Infrared signatures of quantum bounce in collapsing geometry

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We study the radiation profile of the unitarily evolving wave packet constructed for the quantum model of spherically symmetric dust shell collapsing in marginally bound Lemaître-Tolman-Bondi (LTB) model. In this analysis, we consider the quantum model of dust shell collapse in LTB spacetime,[1] where the dust shell collapse to black hole singularity is replaced by a bounce. We identify the observable natural to collapse/expansion character of dust shell, and study the mode decomposition in the quantum model. The incoming/outgoing modes are associated with the eigenfunctions of the Hermitian extension of the operator corresponding to this observable. For the wave packet representing the collapsing and expanding phase of the dust shell, we estimate the contributions of the incoming/outgoing modes. We find that the collapsing and expanding branches do not comprise entirely of incoming and outgoing radiation. The dust shell dynamics is insensitive to the large wavenumber modes as their contribution is negligible. Near the bounce point, the contribution of outgoing (incoming) modes in the collapsing (expanding) branch is substantial and it decreases as the dust shell moves away from the singularity. In the early (later) stage of the collapsing (expanding) phase, the incoming (outgoing) modes dominate. As the dynamics of the dust shell is sensitive to the near-infrared modes of the radiation, the information of the bounce is carried over to infrared modes much before it reaches the observer. In the infrared regime, a flip is observed from largely incoming to largely outgoing radiation as the evolution progresses from collapsing to expanding phase. The information of the short-scale physics is carried over to the longest wavelength in this quantum gravity model.

Keywords: Shell collapse; quantum bounce.

1. Introduction

Ever since the original analysis by Hawking and Unruh,[2,3] the study of particle content of the vacuum and the radiation profile in various geometries have been the frontier research topic in the quantum effects in curved spacetime. These effects are investigated in the context of quantum field theories in the curved spacetime.[4] The key result in this paradigm is that the particle content in the quantum states is not a generally covariant notion, which is the gist of the Hawking-Unruh effect.

The analysis of matter collapse in a consistent quantum gravity model has been addressed in the context of dust shell collapse,[5,6] LTB dust collapse model in canonical quantum gravity[7,8] and loop quantum gravity.[9–11] The radiation profile of dust collapse in the LTB model is studied in[12,13] under the midisuperspace quantization of this model. The Hawking radiation is recovered from the regularized solution of

the Wheeler-DeWitt equation for this model and the non-thermal correction to the spectrum and entropy associated are also computed. In this approach, the Hawking radiation is thought of as the projection of wave functional along the outgoing modes defined in that approach.

There are some issues associated with the aforementioned quantum models. The incoming/outgoing modes defined in this approach are not orthogonal, thus, rendering the notion of incoming/outgoing modes mathematically ill-defined. This issue stems from the fact that the momentum operator in this model is not Hermitian. Apart from that the states in the Kernel of Hamiltonian constraint need not be orthonormal as they are degenerate states with zero eigenvalue.

In this work, we study the radiation profile for the unitarily evolving wave packet constructed for the quantum model for dust shell collapse in a marginally bound LTB geometry.[1] For the collapsing/expanding character of the dust shell, momentum conjugate to areal radius is a good indicator. In the quantum model, we study the mode decomposition viz-a-viz the eigenstates of momentum operator.

2. Minisuperspace Construction of Dust Collapse in Marginal LTB Model

LTB model is an inhomogeneous extension of the FRW model, which is spherically symmetric and sourced by a non-rotational dust of energy density ϵ.[14] The line element for the LTB model is,

$$ds^2 = -c^2 d\tau^2 + \frac{R'^2}{1 + 2f(\rho)} d\rho^2 + R^2 d\Omega^2,$$ (1)

here τ is the dust proper time and $R(\tau, \rho)$ is the areal radius of dust shell with coordinate label ρ at time τ, and $f(\rho)$ is called the energy function. Here, we will restrict ourselves with the marginally bound case of the LTB model for which $f(\rho) = 0$. Einstein's equation for this model is,

$$\frac{F'}{R^2 R'} = \frac{8\pi G\epsilon}{c^2} \quad \text{and} \quad \frac{R\dot{R}^2}{c^2} = F,$$ (2)

where $F(\rho)$ is first integral of Einstein's equation and is equal to twice of the Misner-Sharp(MS) mass[15] for LTB spacetime. This model has a curvature singularity when the cloud collapses to a point i.e., at $R = 0$.

Since the equation of motion which dictates the dynamics of the areal radius (2) depends only on R and F, and not on their spatial derivatives, it implies that the different dust shells are dynamically decoupled for a given mass function. We can write an on-shell action which dictates the dynamics of the outermost dust shell. The dynamics of the full dust cloud is then deduced from the action.[1]

$$\mathcal{S} = -\frac{1}{2} \int d\tau R\dot{R}^2$$ (3)

Here R is the areal radius of the outermost dust shell. The Hamiltonian for this model is,

$$H = -\frac{P^2}{2R} \tag{4}$$

Using Brown-Kuchař dust[16, 17] as the matter source, the Hamiltonian constraint for the model takes the form

$$\mathcal{H} \equiv p_\tau + H \approx 0. \tag{5}$$

Since the momentum conjugate to scalar field τ appears linearly, the Wheeler-DeWitt equation takes a Schrödinger equation like form,

$$i\hbar\frac{\partial\Psi(R,\tau)}{\partial\tau} = \frac{1}{2}R^{-1+a+b}\frac{d}{dR}R^{-a}\frac{d}{dR}R^{-b}\Psi(R,\tau). \tag{6}$$

The Hamiltonian is the product of areal radius and conjugate momentum, which implies that it does not have a unique quantum counterpart and that the model suffers from operator ordering ambiguity which is parameterized by a and b in equation (6). The eigenvalue of the Hamiltonian is interpreted as the ADM energy. The Hilbert space $L^2(\mathbb{R}^+, R^{1-a-2b}dR)$ is chosen, that makes this Hamiltonian Hermitian. The self adjoint extensions of the Hamiltonian (6) are discussed in.[1] A unitarily evolving wave packet is constructed from positive energy eigenstates,

$$\phi_E^1(R) = \frac{2}{\sqrt{3}}E^{\frac{1}{4}}J_{\frac{2|q|}{3}}\left(\frac{2}{3}\sqrt{2E}R^{\frac{3}{2}}\right). \tag{7}$$

by choosing a Poisson-like distribution

$$A(\sqrt{E}) = \frac{\sqrt{2}\lambda^{\frac{1}{2}(\kappa+1)}}{\sqrt{\Gamma(\kappa+1)}}\sqrt{E}^{\kappa+\frac{1}{2}}e^{-\frac{\lambda}{2}E}, \tag{8}$$

$$\psi(R,\tau) = \int_0^\infty d\sqrt{E}\phi_E(R)e^{iE\tau}A(\sqrt{E}), \tag{9}$$

where the parameters of the distribution satisfy $\kappa \geq 0$ and $\lambda > 0$. The expectation value of the Hamiltonian operator with this choice of distribution is inversely proportional to λ. To simplify the expression, a prescription $\kappa = |1+a|/3$ is adopted[1] and the expression for the wave packet reduces to

$$\psi(R,\tau) = \sqrt{3}\frac{R^{\frac{1}{2}(1+a+|1+a|+2b)}}{\sqrt{\Gamma(\frac{1}{3}|1+a|+1)}}\left(\frac{\frac{\sqrt{2\lambda}}{3}}{\frac{\lambda}{2}-i\tau}\right)^{\frac{1}{3}|1+a|+1}e^{-\frac{2R^3}{9\left(\frac{\lambda}{2}-i\tau\right)}}. \tag{10}$$

This choice comes at the cost that the distribution is now a function of operator ordering parameters. Since the dependence of observables on this parameter is contentious, one cannot be sure if it is a genuine signature of operator ordering or just a dependence on the shape of the distribution. In this analysis, we will focus on the radiation profile of the dust shell and address operator ordering ambiguity elsewhere.[18]

3. Mode Decomposition of Wave Packet

In the earlier midisuperspace models, various prescriptions are used to write incoming and outgoing modes. In,[12] the modes are associated with the asymptotic limit of the solutions of the WDW equation on $\mathcal{I}^+$ written in terms of the Killing time. While in,[13] the modes are associated with the positive/negative frequency solutions of the WDW equation after making transformation from the comoving time to the Killing time. In all these prescriptions, the modes are not orthonormal, making them ill-defined for the Bogoliubov analysis. The problem might lie with the fact that the states belong to the Kernel space of the operator and are degenerate and therefore, orthonormality is not guaranteed. Therefore, we need to identify another observable suitable for the mode decomposition.

Classically, the momentum conjugate to areal radius is given by $P = -R\dot{R}$ and we can associate positive momentum with the collapsing phase and negative momentum with the expanding phase of the dust shell. Thus, the momentum operator is a good choice for an observable that depicts the mode decomposition in the quantum model. The model[1] discussed in the previous section is robust enough for us to accommodate Hermitian Hamiltonian and momentum operator. Although the momentum operator on real half line is not self-adjoint, we will work with the Hermitian extension of the momentum operator. The detailed discussion on the self-adjointness and the Hermiticity of the momentum operator can be found in[18] and we have shown that the states are orthogonal in this case, making them suitable as incoming/outgoing modes.

Using the measure R^2 following the quantum scattering theory in spherical polar coordinates, we have a constraint $1 + a + 2b = 0$ on operator ordering parameters. For this model, the expectation value of the general observable in a general wave packet is independent of the parameter b.[18] Therefore it appears as a free parameter in the model and the above constraint does not affect the physical content of the model. In this case, the Hermitian extension of the momentum operator is $\hat{P} = -iR^{-1}\partial_R R$. The eigenfunction of the momentum operator with eigenvalue k is given by e^{ikR}/R, where $k \in \mathbb{R}$. The wave packet can be expressed in the form,

$$\psi(R,\tau) = \int_0^\infty dE \int_0^\infty dk \left(\mathcal{A}(k,E) \frac{e^{ikR+i\tau E}}{R} + \mathcal{A}(-k,E) \frac{e^{-ikR+i\tau E}}{R} \right). \quad (11)$$

The contribution of incoming/outgoing radiation in the dust cloud is estimated by computing the projection of wave packet along incoming($u_{k,E} \equiv e^{ikR+iE\tau}/R$) and outgoing modes($u_{-k,E} \equiv e^{-ikR+iE\tau}/R$) .

$$\tilde{\psi}_k(\tau) = \langle u_{k,E}(\tau) | \psi(R,\tau) \rangle = \int dR\, R^2\, \psi(R,\tau) \frac{e^{-ikR-iE\tau}}{R}, \quad (12)$$

$$= \frac{\sqrt{3}e^{-iE\tau}}{\sqrt{\Gamma(\frac{2}{3}|b|+1)}} \left(\frac{\frac{\sqrt{2\lambda}}{3}}{\frac{\lambda}{2}-i\tau} \right)^{\frac{2}{3}|b|+1} \int dR\, e^{-i\,kR} R^{1+|b|} e^{-\frac{2R^3}{9(\frac{\lambda}{2}-i\tau)}}, \quad (13)$$

$$= C(\tau) \int dR\, e^{-i\,kR} R^{1+|b|} e^{-\frac{2R^3}{9(\frac{\lambda}{2}-i\tau)}}. \quad (14)$$

The wavepacket in k space is normalized which implies $|\tilde{\psi}_k(\tau)|^2$ gives the contribution of modes with wavenumber k to the radiation profile at time τ. At $\tau = 0$, equation (14) can be written as

$$\tilde{\psi}_k(0) = C(0) \left(\int dR \, \cos(kR) \, R^{1+|b|} e^{-\frac{4R^3}{9\lambda}} - i \int dR \, \sin(kR) \, R^{1+|b|} e^{-\frac{4R^3}{9\lambda}} \right), \quad (15)$$

and we can see, $|\psi_k(0)|^2 = |\psi_{-k}(0)|^2$. Thus, at the point of classical singularity, the number of incoming modes is equal to the number of outgoing modes for all k. On the other hand, for finite τ, the equation (14) can be cast into the form,

$$|\tilde{\psi}_k(\tau)|^2 = |C(\tau)|^2 \int dR \int dS \, e^{-ik(R-S)} (RS)^{1+|b|} e^{-\frac{2R^3}{9\left(\frac{\lambda}{2}-i\tau\right)} - \frac{2S^3}{9\left(\frac{\lambda}{2}+i\tau\right)}}. \quad (16)$$

Here, $k \to -k$ is same as $\tau \to -\tau$. Thus, the ratio of incoming to outgoing modes $r_k(\tau) = |\tilde{\psi}_k(\tau)|^2 / |\tilde{\psi}_{-k}(\tau)|^2$ flips after the bounce happens i.e. $r_k(\tau) = [r_k(-\tau)]^{-1}$, which can be seen in Fig. 1. The difference between the number of incoming and outgoing modes at an instant of time and at a fixed k can be written as,

$$\delta_k(\tau) = |\tilde{\psi}_k(\tau)|^2 - |\tilde{\psi}_{-k}(\tau)|^2, \quad (17)$$

$$= -2i|C(\tau)|^2 \int dR \int dS \, \sin(k(R-S))(RS)^{1+|b|} e^{-\frac{2R^3}{9\left(\frac{\lambda}{2}-i\tau\right)} - \frac{2S^3}{9\left(\frac{\lambda}{2}+i\tau\right)}}. \quad (18)$$

$\delta_k(\tau)$ vanishes when $k \to \infty$, as $\lim_{k\to\infty} \sin(kx) = x\delta(x) = 0$. Thus, we expect the ratio to approach unity for large k, which can be seen in Fig. 1. On the other hand, for $k \to 0$ we have

$$\delta_k(\tau) = \frac{4k|C(\tau)|^2}{9^{(|b|+8)/3}} \Gamma \left[\frac{|b|}{3} + 1 \right] \Gamma \left[\frac{|b|+2}{3} \right] \left(\frac{\lambda^2}{4} + \tau^2 \right)^{\frac{2|b|+5}{6}} \sin \left(\frac{1}{3} \tan^{-1} \left(\frac{-2\tau}{\lambda} \right) \right). \quad (19)$$

In the collapsing branch ($\tau < 0$), this function is positive and therefore, the incoming modes are dominating in this case. In the expanding branch ($\tau > 0$), this function is negative and the outgoing modes dominate in this case.

We have plotted the fraction of modes with wavenumber k and the ratio of the incoming to outgoing modes for fixed time in the Fig. 1 with the parameters specifying a narrow wave packet of unit energy. Early in the collapsing phase ($\tau = -10$), most of the contribution comes from the incoming modes and the outgoing modes contribute a small fraction only in the infrared (small k) regime. As we approach the classical singularity, the contribution of outgoing radiation in the collapsing phase keeps on increasing and becomes equals to the contribution of incoming radiation at the bounce point. After the bounce, initially in the expanding phase, there is significant contribution of incoming radiation. As the expansion progresses, this contribution keeps on decreasing to become negligible and contributes only in the infrared regime.

In the early stage of collapse, the ratio of incoming to outgoing modes starts from unity at $k = 0$ and comes back to one again at a finite wavenumber. In between,
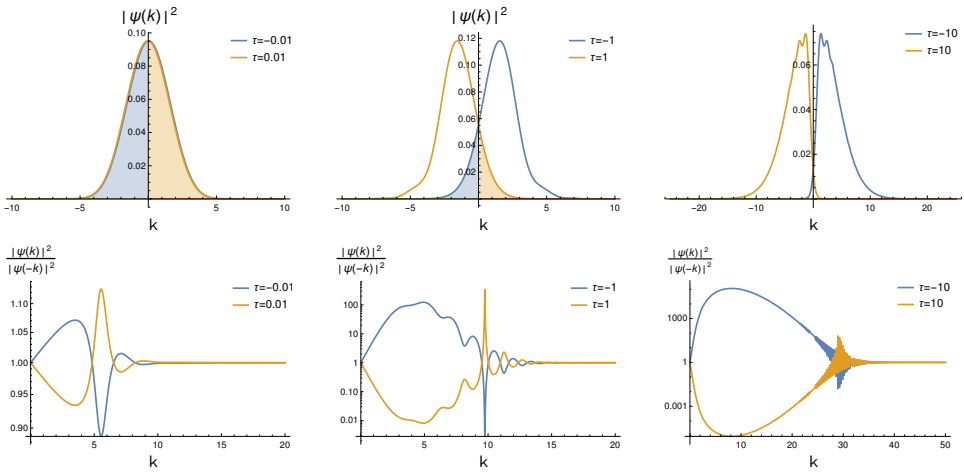
Fig. 1.   Fraction of modes $|\tilde{\psi}_k(\tau)|^2$ and ratio of the incoming to outgoing modes vs k at different times. The blue shaded region gives the contribution of outgoing radiation in collapsing phase and the orange shaded region gives the contribution of incoming radiation in expanding phase.

the ratio attains a maximum and oscillates before settling at unity. Since for large wavenumbers, the contribution is minimal, these modes are insensitive to the dynamics of the dust shell. A ratio greater than one implies that the incoming modes are dominating. As we approach the bounce point, the magnitude of the maximum decreases and the ratio goes to unity at shorter wavenumber. At the bounce point, this ratio is one for all wavenumbers. Apart from this, there exists a crossover window of wavenumbers in which the fraction of outgoing modes is greater than incoming modes in the collapsing phase when the dust shell is closer to the classical singularity. This behavior is inverted for the expanding phase - the ratio attains a minimum before settling at unity.

Therefore, we see that the small wavenumber (infrared) modes are most sensitive to the dynamics of the dust cloud. If one observes in the infrared regime, there is an instantaneous flip from largely incoming to largely outgoing radiation when the dust shell goes from collapsing phase to expanding phase. Moreover, the contribution of the outgoing/incoming radiation in the collapsing/expanding regime is coming from the infrared part of the spectrum only. Thus, one should focus on the infrared regime of the dust shell for a signature of quantum bounce.

We have plotted the radiation profile for the wave packets which represents low energy dust shell broadly peaked ($\lambda = 10$) and high energy dust shells sharply peaked ($\lambda = 0.01$) on the classical trajectory in Fig. 2. We see that for the case of sharply peaked wave packet, the major contribution to radiation profile is coming from incoming (outgoing) modes in the collapsing (expanding) regime even near the classical singularity. Whereas, for low energy dust shells, the contribution of incoming (outgoing) modes in expanding (collapsing) branch is significant even far away from singularity.
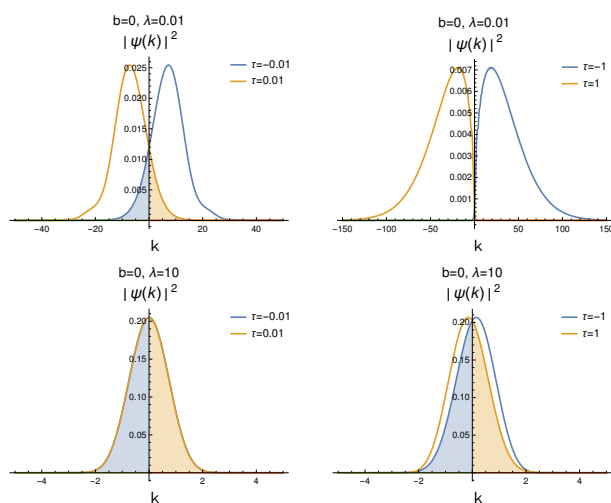
Fig. 2. Fraction of modes $|\tilde{\psi}_k(\tau)|^2$ and the ratio of incoming to outgoing modes for wave at different times. The parameters specify the high energy dust cloud ($\lambda = 0.01$) representing sharper wave packet, and low energy dust cloud($\lambda = 10$) representing broader wave packet.

At the classical singularity, the fraction of infrared modes is directly proportional to the minimal size of the dust shell i.e., the bounce radius.[18]

$$|\tilde{\psi}_{k\to 0}(0)|^2 = \frac{2^{(2|b|-5)/3}3^{2/3}\Gamma\left(\frac{|b|+2}{3}\right)^2\Gamma\left(\frac{2|b|+4}{3}\right)}{\sqrt{2}\pi\Gamma\left(\frac{2|b|+1}{3}\right)\Gamma\left(\frac{2|b|+3}{3}\right)}\bar{R}(0). \tag{20}$$

Thus, the infrared regime of the radiation profile provides a direct estimation of the bounce radius.

4. Conclusions

In this work, we have studied the mode decomposition of wave packet constructed for the Quantum LTB model. We have considered the minisuperspace construction of the dust shell collapse in the LTB model.[1] The classical model of dust shell collapse exhibits black hole singularity which is replaced by bounce from the collapsing phase to expanding phase in the quantum model. We have identified the observable depicting mode decomposition, which is momentum conjugate to areal radius. After identifying the incoming and outgoing modes with the momentum's eigenstates with positive and negative eigenvalues, we have estimated the contribution of the incoming/outgoing modes in the contracting/expanding phase.

We choose the operator ordering parameters for which the Hamiltonian as well as the momentum operator are Hermitian. This is achieved by working with R^2 measure space and choosing the representation which is symmetric with this choice

of inner product. This particular choice puts the constraint $a+2b+1 = 0$ on operator ordering parameters.

We find that at the point of classical singularity or bounce point $\tau = 0$, the number of incoming and outgoing modes is the same. In the contracting branch, apart from the incoming dust, we also have small contribution from the outgoing dust in the infrared regime. As the dust shell continues to move towards the singularity configuration, the contribution of outgoing modes in the infrared regime keeps increasing, culminating in an equal number of incoming and outgoing modes at the point of singularity. This behavior is inverted in the expanding branch. There exists a threshold wavenumber, above which the number of incoming modes is equal to the number of outgoing modes. For the contracting branch, the ratio of incoming to outgoing modes starts from one at $k = 0$, increases to attain a maximum, and then settles back to unity. After the bounce, outgoing modes start to dominate with a small fraction of incoming modes as well. As the dust shell expands, the fraction of the incoming modes keeps decreasing and while the contribution of the outgoing modes keeps increasing. At the later stage of dust shell expansion, most contribution comes from the outgoing modes. Thus, we can conclude that in quantum bounce, incoming radiation is always accompanied by outgoing radiation.

The infrared sector of the wave packet contains significant information about the dynamics of the dust cloud. The major contribution to the incoming/outgoing dust in collapsing/expanding branch comes from the modes with smaller wavenumber. There is a flip from largely incoming to largely outgoing radiation, observed in the infrared regime, as the evolution progresses from the contracting branch to the expanding branch. Therefore, the observer should look at small wavenumber or large wavelength modes to observe if the bounce has happened. Moreover, the fraction of infrared modes near classical singularity is proportional to the bounce radius. Thus, the infrared sector of the process apart from being highly sensitive to the dynamics of the dust cloud is also a direct estimator of the bounce radius, thereby, providing a unique infrared signature of the quantum gravity in the radiation profile of the dust cloud.

Acknowledgments

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