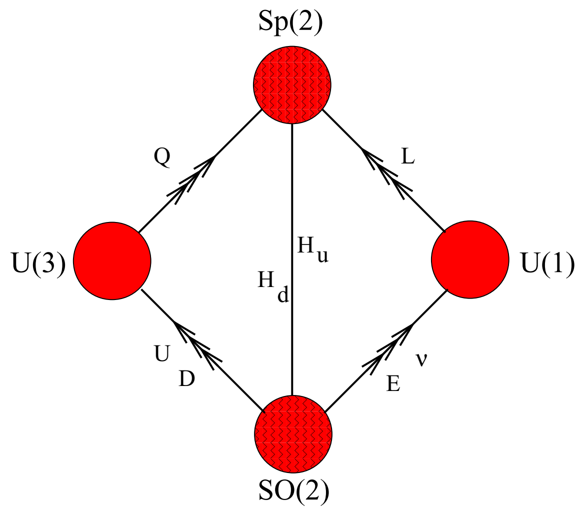

Figure 7: An MSSM-like quiver gauge theory, satisfying all rules for world-volume theories in an unoriented string models.

immediately noticeable is the multitude of Higgs fields, and the fact that supersymmetry is unbroken. Supersymmetry breaking effects may get generated via various mechanisms: via fluxes, nearby anti-branes, non-perturbative string physics, etc. The structure of the SUSY breaking and μ-terms are strongly restricted by phenomenological constraints, such as the suppression of flavor changing neutral currents. However, we see no a priori obstruction to the existence of mechanisms that would sufficiently lift the masses of all extra Higgses and effectively eliminate them from the low energy spectrum.

The presence of the extra Higgs fields is dictated via the requirement (on all D-brane constructions on orientable CY singularities) that each node should have an equal number of in- and out-going lines. To eliminate this feature, it is natural to look for generalizations among gauge theories on orientifolds of CY singularities. Near orientifold planes, D-branes can support real gauge groups like $SO(2N)$ or $Sp(N)$. With this generalization, one can draw a more minimal quiver extension of the SM, with fewer Higgs fields. An example of such a quiver is drawn in fig 7. It should be straightforward to find an orientifolded CY singularity and fractional brane configuration that would reproduce this quiver. The extra $U(1)$ factors in fig 7 can then be dealt with in a similar way as in our dP_8 example.

Chapter 4

Metastable SUSY Breaking in String Theory

In this chapter, we turn our attention to stringy embeddings of the hidden sector.[1] In particular, we provide a geometrical recipe for realizing metastable SUSY breaking. The principal example of the section is the geometrization of the Intriligator-Seiberg-Shih (ISS) metastable SUSY breaking model. To engineer this field theory, we consider a Calabi-Yau geometry with a non-isolated singularity passing through an isolated one in the case that there exists a deformation of the non-isolated singularity. By putting some number of branes on the isolated singularity and some number on the non-isolated one, we can then deform the non-isolated singularity in such a way as to generate an F-term that results in DSB in a way that is analogous to the ISS setup.

Finally, we consider a D-brane realization of the Intriligator-Thomas-Izawa-Yanagida (ITIY) model on a deformed A_3 singularity with an overlapping O3 plane. By placing an appropriate number of fractional branes at contiguous Sp and SO nodes in the corresponding quiver, we find a quantum deformed moduli space that breaks SUSY. In this example non-perturbative contributions from D-brane instantons at the remaining empty Sp node lead to parametrically small R-symmetry breaking effects that restore SUSY far away in

[1] This chapter is based on the paper "On the Geometry of Metastable Supersymmetry Breaking," written in collaboration with D. Malyshev and H. Verlinde. [97]

field space.

4.1 Introduction

As we have alluded to in the introduction, understanding the SUSY breaking hidden sector and finding robust string realizations of gauge theory models that exhibit dynamical SUSY breaking (DSB) is an important facet of string phenomenology. While recent studies have uncovered a growing number of string systems with DSB, there are only a few examples known in which the SUSY breaking mechanism is well understood from both the gauge theory side and the geometric string perspective.

As explained in the introduction, dynamical SUSY breaking assumes that the lagrangian is supersymmetric but that, due to non-perturbative dynamics, the vacuum configuration breaks SUSY at an exponentially low scale [12, 98]. In general, such non-supersymmetric vacuum states need not be the true vacuum of the theory, but may instead represent long-lived metastable states. While controlled examples of metastable vacua in string theory have been known for some time [99], the increased recent interest in their manifestations and properties was sparked by the discovery by Intriligator, Seiberg, and Shih of a metastable SUSY breaking vacuum for SQCD with $N_f > N_c$ massive flavors [14]. Realizations of the ISS mechanism in string theory, as well as other stringy systems with metastable SUSY breaking, have since been found [100, 101, 102, 103]. Another recent advancement has been to merge the calculational power of geometric transitions with insights from field theory to engineer basic field theoretic models of SUSY breaking [104].

In this chapter, we will consider gauge theories on D-branes near a singularity inside a Calabi-Yau manifold. Our goal is to identify a general geometric criterion for the existence of F-type SUSY breaking, and to use this insight to construct simple examples of D-brane systems that exhibit metastable SUSY breaking. F-type SUSY breaking corresponds to the unsolvability of F-term equations

$$\frac{\partial W}{\partial \Phi} \neq 0$$

where $W(\Phi)$ is a superpotential depending on the chiral field Φ. The simplest example of this type is the Polonyi model, consisting of a single chiral field with superpotential $W(\Phi) = f\Phi$ in which SUSY is broken by the non zero vacuum energy $V \sim |f|^2$.

We will assume that the non-perturbative dynamics manifests itself in deformations of a theory with unbroken SUSY. The main purpose of our chapter is to study the consequences of these deformations. In the context of D-branes in IIB string theory, deformations of the superpotential correspond to complex deformations in the local geometry. The deformed geometry still satisfies the Calabi-Yau condition and the D-brane lagrangian is fully supersymmetric but the vacuum configuration of the gauge theory breaks SUSY spontaneously. In the geometric setting, this corresponds to a D-brane configuration that, while submerged inside a supersymmetric background, gets trapped in a non-supersymmetric ground state.

As a simple illustrative example, consider type IIB string theory on a $\mathbf{C}^2/\mathbf{Z}_2$ orbifold singularity [20][60], with N fractional D5-branes wrapped on the collapsed 2-cycle. The corresponding field theory consists of a $U(N)$ gauge theory with a complex adjoint chiral field Φ. Since the $\mathbf{Z}_2$ orbifold locus defines a non-isolated singularity inside $\mathbf{C}^3$, the fractional D5-branes are free to move along a complex line. The location of the N branes along the non-isolated singularity is parameterized by the N diagonal entries of the complex field Φ. As we discuss in more detail in section 2, there exist a deformation of the singularity that corresponds to adding the F-term

$$W = \zeta\, \mathrm{Tr}\Phi$$

to the superpotential. Geometrically, the parameter ζ is proportional to the period of the holomorphic two-form over the deformed 2-cycle. The fractional D-brane gauge theory then breaks SUSY in a similar way to the Polonyi model. This simple observation lies at the heart of many type IIB D-brane constructions of gauge theories that exhibit F-term SUSY breaking.[2] SUSY breaking via D-terms can be described analogously.[3]

[2] F-term SUSY breaking in type IIB D-brane constructions naturally involves deformed non-isolated singularities, that support finite size 2-cycles which D5-branes can wrap [20]. Deformations of isolated singularities correspond to 3-cycles that in type IIB cannot by wrapped by the space-time filling D-branes.

[3] Turning on the FI parameters of the type IIB D-brane gauge theory amounts to blowing up the collapsed

We wish to use this simple geometric insight to construct more interesting gauge theories with DSB, and in particular, with ISS-type SUSY breaking and restoration. When viewed as a quiver theory, the ISS model has two nodes, a "color" node with gauge group $SU(N)$ and a "flavor" node with $SU(N_f)$ symmetry. The "flavor" node has an adjoint field. This suggests that the flavor node must be represented by a stack of N_f fractional branes on a non-isolated singularity. The "color" node, on the other hand, does not have an adjoint, and thus corresponds to branes that are bound to a fixed location. The natural representation for the color node is via a stack of N branes placed at an isolated singularity.

Our geometric recipe for realizing an ISS model in IIB string theory is as follows:

1. Find a Calabi-Yau geometry with a non-isolated singularity passing through an isolated singularity such that there exists a deformation of the non-isolated singularity.

2. Put some number of D-branes on the isolated singularity and some number of fractional branes on the non-isolated singularity. By conservation of charge, the branes can not leave the non-isolated singularity.

3. When we deform the non-isolated singularity, an F-term gets generated that results in dynamical SUSY breaking. The fractional branes have a non-zero volume, and their tension lifts the vacuum energy above that of the SUSY vacuum.

4. There is a classical modulus corresponding to the motion of the fractional branes along the non-isolated singularity. This modulus can be fixed in a way similar to ISS, by the interaction with the branes at the isolated singularity.

Following this recipe we will geometrically engineer, via an appropriate choice of the geometry and fractional branes, gauge theories that are known to exhibit meta-stable DSB. The eventual goal is to fully explain in geometric terms all field theoretic ingredients: the field content and couplings, the meta-stability of the SUSY-breaking vacuum, and the process of SUSY restoration. While in our examples we will be able identify all these ingredients, we will not have sufficient dynamical control over the D-brane set-up to in fact *prove* the

two-cycles of a CY singularity. These blowup modes are Kähler deformations of the geometry, and are somewhat harder to control in a type IIB setup than the complex structure deformations that we use in our study.

existence of a meta-stable state on the geometric side. Rather, by controlling the geometric engineering dictionary, we can rely on the field theory analysis to demonstrate that the system has the required properties.

This wish to have geometrical control over the field theory parameters also motivates why we prefer to work with local IIB D-brane constructions. Although we will work in a probe approximation, in principle we could extend our analysis to the case where the number of branes becomes large. In this AdS/CFT limit, there should exist a precise dictionary between the couplings in the field theory and the asymptotic boundary conditions on the supergravity fields [2]. By changing these boundary conditions one can tune the UV couplings. This in principle allows full control over the IR couplings and dynamics.

The organization of the chapter is as follows. In section 2, as a warm-up, we discuss the F-term deformation of D-branes on $\mathbf{C}^2/\mathbf{Z}_2$. In section 3 we describe the realization of meta-stable supersymmetry breaking via D-branes on the suspended pinch point singularity. We find that supersymmetry restoration involves a geometric transition. In section 4 we give the IIA dual description of the same system and find that it is similar to the IIA constructions of [101, 105]. Finally, in section 5, we present a D-brane realization of the Intriligator-Thomas-Izawa-Yanagida model [107, 108], as an example of a system in which the F-term, that triggers SUSY breaking, is dynamically generated via a quantum deformation of the moduli space.

When the paper that this chapter is based on was close to completion, an interesting paper [104] appeared in which closely related results were reported.[4] In agreement with our observations, in [104] the F-term SUSY breaking takes place due to the presence of fractional D5-branes on slightly deformed non-isolated singularities. One of the main points in [104] was to show that the deformation can be computed exactly in the framework of geometric transitions: this is an important step in finding calculable examples of SUSY breaking in string theory. The main point of this chapter is to identify simple geometric criteria for the existence of SUSY breaking vacua that can have more direct applications in model building.

[4]The IIB string realizations of DSB found in [104] were motivated by the earlier related work [109] in type IIA theory, and by the idea of retrofitting simple systems with DSB, put forward in [110].

4.2 Deformed $\mathbf{C}^2/\mathbf{Z}_2$

The $\mathbf{C}^2/\mathbf{Z}_2$ singularity, or A_1 singularity, is described by the following complex equation in $\mathbf{C}^3$

$$cd = a^2, \qquad (a,b,c) \in \mathbf{C}^3\,. \tag{4.2.1}$$

A D3-brane on $\mathbf{C}^2/\mathbf{Z}_2$ has a single image brane. The brane and image brane recombine in two fractional branes. Correspondingly, the quiver gauge theory for N D3-branes at the A_1 singularity has two $U(N)$ gauge groups. It also has two adjoint matter fields Φ_1 and Φ_2 (one for each gauge group), and two pairs of chiral fields A_i and B_j $i,j = 1,2$ in the bifundamental representations $(N,\overline{N})$ and $(\overline{N},N)$ [20][60]. The superpotential reads

$$W = g\,\mathrm{Tr}\Phi_1(A_1B_2 - B_1A_2) + g\,\mathrm{Tr}\Phi_2(A_2B_1 - B_2A_1)$$

A D3-brane has 3 transverse complex dimensions. The transverse space $\mathbf{C}^2/\mathbf{Z}_2 \times \mathbf{C}$ has a non-isolated A_1 singularity. It is therefore possible to separate the fractional branes. This corresponds to giving different vevs to the two adjoint fields. In the limit of infinite separation one can consider a theory with only one type of fractional brane. This theory consists of a $U(N)$ gauge field with one adjoint matter field and no fundamental matter.

Let us add an F and a D-term

$$W_F = \zeta\,\mathrm{Tr}(\Phi_2 - \Phi_1), \qquad V_D = \xi\,\mathrm{Tr}(D_2 - D_1).$$

The resulting F and D-term equations read

$$\Phi_1A_1 - A_1\Phi_2 = 0, \qquad A_2\Phi_1 - \Phi_2A_2 = 0, \qquad A_1B_2 - B_1A_2 = \zeta,$$

$$\Phi_1B_1 - B_1\Phi_2 = 0, \qquad B_2\Phi_1 - \Phi_2B_2 = 0, \qquad |A_1|^2 + |B_1|^2 - |A_2|^2 - |B_2|^2 = \xi.$$

These equations allow for a supersymmetric solution, provided we set the adjoint vevs to be equal, $\Phi_1 = \Phi_2$. For generic ζ and ξ, some of the A and B fields acquire vevs and break the $U(N)\times U(N)$ symmetry to a diagonal $U(N)$. This corresponds to joining the $2N$ fractional branes into N D3-branes. The space of solutions of the F and D-term equations is the space where the D3-brane moves, which turns out to be a deformed A_1 singularity described by the equation[5]

$$cd = a(a - \zeta), \tag{4.2.2}$$

where $c = A_1 A_2$, $d = B_1 B_2$ and $a = A_1 B_2 = B_1 A_2 + \zeta$ are the gauge invariant combinations of the fields (in the last definition we used the F-term equation for the Φ field).

The F-term coefficient ζ deforms the singularity, the D-term coefficient, or FI parameter, ξ represents a resolution of the $\mathbf{Z}_2$ singularity. In two complex dimensions both the resolution and the deformation correspond to inserting a two-cycle, $E \sim \mathbf{P}^1$, instead of the singular point. The parameters ξ and ζ are identified with the periods of the Kahler form and the holomorphic two-form on the blown up 2-cycle E

$$\xi = \int_E J\,, \qquad\qquad \zeta = \int_E \Omega^{(2)}\,.$$

The non-supersymmetric vacuum state arises in the regime where the vevs of the two adjoint fields Φ_1 and Φ_2 are both different. Geometrically, this amounts to separating the two stacks of fractional branes. The bifundamental fields (A_i, B_i), which arise as the ground states of open strings that stretch between the two fractional branes, then become massive. In the deformed theory, the F-term equations can not be satisfied and SUSY is broken. In the extreme case, where one of the two stacks of fractional branes has been moved off to infinity, so that e.g. $\langle\Phi_2\rangle \to \infty$, the system reduces to the Polonyi model: a single $U(N)$ gauge theory with a complex adjoint Φ_1 and superpotential $W = \zeta\,\mathrm{Tr}\Phi_1$. The vacuum energy $V = N|\zeta|^2$ is interpreted as the tension of the N fractional branes wrapped over the deformed two-cycle.

Strictly speaking the single stack of fractional branes on a deformed singularity is a supersymmetric configuration (one manifestation is that the spectrum of particles in Polonyi model is supersymmetric). In order to break SUSY we really need the second stack of different fractional branes on a large but finite distance. In this case, the SUSY breaking vacuum is not stable due to the attraction between the two stacks of branes.

[5]The general deformations of orbifold singularities of $\mathbf{C}^2$ where found by Kronheimer [111] as some hyperkahler quotients. Douglas and Moore noticed [20] that these hyperkahler quotients are described by the F and D-term equations for D-branes at the corresponding orbifold singularities.

Before we get to our main example of the SPP singularity, let us make a few comments:

1. The gauge theory on N fractional branes on the $\mathbf{C}^2/Z_2$ singularity is an $\mathcal{N}=2$ $U(N)$ theory. If we deform the singularity, then SUSY is broken, whereas in general, $\mathcal{N}=2$ theories are not assumed to have SUSY breaking vacua (see, e.g., Appendix D of [14]). The point is that the SUSY breaking occurs in the $U(1)$ part of $U(N)$ that decouples from $SU(N)$. Moreover the $\mathcal{N}=2$ $U(1)$ theory consists of two non-interacting $\mathcal{N}=1$ theories: a vector boson and a chiral field. Thus the chiral field $\varphi = \mathrm{Tr}\Phi$, responsible for SUSY breaking, is decoupled from the rest of the fields in $\mathcal{N}=2$ $U(N)$ and SUSY is broken in the same way as in the Polonyi model.

2. In general, we consider $\mathcal{N}=1$ theories on isolated singularities that intersect non-isolated singularities. With appropriate tuning of the couplings, the fractional branes wrapping the non-isolated cycles provide an $\mathcal{N}=2$ subsector in the $\mathcal{N}=1$ quiver. Removing the D-branes along the non-isolated singularity reduces the field theory on their world volume to $\mathcal{N}=2$ SYM. For this reason the fractional branes on the non-isolated singularity can be called $\mathcal{N}=2$ fractional branes [112][113]. Similarly to $\mathbf{C}^2/Z_2$ example, the presence of $\mathcal{N}=2$ fractional branes on slightly deformed non-isolated singularity breaks SUSY.

3. The use of $\mathcal{N}=2$ fractional branes is the distinguishing property of our construction from SUSY breaking by obstructed geometry [112][114][115]. The presence of the non-isolated singularity enables the relevant RR-fluxes escape to infinity without creating a contradiction with the geometric deformations. In this way one can avoid the generic runaway behavior (see, e.g., [73] [116]) of obstructed geometries (in our case we still need to take the one loop corrections to the potential into account in order to stabilize the flat direction along the non-isolated singularity).

4.3 ISS from the Suspended Pinch Point singularity

In this section, we will show how to engineer a gauge theory with ISS-type SUSY breaking by placing fractional branes on the suspended pinch point (SPP) singularity. First, however, we summarize the arguments that lead us to consider this particular system.

As we have seen in the previous section, several aspects of the ISS model are quite similar to the $\mathbf{C}^2/Z_2 = A_1$ quiver theory. The term linear in the adjoint in the ISS superpotential is the ζ deformation of the A_1 singularity. Both models have two gauge groups (the global flavor symmetry $SU(N_f)$ in ISS can be thought of as a weakly coupled gauge symmetry). The flavor gauge group is bigger than the color gauge group – this can be achieved in the A_1 quiver by introducing an excess of fractional branes of one type. The vevs of bifundamental fields break $SU(N_f) \times SU(N) \to SU(N)_{diag} \times SU(N_f - N)$. The breaking of $SU(N) \times SU(N) \to SU(N)_{diag}$ corresponds to recombination of N pairs of fractional branes into N (supersymmetric) D3-branes. The vacuum energy is proportional to the tension of the remaining $N_f - N$ fractional branes.

There is however an important difference between the two systems. In ISS it is crucial that the color node $SU(N)$ doesn't have an adjoint field and that all the classical moduli are lifted by one loop corrections. In the $\mathbf{C}^2/\mathbf{Z}_2$ orbifold there is also an adjoint in the "flavor" node. Giving equal vevs to the two adjoints in the $\mathbf{C}^2/Z_2$ quiver corresponds to the "center of mass" motion of the system of branes along the non-isolated singularity. This mode doesn't receive corrections and remains a flat direction.

Thus, the key distinguishing feature of ISS relative to the $\mathbf{C}^2/\mathbf{Z}_2$ model is that the color gauge group $SU(N)$ has no adjoints. For constructing a geometric set-up, we need a mechanism that fixes the position of the N D3-branes. The gauge theories without adjoint fields are naturally engineered by placing D-branes on isolated singularities.

Our strategy will be to find an example of a geometry that has a non-isolated A_1 singularity that at some point gets enhanced by an isolated singularity. The fractional branes on the A_1 will provide the $SU(N_f)$ symmetry; they interact with N branes at the isolated singularity, that carry the $SU(N)$ color gauge group. Such systems are easy to engineer. The most basic examples are provided by the generalized conifolds [117], the simplest of which is the suspended pinch point singularity.[6]

[6] The relevance of generalized conifolds and, in particular, the suspended pinch point was stressed to us by Igor Klebanov. See also [100][118][119] for the earlier constructions of the metastable SUSY breaking vacua in the generalized conifolds.

A similar mechanism of dynamical SUSY breaking for the SPP singularity was previously considered in [113]: SUSY is broken by the presence of D-branes on the deformed A_1 singularity. The essential difference is that in our case the A_1 singularity is deformed without the conifold transitions within the SPP geometry. In fact, we will show that the conifold transition is responsible for SUSY restoration.

4.3.1 D-branes at a deformed SPP singularity

The suspended pinch point (SPP) singularity may be obtained via a partial resolution of a $\mathbf{Z}_2 \times \mathbf{Z}_2$ singularity [120]. It is described by the following complex equation in $\mathbf{C}^4$

$$cd = a^2 b, \qquad\qquad (a,b,c,d) \in \mathbf{C}^4 \,. \tag{4.3.3}$$

There is a $\mathbf{C}^2/\mathbf{Z}_2$ singularity along $b \neq 0$. The quiver gauge theory for N D3-branes at the SPP singularity is shown in figure 4.1. It was derived in [120] by turning on an FI parameter ξ in the $\mathbf{Z}_2 \times \mathbf{Z}_2$ quiver gauge theory, and working out the resulting symmetry breaking pattern. The superpotential of the SPP quiver gauge theory reads

$$W = \mathrm{Tr}\left(\Phi(\widetilde{Y}Y - \widetilde{X}X) + h(Z\widetilde{Z}X\widetilde{X} - \widetilde{Z}ZY\widetilde{Y})\right) \tag{4.3.4}$$

where h is a dimensionful parameter (related to the FI parameter via $h = \xi^{-1/2}$).

As a quick consistency check that this theory corresponds to a stack of D3-branes on the SPP singularity, consider the F-term equations for a single D3-brane. The gauge invariant combinations of the fields are

$$\begin{aligned} a &= \widetilde{X}X = \widetilde{Y}Y \qquad\qquad & c &= X\widetilde{Y}\widetilde{Z} \\ b &= Z\widetilde{Z} & d &= Y\widetilde{X}Z \end{aligned} \tag{4.3.5}$$

where we used the F-term equation for Φ. These quantities (a,b,c,d) satisfy the constraint $cd = a^2 b$, which is the same as the equation for the SPP singularity.

Following our recipe as outlined in the introduction, we now deform the non-isolated A_1 singularity inside the SPP as follows

$$cd = a(a-\zeta)b. \tag{4.3.6}$$

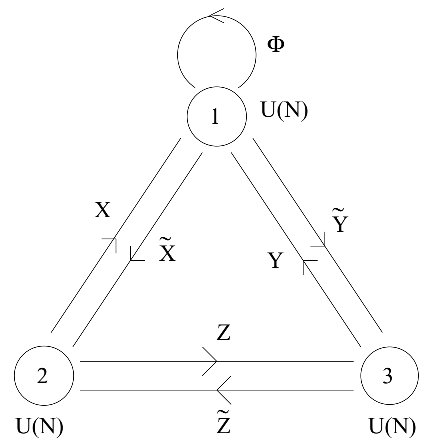

Figure 4.1: Quiver gauge theory for N D3-branes at a suspended pinch point singularity.

This deformation removes the A_1 singularity, replacing it by a finite size 2-cycle. The deformed SPP geometry has two conifold singularities, located at $a = 0$ and $a = \zeta$, with all other coordinates equal to zero. In the field theory, the above deformation corresponds to adding an F-term of the form

$$W_\zeta = -\zeta \mathrm{Tr}(\Phi - h\widetilde{Z}Z). \tag{4.3.7}$$

This extra superpotential term is chosen such that the F-term equations for Φ and Z

$$\widetilde{X}X - \widetilde{Y}Y - \zeta = 0, \qquad \widetilde{Z}(\widetilde{Y}Y - \widetilde{X}X + \zeta) = 0, \tag{4.3.8}$$

are compatible.

The correspondence between (4.3.7) and (4.3.6) is easily verified. Again, consider the gauge theory on a single D3-brane. In view of the deformed F-term equation, the quantity a now needs to be defined via

$$a = \widetilde{Y}Y = \widetilde{X}X + \zeta. \tag{4.3.9}$$

The constraint equation thus gets modified to $cd = a(a - \zeta)b$, which is the equation for the deformed SPP singularity.

As we increase ζ, the two conifold singularities at $a = 0$ and $a = \zeta$ become geometrically separated and the D-branes end up on either of the two conifolds. The field theory should thus contain two copies of the conifold quiver gauge theory. To verify this, consider the vacuum $Y = \widetilde{Y} = \sqrt{\zeta} I$, which solves both the F-term equations (4.3.8) and the D-term equations $|Y|^2 - |\widetilde{Y}|^2 = 0$. These vevs break the gauge group $SU(N)_1 \times SU(N)_3$ to $SU(N)_{diag}$ and give a mass to the Higgs-Goldstone field $Y_- = \frac{1}{\sqrt{2}}(Y - \widetilde{Y})$. Substituting the remaining fields in the superpotential, one finds that the fields Φ and $Y_+ = \frac{1}{\sqrt{2}}(Y + \widetilde{Y})$ are also massive. The surviving massless fields with the superpotential

$$W_{\text{con}} = h(Z\widetilde{Z}X\widetilde{X} - \widetilde{Z}Z\widetilde{X}X) \tag{4.3.10}$$

reproduce the conifold quiver gauge theory.

In general, both X and Y have vevs and the D-branes split into two stacks $N_1 + N_2 = N$ that live on the two conifolds. Note, that the Z field in (4.3.7) corresponds to strings stretching between the two conifolds. The mass of this field is proportional to the length of the string given by the size of the deformed two-cycle.

4.3.2 Dynamical SUSY breaking

A straightforward way to generate dynamical SUSY breaking is to reproduce the ISS model by placing some fractional branes on the SPP singularity. Suppose that there are $N_f = N + M$ fractional branes corresponding to node 1 in figure 4.1, N fractional branes corresponding to node 3, and no fractional branes at node 2. The reduced quiver diagram is shown in figure 4.2. The superpotential for this quiver gauge theory is

$$W = h\,\zeta\,\text{Tr}(\Phi) - h\,\text{Tr}(\Phi Y \widetilde{Y})\,, \tag{4.3.11}$$

which is the same as the ISS superpotential in the IR limit [14], with the $SU(N)$ identified as the "color" group and $SU(N+M)$ as the "flavor" symmetry. The only difference between our gauge theory and the ISS system is that the "flavor" symmetry is gauged. The corresponding gauge coupling is proportional to a certain period of the B-field. We can tune it to be small and treat the gauge group as a global symmetry in the analysis of stability of the vacuum.[7]

[7] In fact, the restriction on the coupling is not very strong, because the SUSY breaking field $\text{Tr}\Phi$ couples only to the bifundamental fields Y, $\widetilde{Y}$ through the superpotential (4.3.11) (see also figure 4.2). Since the

An empty node in the quiver introduces some subtleties, since there might be instabilities or flat directions at the last step of duality cascade leading to this empty node. In the final section of this chapter, we show that this quiver can be obtained after one Seiberg duality from an SPP quiver without empty nodes.

Recall that in the field theory SUSY is broken since the F-term equations for Φ

$$Y\widetilde{Y} = \zeta\, \mathbf{1}_{N+M} \tag{4.3.12}$$

cannot be satisfied by the rank condition. In the vacuum where

$$Y\widetilde{Y} = \zeta\, \mathbf{1}_{N}, \tag{4.3.13}$$

the $SU(N)_1 \times SU(N+M)_3$ gauge symmetry is broken to $SU(N)_{diag} \times SU(M)_3$. The superpotential for the remaining $M \times M$ part of the adjoint field reduces to the Polonyi form

$$W = h\, \zeta\, \mathrm{Tr}_M(\Phi). \tag{4.3.14}$$

The metastable ground state thus has a vacuum energy proportional to $Mh^2\zeta^2$.

We can interpret the SUSY breaking vacuum on the geometric side as follows. Our system contains N fractional branes that wrap one of the conifolds inside the deformed SPP singularity, and $(N+M)$ fractional branes that wrap the 2-cycle of the deformed A_1. The $\Phi = 0$ vacuum corresponds to putting all the $(N+M)$ fractional branes on top of the N branes at the conifold (see fig. 4.2).

The Y modes represent the massless ground states of the open strings that connect the two types of branes. The non-zero expectation value (4.3.13) for $Y\widetilde{Y}$ corresponds to a condensate of these massless strings between N branes wrapping α_3 and N branes wrapping $\alpha_1 = -\alpha_2 - \alpha_3$. As a result of condensation, these two stacks of N fractional branes recombine into N fractional branes wrapping $-\alpha_2$ at the second conifold. The remaining M fractional branes around the deformed A_1 end up in a non-supersymmetric state. The

stabilization of the SUSY vacuum comes from the masses of these bifundamental fields it is sufficient to require that the corrections to the masses due to the gauge interactions are small at the SUSY breaking scale.

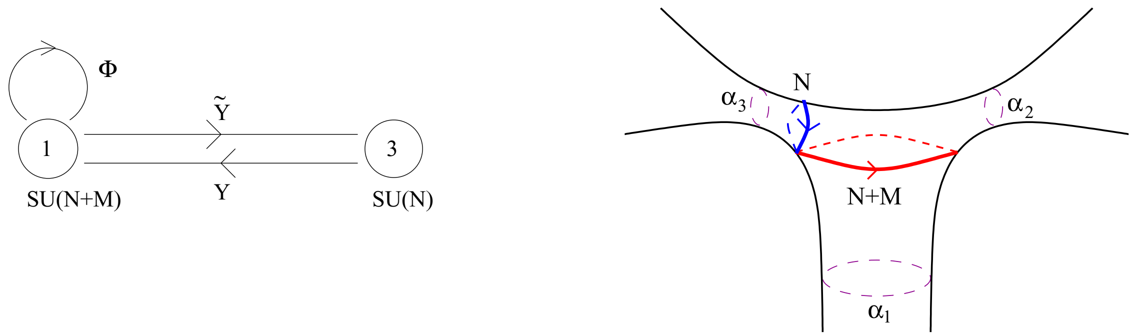

Figure 4.2: A particular combination of fractional branes on the SPP singularity and the corresponding quiver gauge theory that reproduce the ISS model. The cycle α_1 is a non-isolated two-cycle of the deformed A_1 singularity inside the SPP. The cycles α_2 and α_3 denote the isolated two-cycles on the two conifolds that remain after the deformation of the A_1 singularity. The cycles satisfy $\alpha_1+\alpha_2+\alpha_3 = 0$. The N fractional branes wrapping α_3 are supersymmetric. The $N+M$ fractional branes wrapping α_1 break SUSY. This combination of fractional branes corresponds to zero vevs of the bifundamental fields in the ISS.

diagonal entries of the $M\times M$ block in Φ parameterize the motion of the M branes along the deformed non-isolated singularity. The corresponding configuration of branes is represented in figure 4.3.

The stability of the SUSY breaking vacuum is a quantum effect in the field theory—there are pseudo-moduli that acquire a stabilizing potential at one loop [14]. In the D-brane picture this should correspond to the back reaction of the branes that makes the two-cycle at the deformed A_1 singularity grow as one moves away from the conifold. (Alternatively one can think about a weak attraction between the branes.) It would be interesting to derive this directly from SUGRA equations, since it would complete the geometric evidence for the existence of the SUSY breaking vacuum.

4.3.3 SUSY restoration

Let us discuss SUSY restoration in this setup. The SUSY vacuum is found by separating the $(N + M)$ fractional branes on the deformed $\mathbf{C}^2/\mathbf{Z}_2$ singularity from the N fractional branes at the conifold. This separation amounts to giving a vev to Φ. Initially this costs energy. The fields Y and $\widetilde{Y}$ become massive. Below their mass scale, the theory on the N

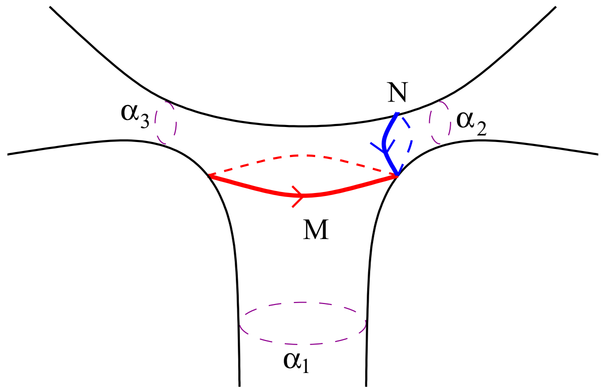

Figure 4.3: In the metastable vacuum. N supersymmetric fractional branes wrap the $-\alpha_2$ cycle of the second conifold. The remaining M fractional branes wrap the non-isolated cycle $\alpha_1 = -\alpha_2 - \alpha_3$ and are weakly bound to the N branes at the conifold. This configuration of fractional branes is obtained from the configuration in figure 4.2 by giving vevs to the bifundamental fields.

fractional branes at the conifold becomes strongly coupled and develops a gaugino condensate. This condensate deforms the conifold singularity, and generates an extra term in the superpotential for Φ that eventually restores SUSY.

On the gauge theory side, the SUSY restoring superpotential term arises due to the fact that the value of the gaugino condensate depends on the masses of Y and $\widetilde{Y}$, and these in turn depend on the vev of Φ. As a result [14], the gaugino condensation modifies the superpotential for Φ to (here $N_f = N + M$)

$$W_{low} = N\Big(h^{N_f}\Lambda_m^{-(N_f-3N)}\det\Phi\Big)^{1/N} - h\zeta\,\mathrm{Tr}\Phi. \tag{4.3.15}$$

Due to the extra term, the F-term equations

$$\frac{\partial W_{low}}{\partial\Phi} = 0 \tag{4.3.16}$$

can be solved. In fact there are $N_f - N = M$ SUSY vacua $\Phi = \Phi_k$, with $k = 1, .., M$.

On the geometric side, the SUSY vacuum is interpreted as the ground state of $N + M$ fractional branes in the presence of a deformed conifold singularity. Suppose that the deformed conifold is the one located at $a = \zeta$. One can describe the situation after the geometric transition by the following equation

$$cd = a((a-\zeta)b + \epsilon_0)\,. \tag{4.3.17}$$

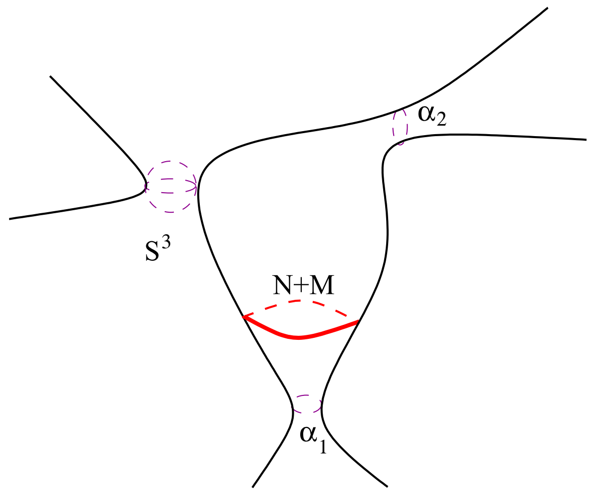

Figure 4.4: To reach the supersymmetric ground state, the $N+M$ fractional branes on the A_1 2-cycle move away from the conifold. The N fractional branes on the conifold then drive the geometric transition: the two-cycle α_3 is replaced by the three sphere S^3. After the transition, the size of the A_1 2-cycle reaches a zero minimum at a new conifold singularity (indicated by the position of α_1).

The original conifold singularity at $a = \zeta$ is now a smooth point in the geometry. However, a new singularity has appeared in the form of an undeformed conifold at $a = c = d = 0$ and $b = \epsilon_0/\zeta$. The D5-branes that were originally stretching between $a = 0$ and $a = \zeta$ can thus collapse to a supersymmetric state by wrapping the zero-size 2-cycle of the undeformed conifold. This process is the geometric manifestation of SUSY restoration in the underlying ISS gauge theory.[8]

Using the geometric dual description, it is possible to rederive the field theory superpotential (4.3.15) and even compute higher-order corrections. The calculation goes as follows, [104]. Let us rewrite the geometry (4.3.17) as:

$$uv = (z-x)((z+x)(z-x-\zeta)+\epsilon_0)\,, \tag{4.3.18}$$

where $z - x = a$, $z + x = b$. Also it is useful to introduce the following notation

$$z_1(x) = x \qquad \widetilde{z}_2(x) = \zeta/2 - \sqrt{(x+\zeta/2)^2 - \epsilon_0}$$
$$z_2(x) = -x \tag{4.3.19}$$

[8] A similar mechanism of SUSY restoration in the case of SPP singularity was anticipated in [112]

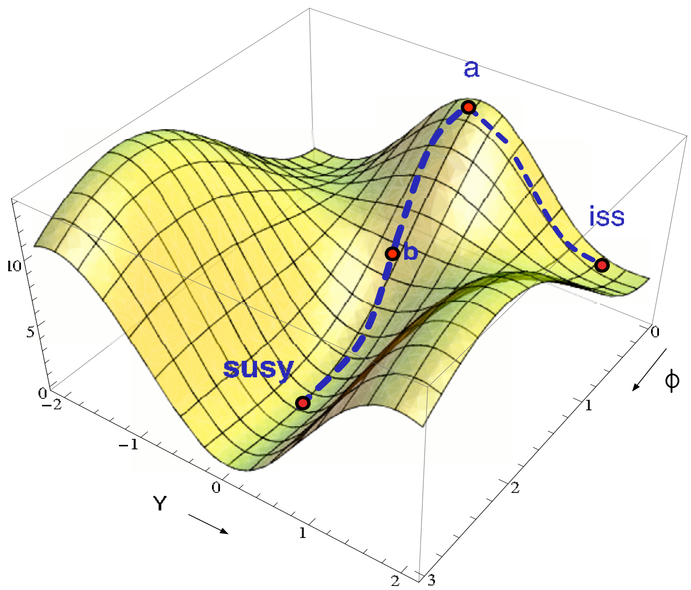

Figure 4.5: The gauge theory potential as a function of the vevs of the adjoint Φ and bifundamental Y. Suppose we start at point **a** corresponding to the situation in fig. 4.2 with the zero vevs of Φ and $Y\widetilde{Y}$. This point is unstable and there are two possibilities. If the bifundamental fields $Y\widetilde{Y}$ get a vev, then we end up in the metastable ISS vacuum in fig. 4.3. If the adjoint field Φ gets a vev, then we follow the path to the SUSY vacuum (an intermediate point on the SUSY restoring path is shown in fig. 4.4).

$$z_3(x) = x + \zeta \qquad \widetilde{z}_3(x) = \zeta/2 + \sqrt{(x+\zeta/2)^2 - \epsilon_0}$$

The conifold singularity is at $z = x = \epsilon_0/2\zeta \equiv x_*$. If initially the fractional D-branes on the deformed A_1 were stretching between $z_1(x)$ and $z_3(x)$, then after the geometric transition, they stretch between $z_1(x)$ and $\widetilde{z}_3(x)$. They can minimize their energy by moving (or tunneling) to the conifold singularity at $z = z_1(x_*) = \widetilde{z}_3(x_*)$.

For the geometric derivation of the superpotential, we take the deformation parameter ϵ_0 to be dynamical, and related to the gaugino condensate via $\epsilon_0 = 2S$.[9] We also identify Φ with the location x of the D5-branes relative to the (deformed) conifold at $a = \zeta$. The superpotential for the gaugino condensate together with the adjoint field is [104]

$$W(S,\Phi) = NS(\log\frac{S}{\Lambda^3} - 1) + \frac{t}{g_s}S + \widetilde{W}(\Phi,S)\,. \tag{4.3.20}$$

[9]The constant 2 appears due to the consistency conditions between the geometric derivation of the superpotential and the KS superpotential for the conifold.

The first two terms comprise the familiar GVW superpotential [121] $W = \int \Omega \wedge G_3$ evaluated for the deformed conifold supported by N units of RR 3-form flux [122][123]. The last term $\widetilde{W}(\Phi, S)$ has a closely related, and equally beautiful, geometric characterization in terms of the integral of holomorphic 3-form

$$\widetilde{W}(\Phi, S) = \int_\Gamma \Omega \tag{4.3.21}$$

over a three chain Γ bounded by the 2-cycle wrapped by the D5 brane.[10] Following [104], we can reduce the integral (4.3.21) for our geometry (4.3.18) to an indefinite 1-d integral

$$\widetilde{W}(x) = \int (\widetilde{z}_3(x) - z_1(x))dx \tag{4.3.22}$$

with $z_1(x)$ and $\widetilde{z}_3(x)$ given in (4.3.19).

Let us show that the geometric expression (4.3.22) reproduces the gauge theory superpotential. In the appropriate limit, $x \gg \epsilon_0, \zeta$, we find from (4.3.22)

$$\widetilde{W}(S, \Phi) = \zeta \mathrm{Tr}\Phi - S \log(\Phi/\Lambda_m)\,. \tag{4.3.23}$$

Here we identify (x, ϵ) with $(\Phi, 2S)$, and use the integration constant to introduce a scale Λ_m. Physically, Λ_m sets the scale of the Landau pole for the IR free theory with $3N < N_f$. Minimizing (4.3.20) with respect to S we find

$$S = \left(\Lambda_m^3 \det(\Phi/\Lambda_m)\right)^{\frac{1}{N}} \tag{4.3.24}$$

If we substitute S back in (4.3.20), we get exactly (4.3.15) (up to an overall sign and after the redefinition $\Phi \to h\Phi$). By expanding the full geometric expression (4.3.22) to higher orders, one can similarly extract the multi-instanton corrections to the superpotential.

Our system in fact has other supersymmetric vacua besides the one just exhibited. These arise because, unlike the ISS-system, the flavor symmetry is gauged. If we move

[10] This contribution to the superpotential is easily understood from the perspective of the GVW superpotential. The D5-brane is an electric source for the RR 6-form potential C_6, and a magnetic source for the RR 3-form field strength $F_3 = dC_2$. If the D5 would traverse some 3-cycle A, this process will induce a jump by one unit in the F_3-flux through the 3-cycle B dual to A, and thereby a corresponding jump in the GVW superpotential. Continuity of the overall superpotential during this process dictates that the D5-brane contribution must take the form (4.3.21).

M fractional branes away from the conifold singularity in figure 4.3, then the N fractional branes wrapping the conifold 2-cycle α_2 may also induce a geometric transition. As in the above discussion, this transition also restores SUSY. For a suitable choice of couplings, the extra SUSY vacuum lies farther away than the one considered above. The ISS regime arises when the coupling of the initial "color" $SU(N)$ gauge group is sufficiently bigger than the coupling of the gauged "flavor" group $SU(N+M)$, $g_3 \gg g_1$. (Note that after the symmetry breaking, the coupling of $SU(N)_{diag} \subset SU(N) \times SU(N+M)$ is of order g_1.) In this section we assumed that we are in this ISS regime.

4.4 Type IIA dual of the SPP singularity

In this section we present the type IIA dual of our discussion of D-branes at the SPP singularity. In particular, we study the F-term deformations in the corresponding system of NS-branes and D-branes and prove that the IIA dual of the SPP singularity is equivalent to the known IIA representations of ISS [101, 105, 106].

D-branes at singularities of CY manifolds in IIB are T-dual to D-branes stretching between NS-branes in type IIA [124][125]. Consider N D3-branes at the SPP singularity described by the following equation in $\mathbf{C}^4$

$$uv = x^2 z. \tag{4.4.25}$$

The resulting space has 6 real dimensions $(x_4, \ldots, x_9)$. Denote $x = x_4 + ix_5$ and $z = x_8 + ix_9$. For $v \neq 0$ one can solve equation (4.4.25) for u. Let $v = re^{i\varphi}$ and denote $x_6 = \varphi$, $x_7 = r$. After T-duality in the compact dimension x_6, we get the configuration of NS branes (blue) and D4-branes (green) in type IIA (this configuration is depicted in figure 4.6 on the left).

The zeros of polynomials on the right hand side of (4.4.25) represent the intersection of NS-branes with the circle in x_6. There is one NS brane at $z = 0$ and two NS$'$ branes at $x = 0$ (we use the prime to distinguish the two NS branes at $x = 0$ from the NS brane at $z = 0$). The NS branes span the following dimensions

$$NS \quad (0\,1\,2\,3\,4\,5)$$

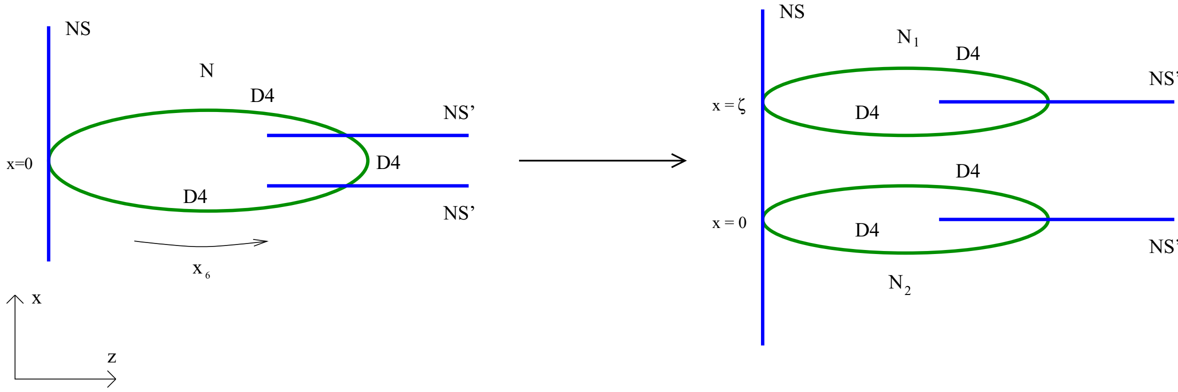

Figure 4.6: On the left there are N D4-branes on the "SPP singularity". After the addition of the F-term ζ, the SPP singularity is transformed to two conifolds at $x = 0$ and $x = \zeta$. The N D4-branes split into $N_1 + N_2 = N$ D4-branes at the two conifolds. (The D4-branes are green and the NS branes are blue.)

$$NS' \quad (0\ 1\ 2\ 3\ 8\ 9)$$

The D4-brane between the two NS$'$ branes can freely move in the z direction. This corresponds to the motion of the fractional D3-branes along the line of Z_2 singularities in the z direction of (4.4.25). The length of the D4 brane in x^6 is mapped, via T-duality, to the period of the B-field on the corresponding shrunken $\mathbf{P}^1$:

$$\Delta x^6 = \int_{\mathbf{P}^1} B \sim \frac{4\pi}{g^2} \tag{4.4.26}$$

The corresponding field theory is the same as the type IIB quiver gauge theory (4.3.4). As we have shown earlier the F-term deformation (4.3.7) corresponds to the deformation of the Z_2 singularity in the SPP

$$uv = x(x - \zeta)z \tag{4.4.27}$$

In the IIA dual picture this corresponds to moving one of the NS' branes from $x = 0$ to $x = \zeta$. This theory has two conifold points: at $x = 0$ and at $x = \zeta$. The corresponding configuration of branes is shown in figure 4.6 on the right.

To get the ISS vacuum we take N_f D4-branes between the two NS$'$ branes and N D4-branes between NS'_2 and NS such that $N_f > N$. The corresponding superpotential is

$$W = \mathrm{Tr}(\zeta\Phi - \Phi\varphi\widetilde{\varphi}) \tag{4.4.28}$$

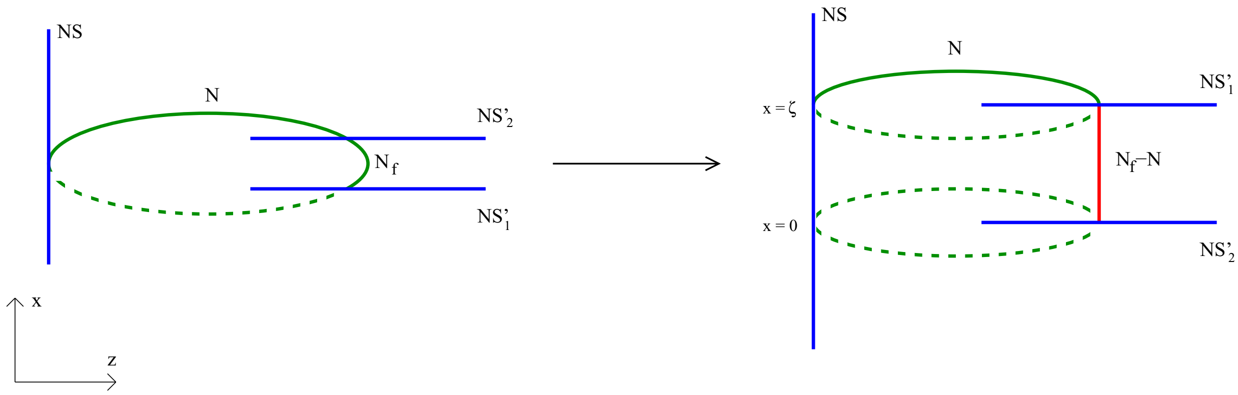

Figure 4.7: On the left there are N_f D4-branes stretching between NS_1' and NS_2' and N D4-branes stretching between NS_2' and NS. After turning on the F-term, ζ, there are N supersymmetric D4-branes (green) stretching between NS_1' and NS and $(N_f - N)$ D4-branes (red) that have non zero size in the x direction and violate the SUSY. The dashed lines represent the empty cycles in the geometry (empty nodes in the corresponding quivers).

The F-term equations for the Φ fields are

$$\varphi\widetilde{\varphi} = \zeta I_{N_f \times N_f}. \tag{4.4.29}$$

The fields φ and $\widetilde{\varphi}$ acquire vevs and break the gauge group as $SU(N_f) \times SU(N) \longrightarrow SU(N)_{diag} \times SU(N_f - N)$. This corresponds to recombination of the D4-branes shown in figure 4.7. The SUSY breaking is due to the $(N_f - N)$ D4-branes stretching a finite distance between $x = 0$ and $x = \zeta$: the tension of these branes creates the vacuum energy. We note, that this configuration of NS-branes and D4-branes is closely related to the constructions of [106] where the $SU(N_f)$ symmetry is slightly gauged compared to the earlier constructions [101, 105] where the $SU(N_f)$ is a flavor symmetry.

4.5 F-term via a Deformed Moduli Space

In the previous sections we introduced the F-terms by hand, assuming that they are generated somewhere else in the geometry and are not affected by the local field theory (see, e.g., the constructions in [104]). In this section we consider an example of F-term generation in the local field theory by a quantum modified moduli space analogous to the Intriligator-

Thomas-Izawa-Yanagida model [107, 108]. Our setup is related to the M-theory example considered in [126].

In order to obtain an ITIY-like model we consider the deformed A_3 singularity in IIB string theory:

$$uv = x^2 z^2. \tag{4.5.30}$$

Recall that the $C^2/Z_4 = A_3$ singularity has the equation $uv = x^4$ in $\mathbf{C}^3$. The corresponding quiver gauge theory [20] for N D3-branes at the A_3 singularity has four $U(N)$ gauge groups, four $\mathcal{N} = 2$ hypermultiplets in bifundamental representations of the gauge groups, and four adjoint fields.

The deformation (4.5.30) corresponds to giving the masses to two adjoint fields on opposite nodes of the C^2/Z_4 quiver. A general derivation of the correspondence between the geometric deformations and the superpotential for the adjoint fields can be found in [127]. Intuitively, an adjoint field gets a mass if the corresponding fractional brane wraps a collapsed two-cycle that has a non-zero volume away from $x = z = 0$. After integrating out the massive adjoint fields, the remaining fields are the four $U(N)$ gauge groups with bifundamental matter between them and two adjoint fields corresponding to the non-isolated Z_2 singularities at $u = v = x = 0$ and $u = v = z = 0$.

Next, let us add an O3 plane located at $u = v = x = z = 0$. We take the action of the O3 plane to be the same as in [128]:

$$u \to v, \quad v \to u, \quad x \to -x, \quad z \to -z \tag{4.5.31}$$

The $U(N)$ gauge groups become $SO(2N+2)$ and $Sp(N)$.

To generate the ITIY model, we occupy two out of the four nodes in the quiver. The corresponding quiver gauge theory is shown in figure 4.8. The N fractional branes corresponding to node 1 give rise to an $Sp(N)$ gauge theory with dynamical scale Λ, while the $N+1$ fractional branes corresponding to node 2 realize an $SO(2N+2)$ theory with dynamical scale Λ'. In our example, the beta function for the $Sp(N)$ gauge group is bigger than for $SO(2N+2)$, i.e. the $Sp(N)$ gauge group confines first. We assume that $\Lambda \gg \Lambda'$ and treat the weakly coupled $SO(2N+2)$ symmetry as global.

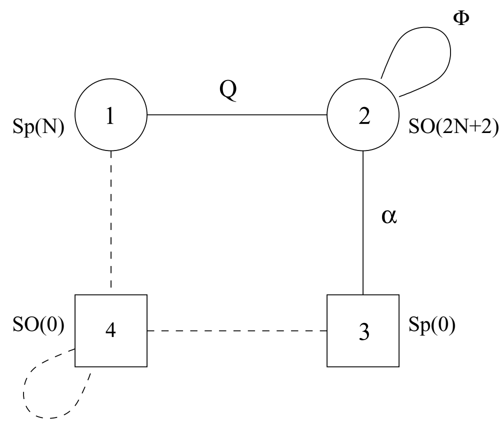

Figure 4.8: Quiver gauge theory that reproduces the ITIY model. It is obtained by putting some fractional branes on an orientifold of the deformed A_3 singularity. The circles represent the occupied nodes, while the squares correspond to the empty nodes in the quiver. The field Q is in the bifundamental representation of $Sp(N) \times SO(2N+2)$. α denotes fermionic zero modes of the D-instantons wrapping the $Sp(0)$ node.

The tree-level superpotential is inherited from the $\mathbf{C}^2/Z_4$ cubic superpotential

$$W = h\Phi_{ij}Q^iQ^j \tag{4.5.32}$$

where Φ is an adjoint of $SO(2N+2)$ and the quarks, Q, transform as bifundamentals of $Sp(N) \times SO(2N+2)$.

Denote the mesons of the $Sp(N)$ gauge group by $M^{ij} = Q^iQ^j$. After the confinement of $Sp(N)$, the theory has a quantum-deformed moduli space of vacua

$$\mathrm{Pf}M = \Lambda^{2N+2} \tag{4.5.33}$$

The superpotential (4.5.32) then becomes

$$\widetilde{W} = h\Phi M + \lambda(\mathrm{Pf}M - \Lambda^{2N+2}). \tag{4.5.34}$$

where λ is the Lagrange multiplier imposing the constraint (4.5.33).

SUSY is broken since the F-term equations for the Φ field cannot be satisfied. Indeed, the deformed moduli space guarantees that

$$-F_\Phi^\dagger = M \sim \Lambda^2 \neq 0. \tag{4.5.35}$$

Note that we needed to introduce the O3 plane in order to (dynamically) break SUSY since otherwise we would have to take baryonic directions $B, \widetilde{B}$ into account in (4.5.33). In the absence of competing effects, the baryons are tachyonic and so our potential would take us to zero vev for M, thus allowing the system to relax to a SUSY groundstate.

In order to get a geometric interpretation of the SUSY breaking, let us solve the F-term equations for the λ and M fields

$$\mathrm{Pf}M - \Lambda^{2N+2} = 0; \tag{4.5.36}$$

$$h\Phi_{ij} + \lambda \mathrm{Pf}M \cdot M^{-1}_{ij} = 0.$$

Then, the superpotential for Φ reads

$$\widetilde{W} = 2h\Lambda^2(N+1)(\mathrm{Pf}\Phi)^{\frac{1}{N+1}}. \tag{4.5.37}$$

Any Φ can be obtained by an $SO(2N+2)$ rotation from a given element Φ_0, $\Phi = O\Phi_0 O^T$, where we take

$$\Phi_0 = \begin{pmatrix} 0 & R \\ -R & 0 \end{pmatrix} \tag{4.5.38}$$

with

$$R = \mathrm{diag}(r_1, ..., r_{N+1}) \tag{4.5.39}$$

The anti-symmetric form of Φ is due to the orientifold projection. Now, plugging (4.5.38) into (4.5.37) and extremizing the resulting potential, we see that

$$V = 4h^2\Lambda^4 \left(\sum_i \frac{1}{|r_i|^2} \right) \prod_j |r_j|^{\frac{2}{N+1}} \geq 4h^2\Lambda^4(N+1) \tag{4.5.40}$$

with the inequality saturated for $r_1 = ... = r_{N+1}$, i.e.

$$\Phi_0 = r \begin{pmatrix} 0 & \mathbf{1}_{N+1} \\ -\mathbf{1}_{N+1} & 0 \end{pmatrix}. \tag{4.5.41}$$

Then $\mathrm{Pf}\Phi = r^{N+1}$ and

$$\widetilde{W} = 2h\Lambda^2(N+1)r. \tag{4.5.42}$$

In other words, this is a Polonyi model in the flat r direction with a set of Goldstone bosons parameterizing the space of broken symmetries $SO(2N+2)/U(N+1)$. In fact, these goldstone bosons will get eaten at the scale Λ since

$$M = \Lambda^2 \begin{pmatrix} 0 & \mathbf{1}_{N+1} \\ -\mathbf{1}_{N+1} & 0 \end{pmatrix}. \tag{4.5.43}$$

as a consequence of satisfying the F-term equations in (4.5.36) (this holds for $\forall r \neq 0$ and therefore holds in the limit $r \to 0$).

Now, by construction, r is uncharged under the $U(N+1)$ group of remaining symmetries. Hence, in particular, we can treat r as a center of mass coordinate of the D-brane system. Then, in analogy with the previous sections, we interpret this superpotential as coming from the complex deformation of the singularity

$$uv = (z - h\Lambda^2)(z + h\Lambda^2)x^2 \tag{4.5.44}$$

Here we take the deformation to be invariant under the O-plane action. In the case of the ITIY model, this geometric interpretation has an important limitation. In the previous constructions we assumed that the deformation parameter ζ is a vev of some field that has a mass much bigger than the scale of ζ, i.e. that we can decouple its dynamics from the D-brane dynamics. For the ITIY, the mass of the M fields is proportional to Λ, i.e. the dynamics of M start to play a role already at the SUSY breaking scale. In other words, the geometric formula (4.5.44) should be trusted only for $x, z, u, v \ll h\Lambda^2$.

Let us now show that SUSY is restored in this model by contributions from the D-instantons wrapping the empty nodes in quiver 4.8. The presence of empty nodes seems rather generic in constructions of the ITIY model from D-branes at singularities. The presence of the O3-plane then allows non trivial D-instanton contributions to the superpotential.[11] In the case of the $Sp(0)$ node, the '+' orientifold projection lifts the additional zero modes of the D-instanton and allows it to contribute to the superpotential [129, 130, 131] (the corresponding D-instanton zero modes are represented by α in figure 4.8), while the

[11] Additional non-perturbative effects in the $U(N+1) \subset SO(2N+2)$ theory are small provided we take Λ' sufficiently small.

'-' sign of the projection on the $SO(0)$ node does not lift the extra zero modes and so no contribution to the superpotential is expected from that node.

Integrating out the fermionic zero modes, α_i, resulting from a Euclidean D1 brane wrapping node 3 gives rise to an exponentially suppressed deformation of the superpotential:

$$W = h\Phi M + \epsilon \mathrm{Pf}\Phi \tag{4.5.45}$$

where the suppression factor is given by:

$$\epsilon \sim e^{-t/g_s} \tag{4.5.46}$$

with t the period of $B^{\mathrm{NS}} + ig_s B^{\mathrm{RR}}$ on the corresponding shrunken 2-cycle. Note that since Φ has $Sp(N)$ non-anomalous R-charge +2, the second term in (4.5.45) breaks the R-symmetry and SUSY will be restored for Φ that satisfies:

$$\Phi \sim \frac{\Lambda^{2+2/N}}{h\epsilon^{1/N}} M^{-1} \sim \left(\frac{\Lambda^2}{h^N \epsilon}\right)^{\frac{1}{N}} \tag{4.5.47}$$

Since ϵ is parametrically small, we can take it such that the model is rendered metastable. For Φ near the origin, however, the stringy instanton contribution will be dominated by the F-term induced by the $Sp(N)$ quantum deformed moduli space. Based on the form of the D-instanton contribution, we are tempted to identify this term with a geometrical transition in (4.5.44). Formally, we can do this and maintain compatibility with the orientifold projection in the limit in which $\Lambda \to 0$ (this reflects the fact that SUSY restoration occurs in an entirely different regime of field space $\Phi \gg \Lambda$):

$$uv = xz(xz - s) \tag{4.5.48}$$

The stability of the SUSY breaking vacuum can be analyzed similarly to [132]. The field r introduced in (4.5.41) is a pseudo-modulus. This pseudo-modulus is lifted upward by corrections to the potential leaving a metastable SUSY-breaking vacuum at the origin, $\Phi = 0$.

One might be worried that contributions from the gauge fields could destabilize the vacuum. The first thing to note is that the r field is not charged under the subgroup

$U(N+1) \subset SO(2N+2)$ unbroken below Λ. The contributions to the potential from the broken $SO(2N+2)/U(N+1)$ gauge sector can be neglected if the corresponding coupling is smaller than the coupling of the matter fields, $g \ll h$, which can be arranged via an appropriate geometric tuning.[12]

4.6 Concluding remarks

In this chapter we presented a simple geometric criterion for the existence of a meta-stable F-term SUSY breaking vacuum in world-volume gauge theories on D-branes. We showed that the basic ingredients of the ISS theory can be realized by placing fractional D-branes on a slightly deformed non-isolated singularity passing through an isolated singularity. We characterized both the meta-stable non-SUSY and stable SUSY vacuum states.

A gap in our study, and an important direction to be explored, is the detailed supergravity analysis of the SUSY breaking vacuum. On the field theory side, the one-loop corrections to the potential are crucial for lifting the classical degeneracy and stabilizing the meta-stable vacuum. In the D-brane picture this corresponds to a weak attraction between the N D-branes at the isolated singularity and the M D-branes at the non-isolated A_1 singularity. This attraction presumably arises due to some back reaction that slightly deforms the 2-cycle of the A_1 singularity, such that its area is minimized near the isolated singularity.

Our construction may be used to introduce SUSY breaking in phenomenological models involving D-branes at singularities of CY manifolds. For example, take the construction of an SM-like theory in terms of D-branes on a del Pezzo 8 singularity considered in the previous chapter [23, 59]. As we argued above, the symmetry breaking towards the SM requires the formation of an A_2 singularity on the del Pezzo 8 surface. This A_2 lifts to a non-isolated singularity on the cone over del Pezzo. The results in this chapter suggest that, if we slightly deform this non-isolated singularity and put a suitable collection of fractional

[12]The couplings of the gauge fields can be tuned by changing the periods of the B-field. If we had an accidental $\mathcal{N}=2$ supersymmetry, then the couplings g and h would be related, but in our $\mathcal{N}=1$ setup they are not protected against independent changes.

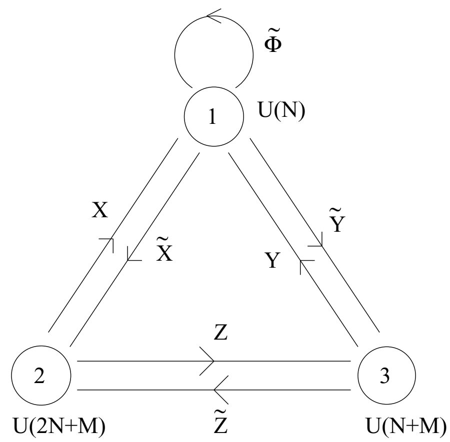

Figure 4.9: Quiver gauge theory of the fractional brane configuration on the SPP singularity that reduces to ISS model after confinement of the $SU(2N+M)$ gauge group at node 2.

branes on it, we can engineer a SUSY breaking hidden sector, with charged matter that interacts with the SM part of the quiver. In this way we may be able to build a semi-realistic phenomenological model.

4.7 ISS quiver via an RG cascade

In this section we show that the ISS quiver in figure 4.2 can be obtained after one Seiberg duality from an SPP quiver in figure 4.9.

This quiver is obtained from the quiver in figure 4.1 by adding M fractional branes to node 3, and $N+M$ fractional branes to node 2, so that the respective ranks of the gauge groups become $N+M$ and $2N+M$. Note, that this theory has an infinite duality cascade that increases the ranks of the gauge groups, i.e. we can suppose that we start in the UV with some big ranks of the gauge groups and after a number of duality steps arrive at quiver 4.9. Let us show that after one more duality at node 2 we reproduce the ISS model.

The theory has gauge group $U(N) \times U(2N+M) \times U(N+M)$, one adjoint under $U(N)$ and three vector-like pairs of bi-fundamentals. The superpotential is given by the sum of (4.3.4) and (4.3.7)

$$W = \mathrm{Tr}\left(-\zeta\widetilde{\Phi} + \widetilde{\Phi}(\widetilde{Y}Y - \widetilde{X}X) + h(Z\widetilde{Z}X\widetilde{X} - \widetilde{Z}ZY\widetilde{Y} + \zeta\widetilde{Z}Z)\right). \qquad (4.7.49)$$

The $SU(2N+M)$ gauge group confines first. This gauge group has $N_f = N_c$ and thus the gauge group after the Seiberg duality is $U(N) \times U(N+M)$. The two $U(1)$ factors can be represented as the overall $U(1)$ (that decouples) and the non-anomalous $U(1)_B$ gauge group. Denote the meson fields as $M_{xx} = \widetilde{X}X$, $M_{xz} = \widetilde{X}Z$, $M_{zx} = \widetilde{Z}X$, and $M_{zz} = \widetilde{Z}Z$. In addition there are two baryons A and B. After the Seiberg duality, the superpotential is

$$\begin{aligned} \widetilde{W} &= \mathrm{Tr}\left(-\zeta\widetilde{\Phi} + \widetilde{\Phi}(\widetilde{Y}Y - M_{xx}) + h(M_{xz}M_{zx} - M_{zz}Y\widetilde{Y} + \zeta M_{zz})\right) \\ &+ \lambda\left(\det\begin{pmatrix} M_{xx} & M_{xz} \\ M_{zx} & M_{zz}\end{pmatrix} - AB - \Lambda^{4N+2M}\right) \end{aligned}$$

Here λ is a lagrange multiplier field. Its constraint equation is the quantum deformed relation between the baryon and meson fields, and dictates that either the baryons or mesons acquire a non-zero vev. We assume that we are on the baryonic branch

$$AB = -\Lambda^{4N+2M}\,. \tag{4.7.50}$$

The vevs of the baryons break the non-anomalous $U(1)_B$. The D-term equations for $U(1)_B$ fix $|A|^2 = |B|^2$.

The adjoint field $\widetilde{\Phi}$, and the meson fields M_{xx}, M_{xz}, and M_{zx} are all massive. So we can integrate them out. The reduced gauge theory has gauge group $SU(N) \times SU(N+M)$, a pair of bi-fundamental fields $(Y, \widetilde{Y})$, and a meson field

$$\Phi = M_{zz} \tag{4.7.51}$$

that transforms as an adjoint under $SU(N+M)$. After integrating out the massive fields, the superpotential (4.7.50) reduces to the ISS superpotential in the magnetic regime [14]

$$\widetilde{W} = h\,\zeta\,\mathrm{Tr}(\Phi) - h\,\mathrm{Tr}(\Phi Y\widetilde{Y})\,,$$

Up to relabeling the nodes 1 and 3, this quiver gauge theory coincides with the quiver in figure 4.2.

Chapter 5

SUSY Breaking Mediation by D-brane Instantons

Here, we formulate and study the idea that Euclidean D-branes can mediate SUSY breaking in string compactifications.[1] The basic idea is to consider the effect of Euclidean branes stretching between two localized collections of space-filling branes that realize, respectively, a "visible" sector and a "hidden" sector where SUSY is broken.

In this setup, it is possible to generate soft terms that are perturbatively forbidden and hence cannot be generated by gauge mediation. Furthermore, there is a natural hierarchy of soft terms that is parameterized by the relative volumes wrapped by the corresponding D-branes. Using this observation, it is possible to write down examples where the geometric conditions for SUSY breaking imply a certain hierarchy of soft terms. We then compare the relative strength of the mediation effects of this mechanism with the effects of gauge mediation, gravity mediation from Kaluza-Klein modes, and anomaly mediation in various regimes.

Finally, we provide a preliminary discussion of the phenomenology of this framework and comment on future directions that could give rise to more control over the soft spectra.

[1]This chapter is based on the paper "SUSY Breaking Mediation by D-brane Instantons," written in collaboration with S. Franco [133].

5.1 Introduction

As we have discussed in previous sections, string theory compactifications provide a natural and consistent laboratory in which to better understand the physics of theories with visible and hidden sectors. In these constructions, the two sectors correspond to different sets of (anti) D-branes separated in the extra dimensions. Various mediation mechanisms can then communicate SUSY breaking. They can be classified according to the string sector involved. We can have, for example: open string mediation (gauge mediation), closed string mediation (gravity mediation) and open/closed mixed mediation (RR p-form topological mediation, which uses RR p-forms to couple $U(1)$ gauge fields in the visible and hidden sectors [136]). More importantly, string theory furnishes a geometrical interpretation of the various SUSY breaking parameters and hence could lead to additional insight into the SUSY breaking physics that is difficult to obtain from field theoretic techniques alone.

As the above examples demonstrate, string theory comes with a whole host of unique objects that can be used in constructing mediation mechanisms. The goal of this chapter is to describe another such mechanism. In particular, we will find that our mechanism has some rather unique properties that do not follow from the standard phenomenological literature on SUSY breaking mediation.

In particular, we will study the effects of Euclidean D-branes localized at a point in the non-compact four dimensions—so-called 'D-brane instantons'— stretching between the hidden and visible sectors.[2] Upon integrating over the massless, charged zero mode strings stretching between the D-instanton and the hidden and visible sectors, we generate operators that couple the two sectors.[3] Roughly speaking, if the hidden sector breaks SUSY, these operators can then generate soft terms for the visible sector fields. In this sense, the D-

[2] In the last year and a half, there has been a surge in the study of D-brane instantons, mainly due to their ability to generate superpotential couplings that are perturbatively forbidden by $U(1)$ symmetries. Many applications have been investigated, such as the cure of runaway directions in models that dynamically break SUSY [95], neutrino masses and mu terms [93, 94, 131, 59, 137], R-symmetry breaking and metastability [97], Yukawa couplings in GUT models [131] and SUSY breaking models with and without non-abelian gauge dynamics [100, 109]. In this chapter, we take the natural step of extending these ideas to the non-supersymmetric realm.

[3] It is important to notice that the strength of D-brane instantons is not related to the strength of any MSSM instanton. Hence, they can be much less suppressed.

brane instantons mediate SUSY breaking. Furthermore, our mediation mechanism has no known field theoretical analog. More generally, we expect D-brane instanton mediation to be present in a variety of string theory constructions. Hence, it deserves to be studied, regardless of whether it is the dominant mediation mechanism or not.

In this chapter, we find the following simple characterization of D-instanton mediation:

- D-instantons can generate soft terms that are perturbatively forbidden by $U(1)$ symmetries and hence cannot be generated by gauge mediation. They can also produce couplings that do not violate any $U(1)$ global symmetry, as in the example in section 4.2 and general models discussed in section 6.

- There is a natural hierarchy of soft terms that is parameterized by the volumes wrapped by the corresponding D-instantons.

- One can easily write down examples where the geometric conditions for SUSY breaking imply a particular hierarchy of soft terms from the mediating instantons.

- D-instanton mediation is sensitive to the details of the SUSY breaking sector. In particular, if the SUSY breaking sector is a D-brane gauge theory that dynamically breaks SUSY, then D-brane instantons generate chiral gauge invariant operators that correspond to their orientation (or, more precisely, the homology cycle they wrap). In particular, some SUSY breaking hidden sectors fail to generate soft terms via this form of mediation since the mediating D-instantons project onto a subspace of vanishing chiral gauge invariants.

From a model building perspective, the last three points are potentially quite interesting. Significantly, they follow rather simply from the fact that our mediators have a clear geometrical interpretation.

Let us briefly summarize the plan of this chapter. In the next section, we will discuss in much greater detail the idea behind D-instanton mediation. Then we will quickly introduce the machinery of toroidal orientifolds with D-branes and D-instantons as a warm-up for some specific examples we then engineer in this setup. We will conclude with a brief and admittedly incomplete phenomenological discussion.

One of the main issues we do not address in this chapter, but should certainly be studied, is the stabilization of compactification moduli, the dilaton and D-brane moduli after SUSY breaking. We have chosen to focus on the question of whether SUSY breaking mediating interactions can be generated by D-brane instantons, under the assumption that it can be disentangled from moduli stabilization. With this as our main goal, we do not try to engineer fully realistic or complete models of D-instanton mediation, but rather we discuss the basics of this mechanism in a few simple and illustrative toy examples. We leave a more detailed analysis to future work.

5.2 General idea

5.2.1 Coupling visible and hidden sectors via D-brane instantons

First, we will review the basics of how to generate chiral operators from Euclidean Dp-branes (Ep-branes for short). Since some of the soft terms (e.g., A-terms, B-terms, etc.) will be generated by products of chiral operators, we will be interested in this well-studied case. However, we will also discuss the possibility of generating non-chiral operators from instanton anti-instanton combinations (or, simply from single, non-BPS instantons [141, 142]) as well, since such operators can give rise to non-holomorphic soft masses.

We begin by imagining a space-filling D-brane sector that the various Ep-branes interact with. This could be a set of fractional branes at a singularity or a set of intersecting D-brane stacks. The physics of the Ep-brane interactions with the D-brane sector can be encoded in an extended quiver diagram of the form shown in Figure 5.1.

The circles denote two of the gauge groups that are part of a larger quiver living on the set of space-filling D-branes. X_{ij} corresponds to a combination of chiral fields transforming in the bifundamental representation of $SU(N)^{(i)} \times SU(N)^{(j)}$. The ranks of both gauge groups must be the same in order to have a non-vanishing instanton contribution. Note

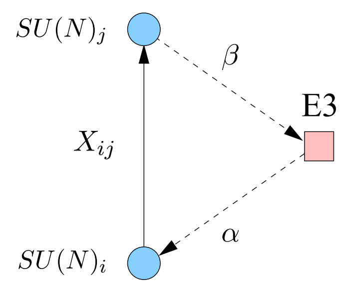

Figure 5.1: Extended quiver diagram for the basic E-brane configuration that generates a superpotential contribution. Dotted arrows indicate charged fermionic zero modes. The figure presents the simplest case, in which the fermionic zero modes couple to a single bifundamental field between a pair of nodes. In the generic situation, charged zero modes can couple to more general operators, associated with an open path in the quiver.

that X_{ij} can simply be a single bifundamental field or, more generally, a product of the form $X_{ij} = X_{i\,k_1} X_{k_1 k_2} \dots X_{k_n j}$, where we sum over the intermediate color indices. In section 6, we discuss this possibility in more detail. The square node in the extended quiver indicates the Ep-brane. Charged fermionic zero modes, represented by the Grassman variables α and β, arise at the intersections between the spacefilling D-branes and the instanton. The instanton action then contains a term of the form

$$L = \alpha_i X_{ij} \beta_j \ , \tag{5.2.1}$$

Let us now discuss the neutral fermionic zero modes—these arise from strings that have both ends on the instanton. We will focus on orientifolded Calabi-Yau compactifications with D-branes, leading to $\mathcal{N} = 1$ SUSY in 4d. Since the Ep-brane breaks 1/2 of the SUSY it therefore has two fermionic zero modes, the goldstinos, living on it—these are represented by the Grassman variables, θ^α. Generically, there are two additional fermionic zero modes on the instanton due to the 'accidental' $\mathcal{N} = 2$ SUSY seen by the Ep-Ep sector. This issue was first discussed and clarified through explicit computations in some orbifold models in [130, 143]. In order to saturate the superspace measure and generate a non-vanishing contribution to the superpotential, there must be only two neutral fermionic zero modes on the instanton. A straightforward way of getting rid of the accidental neutral zero modes is to project them out by placing the instanton on top of an orientifold plane with an $O(1)$ projection.[4] Then, after a straightforward Grassman integration over the charged zero modes, we obtain the following contribution to the 4d effective superpotential

$$W_{inst} = M_s^{3-N} e^{-V_\Sigma/g_s} \det X_{ij} \ . \tag{5.2.2}$$

where V_Σ is the volume in string units of the cycle, Σ, wrapped by the instanton. In this expression and future ones, we omit a numerical multiplicative constant that we assume to be of $\mathcal{O}(1)$.

In orientifold singularities, the $SU(N)^{(i)}$ and $SU(N)^{(j)}$ nodes might be identified by the orientifold. In this case, depending on the charge of the corresponding O-plane, X_{ij} is projected into a 2-index (conjugate) symmetric or antisymmetric representation of the resulting single $SU(N)$ factor. Furthermore, we get a single (anti)fundamental fermionic zero mode α. In this case, the instanton action contains the coupling

$$L = \alpha_a X^{ab} \alpha_b \ . \tag{5.2.3}$$

where X transforms in the $\overline{\square\square}$ or $\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$ representations, while α transforms in the $\square$ representation (of course, we could also have the conjugate representations). We write the color indices a and b explicitly—they should not be confused with the quiver node labels i and j. Integrating over α we get a contribution to the effective potential that takes the form

$$W_{inst} = M_s^{3-N/2} e^{-V_\Sigma/g_s} \sqrt{\det X} \ . \tag{5.2.4}$$

Now, the gauge groups of quiver theories that arise on D-branes are actually $U(N) = SU(N) \times U(1)$. Any operator that does not correspond to a closed oriented path in the quiver is charged under some of these $U(1)$ symmetries. Thus, we conclude that the instanton generated couplings (5.2.2) and (5.2.4) are perturbatively forbidden by these $U(1)$

[4]We can also consider using 3-form fluxes to lift the additional fermionic zero modes — see [129] and [139] for a further discussion of this point. We will later comment on this scenario. Another possibility would be to consider a two-instanton contribution where the extra neutral zero modes are lifted by interactions between the two instantons [140, 141, 142].

symmetries.

Finally, let us also note that it may be possible to generate instanton-induced corrections to the holomorphic gauge kinetic functions of the various nodes in the quiver [144, 145]

$$e^{-V_\Sigma/g_s} W_\alpha W^\alpha \tag{5.2.5}$$

Such corrections can arise from BPS Ep-branes that have additional neutral fermionic (non-Goldstino) zero modes and no massless modes charged under the corresponding $SU(N)$ factor (or, indeed, under any of the other gauge groups present). In the specific examples considered in [144] and [145] such corrections arise in the world-volume theory of D6-branes that interact with E2-branes wrapping cycles with a 1-dimensional 1-homology, i.e., $b_1(\Sigma) = 1$. These E2-branes then have the requisite additional pair of neutral fermionic zero modes (after orientifolding) that allows them to generate corrections to the D6-gauge kinetic functions.

An interesting spectrum of new possibilities arises when we generalize the class of configurations we have just discussed to ones where an instanton can intersect multiple sets of D-branes that are spatially separated. For concreteness, we will specialize our discussion to compact type IIB orientifolds, in which we will study the effect of E3-branes wrapped over compact 4-cycles, Σ. Figure 5.2 shows the simplest situation in which an E3-brane intersects only an O-plane and two D-brane sectors (plus their images). Motivated by our goal of investigating possible SUSY mediating effects in this class of setups, let us denote the two sectors visible and hidden. The hidden sector might involve anti D-branes, although we will not explore this possibility. It is possible for the visible or hidden sector and its image to collapse on top of the O-plane. Our previous discussion, regarding fermionic zero modes on the instanton and their projection via an orientifold, applies to this case without changes.

The configuration can be more complicated than the simplest one, with the E-brane intersecting additional sectors with (anti) D-branes. These extra intersections result in additional insertions of 4d fields. This is an interesting direction that is worth studying. From now on, however, we restrict our discussion to some clean models in which this

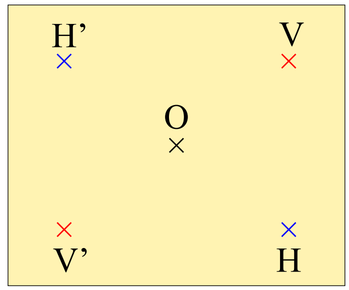

Figure 5.2: The basic configuration for mediation. It consists of visible and hidden sectors V and H, their orientifold images V' and H', and and O-plane O. All of them are intersected by a Euclidean D-brane, depicted in yellow.

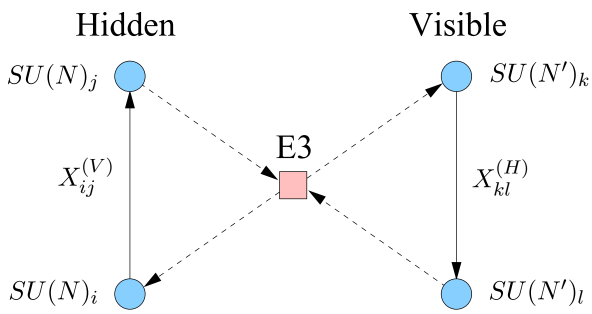

Figure 5.3: Extended quiver diagram for the basic mediating configuration. Dotted arrows indicate charged fermionic zero modes. The figure presents the simplest case in which these fermionic zero modes couple to single bifundamental fields between pairs of nodes in the visible and hiden sectors. Generically, charged zero modes can couple to more general operators, associated with open paths in the quiver.

situation does not arise.

Once again, the configuration can be captured by an extended quiver as shown in Figure 5.3. In this case we have two pairs of nodes, representing pairs of gauge groups in the visible and hidden sectors. In addition, we have fermionic zero modes α_V, β_V, α_H and β_H connecting the instanton to bifundamental operators $X_{ij}^{(V)}$ and $X_{kl}^{(H)}$ via the couplings

$$L = \alpha_V X_{ij}^{(V)} \beta_V + \alpha_H X_{kl}^{(H)} \beta_H \ . \tag{5.2.6}$$

Upon integrating over the E3 zero modes, we generate contributions to the superpotential of the form

$$W_{H/V} = M_s^{3-d_H-d_V} e^{-V_\Sigma/g_s} \mathcal{O}_H \mathcal{O}_V \tag{5.2.7}$$

where

$$\mathcal{O}_V = \det X_{ij}^{(V)}, \qquad \mathcal{O}_H = \det X_{kl}^{(H)} \ . \tag{5.2.8}$$

In the previous discussion, we have implicitly assumed that gauge groups are not identified by orientifold projections. The obvious modifications along the lines of (5.2.3) and (5.2.4) apply if the hidden and/or the visible sectors involve orientifold identifications.

Using similar reasoning, let us also note that by taking instantons that generate the corrections, written in (5.2.5), to the gauge kinetic functions of the space-filling D-branes and allowing them to intersect the hidden sector, we can generate the following operators involving the visible sector field strength superfields

$$W_{H/V} = e^{-V_\Sigma/g_s} \mathcal{O}_H W_{V\alpha} W_V^\alpha \tag{5.2.9}$$

Now that we have discussed the generation of chiral operators from Euclidean D-branes stretching between the hidden and visible sectors, let us also consider the case where we obtain a non-chiral operator. One natural way to generate such an operator in this setup is to consider the contribution of an instanton anti-instanton pair wrapping an orientifold invariant cycle, since we then obtain the four neutral fermionic zero modes that make up the full $\mathcal{N} = 1$ superspace measure, and we know that the opposite GSO projections in the Ep-Dp′ and $\overline{\text{E}}$p-Dp′ sectors will generate factors with opposite chirality.

We then expect the following terms involving the $\overline{\text{E}}$p-Dp′ sector zero modes in the $\overline{\text{E}}$p action[5]

$$L = \overline{\alpha}_V \overline{X}^{(V)} \overline{\beta}_V + \overline{\alpha}_H \overline{X}^{(H)} \overline{\beta}_H \ . \tag{5.2.10}$$

Heuristically, we also expect the following terms involving interactions between the $\overline{\text{E}}$p-Ep states

$$L = (x_-^2 - \frac{1}{2})|\varphi|^2 + i x_{-\mu} \overline{\chi} \sigma^\mu \chi + V \tag{5.2.11}$$

[5] One can rigorously derive such couplings for $\overline{\text{E}}$3 branes in the toroidal orientifold examples we will consider below.

where $x_-^\mu = x_1^\mu - x_2^\mu$ is the distance in the non-compact four dimensional space between the instanton and anti-instanton, χ are fermionic zero modes with one end on each the instanton/anti-instanton, and φ is a bosonic mode stretching between the instanton and the anti-instanton. V is a potential depending on φ, χ, θ whose precise form is not important. Note that the terms appearing in (5.2.11) are analogous to the terms appearing in the corresponding $\overline{\text{D}}$(p+4)-D(p+4) action with φ playing the role of a 'tachyon' in the following sense: about $\varphi = 0$, we can think of the above action as describing an instanton anti-instanton pair, while about the minima with $\varphi \neq 0$ (the precise location of these minima depends on the unspecified potential, V), the above description breaks down since the branes have annihilated. In particular, the zero mode content and interactions described in (5.2.6), (5.2.10), and (5.2.11) are really only an effective description of the physics about $\varphi = 0$ since these modes cease to exist about the minima with $\varphi \neq 0$.

Note, however, that for large x_-^2 we expect the configuration about $\varphi = 0$ to be stable in the sense that it is a local minimum of the φ potential. Indeed, for large x_-^2, we can integrate out the fields φ and χ and so we expect to have a well-defined semi-classical contribution by the instanton anti-instanton pair to the non-chiral operator generated by integrating over the various Ep-Dp′ and $\overline{\text{E}}$p-Dp′ zero modes with the action given by the sum of (5.2.10) and (5.2.6). Physically what is happening is that the only contributions to the operator of interest come from configurations where the instanton and anti-instanton are far apart and hence non-interacting. In particular, we expect to be able to generate a contribution to the Kähler potential of the form

$$K = M_s^{2(1-d_H-d_V)} e^{-2V_\Sigma/g_s} \mathcal{O}_H \overline{\mathcal{O}}_H \mathcal{O}_V \overline{\mathcal{O}_V} \tag{5.2.12}$$

Note that in writing this formula we have assumed that the instantons and anti-instantons wrap the same cycle. Presumably we could also consider instantons and anti-instantons that wrap different cycles. These pairs of branes would generate contributions to the Kähler potential of the form

$$K = M_s^{2(1-d_H-d_V)} e^{-(V_\Sigma + V_{\widetilde{\Sigma}})/g_s} \mathcal{O}_H \overline{\widetilde{\mathcal{O}}}_H \mathcal{O}_V \overline{\widetilde{\mathcal{O}}_V} \tag{5.2.13}$$

where in general $\mathcal{O}_{H,V} \neq \widetilde{\mathcal{O}}_{H,V}$.

Finally, please note that the non-holomorphic operators in (5.2.13) could also be generated by single, non-BPS instantons that wrap non-holomorphic, volume minimizing cycles since these instantons also have the requisite four Goldstino zero modes, $\theta, \overline{\theta}$. In fact, this is the generic case.

5.2.2 Instanton mediation

Let us now discuss in some more detail how this setup results in mediation of SUSY breaking from the hidden sector to the visible sector.

The visible sector consists of D-branes on which the MSSM or another supersymmetric extension of the SM is realized. We denote the chiral superfields in this sector as $\Phi_{V,i}$ and its superpotential as $W_V(\Phi_{V,i})$. The hidden sector is a set of D-branes on which SUSY is broken. Its fields and superpotential are denoted Φ_{H_j} and $W_H(\Phi_{H,i})$ respectively. Let us focus on the case in which SUSY is broken by (some) non-vanishing F-term vev(s), $F_{\Phi_{H,0}}$—note that this state may simply be metastable with a long lifetime.

The mediating instantons described above generate an exponentially suppressed perturbation of the superpotential, $W_{H/V}(\Phi_{V,i}, \Phi_{H,j})$, coupling the two sectors. The total superpotential reads

$$W = W_V(\Phi_{V,i}) + W_H(\Phi_{H,j}) + W_{H/V}(\Phi_{V,i}, \Phi_{H,j}) \ . \tag{5.2.14}$$

This superpotential can give rise to a corresponding non-zero F-term vev(s) for some field(s), F. Note that in general F *need not* be equal $F_{\Phi_{H,0}}$. However, if $W_{H/V}$ is a small perturbation, we expect that $F \sim F_{\Phi_{H,0}}$, and we also expect that the (meta) stability of the SUSY breaking state is not affected. This statement further assumes that some form of mediation—instanton or otherwise—generates masses for the visible sector scalars and that the SUSY breaking sector has all its moduli lifted as well. In this approximation, soft terms arise as follows. We have

$$\left\langle \frac{\partial W_H}{\partial \Phi_{H_0}} \right\rangle \neq 0 \qquad \left\langle \frac{\partial W_{H/V}}{\partial \Phi_{H,V_i}} \right\rangle = \left\langle \frac{\partial W_V}{\partial \Phi_{V_i}} \right\rangle = 0 \ . \tag{5.2.15}$$

Plugging the non-zero F-term into the superpotential, we see that $W_{H/V}$ can give rise to various SUSY breaking terms. Similarly, plugging the non-zero F-term into the instanton/anti-instanton induced perturbation to the Kähler potential described in (5.2.13), will yield additional, non-holomorphic SUSY-breaking terms.[6]

As a final comment, we note that the instanton induced-operators we have written down are generally non-renormalizable. In particular, the non-renormalizable operators will be suppressed by powers of the string scale, M_s. If the F-term vevs are of the scale $F \sim M_s^2$, the suppression by the instanton volume is crucial to obtaining phenomenologically reasonable SUSY breaking scales in the range of several TeV. Of course, it turns out that in many scenarios, including the examples we discuss below, it is possible to have $F \ll M_s^2$. For example, this can happen in D-brane gauge theories that break SUSY dynamically and that have a dynamical scale $\Lambda \ll M_s$ or, for certain values of the moduli, in theories where the SUSY breaking vev is generated by stringy instantons. The mediating instanton then generates an additional suppression with respect to the string scale. Whether the resulting instanton-induced soft terms are an important effect or not depends, as we will see below, on where one sits in the moduli space of the compactification (in particular, in these latter cases, the 4-cycles wrapped by the mediating instantons should be relatively small)

5.2.3 Possible soft terms

In this section we would like to describe more precisely which soft terms we expect to be able to generate via instanton mediation. In order to understand this point, let us first recall the general form of soft terms in renormalizable SUSY gauge theories, of which the MSSM is, of course, an example. For concreteness, consider the following superpotential

$$W = \lambda \Phi^3 + M\Phi^2 + \frac{\tau}{4} W_\alpha W^\alpha \ , \tag{5.2.16}$$

[6] Recall that these terms could also be generated by appropriate non-BPS instantons.

where Φ is shorthand for the various chiral superfields of the theory and W_α is a field strength superfield. In general, the holomorphic soft terms are those terms that can be written as higher components of the superfield couplings. In particular, turning on F-term components of λ and M results in scalar trilinears ϕ^3 and scalar bilinears ϕ^2 called 'A-' and 'B-' terms respectively. Turning on an F-term component in τ generates a gaugino mass.

From the above discussion, it should be clear that instanton mediation can generate both A-terms and B-terms. Indeed, we expect that

- Instanton mediation generates A-terms when a mediating instanton has zero modes that are charged under the gauge groups of two intersecting $SU(3)$ nodes of the visible sector quiver *or* when it has zero modes that transform in the $\overline{\mathbf{6}}$ ($\mathbf{6}$) representation of the gauge group of a visible sector $SU(6)$ node that has an antisymmetric (conjugate antisymmetric) tensor representation after orientifolding.

- Similarly, instanton mediation generates B-terms when one considers the scenarios mentioned in the A-term generation case but with $SU(2)$ nodes instead of $SU(3)$, or $SU(4)$ with an antisymmetric tensor instead of $SU(6)$ with an antisymmetric tensor.

Now, taking into account the couplings given in (5.2.9), we expect that instanton mediation also generates gaugino masses in certain cases.

The remaining soft terms are non-holomorphic. For our purposes, the only interesting non-holomorphic soft terms arise from θ^4 components of the coupling Z in the Kähler potential

$$K = Z\overline{\Phi}\Phi \tag{5.2.17}$$

These soft terms are non-holomorphic soft masses, $\overline{\phi}\phi$. It should be obvious from the above discussion that instanton mediation also generates these terms in some cases. In particular, we find that

- Instanton mediation generates a non-holomorphic mass term for visible sector fields that run between two intersecting $U(1)$ nodes *or* for visible sector fields that transform in the antisymmetric, i.e., trivial representation of an $SU(2)$ node after orientifolding.[7]

Of course, instanton mediation also generically generates higher-dimension terms. However, these terms in the potential are non-renormalizable and hence the corresponding power-law corrections to the 1-PI effective action will not ruin the softness of our mediation mechanism.[8]

5.2.4 Possible hidden sectors and mediation mechanisms

Now that we have described the soft terms that we can potentially generate from our mediation mechanism, let us turn our attention to the possible SUSY breaking sectors that we can include in our setup. String compactifications feature a broad array of possible hidden sectors each of which can have either stable or metastable SUSY breaking vacua. For example, we can have:

- Sectors that realize simple SUSY breaking models without non-abelian gauge dynamics (such as Polonyi, Fayet or O'Raighfertaigh models).
- Hidden sectors that realize an ordinary gauge theory on D-branes with dynamical SUSY breaking.
- A hidden sector with anti D-branes.

Later we present explicit examples of the first two types of hidden sectors in the context of toroidal orientifolds where D-instanton mediation generates various soft terms. We leave a discussion involving the third type of hidden sector to future work. Also note that more

[7] As a brief aside, note that these instanton generated non-holomorphic mass terms could be used, for example, to give a mass to the 'right-handed' scalar neutrino. Furthermore, we should also note that we can obtain exponentially-suppressed non-holomorphic mass terms for the squarks and the other sleptons by taking the corresponding higher-dimensional instanton-generated operators—heuristically of the form $e^{-2V_\Sigma/g_s}\frac{F^2}{M_s^{2n}}\overline{\phi}^n\phi^n$— and contracting $n-1$ pairs of fields or alternatively through diagrams involving A-terms.

[8] One might also worry about soft terms of the form $\overline{\phi}\phi\phi$ which might generate unacceptable quadratic divergences in the effective action. Such divergences will not arise in our setups due to symmetries of our quivers.

than one class of hidden sector can be simultaneously present in a given compactification—for the sake of simplicity, however, all our explicit examples below will have a single type of SUSY breaking hidden sector.

On top of this, more than one mediation mechanisms can act at the same time. For example, gravity and gauge mediation[9] are always present in the sense that one always has open and closed string exchange between the hidden and visible sectors (although the exchange may be suppressed). Different mechanisms become dominant over certain regions of the moduli space. Similarly, various soft terms might get their dominant contribution from different mediation mechanisms. For instance, global symmetries will often prevent perturbative generation of certain soft terms by gauge mediation. These soft terms might instead be generated by D-instantons, and their hierarchy will then encode the relative volumes of the corresponding D-instantons.

5.2.5 Relative dominance of mediation mechanisms

In this section we will give a heuristic sketch of when one can generally expect different mediation mechanisms to become important in a given compactification. From our discussion above, we know that instanton mediation generates soft terms of the form

$$m_\phi^2 \sim \frac{F^2}{M_s^2} e^{-2V/g_s}, \quad A \sim \frac{F}{M_s} e^{-V'/g_s}, \quad b \sim F e^{-\widetilde{V}/g_s} \tag{5.2.18}$$

where we have denoted the different instanton volume factors V, V', and $\widetilde{V}$ to underline the fact that they need not be equal in general.[10] In fact, if we want to avoid generating b that is too large (a typical problem in minimal forms of gauge mediation), then we would need $\widetilde{V} \sim 2V, 2V'$.

Now we note that open and closed string mediation is also generically present in string compactifications. Therefore, a natural question is how strong instanton mediation is relative to these other mechanisms and whether it is dominant in some regime. In order to

[9]We are really using these terms imprecisely as catch-all phrases for closed and open string mediation respectively

[10]We have assumed that m_ϕ^2 is generated by an instanton-anti-instanton pair in 5.2.18.

answer this, we first need to have a very basic understanding of the soft scales that are generated by open and closed string mediation.

Consider closed string mediation first. On general grounds, we expect that mediation from integrating out massive closed string modes generates soft masses at the scale

$$m_{\rm cl}^{\rm soft} \sim \frac{F}{M_P} \tag{5.2.19}$$

where the Planck mass is given by $M_P = \frac{\sqrt{V_Y}}{g_s} M_s$, with V_Y the compactification volume in string units. In fact, (5.2.19) represents an upper bound for massive closed string mediation. Indeed, as argued in [158], this type of mediation can be sequestered by considering compactifications with a hierarchy of length scales. In such cases we obtain

$$m_{\rm cl}^{\rm soft} \sim e^{-d/R} \frac{F}{M_P} \tag{5.2.20}$$

where $d \gg R^{-1}$ is the distance between visible and hidden sectors and R^{-1} is, roughly speaking, a typical mass scale of a mediating bulk KK mode. In fact, string theory naturally accommodates such hierarchies of scale due to the warping associated with large numbers of D-branes—we will not, however, consider such effects in the examples we discuss below although such a study would certainly be worthwhile. It should be clear, however, that by considering, e.g., instantons wrapping 4-cycles that have smaller dimensions transverse to a principle dimension of length $\sim d$, we can arrange for instanton mediation to dominate gravity mediation.[11]

Next, consider open string mediation. Roughly speaking, such mediation is due to open strings that stretch between the hidden and visible sectors and is characterized by a soft scale

$$m_{\rm op}^{\rm soft} \sim \frac{g^2}{16\pi^2} \frac{F}{M_g} \tag{5.2.21}$$

where M_g is a supersymmetric mass associated with the tension of the string

$$M_g = d\, M_s \tag{5.2.22}$$

[11]We should also note that anomaly mediation, due to the superconformal anomaly, is also generically present. However, its contributions are suppressed relative to (5.2.19) by various potentially small couplings.

where d is, again, the distance between the visible and hidden sectors in string units and $g^2/16\pi^2$ is a 1-loop factor of the appropriate D-brane gauge group. These soft terms arise, at least in the simple effective field theory picture (and in a simple form of gauge mediation), from integrating out 'messenger' fields (i.e., the open string modes we have described) that have acquired non-supersymmetric masses of the form $M_g \pm F$ from their couplings to SUSY breaking fields in the hidden sector. For small couplings, g, we can arrange for gauge mediation to be subdominant to instanton mediation. Note that the naive effective field theory picture of open string mediation presumably breaks down for $d \geq 1$ since then string oscillator states become important.

We should also again note that by our above discussion, various mechanisms may be present at once. As we have emphasized above and will see in greater detail below in our explicit examples, different mediation mechanisms—although present—may not even generate certain terms or only generate suppressed terms of a certain type.

5.3 Toroidal orientifolds

For concreteness, in this chapter we focus on Type IIB compactifications using toroidal orientifolds. For a comprehensive and clear explanation of D-branes and instantons at singularities and their embeddings in toroidal orientifolds we refer the reader to [21, 146], to whose notation we adhere. For fast reference, we collect some basic formulas needed for our constructions and give some tips that are useful for identifying D-brane instantons producing desired couplings.

We consider six-dimensional factorized tori of the form $T^6 = T^2 \times T^2 \times T^2$. Before orbifolding, the theory on a stack of n D3-branes is $\mathcal{N} = 4$ $U(n)$ SYM, wich contains $U(n)$ gauge bosons, four adjoint fermions, and six adjoint real scalars. They transform in the **4** and **6** of the $SU(4)$ R-symmetry group, respectively. We quotient by the $\mathbb{Z}_N$ orbifold[12] , which acts on the fermions through the matrix

[12] $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds can be studied analogously.

$$\mathbf{R_4} = \mathrm{diag}(\alpha_N^{a_1}, \alpha_N^{a_2}, \alpha_N^{a_3}, \alpha_N^{a_4}) \ , \tag{5.3.23}$$

with $\alpha_N = e^{i2\pi/N}$ and $a_1 + a_2 + a_3 + a_4 = 0 \ \mathrm{mod}(N)$. From the action on the **4** we can easily derive the action on the **6**, which is given by

$$\mathbf{R_6} = \mathrm{diag}(\alpha_N^{b_1}, \alpha_N^{-b_1}, \alpha_N^{b_2}, \alpha_N^{-b_2}, \alpha_N^{b_3}, \alpha_N^{-b_3}) \ , \tag{5.3.24}$$

where $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$ and $b_3 = a_1 + a_2$. We can combine the scalars into complex coordinates z_s on each T^2, with $s = 1, 2, 3$. In terms of these degrees of freedom, the identification that follows from (5.3.24) is $z_s \sim z_s \alpha_N^{b_s}$. In order to preserve SUSY, we must have $b_1 + b_2 + b_3 = 0 \ \mathrm{mod}(N)$. The $\mathbb{Z}_N$ must act crystalographically on the lattice defining the torus. All possibilities have been classified in [147]. Each T^2 is defined by

$$z_s \sim z_s + r_s \sim z_s + r_s \, \alpha_N \ , \tag{5.3.25}$$

where r_s is the corresponding radius.

In order to completely determine the $\mathbb{Z}_N$ action, we must also specify how its generator, θ, acts on the Chan-Paton (CP) factors of the various (Euclidean) D-branes. This action is encoded in a matrix that, for each kind of brane, takes the form

$$\gamma_\theta = \mathrm{diag}(\mathbf{1_{n_0}}, \alpha \mathbf{1_{n_1}}, \ldots, \alpha^{\mathbf{N-1}} \mathbf{1_{n_{N-1}}}) \ , \tag{5.3.26}$$

where $\mathbf{1_{n_i}}$ denotes the n_i-dimensional identity matrix.

Since we are interested in orientifolds, due to the possibility of then lifting unwanted extra fermionic zero modes of the instantons in our setups, we further quotient by $\Omega(-1)^{F_L} R_1 R_2 R_3$, with Ω the orientation reversal on the worldsheet, F_L the left-moving fermion number and R_s the reflection on each plane, $z_s \to -z_s$. As a result, we obtain 64 O3-planes whose positions on each T^2 are given by

$$0, \qquad \frac{1}{2} r_s, \qquad \frac{1}{2} r_s \alpha_N^{1/2}, \qquad \frac{1}{2} r_s \alpha_N \tag{5.3.27}$$

Depending on the orbifold action, some of these O-planes may also sit on top of orbifold fixed points. Furthermore, each O-plane has two possible RR charges. In this chapter we will only consider the case in which all O3-planes have negative RR charge (and hence lead to SO and antisymmetric projections of vector and chiral multiplets, respectively).

In any compactification, there are global and local consistency conditions corresponding to cancellation of untwisted and twisted RR charges, respectively. Due to the presence of the 64 O3-planes, cancellation of untwisted or global tadpoles reads

$$N_{D3} - N_{\overline{D3}} = 32 \tag{5.3.28}$$

with the net number of other Dp-branes vanishing.[13]

When D3-branes sit on top of an orbifold fixed point that does not coincide with an O-plane, twisted or local tadpole cancellation requires

$$[\prod_{s=1}^{3} 2\sin(\pi k b_s/N)]\mathrm{Tr}\gamma_{\theta^k,3} = 0 \ , \tag{5.3.29}$$

for each element θ^k, $k = 0, \ldots, N-1$, of the orbifold group.

The same expression applies for the case of an anti D3-brane. If the fixed point coincides with an O-plane, the expression changes to

$$[\prod_{s=1}^{3} 2\sin(\pi k b_s/N)]\mathrm{Tr}\gamma_{\theta^k,3} = 4 \ . \tag{5.3.30}$$

These two conditions ensure cancellation of gauge anomalies in the corresponding gauge theories. In the discussion above, we take the convention of counting RR charges in the covering space.

There are various possibilities for locating D-branes: they can sit at orbifold fixed points, O-planes or in the bulk. However, they must be in configurations that are symmetric under both the orbifold and orientifold groups.

[13]For simplicity, we limit our discussion to models with only D3 and anti D3-branes in this chapter. In fact, our explicit examples do not even contain anti D3-branes.

5.3.1 E3-brane instantons

Consider an E3-brane in the class of geometries we have described above. The CP matrix will have the general form

$$\gamma_{\theta,E3} = \mathrm{diag}(\mathbf{1_{v_0}}, \alpha\mathbf{1_{v_1}}, \ldots, \alpha^{\mathbf{N-1}}\mathbf{1_{v_{N-1}}}) \ . \tag{5.3.31}$$

Let us take an instanton wrapping $z_s = const$ and intersecting a stack of D3-branes at an orbifold fixed point. In this case, the fermionic zero modes in the E3-D3 sector were computed in [146]. The result is

$$\begin{array}{ll} a_s \text{ even} & \sum_{i=0}^{N-1}[(n_i, \overline{v}_{i-\frac{1}{2}a_s}) + (v_i, \overline{n}_{i-\frac{1}{2}a_s})] \\ a_s \text{ odd} & \sum_{i=0}^{N-1}[(n_i, \overline{v}_{i-\frac{1}{2}(a_s+1)}) + (v_i, \overline{n}_{i-\frac{1}{2}(a_s+1)})] \end{array} \tag{5.3.32}$$

Furthermore, the instanton has four additional neutral fermionic zero modes due to the accidental $\mathcal{N} = 2$ SUSY of the E3-E3 sector. As discussed above, a simple way of projecting out the two accidental neutral zero modes is by placing the E3-brane on top of an orientifold with an $O(1)$ projection. Since the orientifold acts by conjugating the CP matrix, this determines

$$\gamma_{\theta,E3} = 1 \ , \tag{5.3.33}$$

i.e. $v_0 = 1$ and $v_i = 0$ for $i \neq 0$. Plugging this into (5.3.32), we conclude that the fermionic zero modes connecting such an instanton to the D3-branes transform as follows under the gauge symmetries of the quiver

$$\begin{array}{lll} a_s \text{ even} & \square_{\frac{1}{2}a_s} & \overline{\square}_{-\frac{1}{2}a_s} \\ a_s \text{ odd} & \square_{\frac{1}{2}(a_s+1)} & \overline{\square}_{-\frac{1}{2}(a_s+1)} \end{array} \tag{5.3.34}$$

The last remaining ingredient in our description of the E3-branes is to give an explicit embedding of their worldvolumes in our orientifolded $T^6/\mathbf{Z}_N$ compactification. In order to do this, let us first go to the covering space of the orbifold, the orientifold, and the torus.

The total covering space is $\mathbf{C}^3$, and we will consider E3-branes wrapping divisors

$$t_1\frac{z_1}{r_1} + t_2\frac{z_2}{r_2} + t_3\frac{z_3}{r_3} = v \tag{5.3.35}$$

where we have normalized by the various radii r_s of the T_s^2 and have included arbitrary complex coefficients t_s and v. However, in order for (5.3.35) to represent a divisor wrapped by an E3-brane, it must be compatible with the various geometric projections—let us now run through the list and see how they restrict the complex surfaces wrapped by the E3's.

We define compatibility with the T^6 projection given in (5.3.25) to mean that the E3-brane wraps a non-trivial closed curve, i.e., an element of the 4-homology group, $H_4(T^6)$. This fact requires

$$t_s = n_s, \quad n_s \in \mathbf{Z},\ \forall\, s = \{1, 2, 3\} \tag{5.3.36}$$

with the n_s relatively prime. One easy way to see this is to note that a basis of mutually holomorphic embeddings is given by

$$\Sigma_s = \{z \in T^6 |\ z_s = 0\} \tag{5.3.37}$$

Hence, the integers n_s give the wrapping numbers with respect to this basis. For all the n_s's coprime, the wrapping numbers are simply

$$\omega(\Sigma_s) = |n_\nu|. \tag{5.3.38}$$

The volume of a curve, Σ, wrapped by some E3-brane is then given by

$$V(\Sigma) = \sqrt{\sum_s \omega(\Sigma_s)^2 V(\Sigma_s)^2} \tag{5.3.39}$$

Next, let us consider the action of the orbifold group on E3-branes. The geometric action was given in the discussion immediately following (5.3.24) and is reproduced below

$$z_s \sim z_s \alpha_N^{b_s} \tag{5.3.40}$$

where $\mathcal{N} = 1$ SUSY requires $b_1 + b_2 + b_3 = 0 \bmod(N)$. The cycle wrapped by the instanton may or may not be invariant under the orbifold group. For generic cases in which all the

b_s's are different, the only invariant cycles are given by the Σ_s. In the examples below, we will take the orbifold action to be $b = (-1, -1, 2) \sim (2, 2, 2)$ for $N = 3$ and hence any cycle of the form

$$n_1 \frac{z_1}{r_1} + n_2 \frac{z_2}{r_2} + n_3 \frac{z_3}{r_3} = v \tag{5.3.41}$$

is orbifold invariant so long as

$$v = \alpha_3 v \tag{5.3.42}$$

up to identifications (i.e., up to the toroidal shifts in the z_i). Note that for $b = (-1, -1, 2)$ and $N \neq 3$, however, the only orbifold invariant cycles are given by $n_1, n_2 \in \mathbf{Z}, n_3 = 0$ or $n_1, n_2 = 0$, $n_3 = 1$.

In general, for non-invariant cycles, we must include $N - 1$ additional image E3-branes. We can further divide the orbifold non-invariant cycles into two groups. On the one hand, we have those cycles that go through orbifold fixed points. In this case, the image E3-branes intersect, giving rise to additional neutral fermionic zero modes that must be lifted in order for the E3-brane to contribute to the action. This can be achieved, for example, in compactifications with fluxes. On the other hand, we have non-invariant cycles that do not pass through orbifold fixed points. In this case, the E3-brane images are spatially separated, we do not get additional zero modes and a superpotential is generated.

Finally, let us consider the orientifold action. Since we want an $O(1)$ instanton (at least for generating holomorphic soft terms), the cycle wrapped by the E3-brane must be mapped to itself under the geometric part of the orientifold action

$$z_s \to -z_s \tag{5.3.43}$$

Plugging this action into (5.3.35), we see that the cycle is invariant if and only if

$$v = -v \tag{5.3.44}$$

up to identifications.

As an aside, note that the volumes of the orientifold and orbifold invariant cycles are then given in the quotient space by (5.3.39) divided by a numerical factor

$$V(\Sigma)|_{\text{quot}} = \frac{1}{(4N)^3}\sqrt{\sum_i \omega(\Sigma_i)^2 V(\Sigma_i)^2} \tag{5.3.45}$$

where the factor of 4^3 is due to the orientifold and the factor N^3 is due to the orbifold. The volume of orientifold invariant cycles that are not invariant under the orbifold group are given by (5.3.45), but without the $1/N^3$ factor.

Now, given an instanton wrapping a particular cycle characterized by some coefficients, n_s, subject to the constraints just discussed, we would like to understand which operators are generated when the instanton intersects a spacefilling D3-brane. By simple worldsheet CFT arguments, the resulting R-sector fermionic zero modes must have Dirichlet-Dirichlet boundary conditions and hence are in the complex dimension transverse to both the E3 and the D3. For concreteness, then, we see that an E3-brane wrapping $z_s = const$ must couple to the adjoint, Φ^s, of the D3-brane. If this D3-brane sits at an orbifold fixed point, then the E3-D3 zero modes couple to the bifundamental X^s_{ij} determined by (5.3.34).

More generally, instantons with worldvolume given by a linear combination with various coefficients, n_s, non-zero generate couplings to operators made out of the corresponding combinations of fields. In other words, the orientation of the instanton selects the type of bifundamentals that form the operator. After performing the zero mode integral, the orientation of the instanton then picks out a chiral gauge invariant of the same orientation.

As a final point, let us consider more specifically the possible forms of the cycles wrapped by *mediating* instantons. Given a hidden and a visible sector, more than one instanton can connect them. For simplicity, let us consider the situation in which one of the sectors is located at the origin, as will happen in our examples below. This implies that we can set $v = 0$ in (5.3.35). The position of the other sector is $(h_1 r_1, h_2 r_2, h_3 r_3)$, with $h_s \in \mathbb{C}$. As explained above, we can discard orbifold non-invariant 4-cycles that go through fixed points. In the absence of a mechanism that lifts the additional zero modes (as is the case in our examples), they do not generate mediating interactions.

For concreteness, let us now assume the orbifold form $b = (-1,-1,2) \sim (2,2,2)$ for $N = 3$. For setups with $(h_1, h_2, h_3) \sim (0, 0, h_3^0)$—where '$\sim$' means, 'up to identifications of the T^6 action'—we see that the orbifold invariant cycles going through both the visible and hidden sectors are given by

$$n_1 h_1 + n_2 h_2 + n_3 h_3 = 0 \tag{5.3.46}$$

where we generate solutions n_i by substituting $h_{1,2} = 0, h_3 = h_3^0$ and also substituting values related to these by the action of the T^6. If all the $h_i \neq 0$, then a similar discussion applies.

Any solution to this equation with $n_1, n_2, n_3 \in \mathbb{Z}$ defines a cycle wrapped by a mediating instanton. In general, there is more than one such solution. In practice, due to the exponential suppression, we are only interested in the solutions with the smallest volumes.

5.3.2 The $\mathbb{Z}_3$ orientifold

A simple way to achieve three generations and generate a crude zeroth order approximation to the Standard Model in the context of D-brane at singularities is via a $\mathbb{Z}_3$ orientifold. Consequently, we will base our explicit examples on this case, keeping in mind that other geometries allow for more realistic visible sectors and interesting hidden sectors (see for example [21, 22, 23, 148]). For later reference, we devote this subsection to a more detailed presentation of the $\mathbb{Z}_3$ case. A similar general discussion of some of the results for the $\mathbb{Z}_3$ orbifold appears in [149].

We take the (SUSY) orbifold action on the fermions to be given by $(a_1, a_2, a_3, a_4) = (1, 1, -2, 0)$. From this data, we determine the action on the three complex planes to be given by $(b_1, b_2, b_3) = (-1, -1, 2)$. This model contains 27 orbifold fixed points. Their positions are (z_1, z_2, z_3), with $z_s = 0, \frac{1}{\sqrt{3}} e^{i\pi/6}, \frac{1}{\sqrt{3}} e^{-i\pi/6}$. In what follows, we refer to these points as 0, $+1$ and -1 respectively. Out of the 64 O3-planes, only the one at the origin, $(0, 0, 0)$, coincides with an orbifold fixed point. Note that the orientifold action leaves $z_s = 0$ invariant and interchanges $z_s = \pm 1$.

Given this discussion, we can write down the general solutions to the twisted tadpole cancellation equations (5.3.29) and (5.3.30) at orbifold fixed points. Since the orbifold fixed points at $(z_1, z_2, z_3) \neq (0, 0, 0)$ do not coincide with an O-plane, they must satisfy (5.3.29). The general solution reads

$$\gamma_\theta = \mathrm{diag}(\mathbf{1_N}, \alpha \mathbf{1_N}, \alpha^{\mathbf{2}} \mathbf{1_N}) \tag{5.3.47}$$

The resulting gauge theory has a $U(N) \times U(N) \times U(N)$ gauge group and matter content

	$U(N) \times U(N) \times U(N)$
X_{01}^s	$(\square, \overline{\square}, 1)$
X_{12}^s	$(1, \square, \overline{\square})$
X_{20}^s	$(\overline{\square}, 1, \square)$

(5.3.48)

with $s = 1, 2, 3$ and with the subindices indicating the gauge groups under which bifundamental fields transform. The overall $U(1)$ is anomaly free but decouples since all fields are neutral under it. The other two linear combinations of $U(1)$'s have mixed anomalies and become massive via the $B \wedge F$ couplings of the Green-Schwarz mechanism. The superpotential is

$$W = \epsilon_{stu} X_{01}^s X_{12}^t X_{20}^u \ , \tag{5.3.49}$$

where we have suppressed color indices for simplicity.

The origin $(z_1, z_2, z_3) = (0, 0, 0)$ is an orientifold singularity and hence (5.3.30) holds. The most general solution is

$$\gamma_\theta = \mathrm{diag}(\mathbf{1_N}, \alpha \mathbf{1_{N+4}}, \alpha^{\mathbf{2}} \mathbf{1_{N+4}}) \tag{5.3.50}$$

This results in a gauge theory with an $SO(N) \times U(N+4) = SO(N) \times SU(N+4) \times U(1)$ gauge group, with matter transforming as

	$SO(N) \times SU(N+4) \times U(1)$
$\overline{Q}^s$	$(\square, \overline{\square})_{-1}$
A^s	$(1, \square\!\square)_2$

(5.3.51)

where $s = 1, 2, 3$. There are mixed anomalies and the $U(1)$ factor becomes massive due to $B \wedge F$ couplings. The superpotential is given by

$$W = \epsilon_{stu} \overline{Q}^s A^t \overline{Q}^u \ . \tag{5.3.52}$$

The gauge theory of N D3-branes sitting on an O-plane that is not at an orbifold fixed point is $\mathcal{N} = 4$ SYM with $SO(N)$ gauge group and three antisymmetric chiral fields A^n. The superpotential in this case is

$$W = \epsilon_{stu} A^s A^t A^u \ . \tag{5.3.53}$$

Let us now consider the couplings generated by an E3-brane instanton. Equation (5.3.34) gives the fermionic zero modes between the instanton and the (fractional) D3-branes in either the hidden or visible sectors. Specializing to the case at hand, we get fermionic zero modes transforming in the $\square_1$ and $\overline{\square}_2$ representations for instantons wrapping $z_s = 0$, for $s = 1, 2, 3$. The subindices 1 and 2 of the representations denote the quiver nodes associated with the α and α^2 blocks of the CP matrix. As a result, any instanton corresponding to a general linear combination of the form (5.3.35), generates a coupling of the form

$$W_{inst} = \det X_{12} \qquad \text{or} \qquad W_{inst} = \sqrt{\det A} \ , \tag{5.3.54}$$

where the second possibility corresponds to the case in which nodes 1 and 2 are identified by the orientifold. X_{12} and A in the previous expressions correspond to the linear combinations of X_{12}^s and A^s that are determined by (5.3.35). We see that the instanton generates couplings involving only fields connecting nodes 1 and 2 (before orientifolding) in both hidden and visible sectors. For this reason, we are interested in hidden sectors in which operators made out of X_{12} or A have a non-zero F-term.

Similar reasoning applies to the case of D3-branes on O-planes that are not orbifold fixed points. In fact, we can simply take the general expressions for $\mathbb{Z}_N$ orbifolds and set $N = 0$. The instanton generated superpotential is, once again,

$$W_{inst} = \sqrt{\det A} \ , \tag{5.3.55}$$

with A the linear combination of antisymmetrics associated with the specific instanton embedding.

For D3-branes over orientifolds (either on orbifold singularities or not), the instanton generated coupling is non-zero only when the relevant $SO(n)$ or $SU(n)$ gauge group has even n,[14] since the determinant of an odd order antisymmetric matrix vanishes. This is true if the corresponding sector is either the visible or the hidden sector. For even n, $\det A$ is a perfect square, and the following way of writing its square root is sometimes convenient

$$\sqrt{\det A} = \frac{1}{2^n\, n!}\epsilon^{a_1 \dots a_n} A_{a_1 a_2} \dots A_{a_{n-1} a_n} := \mathrm{Pf}(A)\ . \tag{5.3.56}$$

5.4 Examples

In this section we present explicit models of D-instanton mediation involving different classes of hidden sectors. Our goal is to provide illustrative examples that show how D-brane instantons can communicate SUSY breaking rather than constructing models with fully realistic visible sectors. In sections 5 and 5.6 we discuss some possible directions for constructing more elaborate models.

5.4.1 Dynamical SUSY breaking hidden sector

In this subsection we present a model in which the hidden sector breaks SUSY via some non-abelian gauge dynamics.

We consider the $\mathbb{Z}_3$ orientifold of section 5.3.2. We place the hidden sector on top of the orientifold singularity at the origin, with CP matrix given by taking $N = 0$ in (5.3.50), i.e.

$$\gamma_{\theta,3} = \mathrm{diag}(\alpha \mathbf{1_4}, \alpha^{\mathbf{2}} \mathbf{1_4}) \tag{5.4.57}$$

The resulting gauge theory has an $SU(4) \times U(1)$ gauge symmetry and three fields A^i in the $\square\!\square_2$. The $\overline{Q}^s$'s from (5.3.51) are absent and hence there is no tree-level superpotential.

[14] The SO case refers to the situation in which the D3 branes are not at an orbifold fixed point, while the SU case refers to the situation in which the D3 branes are at an orbifold fixed point, and two of the resulting SU groups are identified by the orientifold.

Let us forget about the $U(1)$ factor for the moment. This model has been considered in [149]. It can be alternatively viewed as an $SO(6)$ gauge theory with three flavors of quarks in the vector representation. The theory is strongly coupled in the IR.[15] The low energy theory has two physically inequivalent phase branches [157]. On one of them, a non-perturbative superpotential is generated

$$W_{H,np} = 8\frac{\Lambda^9}{\det M} \ , \tag{5.4.58}$$

with M, the symmetric (over the $SU(3)$ flavor indices) meson matrix. SUSY is broken on this branch with runaway along M. The previous expression can be written in terms of the antisymmetrics of $SU(4)$. For diagonal M, we obtain

$$W_{H,np} = 8\frac{\Lambda^9}{\prod_s \mathrm{Pf}(A^s)} \ . \tag{5.4.59}$$

Reintroducing the $U(1)$ factor, the scalar potential contains a D-term contribution of the form

$$V_{U(1)} = \frac{1}{\lambda}(\sum_s 2|A^s|^2 - \xi)^2 \ , \tag{5.4.60}$$

where ξ is a *dynamical* FI term, which is related by SUSY to the $B \wedge F$ coupling that makes the $U(1)$ massive. If there is a mechanism stabilizing all Kähler moduli, ξ becomes a fixed parameter and (5.4.60) cures the runaway, producing a non-SUSY vacuum. For the purpose of illustration, we content ourselves with the fact that, regardless of whether the runaway is stabilized or not, there is a non-vanishing F_M on this branch.[16] The second branch has no non-perturbative superpotential and a quantum moduli space of vacua. The theory confines without chiral symmetry breaking. Since there is no non-perturbative superpotential, SUSY is not broken on this branch.

[15] The beta function for the inverse coupling, computed in both pictures, is equal to 9.

[16] This model has also been considered in [146] in connection with SUSY breaking, although with a completely different approach. In that case, a mass term for the antisymmetrics is generated by a D-brane instanton and SUSY breaking results from its interplay with a fixed non-vanishing ξ.

Our visible sector is a trinification model, a simple extension of the SM that has been investigated in the model building literature [150]. We place it at $(1,0,0)$, with its orientifold image at $(-1,0,0)$. Its CP matrix corresponds to taking $N=3$ in (5.3.47). We get

$$\gamma_{\theta,3} = \mathrm{diag}(\mathbf{1_3}, \alpha\mathbf{1_3}, \alpha^{\mathbf{2}}\mathbf{1_3}) \tag{5.4.61}$$

This produces an $SU(3)\times SU(3)\times SU(3)$ gauge theory with

	$SU(3)\times SU(3)\times SU(3)$
X^s_{01}	$(\Box,\overline{\Box},1)$
X^s_{12}	$(1,\Box,\overline{\Box})$
X^s_{20}	$(\overline{\Box},1,\Box)$

(5.4.62)

with $i=1,2,3$ and superpotential is

$$W_V = \epsilon_{stu} X^s_{01} X^t_{12} X^u_{20} \ , \tag{5.4.63}$$

The model is not fully realistic. For example, it does not contain the higgs fields that are necessary to break two of the $SU(3)$'s down to $SU(2)\times U(1)$.

Assuming $r_1 \sim r_2 \sim r_3$, the two smallest-volume mediating instantons wrap $z_2 = 0$ and $z_3 = 0$. There are additional orbifold invariant 4-cycles connecting the hidden and visible sectors, but contributions from instantons wrapping these cycles are highly suppressed since they have larger volume. From (5.3.49) and (5.3.52), our leading-order mediating instantons generate the following superpotential

$$W_{H/V} = e^{-V_{\Sigma_2}/g_s}\sqrt{\det A^2}\,\det X^2_{12} + e^{-V_{\Sigma_3}/g_s}\sqrt{\det A^3}\,\det X^3_{12} \ . \tag{5.4.64}$$

where the orientation of the mediating instantons projects onto gauge invariants of the SUSY breaking hidden sector theory that acquire F-term vevs on the branch with the non-perturbative superpotential given in (5.4.59). Therefore, (5.4.64) gives rise to the following A-terms

$$V_{soft} = e^{-V_{\Sigma_2}/g_s} F^*_{\mathrm{Pf}(A^2)} \det X^2_{12}|_{\theta=0} + e^{-V_{\Sigma_3}/g_s} F^*_{\mathrm{Pf}(A^3)} \det X^3_{12}|_{\theta=0} + c.c. \ , \qquad (5.4.65)$$

where specializing for $\theta = 0$ indicates that we take the scalar component of the visible sector chiral superfields. For simplicity, in this section and the next one, we omit obvious powers of M_s and the dynamical scale Λ of the hidden sector, which are necessary for expressions to have the correct dimensionality. Note that these A-terms correspond to couplings between the Higgs fields and the sleptons. Analogous Yukawa couplings are also generated non-perturbatively by other D-brane instantons (see discussion below).

Our hidden sector and visible sector (plus image) involve 26 D3-branes. We can cancel untwisted tadpoles (5.3.28) without spoiling the nice features of our model by placing the 6 additional D3-branes in sets of two over the O3-plane at $(\frac{1}{2}r_1, \frac{1}{2}r_2, \frac{1}{2}r_2)$ and its two $\mathbb{Z}_3$ images.

5.4.2 Polonyi hidden sector

Another exciting direction is the possibility of having a simple field theory hidden sector that breaks SUSY without involving non-abelian gauge dynamics, along the lines of [109].[17] A remarkably simple possibility is to engineer a Polonyi model. This construction is very general and can easily be part of more complicated setups. Because of this, we consider it deserves to be discussed first, independently of the details of the full compact model.

Engineering a Polonyi model

The configuration we want to consider consists of an O3-plane and a single D3-brane separated from it, with an E3-brane connecting them. Without loss of generality, we can assume that the E3-brane wraps the $z_1 = 0$ cycle.[18] The setup is sketched in Figure 5.4, where we have also included the image D3-brane.

[17] In a similar spirit, another realization of a Polonyi model involving D-brane instantons appears in [138].

[18] It is straightforward to extend our argument to the case in which the instanton is defined by some linear combination of the of the form (5.3.35).

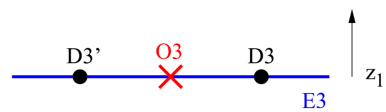

Figure 5.4: The basic configuration realizing a Polonyi model. It consists of an O3-plane and a D3-brane away from it, connected by a finite size E3-brane with $O(1)$ CP projection.

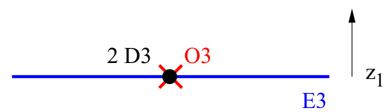

Figure 5.5: A Polonyi model is also obtained when two D3-branes sit on top of an O3-plane. A finite size E3-brane with $O(1)$ CP projection generates the superpotential.

The gauge theory on the D3-brane is $\mathcal{N}=4$ $U(1)$ SYM with three chiral superfields Φ^s. The Φ^s transform in the "adjoint" representation, which is trivial for $U(1)$ (i.e. they are neutral fields). As a result, the beta function for the gauge coupling is zero and we can tune the gauge coupling to be arbitrarily small. In addition, the $\mathcal{N}=4$ superpotential (5.3.53) vanishes.

If the CP projection on the E3-brane is $O(1)$, it induces a coupling

$$W = e^{-V_{\Sigma_1}/g_s}\, \Phi^1 \ . \tag{5.4.66}$$

This is precisely a Polonyi model superpotential and SUSY is broken by $F_{\Phi_1} \neq 0$. Of course, other instantons contribute to the superpotential. However, by considering the effect of only the z_1 instanton, we are implicitly assuming that $r_1 \gg r_2, r_3$. As usual in Polonyi models, Φ_1 is a classically flat direction. Its stability depends on the details of the full model. The same comments apply to the construction we present below. This is a question that certainly deserves more study in our concrete setups.

Let us now investigate what happens if we collapse the D3 and D3$'$ on top of the O3-plane. To avoid a chiral theory on the D3-branes, we further assume that the O3-plane is not at an orbifold fixed point. The configuration is shown in Figure 5.5.

The resulting gauge theory is $\mathcal{N}=4$ $SO(2)$ SYM, with three chiral superfields A^s in the antisymmetric representation. Once again, the beta function for the gauge coupling vanishes and can thus be tuned to any desired value.[19] As before, we exploit this fact to make the gauge coupling small so it can be neglected. The antisymmetric representation of $SO(2)$ is trivial and the A^s have the general form

$$A^s_{ab} = \phi_s \, \epsilon_{ab} \ . \tag{5.4.67}$$

Then, the $\mathcal{N} = 4$ superpotential (5.3.53) vanishes. If the E3-brane has an $O(1)$ CP projection, it generates a coupling

$$W = e^{-V_{\Sigma_1}/g_s} \sqrt{\det A^1} = e^{-V_{\Sigma_1}/g_s} \, \phi_1 \tag{5.4.68}$$

A square root appears in (5.4.68) as opposed to (5.4.66) because, in this case, the E3-D3 and D3-E3 fermionic zero modes are identified by the orientifold projection. [20]

In our opinion, this simple realization of a Polonyi hidden sector is interesting in its own right, independently of which mechanism mediates SUSY breaking.

Notice that the superpotential terms generated in this section are not perturbatively forbidden by any $U(1)$ symmetry. The appearance of a non-perturbative superpotential determined by the zero of the E-brane embedding is an example of Ganor's zeros [90, 91]. In [90] the ADS superpotential of $N_f = N_c - 1$ SQCD was generated along these lines. In that case, the effect is due to a gauge theory instanton since the flavor D7-branes and the E3-brane are wrapped over the same 4-cycle. In our setup, D7-branes are not present and the effect is purely stringy. In section 6, we discuss similar operators in more general models.

[19] This can be understood as follows. The antisymmetric representation of $SO(N)$ is the same as the adjoint representation. $C(adj) = (N-2)$ for $SO(N)$, and hence vanishes in this case. As a result, the beta function also vanishes. An equivalent way of thinking about the beta function is that $SO(2) = U(1)$ and the antisymmetric representation corresponds to a neutral field.

[20] It is interesting to consider what happens for a single D3-brane on top of the O-plane. The coupling (5.2.3) vanishes identically due to anti-commutativity of α which, in this case, is one dimensional.

A full model: visible sector and mediation

We now use a hidden sector of the type just described as a part of a simple compactification. We consider the $\mathbb{Z}_3$ toroidal orientifold of section 5.3.2. In this case, we will use D-brane instantons not only for mediating, but also for generating SUSY breaking. This fact gives rise to constraints on the relative sizes of the tori that are necessary for the model to work.

We engineer a Polonyi hidden sector by placing two D3-branes on top of the O3-plane at $(\frac{1}{2}r_1, \frac{1}{2}r_2, \frac{1}{2}r_3)$ (plus its two $\mathbb{Z}_3$ images). A Polonyi superpotential

$$W = e^{-V_{\Sigma_1}/g_s}\phi_1 \tag{5.4.69}$$

is generated by the E3-brane at $z_1/r_1 = 1/2$. This term is the dominant contribution to the superpotential involving hidden sector fields provided that $r_1 \gg r_2, r_3$ since this condition guarantees that $V(\Sigma_1) \ll V(\Sigma_{2,3})$. Notice that although the cycle wrapped by this instanton is not invariant under the orbifold group, it does not intersect its images and hence there are no extra zero modes. Following the discussion in section 5.3.1, it generates a non-vanishing contribution.

Next, let us think about where to locate the visible sector. Note that in order not to spoil the generation of the Polonyi superpotential, the instanton wrapping $z_1/r_2 = 1/2$ must not intersect the visible sector. A particularly simple choice, then, is to locate the visible sector on top of the orientifold singularity at the origin, $(0,0,0)$. As our visible sector, we will choose an interesting GUT-like model that can be engineered as follows [146]. We take $N = 2$ in (5.3.50)

$$\gamma_{\theta,3} = \mathrm{diag}(\mathbf{1_2}, \alpha\mathbf{1_6}, \alpha^{\mathbf{2}}\mathbf{1_6}) \ . \tag{5.4.70}$$

This gives rise to a theory with $U(6) \times O(2)$ gauge group and chiral multiplets transforming as

$$3\,(\overline{15}, 0) + 3\,(6, +1) + 3\,(6, -1) \ . \tag{5.4.71}$$

Under the $SU(5)$ subgroup of $U(6)$, these representations decompose as $\overline{15} = \overline{10} + \overline{5}$ and $6 = 5+1$, giving rise to three SM generations $(\overline{10}+5)$ and three sets of higgs fields $(5+\overline{5})$.[21]

As it stands, this model is not fully realistic since it does not contain the Higgs field necessary for breaking the GUT group.

Let us now consider the mediating instantons. In order to couple to F_{ϕ_1}, the embedding equation of a mediating instanton must involve z_1—i.e., the orientation of the instanton must project onto the SUSY breaking part of the hidden sector. However, since the hidden sector is located at the origin, the equation must also involve z_2 and/or z_3. Notice that this geometric fact immediately implies that the mediation term will be a small perturbation of the Polonyi superpotential, since an equation involving $z_{2,3}$ requires that the instanton wrapping numbers on the much larger cycles $\Sigma_{2,3}$ cannot both be zero. Hence, a simple choice for a cycle wrapped by a mediating instanton is to take

$$z_1/r_1 - z_2/r_2 = 0 \tag{5.4.72}$$

with volume

$$V(\Sigma_{(1,-1,0)}) = \frac{1}{12^3}\sqrt{V(\Sigma_2)^2 + V(\Sigma_1)^2} = \frac{1}{12^3}\Big(V(\Sigma_2) + V(\Sigma_1) \cdot \frac{V(\Sigma_1)}{2V(\Sigma_2)} + ...\Big) \tag{5.4.73}$$

where we explicitly see that the mediating instanton will have large volume compared to the Polonyi instanton since $V(\Sigma_2) \gg V(\Sigma_1)$.

What about additional instantons connecting the two sectors? In general, the answer is quite complicated but can be worked out in detail. However, for simplicity, we will further assume that

$$V(\Sigma_1)\epsilon_{2,3} \gg 1 \tag{5.4.74}$$

where

$$\epsilon_{2,3} = \frac{V(\Sigma_1)}{V(\Sigma_{2,3})} \ll 1 \tag{5.4.75}$$

Note that (5.4.74) is assumed in addition to the assumption that $V(\Sigma_1) \ll V(\Sigma_{2,3})$. We can explain the motivation for (5.4.74) in a general context. Consider two instantons with

[21] The presence of a couple of copies of each MSSM or SUSY GUT higgs, although not unavoidable, is a usual feature in D-brane realizations.

comparable volumes $V \sim V'$. The relative suppression of their contributions is given by $e^{-V'}/e^{-V}$. We see that small differences in the volume are exponentially enhanced. Equation (5.4.74) amounts to requesting that $e^{-V'}/e^{-V} \ll 1$. Under these conditions, the four leading-order mediating instantons in this approximation wrap

$$\begin{aligned} \frac{1}{r_1}z_1 \pm \frac{1}{r_2}z_2 = 0 \\ \frac{1}{r_1}z_1 \pm \frac{1}{r_3}z_3 = 0 \end{aligned} \tag{5.4.76}$$

Thus, the superpotential from each of these mediating instantons is

$$W_{H/V}^{\pm 2,\pm 3} \sim e^{-V_{\Sigma_{\pm 2,\pm 3}}/g_s} \left(\frac{\phi_1}{r_1} \pm \frac{\phi_{2,3}}{r_{2,3}}\right) \sqrt{\det\left(\frac{A_1}{r_1} \pm \frac{A_{2,3}}{r_{2,3}}\right)} \tag{5.4.77}$$

where $\overline{A}_i$ are the three $(\overline{15}, 0)$ fields in the visible sector. While $\phi_1/r_1 \ll \phi_2/r_2$ in the hidden sector piece, it is only ϕ_1 that gets a non-vanishing F-term and contributes to the soft terms. We get the following A-terms from each of the instantons

$$V_{soft} \sim e^{-(V_{\Sigma_1}+V_{\Sigma_{\pm 2,\pm 3}})/g_s} \frac{F_{\phi_1}^*}{r_1} \epsilon^{abcdef} \widetilde{A}_{ab}\widetilde{A}_{cd}\widetilde{A}_{ef}|_{\theta=0} + c.c. \ , \tag{5.4.78}$$

where the cycle wrapped by the Polonyi instanton is Σ_1 and where we have defined $\widetilde{A} = (A_1/r_1 \pm A_{2,3}/r_{2,3})$. For simplicity, we have omitted an obvious r_1 and $r_{2,3}$ dependent normalization of (5.4.77) and (5.4.78). As an aside, note that these A-terms contain couplings between the Higgs fields and the U-type squarks. The corresponding Yukawa couplings are also generated by D-instantons (see discussion below).

The configuration is still missing 12 D3-branes in order to cancel untwisted tadpoles. A simple way of completing the model without spoiling the features we have just discussed is by placing 6 D3-branes with CP matrix

$$\gamma_{\theta,3} = \mathrm{diag}(\mathbf{1_2}, \alpha\mathbf{1_2}, \alpha^{\mathbf{2}}\mathbf{1_2}) \tag{5.4.79}$$

at each of the $(\pm 1, 0, 0)$ orbifold fixed points.

5.5 Phenomenology and instanton orientation

The visible sectors we have discussed in the models above are deliberately simple and, as a result, unrealistic. For example, as we have mentioned, we do not even have all the Higgs fields necessary to break to the SM gauge group. However, motivated by the fact that our theories contain three generations of matter with various Yukawa couplings (among them the Yukawa couplings of the MSSM) and noting that our instantons generate various A-terms for the visible fields, we are led to ask a very simple question: are the A-term matrices, $\mathbf{A}_i$, and Yukawa coupling matrices, $\mathbf{Y}_i$, aligned? The main phenomenological motivation for this question is that alignment of these matrices guarantees suppression of potentially troublesome Flavor Changing Neutral Currents (FCNCs) contributions from the A-terms. In particular, if $\mathbf{A}_i \sim k_i \mathbf{Y}_i$ we say the matrices are aligned. This implies that the A-term contributions to the FCNC processes responsible for reactions like $K^0 \to \overline{K}^0$ are highly suppressed (though not absent).

Before proceeding, we should make two clarifying points. First, even if we can align the A-terms and Yukawas, we should emphasize that there are still other soft terms that could generate FCNCs, like the non-holomorphic part of the squark mass matrix. Since we have only discussed non-holomorphic mass generation by instantons in the case of squarks and sleptons charged under abelian symmetries (note however footnote 8), we will simply assume that the physics responsible for the non-holomorphic squark mass generation in the examples is flavor blind. Finally, let us also point out that demanding $\mathbf{A}_i \sim k_i \mathbf{Y}_i$ is generally a sufficient but not necessary condition for suppressing FCNC contributions from A-terms. Indeed, we will also consider the less restrictive condition that the A-terms and Yukawas are simply mutually diagonalizable. This scenario also leads to suppression of FCNC contributions under a rather broad set of conditions.

Giving a precise answer to the question of whether or not the A-terms and Yukawas are diagonal in the same basis or, more restrictively, whether $\mathbf{A}_i \sim k_i \mathbf{Y}_i$ in our setups depends on stabilizing the various moduli of our compactification. However, as we will see, we can give an interesting heuristic answer to this question with no additional assumptions beyond those we have already made. Furthermore, this discussion will point us to other potentially interesting constructions. In particular, we will continue to assume that the complex structure moduli dependence of the instanton-induced operator coefficients can be treated as insignificant $\mathcal{O}(1)$ factors and that the volume (Kähler) moduli can indeed be dynamically set to the rough values and hierarchies we take.

Let us focus our discussion on the example with the $U(6) \times SO(2)$ visible sector and Polonyi hidden sector. Furthermore, we will focus on the same region of moduli space as in the discussion above. Namely, we will assume a particular hierarchy $r_1 \gg r_{2,3}$, so that

$$1 \ll V(\Sigma_1) \ll V(\Sigma_{2,3}) \ll V(\Sigma_1)^2 \tag{5.5.80}$$

This corresponds to a region of moduli space where it costs a significant amount of action to go from an instanton with a particular set of wrapping numbers to a configuration with one of the wrapping numbers increased by one.

Now, let us discuss the Yukawa couplings of the $U(6) \times SO(2)$ visible sector. Note that the Yukawa couplings of the visible sector are of two types. The first type are perturbative couplings from the tree level quiver superpotential and are of the form

$$W_{\text{tree}} = Y_{ijk} A^i \overline{Q}^j \overline{Q}^k \tag{5.5.81}$$

The second type of terms are non-perturbative Yukawa couplings generated by D-instantons and are of the form

$$W_{\text{np}} \sim e^{-V_1/g_s}(A^1)^3 + e^{-V_2/g_s}(A^2)^3 + e^{-V_3/g_s}(A^3)^3 \sim e^{-V_1/g_s}(A^1)^3 \tag{5.5.82}$$

in our approximation. We have used the shorthand $(A^1)^3 = \epsilon^{a_1 b_1 a_2 b_2 a_3 b_3} A^1_{a_1 b_1} A^1_{a_2 b_2} A^1_{a_3 b_3}$, etc. As mentioned in the example, these non-perturbative Yukawas give rise to the u-type quark couplings [146].

By comparing (5.4.78) with the Yukawa couplings we have just described above, it should be clear that the A-term and Yukawa matrices are highly non-aligned. Indeed, it is not hard to see why this is the case. First of all, the mediating instantons do not generate A-terms that correspond to the perturbative Yukawas since the perturbative superpotential comes from closed paths in the quiver, while the instanton-induced terms come from open paths (which we define to include two-tensor field loops at the same node). Note that this lack of A-terms corresponding to the perturbative Yukawa couplings is not a problem since it does not affect the mutual diagonalizability of the A-terms and Yukawas. Furthermore, if we want, we can presumably generate such terms by going to regions of the moduli space where instanton mediation and e.g. gauge or anomaly mediation are comparable in strength—these other mediation mechanisms will generate the 'perturbative' A-terms in a flavor blind way.[22]

Thus it remains only to discuss the mutual diagonalizability and alignment of the instanton-induced A-terms with the non-perturbative Yukawas. Notice that these couplings, like the corresponding A-terms, cannot be generated perturbatively since they violate the anomalous $U(1)$ factor of the $U(6)$ node, and so they must be generated by a non-perturbative effect like D-brane instantons. Examining our above results, it should be obvious that though the A-terms and non-perturbative Yukawas are not aligned, they are mutually diagonal![23]

Let us press on and try to understand the lack of alignment between the A-terms and the Yukawas. This goal is useful because a better understanding of this lack of alignment will lead us to a slightly more interesting characterization of D-brane instantons that may serve as a simple guide in building more complicated models.[24]

To that end, note that the non-perturbative Yukawas we have written above are generated by the instantons wrapping the cycles $z_i = 0$ with the dominant contribution coming from the instanton wrapping $z_1 = 0$. We can then see a more general geometric reason for the lack of alignment in the non-perturbative sector of the theory: since the orientation of the instanton picks out the flavor of the fields it couples to, in order to have alignment of the A-terms and the Yukawa couplings, the orientations of their generating instantons must

[22] We will briefly discuss the possibility of also generating such soft terms via instantons in the next section.
[23] A similar conclusion applies to our first model.
[24] Another motivation is that we could presumably have considered moving the visible sector in the first example from $(1, 0, 0)$ to $(1, 1, 1)$ while keeping the hidden sector fixed at the origin. If we again assumed $r_1 \sim r_2 \sim r_3$, then the A-terms and Yukawas would not have been mutually diagonal.

also align. It is rather easy to see this is not possible by the following simple argument. Suppose we could choose a Yukawa generating instanton to align with an A-term generating instanton. Then, the two instantons would share a common normal T^2 which we denote T^2_N. On T^2_N, the Yukawa and mediating instanton worldvolumes are localized at points x_Y and $x_M \neq x_Y$ respectively. Since the mediating instanton intersects both the hidden and visible sectors, both of these sectors must also be localized at x_M.[25] Hence, the Yukawa instanton cannot intersect the visible sector and no term is generated. Still, one might hope that it is possible to approximately align the instantons and hence circumvent our previous argument.

The situation seems better when some of the cycles are small since then one could hope to find greater alignment by considering instantons that differ by wrappings on these small cycles. This is not the case if one of the two instantons does not wrap the small cycle. For example, consider $r_1 \gg r_2$ and two instantons, wrapping $z_1 = 0$ and $z_1/r_1 + n\, z_2/r_2 = 0$ (with $n \in \mathbb{Z}$), respectively. While it is true that not only both cycles are almost aligned but also their volumes are very similar, the generated A-terms and Yukawa couplings are very different. This is because the first instanton generates a term involving only A^1, while the second one gives rise to a contribution which mostly depends on A^2.

5.5.1 A broader definition of instanton orientation

One potentially interesting solution to the lack of alignment between the mediating and Yukawa instantons is to realize that we have been considering particularly simple instantons—those that are invariant under the orientifold and therefore carry an $O(1)$ CP bundle. These instantons are interesting since the orientifold lifts additional neutral fermionic zero modes that would otherwise lead to a vanishing contribution to the A-terms and Yukawa couplings. However, if we are willing to consider, for example, using fluxes to lift the extra neutral zero modes of the $E3$ branes, then we are free to consider instantons with a $U(1)$ CP bundle. In

[25] This last statement need not apply in cases involving D7 branes.

particular, these instantons can have non-trivial CP orientation given by

$$\gamma_{\theta,E3} = \alpha^j \tag{5.5.83}$$

Therefore, such an instanton carries two orientations: the geometrical orientation we have discussed above and the CP orientation just described.[26] We can use this richer structure to align the instantons geometrically by noting that

- The (untwisted) geometrical orientation controls which flavors the instanton couples to.
- The CP orientation controls which gauge nodes the instanton interacts with.

Hence a simple way to potentially align A-terms and Yukawa couplings is to take their generating instantons to wrap the *same* cycle but give the instantons different CP orientation. This means that the instantons will couple to different nodes in both the hidden and visible sector. By considering an orientifolded visible sector, it is possible to identify the different nodes so that the operators the instantons generate are the same in the visible sector. If, however, we take the hidden sector to be non-invariant under the orientifold, then we could imagine the situation in Figure 5.6, where the mediating instanton couples to fields responsible for SUSY breaking, while the Yukawa instanton couples to empty nodes and hence generates a term without hidden sector fields (note that there are no additional zero modes going between the Yukawa instanton and the hidden sector and so one does not have to worry about a vanishing contribution to the effective action). This strategy may work in higher-order orbifolds and their partial resolutions or in situations where one considers multiple orbifolds of a given space. It would be interesting to find an explicit construction realizing this idea.

[26] Strictly speaking, the CP orientation represents the cycle wrapped by the instanton in the twisted homology of the singularity.

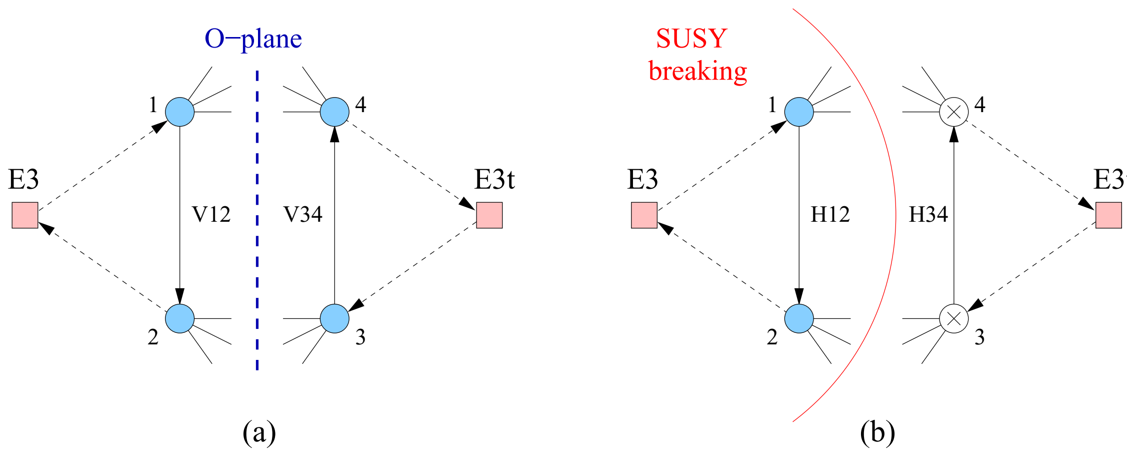

Figure 5.6: The zero mode structure of a $U(1)$ mediating instanton (E3) and a Yukawa-generating instanton ($\widetilde{\rm E}3$) wrapping the same cycle but with different CP orientations. Figure (a) represents the interaction structure in the visible sector where the interactions are identified by the orientifold plane while Figure (b) represents the interaction structure in the hidden sector.

5.6 Further possibilities

We have deliberately kept the previous examples as simple as possible, only using D3-branes on toroidal orientifolds. There are various refinements that can be introduced in order to obtain more interesting models. We now mention a few of them. It would certainly be interesting to explore model building along these more general lines.

One such extension consists of considering not only orbifold but more general singularities. A practical way of generating many examples of this sort consists of starting from a large orbifold group, for example $\mathbb{Z}_M \times \mathbb{Z}_N$, and then partially resolving some of the (orientifold) singularities. We can take this approach to generate more general visible and hidden sectors. Consistency is not affected by partial resolution, since cancellation of twisted and untwisted tadpoles is preserved in the process. The resulting turning on of background values for the twisted Kähler moduli might also have some interesting effects on the instanton dynamics.

Another illuminating avenue might be to consider more general compact geometries than T^6, since in the case of T^6 we have a very simple untwisted cohomology structure.

This limited structure can be interesting in the sense that we can then see clear and rather simple connections between completely different physics as in the second example where the need to break SUSY required a certain hierarchy of scales that then was imprinted on the A-terms and the u-type Yukawas.[27] On the other hand, such a geometry may be unduly restrictive when trying to generate different models of phenomenological interest that avoid troublesome aspects like large FCNCs.

Another possibility is that the compactification might also involve some anti D-brane sectors, which give rise to additional sources of SUSY breaking.

A further extension might be to introduce D7-branes (and anti D7-branes as needed to cancel untwisted tadpoles). D7-branes have various useful applications. For example, they are necessary in simple supersymmetric extensions of the SM based on D-branes at singularities [21]. They can also give rise to simple metastable SUSY breaking hidden sectors [113, 153].

Also, it should be rather simple to construct models that generate B-terms as well. It might then be interesting to study the $\mu/B\mu$ problem in this context.

An direction worth pursuing is to consider more general kinds of E3-brane instantons than those we have considered in the examples. One avenue is to turn on fluxes as a means of both stabilizing the complex structure moduli of the geometry and of lifting the accidental zero modes of the instantons. Also, considering E3-branes with non-trivial gauge bundles would also potentially be interesting, and one could then make contact, via T-duality, with the study of instanton stability and dynamics across lines of marginal stability in the moduli space discussed in [142]. A detailed investigation of these topics may lead to a richer set of examples of instanton-generated soft terms and also to a better understanding of instanton-mediation in the closed string picture.

We have focused on D-brane instantons wrapping 4-cycles of the form (5.3.35), which produce operators made out of a some linear combination of bifundamental fields connecting a single pair of nodes in the quiver. In general compactifications, as we mentioned briefly in

[27] In a very limited sense, the conditions required to break SUSY in the hidden sector of this model 'explain' the relatively large top quark Yukawa coupling!

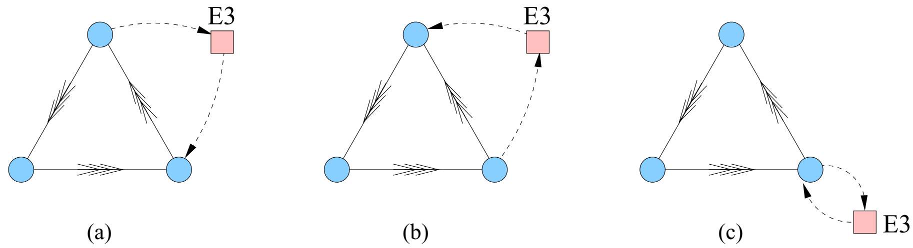

Figure 5.7: The three classes D-brane instantons on $\mathbb{C}^3/\mathbb{Z}_3$. Notice the opposite orientation of the fermionic zero modes between (a) and (b). While class (a) couple to single bifundamentals, class (b) couple to linear combinations of products of two of them. Class (c) are completely non-chiral.

section 2.1, we can expect to generate operators of the form (5.2.2) (and generalizations for cases with orientifold identifications), for X_{ij} being an arbitrary oriented path in the quiver $X_{ij} = X_{i\,k_1} X_{k_1 k_2} \ldots X_{k_n j}$. These more general operators expand the range of model building possibilities, for example relaxing the conditions for quadratic and cubic superpotential terms listed in section 2.3. In local constructions, i.e. leaving aside the issue of how 4-cycles are completed in a compactification, the question of which 4-cycles are wrapped by the corresponding instantons can be understood in detail. A systematic construction of such embeddings for toric singularities can be found in [159] (see also appendix A of [160] for a relevant discussion in the related context of flavor D7-branes). Figure 5.7 shows the extended quivers for general instantons in a local $\mathbb{C}^3/\mathbb{Z}_3$ singularity. In this chapter, we have considered the first possibility.

For closed paths in the quiver, i.e. for $i = j$, we generate the determinant of a "mesonic" operator X_{ii}.[28] These operators are not perturbatively forbidden by global $U(1)$ symmetries, since they are neutral under all of them. The corresponding instanton contains vector like fermionic zero modes α and β (see e.g. Figure 5.7.c), whose mass is controlled by X_{ii} according the action term (5.2.1). In other words, X_{ii} measures the distance between the D3 and the E3. The couplings in [161] are examples of such "mesonic" operators. Our mechanism can be regarded as the open string channel interpretation of the closed string gravitational exchange in [161].

[28]Notice that we do not sum over initial and final $SU(N)^{(i)}$ color indices.

5.7 Conclusions

In this chapter we have described a new way of generating soft terms in string compactifications. It is quite interesting to note that instanton mediation has aspects of both open and closed string mediation. On the one hand, it is not sensitive to global $U(1)$ symmetries that constrain (low energy effective) open string mediation, but on the other it is sensitive to the chiral gauge invariants of the various sectors in the theory—in particular, certain hidden sector theories seemingly can never communicate their SUSY breaking to the visible sector via instantons since in these cases instantons project onto trivial chiral gauge invariants.[29]

In any case, we hope to have given a flavor of instanton mediation in this chapter, and we leave it to future work to resolve the various outstanding questions we have raised and find more complete realizations of the ideas we have discussed. Above all, though, we simply hope to have illustrated the point that instanton-mediated physics between various D-brane sectors is rather generic in string compactifications and may serve as a phenomenological constraint on string model building.

[29] An example of this statement is the $SU(5)$ model considered in [151]. In this case, the simple $O(1)$ instantons we have focused on would project onto gauge invariants that correspond to determinants of 5×5 anti-symmetric matrices that must trivially vanish.

Chapter 6

Conclusions

In this thesis we have examined various aspects of SUSY breaking and its mediation to the visible sector. In the second chapter, we focused on a class of purely field theoretical constructions for mediating SUSY breaking called, “gauge mediation.” These theories are particularly attractive because they naturally solve the problem of FCNCs.

By further developing a general framework involving current correlators to describe theories of gauge mediation, we were able to deepen our understanding of these theories. For example, we proved the finiteness of the various soft parameters generated by gauge mediation; we showed that by using only weakly coupled models, one could cover the parameter space associated to these current correlators; finally, we understood how the existence of a non-vanishing supertrace destroyed the calculability of theories of weakly coupled messengers. The main result of our work is twofold. First, we have shown that the soft spectrum generated by gauge mediation is much more diverse than previously thought, while still solving the problem of FCNCs and having distinctive sum rules for the squarks that hold even at strong coupling. Second, we have given clear criteria for understanding when one can trust the results of computations in certain broad classes of theories of gauge mediation.

Still, there are many outstanding questions raised by our work. For example, we only defined the supertrace in weakly coupled theories. One natural question is whether the concept of the supertrace can be generalized to strongly coupled theories as well. Although we will not describe it further in this thesis, the answer to this question seems to be yes, and it likely involves correlation functions of the supercurrent multiplet of the hidden sector. This quantum definition of the supertrace may then be naturally related to various anomalies of the hidden sector.

Another outstanding question raised by our work is whether one can prove the finiteness of the soft masses generated in gauge mediation without appealing to the existence of a special discrete symmetry to eliminate possible infrared (IR) divergent contributions from massless particles. It is then interesting to ask whether our results still hold in the absence of such symmetries. Work that will soon be published with Z. Komargodski shows that the answer to this question is a resounding yes.

Related to the issue of the IR behavior of the various current correlators is the question of how the detailed properties of the hidden sector vacuum affect the soft observables. One intriguing subquestion in this vein is to understand how to describe scenarios in which the symmetry responsible for mediating the SUSY breaking is Higgsed. In particular, it would be interesting to understand what one can say in general about such situations. Surprisingly, the general description of theories of gauge mediation with spontaneously broken symmetries seems to be much more powerful than the description we gave above in the un-Higgsed case. Indeed, work soon to be published with Z. Komargodski shows that for spontaneously broken gauge symmetries one has considerable analytic control over the leading order in g corrections to the soft masses and A-terms even in strongly coupled theories.

Interestingly enough, the question of Higgsed symmetries leads us naturally to our construction of a stringy SSM-like theory in Chapter 3. Indeed, as we saw from our model and the discussion surrounding it, D-brane realizations of the SSM often contain additional $U(1)$ symmetries, some of which will be Higgsed (as we have discussed in Chapter 3, other symmetries will have their gauge bosons receive a mass through the Stückelberg mechanism). It would then be interesting to study the effects of gauge mediation involving these

Higgsed symmetries on the soft spectrum of the light fields. Some work in this direction has already been done, but we would like to use our general formalism of gauge mediation to further understand these setups.

On the purely string theoretical side, there are also many open problems. For example, given our work in Chapters 3 and 4, we would like to study complete local constructions involving a SUSY breaking hidden sector and an SSM. As we discussed in the conclusions to Chapter 4, our construction in Chapter 3 could naturally accommodate just such a possibility. Also, to get a better handle on the SUSY breaking dynamics of the hidden sector, it would be useful to have a better understanding of the back reaction of the hidden sector branes and the gravity side of the relevant gauge-gravity dualities.

Finally, it would be interesting to construct complete compactifications that include our local string constructions. In principle, one could also imagine studying SUSY breaking mediation from D-brane instantons as we studied in Chapter 5 but with more realistic visible sectors. Perhaps by understanding how our constructions and ideas are completed into full UV compactifications, we will actually find more detailed geometrical and topological constraints from phenomenological inputs. We have already seen in Chapter 3 that the requirement of having only one massless $U(1)$ symmetry gives us some information about the compactification topology. As we have hinted at in Chapter 5, D-brane instanton effects may give us even more information.

In any case, much work remains to be done, and much remains to be explored. Indeed, while SUSY stabilizes the hierarchy between the weak and Planck scales, the huge energy expanse remains. Almost certainly, many interesting phenomena await in this vast domain. It seems entirely plausible that SUSY will give us some of the most important tools to use in trying to understand this brave new world of high energy quantum physics.

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