Gravity: local or nonlocal

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Abstract. While gravity is often imagined to have the characteristics of a local gauge theory, no such gauge theory has so far been constructed, with all attempts at quantum gravity leading to nonrenormalizable infinities. All known gauge theories, however, have both local and nonlocal aspects, and it is worth exploring how far we can describe gravity using a nonlocal starting point. It is suggested that this leads to a local theory which privileges inertia rather than gravity. Such a theory based on a repulsive force, would be renormalizable, and it would also have consequences that can be linked to cosmological redshift, dark energy and possibly also dark matter. It further suggests generic connections between gravity and inertia and between general relativity and Newtonian gravity.

1. Introduction

Gravity is generally assumed to be described by a local gauge theory like the other three fundamental forces but no one has been successful so far in creating the quantum version that this would imply. The main difficulty seems to lie in the fact that gravity requires an attractive force between identical particles with corresponding spin 2 boson, where the other gauge theories require a repulsive force with spin 1 boson. Gauge theories with spin 1 bosons are renormalizable, whereas those with spin 2 bosons are not, leading to nonrenormalizable infinities in the quantum field integrals. The assumption that gravity is intrinsically a local theory arises from the success of general relativity in predicting a whole series of relativistic consequences, including perihelion precession, light deflection, time delay, redshift, gravitomagnetic effects and gravitational waves. Local and nonlocal are not separate concepts, however. Each implies the other. Quantum theories may start off using a local description, for instance, but they end up predicting a series of nonlocal consequences. And some phenomena with a clearly nonlocal origin can have massive local consequences, as Pauli exclusion does in the case of the structure of matter and the lifetime of stars.

The clear relationship between local and nonlocal interpretations is evident in the author's own nilpotent representation of relativistic quantum mechanics [1-2]. It is also clear from this representation that the kind of negative energy and attractive forces associated with gravity are aspects of vacuum nonlocality rather than of discrete locally described particle states. In addition, there is a strong indication of gravity/gauge theory correspondence between the four interactions, and a local/nonlocal connection between gravity and inertia. All these indications suggest that gravity might be considered initially using a nonlocal description leading to a localised (repulsive) force of inertia in contrast to the local description leading to nonlocal vacuum effects which we find in the three known gauge forces. In this description, the experimental consequences of the general relativistic field equations will be exactly as predicted,

but the equations will not break down in strong fields, as many theorists currently imagine. Among other effects (as already predicted) there will be an inertial 'dark energy' making up 67% of the total energy of the universe.

2. Newtonian gravity and general relativity

General relativity (GR) has an anomalous relation with Newtonian gravity (NG). It uses NG as a physical limit but claims to be qualitatively different in that it replaces the idea of a 'physical' force or field with a degree of space-time curvature. This supposed qualitative difference has been a barrier to understanding the connection, but it is an unnecessary barrier, and has prevented a proper integration of NG into GR. In 1923 and 1924, however, Cartan showed how Newtonian gravity or any other field theory could be represented by a tensor equation of the same form as GR [3-4]. In the notation of the time this became

$$R_{ab} = (8\pi T_{ab} - g_{ab}T/2)$$

for GR, and

$$R_{ab} = 8\pi T_{ab}$$

for NG. The difference isn't in the curvature but the local space-time connection assumed by GR, which occurs in the second term of the GR equation.

Locality assumes interactions between discrete centres of force or particles acting at the speed of light (c). This is certainly absent in Newtonian theory which makes the default assumption that interactions are instantaneous, and so is in this sense 'nonlocal'. But gravity has always come associated with another concept, inertia. In Newtonian theory, inertia and gravity are associated only through being concerned with the same mass, but, in GR, inertia is used as part of the construction of the space-time curvature associated with gravity. The two concepts are inextricably linked within the structure of GR, by contrast with NG. The curvature terms in GR are then introduced using an argument that links the gravitational effect to an inertial one. Interestingly, however, the universe does not exhibit curvature on a large scale where gravity is strongest.

The separate realms of locality and nonlocality are best seen in quantum theory. The electric, strong and weak theories are gauge theories, with sources that are discrete, point-like particles, interacting through other, intermediate, particles called gauge bosons. In the background is a continuous field, of which the particles are quanta, and the real particles also interact with virtual particles from this field, leading to renormalization of the quantities involved in the local interactions. In such theories, local and nonlocal are not separate concepts. Each implies the other. Quantum theories, for instance, frequently start off using a local description, but they end up predicting a series of nonlocal consequences. At the same time, some phenomena with a clearly nonlocal origin can have massive local consequences, as Pauli exclusion does in the case of the structure of matter and the lifetime of stars.

3. Local and nonlocal in gauge theories

The clear relationship between local and nonlocal interpretations is evident in my own nilpotent representation of relativistic quantum mechanics [1-2]. Here, the amplitude of any fermion is always of the generic form

$$(\pm i\boldsymbol{k}E \pm i\boldsymbol{p} + j m) \times \text{phase factor}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are quaternion units and $\mathbf{p}$ is a multivariate vector (or complexified quaternion). The bracket

$$(\pm i\boldsymbol{k}E \pm i\boldsymbol{p} + j m)$$

indicates locality and speed of light transmission because it is a square root of Einstein's energy-momentum-mass equation

$$E^2 - p^2 - m^2 = 0$$

It also indicates Pauli exclusion because no two amplitudes of this form can be the same without creating a zero combination state. The velocity of light term is inside the bracket (here $c = 1$). Nonlocality, however, is indicated by combinations and superpositions because these are *outside the bracket*. Pauli exclusion is one example because it is about combination. So, this is clearly nonlocal.

Interference effects are based on superpositions and these are also nonlocal. The bracket gives us a clear boundary between local and nonlocal realms, but its existence means that both are important to the complete picture. Local interactions are expressed by inserting potential terms into the bracket as part of the $i\mathbf{k}E$ and $i\mathbf{p}$ terms, or their differential equivalents, $-\mathbf{k}\partial/\partial t$ and $-i\mathbf{i}\nabla$. These, however, often arise from originally nonlocal representations.

Nilpotency, as has been stated, can be taken as an expression of the nonlocal Pauli exclusion. It also implies, however, that the field of a point source must incorporate a Coulomb potential. This comes from converting the Dirac nilpotent differential operator to polar coordinate form using Dirac's prescription

$$\nabla \rightarrow \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm i\frac{j + 1/2}{r}$$

$$(i\mathbf{k}E - i\mathbf{i}\nabla + \mathbf{j}m) \rightarrow \left(i\mathbf{k}E - i\mathbf{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + 1/2}{r}\right) + \mathbf{j}m\right)$$

The presence of the extra $1/r$ in the momentum or $\partial/\partial r$ term means that a nilpotent amplitude solution is impossible unless there is a compensating A/r or Coulomb term added to $i\mathbf{k}E$. So, nilpotency alone determines that any particle-particle interaction demands a minimum of a Coulomb potential.

$$\left(i\mathbf{k}\left(E - \frac{A}{r}\right) - i\mathbf{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + 1/2}{r}\right) + \mathbf{j}m\right)$$

The term is found in some form in all four interactions: gravity, electric, strong and weak. The last two interactions, however, have additional terms with different origins.

In the case of the strong interaction, the nilpotent amplitude of the baryon is of the form

$$(i\mathbf{k}E \pm i\mathbf{i}p_x + \mathbf{j}m)\,(i\mathbf{k}E \pm i\mathbf{j}p_y + \mathbf{j}m)\,(i\mathbf{k}E \pm i\mathbf{k}p_z + \mathbf{j}m)$$

where there is a superposition of six terms, each with a single nonzero value of $\pm i\mathbf{i}p_x$, $\pm i\mathbf{j}p_y$ or $\pm i\mathbf{k}p_z$. There are six phases in which the momentum 'transfers' nonlocally between these states at a rate which does not depend on any physical distance between the components. This is equivalent *locally* to a constant force or a *linear* potential, which, added to the minimally required Coulomb force, gives the form of the strong interaction [1-2].

A template for the weak interaction can be seen in the superposition of the four states in $(\pm i\mathbf{k}E \pm i\mathbf{p} + \mathbf{j}m)$:

fermion spin up	$(i\mathbf{k}E + i\mathbf{p} + \mathbf{j}m)$
fermion spin down	$(i\mathbf{k}E - i\mathbf{p} + \mathbf{j}m)$
antifermion spin down	$(-i\mathbf{k}E + i\mathbf{p} + \mathbf{j}m)$
antifermion spin up	$(-i\mathbf{k}E - i\mathbf{p} + \mathbf{j}m)$

The four states combine the real particle and antiparticle vacuum, and the switching between them or *zitterbewegung*, which fixes the particle's localised or discrete invariant mass, determines that the source is an effective dipole or multipole. Adding a dipole or multiple potential to the minimal Coulomb in the $i\boldsymbol{k}E$ term leads to a harmonic oscillator solution which has the characteristics of the weak interaction, where it represents the creation and annihilation of states. All weak interactions have the same form and follow the same transition from nonlocal to local [1-2].

Clearly any changes in the nonlocal conditions outside the nilpotent bracket will cause changes in the local potentials *inside* the bracket. The effect can, of course, go the other way. Interference fringes from Young's slits are an effect of a nonlocal superposition outside the bracket. If we block one of the slits (effectively using local potentials), we will remove the nonlocal superposition. In principle, quantum gauge theories can be seen as providing a local interaction between particles, governed by specific forms of scalar and vector potentials added to $i\boldsymbol{k}E$ and $i\mathbf{p}$. At the same time there is always a nonlocal interaction with a continuous vacuum, though we think of the interactions between discrete particles as real and those with continuous vacuum as virtual.

4. Local and nonlocal in gravity

With gravity it is not so obvious. Gravity is generally assumed to be a local gauge theory like the other three fundamental forces. This assumption arises from the success of GR in predicting a whole series of relativistic consequences of locality, including perihelion precession, light deflection, time delay, redshift, gravitomagnetic effects and gravitational waves. But no one has been successful so far in creating the quantum version that this would imply. The main difficulty seems to lie in the fact that, in addition to negative potential energy, gravity requires an attractive force between identical particles with corresponding spin 2 boson, where the other gauge theories require a repulsive force with spin 1 boson. Gauge theories with spin 1 bosons are renormalizable, whereas those with spin 2 bosons are not, leading to unrenormalizable infinities in the quantum field integrals.

We might, however, imagine that gravity and inertia represent the local and nonlocal aspects of a single physical phenomenon. Inertia is clearly local as it concerns localised discrete masses. But it would seem that the kind of negative energy and attractive forces associated with gravity might be aspects of vacuum nonlocality rather than of discrete locally described particle states. This would be exactly in line with a theory of gravity / gauge theory correspondence in which each is dual to the other. If this is valid, then gravity might be considered initially using a nonlocal description leading to a localised (repulsive) force of inertia, in contrast to the local description leading to nonlocal vacuum effects which we find in the three known gauge forces. The experimental consequences of the general relativistic field equations will be exactly as predicted, but there will be no breakdown in strong fields.

Gravity is, of course, incredibly weak, much weaker than the electric, strong and weak interactions – by a factor of 1 in 10^{42} in the electrical case. However, the 'cosmological constant' (Λ) or vacuum calculated from applying quantum theory is about 123 orders of magnitude too high. If the universe is composed of $\sim 10^{123}$ 'bits' of information as Seth Lloyd has proposed [5-6], then the discrepancy is as large as it could conceivably be. It is not even 'not even wrong'! What could be the explanation?

This may be an opportunity rather than a problem, because Λ may be an expression of continuous vacuum nonlocality, rather than discrete, local 'quantum gravity'. Gravity would then be weak because it requires the whole observable universe to act to produce something comparable to what single particles do with the other forces. In this case its transmission would be instantaneous, just as Newton assumed it to be, and the negative energy of gravitational interaction would suggest vacuum, as it does for fermions. Nonlocal forces could also carry the correlation information required by quantum mechanics.

Instantaneous transmission is now known to be true for the static gravitational field as well as the electrostatic field. These fields don't propagate. There is no c dependence, and this is necessary to maintain Lorentz invariance. We note that the equations $\nabla.\mathbf{g} = 0$ and $\nabla.\mathbf{E} = 0$ don't include c, and it is significant that the mass, charge and angular momentum are the three things that we can know about a black hole, because knowledge of these doesn't require c-related information. In the nilpotent version of quantum mechanics, the static fields have a nonlocal aspect to maintain Pauli exclusion. We may also imagine it to be true for the static Coulomb parts of the strong and weak interactions – the weak case could be the actual 'mechanism' for Pauli exclusion, as this only applies to fermions and these are the only particles with weak field sources or charges.

5. Gravitomagnetic equations

At this stage, we don't know how c comes into the theory. The force law itself doesn't indicate whether it comes from locality or nonlocality. Essentially, we have to construct the other 'Maxwell equations', which contain the c-dependent terms. So, how do we get locality in this case from nonlocality? If we imagine for a moment that gravity really is nonlocal as in Newtonian theory, with its continuous field extending across the universe, filling the whole of space, and allowing for instantaneous interaction, then we couldn't measure it directly, because we need local discrete sources and speed-of-light transmission for measurement, and, with any local description, we will end with a Lorentzian space-time structure, and a noninertial frame of reference [7-10].

The usual laws of dynamics apply only to inertial frames, so, with any noninertial frame, such as the rotating Earth, we assume that the measurement frame is inertial and wait for the consequences. These turn out to be the appearance of fictitious noninertial forces, that is the well-known centrifugal and Coriolis forces. The velocity of light is then reintroduced through the measurement process, with the added complication of c being affected by gravity. It is entirely possible, then, that the 'relativistic corrections' to gravity could be added as a result of the local measurement process, without being generated directly by gravity at all. Ontology is replaced by epistemology though the mathematical structure looks the same, and the effects we have observed so far will be the same. This would be in line with GR, which privileges inertia over gravity in deriving its effects, with the geometric structure derived from specifically inertial arguments.

In effect, we have a redefinition of the coordinates of measured space time, as used in light-bending – which is nothing to do with gravity being *generated* by curvature, but everything to do with gravity producing apparent curvature through the local *inertia*. It is a kind of aberration effect, an 'aberration of space-time', the measured coordinate systems being rotated by the fictitious forces, exactly as in GR. Such a model would allow us to incorporate a version of *Mach's principle* fully into physical theory, and make sense of the relation between gravity and the other forces in quantum terms. As we will show, two well-known effects – redshift and dark energy – become immediate consequences, without any prior assumption of a cosmological model. In fact, they would determine possible cosmological models.

We provide, instead, a *physics* explanation for cosmological redshift and possibly also background radiation, with numerical results. At present, these fundamental phenomena are treated as a kind of archaeology, not physics. 'That's just how it happened.' Our procedure requires us to extend the equation of gravitational force by adding three more *gravitomagnetic* equations. A rotational component causes the gravitomagnetism, as it does with the ordinary magnetic field, and this will be the rotation of the coordinate system due to the inertial forces or the aberration of space-time. Any observation of a gravitating system will then produce an otherwise unexplained rotation, exactly as we observe in stars, planets, galaxies, etc. The inertial force effect will be defined as precisely that which we would expect to obtain from a

relativistic calculation.

It is an extremely simple calculation to add a Newtonian potential $-GM/r$ to a light-ray geodesic defined in the absence of a gravitational field to find the degree of rotation of the coordinate system, and so determine the inertial force effect to be added to create the new geodesic equation expected under these conditions. Extra terms can now be interpreted as an inertial rotation of the coordinate system, or, equivalently, the action of centrifugal and Coriolis forces. The effects of creating a space-time connection and of acting upon it by gravitational fields occur together and are equivalent to the relativistic time dilation and space contraction.

These are well known. The crucial case is the bending of a light ray in a gravitational field because any local coordinate system will have a local rotation from the non-local Euclidean one, exactly as is observed when a light ray passes through a gravitational field. The *geodesics* of such a system (i.e. the lines it 'thinks' are straight) will be exactly those of such light rays, and will *define* the coordinate rotation. Despite several assertions (some recent) that this effect can only be derived from the GR field equations, there have been many derivations from simpler starting points using special relativity [1,11-14]. Essentially time is dilated by $1/(1-GM/rc^2)$ while length is contracted by the same factor, so producing a factor of $(1-GM/rc^2)^2$ in the light ray geodesic. Many other effects follow automatically. The equation for the straight-line light ray:

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 = 0$$

leads immediately to the deflected light ray geodesic:

$$\left(\frac{dr}{dt}\right)^2 + \left(1 - \frac{2GM}{rc^2}\right) r^2\left(\frac{d\phi}{dt}\right)^2 = 0$$

while the equation for a standard orbit:

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 = \frac{GM}{r} - E$$

becomes that for an orbit with deflected geodesic

$$\left(\frac{dr}{dt}\right)^2 + \left(1 - \frac{2GM}{rc^2}\right) r^2\left(\frac{d\phi}{dt}\right)^2 = \frac{GM}{r} - E$$

For local effects that are inertial rather than directly gravitational, the relativistic effects connected with gravity will be extrinsic effects of measurement rather than intrinsic, that is epistemological rather than ontological. In this case, the gravitational field will remain linear, with no unmanageable infinities because curvature will be an effect of gravity rather than its cause. Quantization will be equally extrinsic, with fictitious inertial repulsion, rather than real gravitational attraction, and this will be mediated by spin 1, not spin 2, bosons. Gravitomagnetism will also be extrinsic, with Maxwell-type equations in which the inertial rotation takes the place of magnetic field. Rotation (involving a conserved angular momentum) will be a consequence of the process of *observation* of gravitating systems, explaining the so far unexplained significant intrinsic rotations of gravitating systems. The fictitious inertial forces, both centrifugal and Coriolis, associated with the observation of gravitating systems, may be expected to have directly observable effects, possibly explaining both 'dark energy' and 'dark matter'.

Adapting an argument by Kolbentsvedt [15, 1-2], we suppose an object of mass M moves with velocity **u** in the positive x-direction in the frame of the laboratory, at the same time as a

particle of mass m moves with velocity $\mathbf{v}$ under its gravitational influence. The Lagrangian $\mathcal{L}_0$ of a particle of mass m, in the rest frame of mass M, can be found from the variational principle:

$$\delta \int (-mds) = \delta \int \mathcal{L}_0 dt_0 = 0$$

where

$$ds_0^2 = \gamma^{-2} dt_0^2 - \gamma^2 dr_0^2$$

is the line element in the rest frame, and

$$\gamma^2 = 1 - \frac{2GM}{rc^2} = 1 - \frac{2\phi}{c^2}$$

If we integrate for the rest frame,

$$\mathcal{L}_0 dt_0 = -m \left(\gamma^{-2} c^2 dt_0^2 - \gamma^2 dr_0^2 \right)^{1/2}$$

Transforming to the laboratory frame, and neglecting higher order terms, now yields

$$\mathcal{L} dt = -m \left[(c^2 + 2\phi)(dt - udx)^2 - (1 - 2\phi)(dx - udt)^2 - dy^2 - dx^2 \right]^{1/2}$$

All this leads, when we again neglect the higher order terms, and divide by dt, to the Lagrangian

$$\mathcal{L} = -\left(c^2 - v^2 + 2\phi - 8\phi \frac{\mathbf{u}.\mathbf{v}}{c^2} \right)^{1/2}$$

For $u \ll v \ll c$, series expansion which approximates to

$$\mathcal{L} = -\left(\frac{1}{2} mv^2 - m\phi + 4m\phi \frac{\mathbf{u}.\mathbf{v}}{c^2} \right)^{1/2}$$

when we neglect higher order corrections and the rest energy term mc^2. We may compare this expression with the standard Lagrangian for a particle of mass m and charge q moving with speed $v \ll c$ in an electromagnetic field, which is determined by scalar potential ϕ and vector potential $\mathbf{A}$

$$\mathcal{L} = -\left(\frac{1}{2} mv^2 - q\phi + q\phi \frac{\mathbf{A}.\mathbf{v}}{c^2} \right)^{1/2}$$

The two equations are clearly analogous; $4\phi\mathbf{u}/c$ becomes the gravitational equivalent of the electromagnetic vector potential $\mathbf{A}$, and the rotational analogue of the magnetic field term $\mathbf{B} = \nabla \times \mathbf{A}$ becomes the 'gravitomagnetic' field

$$\boldsymbol{\omega} c = \nabla \times \left(4\phi \frac{\mathbf{u}}{c} \right) = \left(4\phi \frac{\mathbf{u}}{c} \right) \times (\nabla \phi) = 4 \frac{\mathbf{u}}{c} \times \mathbf{g}$$

We can then set out a series of 'Maxwell equations':

$$\nabla.\mathbf{g} = 4\pi G\rho$$
$$\nabla.\boldsymbol{\omega} c = 0$$
$$\nabla \times \mathbf{g} = -c\frac{\partial \boldsymbol{\omega}}{\partial t}$$
$$\nabla \times \boldsymbol{\omega} c = 4\pi G\rho\mathbf{v} + c\frac{\partial \mathbf{g}}{\partial t}$$

Such equations can certainly be derived from GR, and have received experimental confirmation with the discovery of gravitational waves, but the earliest version predates that theory and even Einstein's special theory by many years [16-18].

6. Mach's principle and dark energy

GR, as previously interpreted, though based initially on inertial arguments, never completely succeeded in resolving the relation between gravitation and inertia. It never succeeded in accommodating Mach's principle, or the idea that the inertial mass of any object is due to its interactions with the rest of the matter in the universe. Sciama tried to overcome this in 1953, though he later abandoned his argument [19-20]. By extending this argument on the basis of our new understanding of how gravitomagnetism and inertia are related, we are led to a remarkable prediction with a close relation to some significant experimental results discovered at a much later period.

The true connection between gravity and inertia, both linked to mass, is one of the most profound questions in physics. Inertia (resistance to motion) seems to be concerned with whether accelerated motion is absolute or relative. Newton devised an experiment with a water bucket to show the difference between absolute and relative rotation, with a concave surface of the water in the bucket indicating absolute rotation. The absolute rotation may be defined as relative to the fixed stars, or the bulk of matter in the universe. The question is: do the fixed stars actually *cause* it?

Mach's principle suggests that the inertia of a body – its mass or resistance to accelerated motion ($F = ma$) – is determined in some way *by an interaction with the rest of the matter in the universe*. If it is, then it must come from a 'magnetic' component. Does gravity have a 'magnetic' inertial component and an acceleration-dependent inductive force analogous to that which occurs in electromagnetic theory? That is, a force like

$$F = \frac{G}{c^2 r} m_1 m_2 \sin\theta \frac{dv}{dt}$$

Sciama considered the possibility of explaining Mach's principle using such an inductive force (in his model a real one). The inertia of a body of mass $m = m_1$ would then be attributed to the inductive action of the total mass $m_u = m_2$ within the *observable universe*, specified by radius r_u, so making the inductive force equation equivalent to $F = Kma$, with K a constant.

We make an additional supposition that the continuous mass-field we need for physics provides a standard by which we can define a unit inertial mass nonlocally for the *entire universe*, in the same way as the near-constant gravitational field **g** provides a way of defining a unit mass at the Earth's surface. We imagine that mass of the 'Hubble universe' m_u defines a radial inertial field of constant magnitude from the centre of a local coordinate system, and, at the same time, use the principle of equivalence (equivalence of mass from inertia and gravitation), to equate this to the *nonlocal* gravitational field (Gm_u/r_u^2), which, independently of the local coordinate system, defines a unit of gravitational mass within the same event horizon. Using isotropy to remove the angular dependence, we obtain:

$$\frac{Gm_H}{c^2 r}\frac{dv}{dt} = \frac{Gm_H}{r_H^2}$$

This leads to an acceleration

$$a = \frac{dv}{dt} = v\frac{dr}{dt} = \frac{c^2 r}{r_H^2}$$

which can be integrated with respect to r, to give

$$a = \frac{dv}{dt} = \frac{c^2 r}{r_H^2}$$

where H_0 is the Hubble constant, and the acceleration is now

$$a = \frac{v^2}{r_u} = H_0^2 r$$

We can continue with $da/dt = H_0^3 r$, etc. Perhaps at some future period, our measurements will become sufficiently accurate to detect at least the first higher order differential. If gravity is nonlocal, then this inertial acceleration is a fictitious one, which describes the effects on the coordinate system produced by using a localized 4-D Lorentzian space-time to model the instantaneous interaction.

We can express the physics in terms of two forces; gravity, which is attractive, and the fictitious inertial repulsion. Combining them gives

$$F = \frac{GM}{r^2} - H_0^2 r = \left(\frac{4}{3}\pi G\rho - H_0^2 \right) r$$

If we add the inertial term to standard gravitational theory we obtain

$$\nabla^2 \phi = 4\pi G \left(\rho - \frac{3H_0^2}{4\pi G} \right) = 4\pi G \left(\rho + \frac{3P}{c^2} \right) = 4\pi G(\rho + 3\rho_{vac})$$

with the additional term equivalent to a 'dark' energy density or negative pressure

$$-P = \frac{H_0^2 c^2}{4\pi G}$$

and cosmological constant

$$\Lambda = 8\pi G\rho_{vac} = 2H_0^2$$

Defining the critical density for a 'flat' universe as

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

we find that

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{2}{3}$$

The 'vacuum' or 'inertial' component of the universe's energy (the so-called dark energy contribution) appears to be 2/3 of the total or 67 %, which is within the limits of current observations, with the experimental value, which, as estimated from the Planck data, is of order 68 % [21-22]. The force law involving this component suggests a linear increase with distance ('Hooke's law'), exactly as assumed in dark energy calculations. It also has the classic mv^2/r form expected of *centrifugal force*. The calculations, however, precede the observations by about twenty years [11, 23].

Also, although we derived it here from a 'gravitomagnetic' component, we may note that it is also equivalent to modifying the Newtonian force law by a length contraction factor of $(1–v^2/c^2)$. The inertial force then has exactly the form that would emerge from making relativistic corrections to the distance terms in Newton's law of gravitation. Perhaps not surprisingly, this works out the same as the gravitomagnetic calculation. If it is the origin of the dark energy, then we already have a relativistic correction to gravity in inertia, and so Newtonian theory becomes completely integrated into GR through adding the inertial component. (We may note that, in the current solutions, the curvature term is not present at the same time as the cosmological constant) [7-10].

7. Dark matter and other considerations

When we equate kinetic and potential energies, a Machian origin for inertia leads to

$$Gm_u = \frac{1}{2}c^2 r_u$$

with a universal inertial field equal in magnitude to Gm_u/r_u^2, defining the unit of inertial mass. Like centrifugal accelerations of any origin, this measure of inertia is valid for all physical objects, whether static or moving. In the case of objects which are themselves in motion, we might expect an additional fictitious (Coriolis) acceleration due to the rotation of the coordinate system. This would be perpendicular to both the real velocity of the object and the axis of rotation of the coordinate system. In magnitude, this effect, which incorporates equal components from the changes of velocity in space and time, would be twice that of the centrifugal acceleration.

The magnitude of a universal or nonlocal inertial Coriolis effect for a rotating or gravitational orbiting system, such as a galaxy, might then be of order $2 \times \frac{1}{2}c^2 r_u/r_u^2 = c^2/r_u$. Evidence might be sought from the flattening of the rotation (v–r) curves of galaxies, which seems to hover (though very approximately) round the point where $v^2/r = c^2/r_u$, as if the Coriolis acceleration of the coordinate system provided an inertial upper limit to the galactic size. Such an explanation would produce something like a Tully-Fisher relation between mass and limiting velocity v with the mass proportional to v^4. Perhaps, such a Coriolis effect could be the ultimate source of the galactic rotation.

Sciama thought that the inertial force he was investigating was 'real'. However, if it is, then the Earth does not merely *appear* to be stationary in space, but must actually be so. An unpublished proposition by Newton argued that an (inertial) force opposing the effect of gravity due to the rest of the universe would make the Earth part of a Tychonic system. It would be a truly stationary body round which the Sun orbited with the rest of the solar system [14, 24]. The only argument against this is that it would make the Earth 'special' when it was likely that many of the fixed stars were also centres of solar systems similar to our own. A fictitious inertial force, however, would remove this possibility, as it would be merely an effect of observation and not of intrinsic physical action.

This Newtonian observation is one of many that should lead us to question the widely-held assumption that NG and GR are rival theories that occupy entirely separate realms of intellectual space and have no intimate connection. This is despite the fact that GR uses NG as an arbitrarily-defined limiting case, with no explanation why this should happen, and that its purely mathematically-defined curvature equations only acquire physical meaning when they are linked with the Newtonian potential. In fact, despite the revolutionary rhetoric of 1919, there are many generic connections between the two theories, and exploring these will give us a better insight into how each is constructed.

The special relativistic space-time connection certainly separates the theories, but in other respects there are many similarities. In addition to the Cartan argument mentioned in section 2, there is precedent for the use of geometry instead of force in Newton's *Principia*, and, in particular, for the use of a geodesic to define the action of a force in the Corollaries to Book I, Proposition 44 [14]. However, even the more 'physical' interpretations of the theories are connected, as has long been recognised, through the use of a gravitational refractive index and an aether-density mechanism. Newton never found an explanation of gravity, but at various times tried to imagine an aether density mechanism in which the aether was denser in the regions less dense with material bodies. The denser aether then pushed material bodies towards places denser with other ones. Einstein was also, at times, inclined to use the language of aether to explain his geometrical theory [11].

The aether-density theory is connected with the analogous phenomenon of optical refraction in that the speed of light (and, in relativity, the measurement of time) changes with the density

of the medium. While Newton's version of this theory has the gravitational field *displacing* the medium, Einstein's theory has the gravitational field *as* the medium. One author, Eric Baird, has proclaimed that this is because Newton had the 'wrong' explanation of refraction, with light particles speeding up in the denser medium rather than slowing down as waves would do [25]. In fact, as wave-particle duality shows, both these conditions apply, with the canonical *momentum* of the photons increasing in the denser medium, as the 'speed of light' (interpreted as the phase velocity of the waves) decreases, dilating time in the process [13]. However this is interpreted, it is interesting that Newton's medium acts according to *inertia*, repelling rather than attracting, suggesting that the gravitation-inertia link has a connection with wave-particle duality.

In NG, a 'God-centred theory', the gravitational force is described in ontological terms, whereas inertia is epistemological. Only the concept of mass connects them. In GR, an observer-centred theory, the ontology is interpreted through epistemology, and gravity and inertia are inextricably linked, though the theory is frequently described as an explanation of gravity. The result is that we have interpreted it as a theory that will blow up in strong fields, with a quantum gravity that will be forever nonrenormalizable. However, if we can extricate the local and nonlocal parts as we can with the three gauge theories, then we can overcome these problems, and at the same time gain a new insight into phenomena such as redshift and dark energy.

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