PROCEEDINGS OF SCIENCE

Recent developments in FeynHiggs

Henning Bahl*
Max-Planck Institut für Physik, München
E-mail: hbahl@mpp.mpg.de

We discuss recent developments in FeynHiggs a computer code for predicting the Higgs boson masses in the Minimal Supersymmetric Standard Model (MSSM). In especially, we describe how the existing fixed-order calculation was combined with a resummation of large logarithms using an EFT approach. This allows for a precise prediction for low and high SUSY scales. Furthermore, we discuss how the determination of the propagator poles induces higher order terms which would cancel in a more complete calculation.

Corfu Summer Institute 2017 "School and Workshops on Elementary Particle Physics and Gravity"
2-28 September 2017
Corfu, Greece

*Speaker.

https://pos.sissa.it/

PoS(CORFU2017)128

1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) is one of the most common models of beyond Standard Model physics building upon the concept of Supersymmetry (SUSY). Its enriched Higgs sector consists out of two Higgs doublets leading to five physical Higgs bosons. In the MSSM with real parameters, these are the CP-even h and H bosons, the CP-odd A boson and the charged $H^{\pm}$ bosons. One of the CP-even Higgs bosons has to play the role of the Higgs boson discovered at the LHC by the ATLAS and CMS experiments [1, 2]. It is a distinct feature of the MSSM that the mass of this SM-like Higgs boson is predictable in terms of the model parameters. Therefore, the Higgs boson mass can be used as a precision observable to constrain the available parameter space.

Since the Higgs mass is heavily affected by quantum corrections, much work has been dedicated to the calculation of higher order corrections. Various techniques have been employed for that: Fixed-order calculations allow to capture all corrections at a given order and are therefore expected to be precise for low SUSY scales; in contrast, effective field theory (EFT) calculations allow to resum large logarithmic contributions yielding precise results for high SUSY scales. In `FeynHiggs` [3, 4, 5, 6, 7, 8, 9], both techniques are combined to obtain precise predictions also for intermediary scales. Here, we review the work presented in [9].

2. Fixed-order calculation

The fixed-order approach is based on calculating the Higgs self-energies taking into account contributions from SM particles, extra Higgs bosons as well as their superpartners. This allows to capture all effects at a given order and allows to easily take into account different mass hierarchies. If however some or all of the non SM particles are heavy, logarithmic contributions can become numerically large spoiling the convergence of the perturbative expansion.

The self-energy corrections in `FeynHiggs` comprise full one-loop and two-loop corrections of $\mathcal{O}(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2)$ [3, 4, 10, 11, 12, 5, 13, 14, 6, 15, 7, 16] obtained in the limit of vanishing external momentum. They have been derived employing a mixed on-shell (OS) and $\overline{\text{DR}}$-scheme (more details can be found in [6]).

Having calculated the renormalized Higgs-boson self-energies, the physical masses of the CP-even Higgs bosons are then obtained by finding the poles of the propagator matrix with its inverse given by

$$\Delta_{hH}^{-1}(p^2) = i\begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{MSSM}}(p^2) & \hat{\Sigma}_{hH}^{\text{MSSM}}(p^2) \\ \hat{\Sigma}_{hH}^{\text{MSSM}}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}^{\text{MSSM}}(p^2) \end{pmatrix}, \tag{2.1}$$

where m_h denotes the tree-level mass of the h boson; m_H, the tree-level mass of the H boson. $\hat{\Sigma}_{hh,hH,HH}^{\text{MSSM}}$ are the corresponding renormalized MSSM self-energies. Finding the poles is equivalent to solving the equation

$$\left(p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{MSSM}}(p^2)\right)\left(p^2 - m_H^2 + \hat{\Sigma}_{HH}^{\text{MSSM}}(p^2)\right) - \left(\hat{\Sigma}_{hH}^{\text{MSSM}}(p^2)\right)^2 = 0. \tag{2.2}$$

In the decoupling limit, $M_A \gg M_Z$, the reduced equation

$$p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{MSSM}}(p^2) = 0 \tag{2.3}$$

allows to determine the mass of the lightest Higgs boson up to corrections suppressed by powers of M_A.

Solving Eq. (2.3) iteratively we obtain a simple expression for the physical mass of the lightest Higgs,

$$(M_h^2)_{\rm FD} = m_h^2 - \hat{\Sigma}_{hh}^{\rm MSSM}(m_h^2) + \hat{\Sigma}_{hh}^{\rm MSSM\prime}(m_h^2)\hat{\Sigma}_{hh}^{\rm MSSM}(m_h^2) + \ldots \tag{2.4}$$

with the prime denoting the derivative of the self-energy with respect to the momentum squared and the ellipsis standing for terms involving higher-order derivatives and products of differentiated self-energies.

3. EFT calculation

As mentioned above, the fixed-order approach suffers from large logarithms if some or all of the non SM particles are heavy. In such scenarios, EFT methods are beneficial. They allow to resum large logarithms incorporating contributions beyond the order of fixed-order calculations. If no higher-dimensional operators are included in the effective Lagrangian, terms suppressed by a heavy scale are however neglected. Therefore, EFT calculations become unreliable for low SUSY scales.

In the simplest EFT framework, all non SM particles are integrated out from the full theory at a common mass scale $M_{\rm SUSY}$. Below $M_{\rm SUSY}$ the SM remains as the low-energy EFT. The couplings of the EFT are determined by matching to the MSSM at the scale $M_{\rm SUSY}$. In the case of the SM as the EFT below $M_{\rm SUSY}$ this concerns only the effective Higgs self-coupling λ, all the other couplings are fixed by matching them to physical observables at the low-energy scale. Renormalization group equations (RGEs) are used to evolve the couplings between the high scale $M_{\rm SUSY}$ and the low scale, which is typically chosen to be the top mass M_t.

The effective Higgs self coupling at the top mass scale, $\lambda(M_t)$, is then used to determine the SM $\overline{\rm MS}$ Higgs mass

$$(m_h^{\overline{\rm MS},\rm SM})^2 = 2\lambda(M_t)v_{\overline{\rm MS}}^2 \tag{3.1}$$

with $v_{\overline{\rm MS}}$ being the $\overline{\rm MS}$ vev (at the scale M_t).

The physical Higgs mass is then obtained by solving the corresponding pole equation

$$p^2 - (m_h^{\overline{\rm MS},\rm SM})^2 + \tilde{\Sigma}_{hh}^{\rm SM}(p^2) = 0, \tag{3.2}$$

with the renormalized SM Higgs boson self-energy $\tilde{\Sigma}_{hh}^{\rm SM}$.

We again can obtain a compact expression for the physical Higgs mass expanding perturbatively around the tree-level mass m_h^2 of the MSSM,

$$(M_h^2)_{\rm EFT} = 2v_{\overline{\rm MS}}^2\lambda(M_t) - \tilde{\Sigma}_{hh}^{\rm SM}(m_h^2) - \tilde{\Sigma}_{hh}^{\rm SM\prime}(m_h^2)\cdot\left[2v_{\overline{\rm MS}}^2\lambda(M_t) - \tilde{\Sigma}_{hh}^{\rm SM}(m_h^2) - m_h^2\right] + \ldots. \tag{3.3}$$

In `FeynHiggs`, full one-loop matching conditions, the dominant two-loop matching conditions of $\mathcal{O}(\alpha_s\alpha_t, \alpha_t^2)$ and three-loop RGEs are implemented. Therefore, all leading, next-to-leading and $\mathcal{O}(\alpha_s, \alpha_t)$ next-to-next-to-leading logarithms are resummed. In addition to the simple single scale scenario, mentioned above, the possibility of an intermediary electroweakinos scale allows to also handle scenarios with low mass electroweakinos accurately.

4. Hybrid calculation

To profit from the advantages of both approaches – high precision for low scales in the case of the fixed-order calculation and high precision for high scale in the case of the EFT calculation –, both are combined in `FeynHiggs`. More specifically the logarithms resummed in the EFT approach are added to the fixed-order result by adapting Eq. (2.3),

$$p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{MSSM}}(p^2) + \Delta\hat{\Sigma}_{hh}^2 = 0. \tag{4.1}$$

Here, the quantity $\Delta\hat{\Sigma}_{hh}$ contains all logarithms resummed in the EFT approach as well as a subtraction term ensuring that the logarithms already contained in $\hat{\Sigma}_{hh}^{\text{MSSM}}$ are not counted twice,

$$\Delta\hat{\Sigma}_{hh}^2 = -\left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\log}. \tag{4.2}$$

The subscript 'log' is used to indicate that only logarithmic contributions are taken into account. Using this expression and expanding around the tree-level mass m_h^2, the physical Higgs mass is given by

$$\begin{aligned}(M_h^2)_{\text{FH}} = m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM}}(M_h^2) + \left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} + \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\log} = \\ = m_h^2 + \left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}} \\ - \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\left(\left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}}\right) \\ + \dots .\end{aligned} \tag{4.3}$$

Analogously to the label 'log', the label 'nolog' is used to indicate that only terms not involving large logarithms are taken into account.

Here, we assumed that the same renormalization scheme is used for input parameters of the fixed-order and the EFT calculation. If this is not the case, a parameter conversion is necessary. This is discussed in more detail in [8].

5. Comparison of hybrid and pure EFT calculation

For large SUSY scales suppressed terms become negligible. Therefore, we expect the hybrid approach and the pure EFT approach to yield the same result. I.e., the large logarithms contained in both results should coincide.

The logarithms of the EFT approach are given by (see Eq. (3.3))

$$(M_h^2)_{\text{EFT}}^{\log} = \left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} - \hat{\Sigma}_{hh}^{\text{SM}\prime}(m_h^2)\left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} + \dots, \tag{5.1}$$

the logarithms of the hybrid approach by (see Eq. (4.3))

$$\begin{aligned}(M_h^2)_{\text{FH}}^{\log} = \left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} + \left[\hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\right]_{\log}\left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}} \\ - \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\left[2v_{\overline{\text{MS}}}^2\lambda(M_t)\right]_{\log} + \dots .\end{aligned} \tag{5.2}$$

Next, we split up the MSSM self-energies into a SM part and a non-SM part,

$$\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) = \hat{\Sigma}_{hh}^{\text{SM}}(m_h^2) + \hat{\Sigma}_{hh}^{\text{nonSM}}(m_h^2). \tag{5.3}$$

This is allowed in the decoupling limit ($M_{\text{SUSY}} = M_A \gg M_t$), since the couplings of the lightest Higgs become SM-like.

With this relation, we yield

$$(M_h^2)^{\log}_{\text{FH}} - (M_h^2)^{\log}_{\text{EFT}} = \left[\hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)\right]_{\log}\left[\hat{\Sigma}^{\text{MSSM}}_{hh}(m_h^2)\right]_{\text{nolog}} - \hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)\left[2v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\log} + \ldots . \tag{5.4}$$

We observe that this difference originates from the momentum dependence of the non-SM contributions to the Higgs self-energy. As shown explicitly at the two-loop level in [9], this difference cancels out when subloop renormalization contributions to the two-loop self-energy are taken into account. I.e., the vev-counterterm generates a contributions proportional to $\hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)$ compensating the difference. Probably, also at higher orders the difference is cancelled in a similar manner. A very similar analysis can also be performed for non-logarithmic terms.

In `FeynHiggs`, the pole equation Eq. (4.1) was solved numerically up to version `2.13.0`. Therefore, terms also beyond the order of the included two-loop corrections were induced. In consequence, the difference discussed above was compensated at $\mathcal{O}(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2)$ but not beyond. E.g., terms of $\mathcal{O}(\alpha_t\alpha_{\text{ew}})$ were not compensated. It is however easy to enforce this compensation by just eliminating all terms proportional to $\hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)$ which are not of $\mathcal{O}(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2)$ in the final equation for the Higgs mass. This can be achieved by an iterative solution of the pole equation instead of a numerical one (implemented from version `2.14.0` on).

6. Numerical results

In this section, we show same example results for a simple single scale scenario with $\overline{\text{DR}}$ input parameters. All soft masses, the Higgsino mass paramater μ and M_A are set equal to M_{SUSY}. All trilinear couplings are set to zero (except of the one of the stop sector) and $\tan\beta$ is chosen to be equal 10.

In the left plot of Fig. 1, we investigate the numerical effect of the uncompensated terms arising from the determination of the propagator pole. Up to `FeynHiggs2.13.0` these terms were taken into account. In the new `FeynHiggs2.14.0`, we improved the pole mass determination to prevent these terms from appearing. As we observe, this leads to downwards shift of the Higgs mass. This shifts grows logarithmically with M_{SUSY} reaching values of up to ~ 2 GeV for $M_{\text{SUSY}} \sim 20$ TeV.

In the right plot of Fig. 1, we show a comparison of `FeynHiggs` with improved pole determination to the pure EFT code `SUSYHD` [17]. The discrepany we observe for low scales (below 1 TeV) is caused by suppressed terms which are included in `FeynHiggs` by means of the fixed-order calculation, whereas they are missed in `SUSYHD`. For higher scales, these terms become negligible and both the results of both codes agree very well especially for vanishing stop mixing. The small discrepancy for high scales (~ 20 TeV) and $X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 2$ is caused by a different parametrization of non-logarithmic terms (for more details see [9]).

7. Conclusions

Different methods are used to calculate the Higgs boson masses in the MSSM. Fixed-order

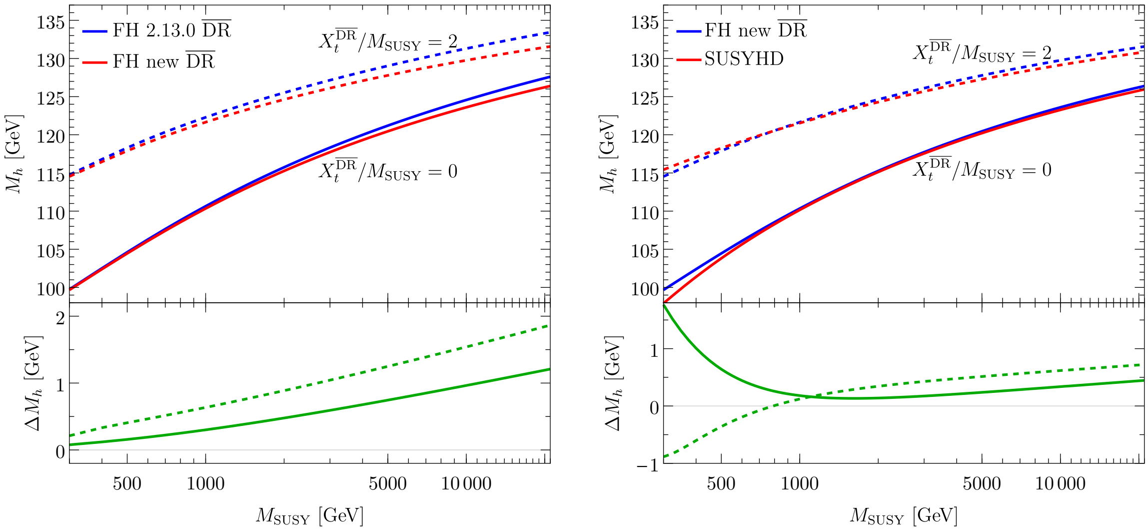

Figure 1: M_h is shown in dependence of M_{SUSY} for $X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 0$ (solid) and M_{SUSY} for $X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 2$ (dashed). Left: The results of `FeynHiggs` with (red) and without (blue) the improved pole mass determination are compared. Right: The results of `FeynHiggs` with new pole mass determination (blue) and `SUSYHD` (red) are compared. In the bottom panels, the difference between the blue and red curves is shown.

calculations are accurate for low scales. They however become inaccurate for high SUSY scales due to large logarithms spoiling the convergence of the perturbative expansion. These logarithms can be resummed using EFT techniques. Without taking into account higher dimensional operators, EFT calculations miss in contrast suppressed terms and are therefore inaccurate for low scales.

We described how both methods have been combined in the code `FeynHiggs` to obtain a prediction accurate also for intermediary scales. Furthermore, we compared the logarithms obtained in our hybrid approach with those of a pure EFT calculation. We found that the non-zero difference between both arising through the determination of the propagator pole is cancelled by subloop-renormalization contributions. We adapted the determination of the propagator poles to ensure that uncancelled terms beyond the order of the fixed-order calculation do not appear in the final result.

In our numerical investigation, we found the terms arising from the determination of the propagator pole to shift the Higgs mass downwards by up to 2 GeV. Taking this effect into account, we found very good agreement of `FeynHiggs` with the pure EFT code `SUSYHD` for scales above 1 TeV, where suppressed terms become negligible.

Acknowledgements

I thank Sven Heinemeyer, Wolfgang Hollik and Georg Weiglein for the collaboration on this project. Moreover, I wish to thank the organizers of the workshop for the invitation and the hospitality during the stay.

References

[1] ATLAS collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard*

Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. **B716** (2012) 1–29, [1207.7214].

[2] CMS collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. **B716** (2012) 30–61, [1207.7235].

[3] S. Heinemeyer, W. Hollik and G. Weiglein, *FeynHiggs: A Program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM*, Comput. Phys. Commun. **124** (2000) 76–89, [hep-ph/9812320].

[4] S. Heinemeyer, W. Hollik and G. Weiglein, *The masses of the neutral CP-even Higgs bosons in the MSSM: Accurate analysis at the two loop level*, Eur. Phys. J. **C9** (1999) 343–366, [hep-ph/9812472].

[5] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Towards high precision predictions for the MSSM Higgs sector*, Eur. Phys. J. **C28** (2003) 133–143, [hep-ph/0212020].

[6] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *The Higgs boson masses and mixings of the complex MSSM in the Feynman-diagrammatic approach*, JHEP **02** (2007) 047, [hep-ph/0611326].

[7] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *FeynHiggs: A program for the calculation of MSSM Higgs-boson observables - Version 2.6.5*, Comput. Phys. Commun. **180** (2009) 1426–1427.

[8] H. Bahl and W. Hollik, *Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations*, Eur. Phys. J. **C76** (2016) 499, [1608.01880].

[9] H. Bahl, S. Heinemeyer, W. Hollik and G. Weiglein, *Reconciling EFT and hybrid calculations of the light MSSM Higgs-boson mass*, 1706.00346.

[10] G. Degrassi, P. Slavich and F. Zwirner, *On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing*, Nucl. Phys. **B611** (2001) 403–422, [hep-ph/0105096].

[11] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *On the $O(\alpha_t^2)$ two loop corrections to the neutral Higgs boson masses in the MSSM*, Nucl. Phys. **B631** (2002) 195–218, [hep-ph/0112177].

[12] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *On the two loop sbottom corrections to the neutral Higgs boson masses in the MSSM*, Nucl. Phys. **B643** (2002) 79–92, [hep-ph/0206101].

[13] A. Dedes, G. Degrassi and P. Slavich, *On the two loop Yukawa corrections to the MSSM Higgs boson masses at large* $\tan\beta$, Nucl. Phys. **B672** (2003) 144–162, [hep-ph/0305127].

[14] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *High-precision predictions for the MSSM Higgs sector at $O(\alpha_b\alpha_s)$*, Eur. Phys. J. **C39** (2005) 465–481, [hep-ph/0411114].

[15] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *The Higgs sector of the complex MSSM at two-loop order: QCD contributions*, Phys. Lett. **B652** (2007) 300–309, [0705.0746].

[16] W. Hollik and S. Paßehr, *Higgs boson masses and mixings in the complex MSSM with two-loop top-Yukawa-coupling corrections*, JHEP **10** (2014) 171, [1409.1687].

[17] J. P. Vega and G. Villadoro, *SusyHD: Higgs mass determination in supersymmetry*, JHEP **07** (2015) 159, [1504.05200].