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**Article**

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Article

# On the Negative Result Experiments in Quantum Mechanics

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**Abstract:** We comment on the so-called negative result experiments (also known as null measurements, interaction-free measurements, and so on) in quantum mechanics (QM), in the light of the new general understanding of the quantum-measurement processes, proposed recently. All experiments of this kind (null measurements) can be understood as improper measurements with an intentionally biased detector set up, which introduces exclusion or selection of certain events. The prediction on the state of a microscopic system under study based on a null measurement is sometimes dramatically described as “wave-function collapse without any microsystem-detector interactions”. Though certainly correct, such a prediction is just a consequence of the standard QM laws, not different from the situation in the so-called state-preparation procedure. Another closely related concept is the (first-class or) repeatable measurements. The verification of the prediction made by a null measurement requires eventually a standard unbiased measurement involving the microsystem-macroscopic detector interactions, which are nonadiabatic, irreversible processes of signal amplification.

**Keywords:** particles; quantum measurement; null measurement; wave-function collapse



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## 1. Introduction

A typical quantum-mechanical measurement process involves an interaction between a microscopic quantum system with a macroscopic experimental device, which is capable of faithfully capturing the quantum state of the microscopic system—object of the “measurement”—and of recording the result in the form of a classical state of matter. The process typically involves a non-adiabatic, irreversible process of signal amplification (such as a chain ionization and fixture of images on a photographic film). Critical discussions on earlier attempts for constructing a theory of measurements can be found in [1]; many original papers are collected in [2]. See also [3,4]. Quantum-measurement processes have been analyzed recently, with a few new key observations [5,6].

Such a physical characterization of a quantum measurement was challenged by a series of Gedanken (as well as real) experiments [7,8] of a particular kind, in which some negative result (or null) measurement, hence without any microsystem-macroscopic-device interactions typical of the standard quantum measurements, allows one to acquire nontrivial information on the quantum state of the system under study. The predicted state,  $|\psi'\rangle$ , is necessarily a restriction—namely, a projection of the original vector in the Hilbert space onto a vector in a space of a smaller dimension—of the original wave function  $|\psi\rangle$ . This fact was somewhat dramatically expressed as “an interaction-free measurement leading to a wave-function collapse”. These negative result experiment arguments have been presented, and believed by some, as counter-examples to the physical characterization of the quantum-measurement processes, i.e., non-adiabatic, irreversible processes involving micro system-macroscopic device interactions.

The aim of this short note is twofold. The first is to review the characteristics of a typical quantum-measurement processes, based on a few novel observations made

recently [5,6,9]. The second is to give a simple, more sober interpretation of these null measurements: they all correspond to specific, biased detector set-ups which select or exclude certain possible events. The prediction made on the quantum state after a null measurement, certainly correct, is just a consequence of the standard quantum mechanical laws: they do not in any way disprove the physical picture of a typical quantum-measurement process. Indeed, verification of the predictions made by a null measurement on the system being studied eventually requires a proper, unbiased quantum measurement, involving microsystem-macroscopic device interactions.

Even though the main subject of this note [7,8] is a rather old one (see, however, [10,11] for recent discussions), and has been discussed extensively (see [1] and the references cited in [10,11]), it touched some of the most subtle aspects of the interpretations of the quantum mechanical predictions. It is thus not entirely pointless today, perhaps, to revisit the essential aspects of these negative result experiments, correct any inappropriate interpretations, and to ensure that our understanding of the QM laws is crystal clear.

## 2. Solution of the Quantum-Measurement Problem in a Nutshell

A measurement of a quantity  $F$ , conducted on a quantum state  $\psi$

$$|\psi\rangle = \sum_n c_n |n\rangle, \quad \sum_n |c_n|^2 = 1, \quad (1)$$

where  $|n\rangle$  is the eigenstate of  $F$  with eigenvalue,  $f_n$ , used to be schematized as

$$|\psi\rangle \otimes |\Phi_0\rangle \otimes |Env_0\rangle = \left( \sum_n c_n |n\rangle \right) \otimes |\Phi_0\rangle \otimes |Env_0\rangle \quad (2)$$

$$\rightarrow \sum_n c_n |n\rangle \otimes |\Phi_n\rangle \otimes |Env_0\rangle \quad (3)$$

$$\rightarrow \sum_n c_n |n\rangle \otimes |\Phi_n\rangle \otimes |Env_n\rangle, \quad (4)$$

where  $|\Phi_n\rangle$  represents the detector state, with  $n$  has recorded the result  $F = f_n$  and  $|Env\rangle$  stands for everything else, the state of the experimentalist and the rest of the world. The index 0 indicates a neutral state, whereas the index  $n$  stands for the measurement result,  $F = f_n$ . Such a formula is found in many textbooks of quantum mechanics (QM), and (basically) in all past discussions [1,2].

Actually, the Formulae (2)–(4) are incorrect in several accounts [5,6].

(i) First, the factorized form for the wave function expressed by the symbol  $\otimes$  in the state after the measurement is not valid. As for the state before the measurement in the first line, (2), factorization of  $|\psi\rangle$  is correct: it must be so in any ideal experiment. Factorized form  $|\Phi_0\rangle \otimes |Env_0\rangle$  is instead incorrect; see below and [5]. In general, in the aftermath of a measurement, the microscopic system becomes entangled with the device and with the environment, typically in an uncontrolled manner. In an exceptional class of the so-called repeatable (or the first-class) experiments, the microscopic system under study remains factorized and intact. These processes are indeed closely related to the “negative-result experiments” as well as with the state preparation procedure, as discussed below, Section 3, even though the information about its original state is faithfully recorded by the detector.

(ii) Secondly, the experimental device,  $|\Phi\rangle$ , typically a macroscopic body at finite temperatures, and entangled with the rest of the world (environment), is itself in a decohered, mixed state [9,12–18]. This is so even before the measurement, let alone after the measurement.

This means that the expression (3) should not be considered a pure state (a coherent superposition); it is a mixture. This observation is sufficient to explain away a “puzzle” recently discussed in [19].

Taking these points into account, it was proposed [5] that the detector-environment entangled state be denoted as  $|\Phi; Env\rangle$ . The typical measurement process thus looks more like

$$|\psi\rangle \otimes |\Phi_0; Env_0\rangle = \left( \sum_n c_n |n\rangle \right) \otimes |\Phi_0; Env_0\rangle, \quad (\text{before}) \quad (5)$$

$$\rightarrow \sum_n c_n |\tilde{\Phi}_n; Env_0\rangle, \quad (\text{after}) \quad (6)$$

$$\rightarrow \sum_n c_n |\tilde{\Phi}_n; Env_n\rangle, \quad (\text{later}), \quad (7)$$

where  $|\tilde{\Phi}_n; Env_0\rangle$  with the symbol  $\tilde{\Phi}_n$  stands for the microsystem detector-entangled state (see [5] for more discussion) with a macroscopic marking of the recording,  $F = f_n$ .

The second stage of the process, (7), in which the experimentalist sees the result of the measurement on her computer screen, the others read about it in Physical Review, etc., is totally irrelevant, in spite of many sophisticated and sometimes philosophical discussions made in the past (see [2] for some). The essential features of the process, (5)  $\rightarrow$  (6), are the following:

(a) *Each term in (6) containing  $|\tilde{\Phi}_n; Env_0\rangle$  is a complicated mixed state (point (ii) above), representing the microsystem-detector-environment entangled state (point (i) above), with a well-defined macroscopic marker of the measurement result,  $F = f_n$ . It is an eigenstate of the operator  $F$ . Namely,*

$$F |n\rangle = f_n |n\rangle \quad (8)$$

$$\rightarrow F |\tilde{\Phi}_n; Env_0\rangle = f_n |\tilde{\Phi}_n; Env_0\rangle. \quad (9)$$

The relation between (8) and (9) defines a good, faithful measurement.

(b) *A key observation [5] is that, reflecting the pointlike nature of the fundamental entities of our world, each measurement process is a spacetime pointlike event (or triggered by one). This entails that the wave functions corresponding to the different terms in (6) have no overlapping spacetime supports. Thus, not only the orthonormality*

$$\langle \tilde{\Phi}_m; Env_0 | \tilde{\Phi}_n; Env_0 \rangle = \delta_{mn}, \quad (10)$$

holds, but also a dynamical diagonalization

$$\langle \tilde{\Phi}_m; Env_0 | G | \tilde{\Phi}_n; Env_0 \rangle = G_n \delta_{mn}. \quad (11)$$

occurs for any local operator  $G$ . Note that  $G_n$  in (11) is defined by (11) itself: it is unrelated to the eigenvalues of the operator  $G$  in the isolated microscopic system before the experiment.

The diagonalization (11) is of utmost importance.

*Before the measurement*, in the state (5), the expectation value of a generic quantity  $G$  is given by

$$\{ \langle \Phi_0; Env_0 | \otimes \langle \psi | \} G \{ |\psi\rangle \otimes |\Phi_0; Env_0\rangle \} = \langle \psi | G | \psi \rangle = \sum_{m,n} G_{mn} c_m^* c_n \equiv \text{Tr} \rho^{(0)} G, \quad (12)$$

( $G_{mn} \equiv \langle m | G | n \rangle$ ) meaning that the system is described by a density matrix

$$\rho_{nm}^{(0)} = c_n c_m^*, \quad (13)$$

i.e., by the pure state,  $|\psi\rangle = \sum_n c_n |n\rangle$ . For  $G = F$ , the variable whose eigenstates are taken as the basis  $\{|n\rangle\}$ , one finds of course the standard formula

$$\langle \psi | F | \psi \rangle = \sum_n f_n |c_n|^2. \quad (14)$$

After the measurement, according to (11), the expectation value of a generic variable  $G$  taken in the “state” (6), is given (by using (11)) by

$$\left( \sum_n c_m^* \langle \Phi_m; Env_0 | \right) G \left( \sum_n c_n | \tilde{\Phi}_n; Env_0 \rangle \right) = \sum_n |c_n|^2 G_n . \quad (15)$$

That this holds for *any* operator  $G$  means that the density matrix of the system has been effectively reduced to a diagonal form

$$\rho^{(0)} \xrightarrow{(measurement)} \rho^{(1)} = \begin{pmatrix} |c_1|^2 & & & \\ & |c_2|^2 & & \\ & & \ddots & \\ & & & |c_n|^2 \\ & & & & \ddots \end{pmatrix} . \quad (16)$$

By paraphrasing the “environment-induced superselection rule” [1], we may call (16) the measurement-induced superselection rule.

- (c) The fact that the wave functions of the different terms in (6) have no overlapping spacetime support means that the aftermath of each measurement event is a single term in (6). A related fact is that the detector-environment “state”  $|\Phi_0; Env_0\rangle$ , even if it might look identical macroscopically, it can never be the same quantum state at two different measurement instants. The time evolution of the macroscopic number of molecules and atoms in the detector and environment means that the “state” just before each experiment  $|\Phi_0; Env_0\rangle$  is a unique and distinct quantum state, actually carrying a hidden index “(n)” of each measurement. That is, the time evolution in *each single measurement* is,

$$\left( \sum_n c_n |n\rangle \right) \otimes |\Phi_0; Env\rangle \xrightarrow{} |\tilde{\Phi}_m; Env\rangle : \quad (17)$$

i.e., with a single term present the instant after the measurement (e.g., with  $F = f_m$ ). This fact is often (improperly) described as a “wave-function collapse”. The words evoke in our mind an image of some distribution suddenly contracting, which does not correspond to any real physical process. The wave function is itself not an observable.

- (d) A second crucial consequence of our description of the measurement process, (5), (6), concerns the *repeated* measurements. For the measurement of the quantity  $F$ , it follows from (9), (10), and (15) that the expectation value is given by

$$\left( \sum_n c_m^* \langle \tilde{\Phi}_m; Env_0 | \right) F \left( \sum_n c_n | \tilde{\Phi}_n; Env_0 \rangle \right) = \sum_n |c_n|^2 f_n , \quad (18)$$

where  $f_n$  are the eigenvalues of  $F$ . This means that the relative frequency for finding the result,  $F = f_n$ , has been found to be given by

$$\mathcal{P}_n = |c_n|^2 . \quad (19)$$

The derivation of the “wave-function collapse” (17) and of the formula for the relative frequency (19), amount to the solution of the quantum-measurement problem.

One might object that we have just reproduced the standard Born rule. This is not quite so. Unlike the latter, our description explains why and how the “wave-function collapse” occurs, and yields the rule (19) as the result of physical measurement processes involving the microsystem-detector-environment interactions. Even though they look similar, the difference in the meaning of (14) and (18) is crucial. Last, by eliminating the fundamentally obscure concept of “probability” inherent to Born’s rule, and by replacing it

by the (normalized) relative frequency for various outcomes in repeated experiments, it leads to a more natural interpretation of the QM laws [5,6]. For instance, the concept of the “wave function of the universe” makes perfect sense now, whereas in the traditional interpretation of QM based on Born’s rule, one cannot avoid falling into a conundrum of having nobody outside the universe, observing it and making repeated experiments on it. Note that the cosmologists today are adopting this new interpretation of QM laws, naturally, when they discuss the structure formation in the early universe, through the density quantum fluctuations at a certain stage of the inflation.

### 2.1. A Secret Key

An alert reader must have noticed the following subtlety. We affirmed that each of the microsystem-detector-environment entangled state  $|\tilde{\Phi}_n; Env_0\rangle$  is a complicated mixed state, involving the myriad of microscopic processes, such as the scattering of air molecules against the  $\sim O(10^{25})$  atoms and molecules composing the detector, the emission of the infrared photons from the latter, and so on. Nevertheless, we used it as an ordinary wave function (i.e., a pure state), to evaluate the expectation values, (15), (18). Is it consistent?

Actually, here lies a secret key in the whole discussion. What might not be widely appreciated is the fact that there are no differences in principle between the concepts of the pure and mixed states. Consider any (pure) quantum state  $\Psi(\{\mathbf{r}_i, s_i\}, \{\mathbf{r}'_k, s'_k\})$  where  $\{\mathbf{r}_i, s_i\}$  and  $\{\mathbf{r}'_k, s'_k\}$  are the position and spin component of the particles composing the whole system. We (the physicists) are assumed to have access only to the subsystem ( $A$ ) containing the degrees of freedom  $\{\mathbf{r}_i, s_i\}$ . The rest of the world ( $B$ ) described by  $\{\mathbf{r}'_k, s'_k\}$  is off-limits. By introducing an orthonormal (ON) set of states describing the system  $A$ ,  $\{|n\rangle^{(A)}\}$ , and expanding the coefficient in ON states of  $B$ ,  $\{|N\rangle^{(B)}\}$ , a generic state has the form

$$|\Psi\rangle = \sum_{n,N} c_{n,N} |n\rangle^{(A)} |N\rangle^{(B)}. \quad (20)$$

The expectation value of any variable  $G$  pertinent to  $A$  is then

$$\langle \Psi | G | \Psi \rangle = \text{Tr } G \rho, \quad G_{mn} = \langle m | G | n \rangle, \quad \rho_{nm} = \sum_N c_{n,N} c_{m,N}^* \quad (21)$$

where  $\rho$  is the density matrix. It is perfectly correct, however, to use the wave function (20), or (6), e.g., in (15) or in (18), when the sum over the system  $B$  is indeed implied in the calculation. A similar idea was used in [9] to explain why the Ehrenfest theorem can be used to derive Newton’s equations for the center of mass of a macroscopic body at finite body temperatures, which is in a decohered, mixed state.

To sum up, the concept of a mixed state (versus a pure state) is a relative one, depending on which part of the world ( $A$ ) is accessible to us. The ignorance about the rest of the universe ( $B$ ) is parametrized by the density matrix. More interestingly, two (or more) groups of physicists studying a common event such as a multiparticle decay, located in spacetime regions which are spacelike separated, might find apparently paradoxical outcomes involving various quantum correlations. These phenomena of “quantum nonlocality”, as fascinating and mysterious as they are and might continue to appeal to the human mind, are today fully understood. The peculiar “subjectivity” of the quantum-mechanical laws also hinges upon these circumstances.

### 2.2. A Remark

One of the characteristics (or requirements) of a proper quantum measurement, which every experimentalists know well, is that the device must not have any bias, i.e., it should be equally effective to register all possible outcomes,  $f_n$ . For, if it were not so, the experimental average for the frequency times various possible results would not match the theoretical prediction, (19), even after many repeated measurements.

An exception occurs when there are only two possible outcomes, either  $F = f_1$  or  $F = f_2$ . In this case, the device which is capable of measuring only one of the possible results, e.g.,  $f_1$  (the so-called yes–no experiment) is sufficient to give unbiased measurement results. Event by event, the detection of  $f_1$  (yes) means the wave function collapse (17) with  $F = f_1$ ; the non-detection of  $f_1$  (no) implies that the state is  $|\psi\rangle = |2\rangle$ , even if the measurement  $F = f_2$  has not yet been actually conducted. The negative result experiments reviewed below rely on an analogous logic.

### 3. Negative Result Experiments

In this section, a few well-known examples of the negative result experiments are reviewed and their essential features critically analyzed.

#### 3.1. Renninger

A pair of collections of particle detectors, each covering one of the hemispheres, surround a radioactive nucleus in the center, which emits an  $\alpha$  particle, e.g., in the S-wave. In the original article by Renninger [7], an excited atom is used and what is emitted is a photon. Nothing essential changes by replacing the atom in an excited level by an unstable nucleus, however. If one of the shells does not observe  $\alpha$ , its wave function has been “collapsed to the other hemisphere”, without any interactions between the  $\alpha$  particle and the experimental device!

To make the puzzle look sharper, the detectors in the second hemisphere (call  $\Phi^{(lower)}$ ) may be set at a radius much larger than the first. If the half-life time of the nucleus is  $\tau$ , the nucleus has most likely decayed ( $\alpha$  has been emitted) by the time  $t = 30\tau$ . Only in an exceptional one out of  $e^{30}$  repeated experiments, on average, the nucleus will be found still undecayed, without  $\alpha$  emission. If the second detector is set at the distance

$$R > 30c\tau, \quad (22)$$

then, by the time  $t = 30\tau$ , if the first detector has not detected  $\alpha$ , then most likely it is still traveling towards the lower-hemisphere. So it might appear as if “the wave function had collapsed”, without any particle-detector interactions having taken place. The “paradox” is only apparent, and its origin can be traced to the misconception that the wave-function collapse is a sort of real physical process in itself, as noted already.

The metastable nucleus which  $\alpha$ -decays in an  $S$  state can be expressed by

$$|\Psi\rangle = |\Psi^{(0)}(t)\rangle + |\Psi'(t)\rangle |\alpha\rangle, \quad (23)$$

At the time  $t = 30\tau$  the nucleus has most certainly decayed, so let us concentrate on the second term of (23). The  $\alpha$  particle is described by an S-wave function. This is really the reduced one-body description of the  $\alpha$  particle-nucleus,

$$|\Psi\rangle = \sum_i c_i |i\rangle, \quad (24)$$

where  $i = 1, 2, \dots, N$ ,  $N \gg 1$ , represent the uniformly discretized cells of the  $4\pi$  solid angle. The S-wave nature of the wave function means that

$$\forall i \quad c_i = \frac{1}{\sqrt{N}}. \quad (25)$$

This is simply a discretized version of the S-wave wave function,

$$\Psi(\mathbf{r}) = f(r) \sim \frac{e^{ikr}}{r}, \quad (26)$$

independent of the angular variables. The standard measurement of the angular distribution is conducted with

$$|\psi\rangle \otimes |\Phi_0\rangle = \sum_i c_i |i\rangle \otimes |\Phi_0\rangle. \quad (27)$$

where the detector  $\Phi_0$  is uniformly sensitive over the  $4\pi$  solid angle. It should not have any bias as to which angular direction the particle is eventually measured.

The experimental set-up by Renninger, instead, has a nonuniform  $\Phi_0$ . Namely,

$$|\psi\rangle |\Phi_0\rangle = \sum_{i=1}^{N/2} c_i |i\rangle \otimes |\Phi_0^{(upper)}\rangle + \sum_{i=N/2+1}^N c_i |i\rangle \otimes |\Phi_0^{(lower)}\rangle. \quad (28)$$

where  $\Phi_0^{(upper)}$  and  $\Phi_0^{(lower)}$  are the first and second groups of detectors at the upper and lower hemispheres, respectively. The detector  $\Phi_0^{(upper)}$  ( $\Phi_0^{(lower)}$ ) is insensitive to the  $\alpha$  particle flying towards the lower (upper) hemisphere, and thus the cross terms such as  $\sum_{i=1}^{N/2} c_i |i\rangle \otimes |\Phi_0^{(lower)}\rangle$  are absent.

In case (22), the two detectors are such that up to  $t = 30\tau$ , the lower detector is insensitive to the  $\alpha$  particle, i.e., is not able to detect it. Only the first is sensitive, and only to the  $\alpha$  particle traveling towards the upper hemisphere.

If the experiment is repeated, half of the times the detector  $\Phi_0^{(upper)}$  will record  $\alpha$  at time  $t \leq 30\tau$ , and the other half of the times it will not. This is what this particularly biased detector will produce, in accordance with the QM prediction.

In a second type of event, where the first detector  $\Phi_0^{(upper)}$  does not detect the  $\alpha$  particle (“the negative-result measurement”), it is correct *for us to infer that* the wave function is reduced to the second term of (28),

$$|\psi\rangle |\Phi_0\rangle \rightarrow \sum_{i=N/2+1}^N c_i |i\rangle \otimes |\Phi_0^{(lower)}\rangle. \quad (29)$$

In these events, the  $\alpha$  particle will be detected by  $\Phi_0^{(lower)}$  at a later time,

$$t \sim \frac{R}{c} \gg 30\tau, \quad (30)$$

as will be eventually verified by the standard  $\alpha$ - $\Phi_0^{(lower)}$  detector interactions.

The negative result experiment such as this could look somewhat paradoxical. A phrase such as “*for us to infer that...*” might indeed appear to indicate that the “wave function collapse”,  $|\psi\rangle \rightarrow \sum_{i=N/2+1}^N |i\rangle$ , has been caused by the human mind—the realization of the non-detection fact. The wave-function collapse, so it might seem, does not need any microsystem-macroscopic device interactions, as those assumed in [20,21]. These questions were at the center of ardent debates, partially reignited by Renninger’s work (see, for example, [22,23]).

What happens actually in the Renninger experiment is that, in each decay event, the  $\alpha$  particle is emitted either towards the upper hemisphere or towards the lower hemisphere, with the same relative frequencies, if the experiment is repeated. That is all.

The aim of revisiting these old, and after all simple, issues nonetheless, was to illustrate how the wrong wordings and the misconception about the “wave-function-collapse” have led to nonexistent, and hence unsolvable, problems in the past discussions on QM.

### 3.1.1. State Preparation

The deduction (29) in the case of Renninger’s negative result experiment is not essentially different from the preparation of a collimated atomic beam, by using two successive slits, such that the particles which have passed both slits have a more or less well-defined

momentum direction, so that one can predict that the particle, left freely propagating, will be detected in the direction of a straight line connecting the two successive slits, within some error (taking into account the diffraction effects).

The analogy may be made even closer, by making the upper detector  $\Phi^{(upper)}$  of Renninger cover 99% of the  $4\pi$  solid angle, leaving a small hole in the south-pole direction. Then, in the very rare event (one in 100) in which  $\Phi^{(upper)}$  has not recorded  $\alpha$ , it can be predicted that the  $\alpha$  will be detected in the direction of the south-pole direction by  $\Phi^{(lower)}$ , later. Instead of the beam preparation by using two slits, here, one uses just one slit and the selection of the (non-observation, null measurement) event.

### 3.1.2. $\alpha$ Particle Tracks in a Cloud Chamber

Renninger's process brings us back to one of the oldest "puzzles" in QM: why does an  $\alpha$  particle emitted by a metastable nucleus, described by a spherically symmetric wave function, (26), produce *each* an (almost) straight-line track, instead of a sequence of ionization blots distributed all over  $4\pi$  angular directions? The answer has been given by a standard perturbation analysis made by Mott [24], and we are not here to discuss it anew.

The reason why we brought up this historical issue here, in spite of little direct logical connection with the negative experiment problems, is this. The observation of an  $\alpha$  particle track in a Wilson chamber is a quantum measurement of the angular distribution of its momentum. It is a *measurement of intermediate type*, between the general one ((6) and (17)), and the repeatable experiments ((36) and (37) below). In the latter special type of experiments, the state of the microsystem under study remains factorized and intact as a pure state: the measurement process can be used as a state preparation.

In the former, more general type of measurements, the microscopic system becomes entangled with the device and with the environment in the process, and the information about its quantum state becomes lost completely, in general, after the measurement event (see (17)).

An  $\alpha$  particle track in a cloud chamber starts when  $\alpha$  hits the first atom, ionizing it. The  $\alpha$ -electron scattering process is a spacetime pointlike event, triggering the measurement event. The state of the  $\alpha$  particle is only slightly affected by the  $\alpha$ -electron scattering. The large mass ratio,  $m_\alpha/m_e \sim 8000$ , means that the momentum of the  $\alpha$  particle is almost unaffected by the  $\alpha$ -electron scattering, as is obvious from kinematics and as explicitly verified by concrete QM calculations [24], as it proceeds along an almost straight path, hitting and ionizing a sequence of atoms on its way.

In conclusion, the explanation of the (apparent) wave function collapse, (17), i.e., that a measurement process is effectively a spacetime local event, is valid also in the intermediate type of measurements, such as the Mott process, momentum measurements by using the magnetic fields, particle tracks in the vertex detectors, and so on.

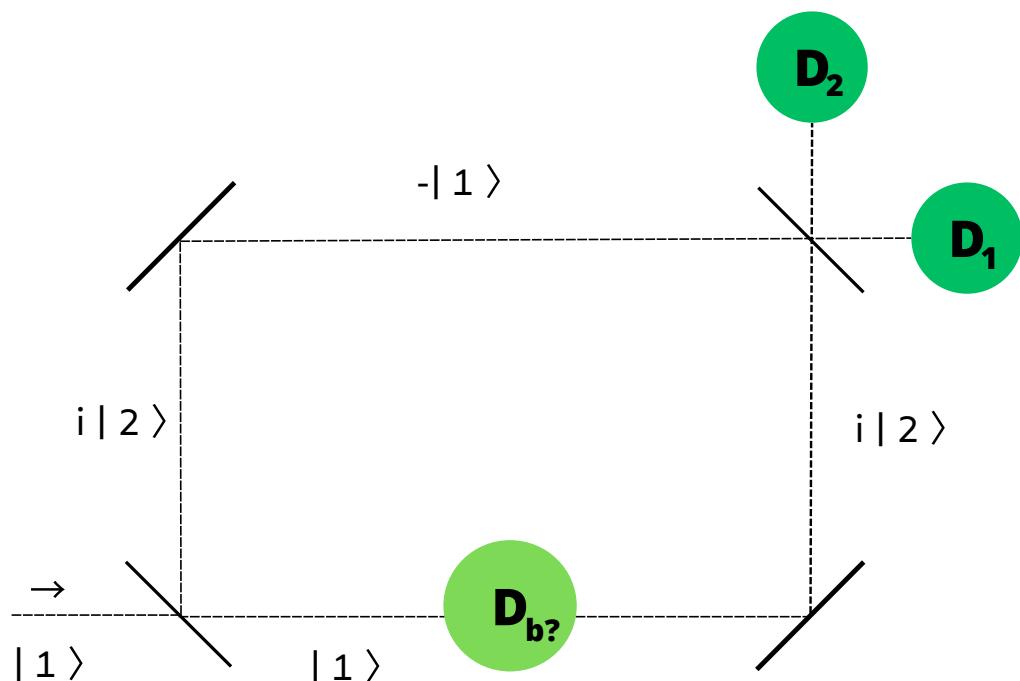
### 3.2. Elitzur–Vaidman Bomb Tester

A more sophisticated, amusing set-up is the so-called Elitzur–Vaidman bomb tester experiment [8]. A single photon is sent to Mach–Zehnder interferometry. See Figure 1.

A bomb which is either real or fake is introduced in the lower path. After the passage through the first half mirror (the lower left corner in Figure 1) the original right moving photon  $|1\rangle$  is converted to the superposition

$$|1\rangle \rightarrow |\gamma\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle), \quad (31)$$

where  $|2\rangle$  is the wave packet of the photon (reflected and) moving upwards. We follow the notation and convention of [8]. We recall that the photon, upon reflection, acquires a phase shift of  $\frac{\pi}{2}$ .



**Figure 1.** Elitzur–Vaidman bomb-tester experiment. The photon enters from the lower left corner to a Mach–Zehnder interferometry. The detection of the photon at the detector  $D_2$  implies that the bomb is real, but that the photon has not interacted with the bomb.

In the case that the bomb is a fake (it is assumed [8] that in that case the photon passes the region unaffected), the photon (31) goes through the two fully-silvered mirrors (at the upper-left and lower-right corners of Figure 1), and is in the linear superposition,

$$|\gamma'\rangle = \frac{1}{\sqrt{2}}(i|2\rangle - |1\rangle), \quad (32)$$

before entering the second half-silvered beam splitter (in the upper right corner in Figure 1). Going through it, the wave packet  $|1\rangle$  is transformed as in (31), whereas  $|2\rangle$  goes into the state,

$$|2\rangle \rightarrow |\gamma''\rangle = \frac{1}{\sqrt{2}}(i|1\rangle + |2\rangle). \quad (33)$$

Substituting (31) and (33) into (32), we see that the state of the photon after the final beam splitter is

$$|\gamma'''\rangle = -|1\rangle, \quad (34)$$

which is purely right-moving. Due to the interference effects the  $|2\rangle$  component coming from the two terms of the original split-photon state (31), has been canceled. This is one of the beautiful features of the Elitzur–Vaidman experiment. The interference effect, characteristic of the wave aspect of quantum particles, is here seen in a single photon event. Typically, the interference effects in QM, instead, manifest after many identical experiments are repeated, such as in [25]. Only the detector  $D_1$  is triggered (the photon detected) by  $|\gamma'''\rangle$ , accordingly.

In the case it is a real one, the bomb is a detector inserted in the lower horizontal section. It is a quantum-measurement device to measure the state of the photon in  $|\gamma\rangle$ , (31). It is, however, a biased detector, capable of registering only the photon traveling in the lower path,  $|1\rangle$ . It is thus completely analogous to the upper-half detectors in the Renninger set-up, (28), in which the second, lower-hemisphere detectors are set at a large distance. There are two possible outcomes for each incident photon: either detection (explosion), or

non-detection (no explosion). In the first case, the photon simply gets lost, and neither the detector  $D_1$  nor  $D_2$  will register the photon.

In the second case—a sort of null measurement—the wave function (31) is reduced as

$$|\gamma\rangle \rightarrow |2\rangle, \quad (35)$$

in each such event. The photon is then reflected by the mirror at the upper-left corner ( $|2\rangle \rightarrow i|1\rangle$ ), and arriving at the final beam splitter, is transformed again as in (31). It is detected by detector  $D_1$  half of the times, and by  $D_2$  the other half of times, if the experiment is repeated.

In conclusion, detection of the photon by detector  $D_2$  implies that the bomb is real. The interesting point is that we know that this is so, but we know also that it has not exploded, i.e., the photon has not interacted with the bomb.

Even though the phenomenon might look quite remarkable, and is certainly not expected in classical physics, everything follows from the standard QM laws. If any, as emphasized by Elitzur and Vaidman themselves [8], this process is interesting as a particular, peculiar manifestation of quantum nonlocality. The situation here might look rather different from the more familiar examples of quantum nonlocality associated with entangled pairs of photons, electrons, etc. Actually, quantum nonlocality manifests itself whenever a microscopic system is in a pure quantum state with wave function having spatial support of a macroscopic extension ((31) here). As noted in [5], quantum nonlocality is due to the absence in QM of any fundamental constant with the dimension of a length.

### 3.3. Modified Stern–Gerlach Set-Up

The process (35)—the negative result event—can also be regarded as a particular realization of the so-called repeatable experiment. A repeatable measurement is an exceptional class of experiments in which the microscopic system under study remains factorized (and intact) after the measurement, i.e., as

$$\left( \sum_n c_n |n\rangle \right) \otimes |\Phi_0\rangle \longrightarrow \sum_n c_n |n\rangle \otimes |\Phi_n\rangle, \quad (36)$$

or focusing on a single experiment with the result,  $F = f_m$ ,

$$\left( \sum_n c_n |n\rangle \right) \rightarrow |m\rangle. \quad (37)$$

A simple example of the repeatable measurement is a variation in the Stern–Gerlach (SG) experiment. In the standard SG set-up (Figure 2), an incoming beam of spin  $\frac{1}{2}$  atom (e.g.,  $A_g$ ), traveling in the  $\hat{x}$  direction, is sent into a region of the inhomogeneous magnetic field, with a gradient,

$$\partial B_z(z) / dz \neq 0. \quad (38)$$

The incident wave packet is divided into two,

$$|\psi\rangle = c_1 |\uparrow\rangle + c_2 |\downarrow\rangle, \quad |c_1|^2 + |c_2|^2 = 1, \quad (39)$$

with the spin-up wave packet  $|\uparrow\rangle$  deflected upwards and the spin down component  $|\downarrow\rangle$  downwards, as the atom proceeds towards the  $\hat{x}$  direction. On the screen, they leave the two groups of blots whose intensities (the numbers of atoms) are proportional to  $|c_1|^2 : |c_2|^2$  after many atoms have been registered.

Though the Stern–Gerlach process is discussed in every textbook on quantum mechanics, there is some subtlety which is sometimes overlooked due to the fact that the magnetic field satisfies Maxwell's equations  $\nabla \cdot \mathbf{B} = 0$ ,  $\nabla \times \mathbf{B} = 0$ . The (apparent) puzzle is why, in spite of the fact that the condition  $\nabla \cdot \mathbf{B} = 0$  implies that the inhomogeneity (38) means an

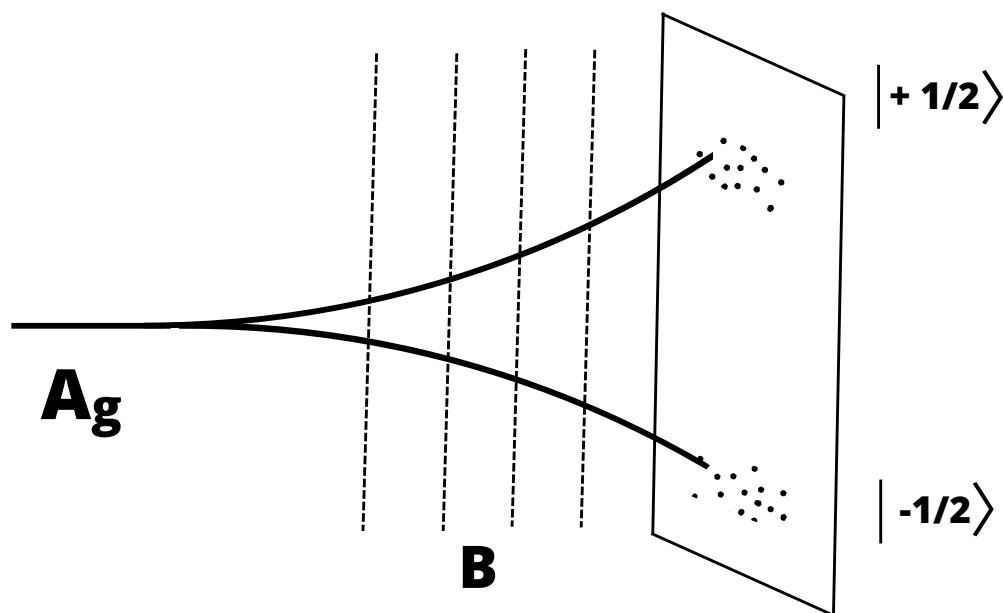
inhomogeneity  $\partial B_y / \partial y$  of the same magnitude (if  $B_x = 0$ ), the net effect is the deflection of the atom towards the  $\pm \hat{z}$  direction only. The explanation (the rapid spin precession and the cancellation of the forces in the  $x - y$  directions) with the discussion on the characteristics of the appropriate magnetic fields, has been given in [26,27].

In a possible variation in the Stern–Gerlach set-up (Figure 3), a detector  $D$  is inserted in the region where the lower wave packet passes [23]. The screen behind the region of the inhomogeneous magnetic field is eliminated.  $D$  is analogous to the set of detectors in the upper hemisphere in Renninger’s set-up: it is a biased detector, capable of capturing and recording only the atoms in the spin-down state. For each single incident atom,  $D$  either registers it (yes) or does not (no). In the first case, the spin has been measured to be in the state  $s_z = -\frac{1}{2}$ , but the atom itself is lost in the complicated atom-detector interactions.

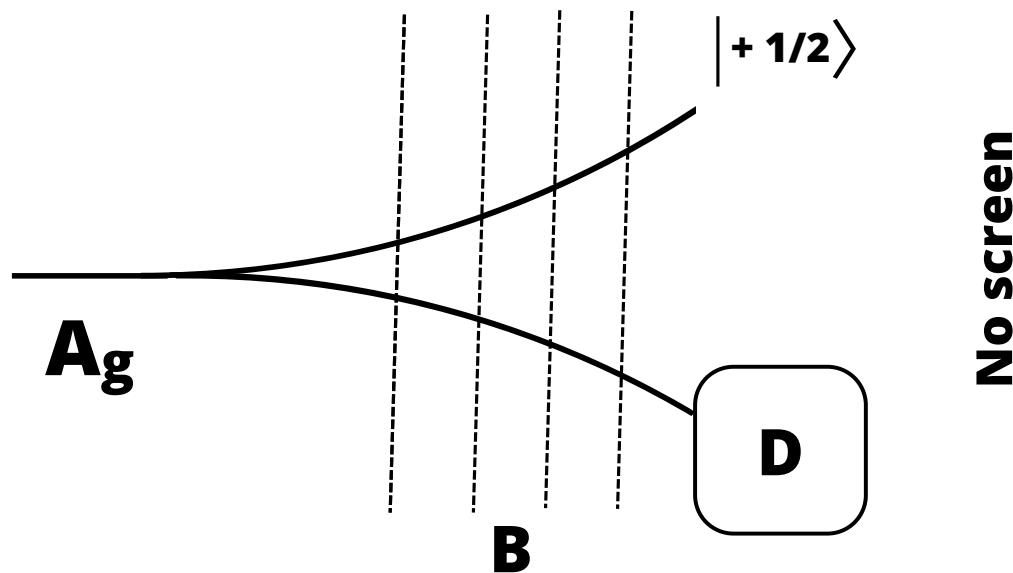
In the negative answer case (null measurement), no atom-detector interactions have taken place; nonetheless, its spin state is determined to be  $s_z = +\frac{1}{2}$ . The atom is in the pure  $|\uparrow\rangle$  state, and it can be used as the initial condition for subsequent analyses, for instance, with another SG set-up with the magnets oriented in another direction, etc.

The whole discussion can be readily generalized to the case of atoms with spin  $1, \frac{3}{2}$ , etc. by appropriately enlarging the set of detectors, so as to extract and prepare the state of any chosen spin state  $|s_z = m\rangle$  through interaction-free, null measurements.

The modified SG set-up we considered in this section can thus be seen as a simple, prototype version of Renninger’s negative result experiment [23], as an example of the repeatable measurement, or as a typical “state preparation” process, illustrating well the fact that these three concepts are closely related to each other. We have already seen a similar connection also in Section 3.1.1.



**Figure 2.** The standard SG set-up.



**Figure 3.** The modified SG set-up.

#### 4. Reflections

It is essential, in all negative result experiments discussed in Section 3, that a very weak flux of the incident particles is used, such that processes with a *single incident*  $\alpha$  particle, a photon, or an atom, are studied. Also, the experimental control must be good enough so that the expected time of arrival of each particle at the (biased) detector is known with reasonable precision. The reason is that, if it were not so, the non-observation of a certain event would not lead to any useful conclusion, as, e.g., the particle may not yet have arrived, or has already passed, or is unknown when it will arrive, and so on.

*In other words, an ideal null measurement is a spacetime local event, albeit a virtual (i.e., missed) one. In the Renninger experiment, even though the spontaneous  $\alpha$  emission is a spacetime local event [5], the exact instant  $\alpha$  is emitted cannot be predicted, being a manifestation of quantum fluctuations. This is the reason why one must construct the argument by considering a lap of time (e.g.,  $t \leq 30\tau$ ), to make sure that the nucleus has decayed and the  $\alpha$  emission has taken place—with certainty,  $1 - O(e^{-30})$ . It represents the other side of the same medal of the standard quantum-measurement processes, each of which is a local spacetime event at its core [5]. This latter fact is the origin of the apparent “wave-function collapse”, as reviewed in Section 2.*

Another important reflection is that the discussions of Section 3 illustrate nicely the well-known fact about QM, i.e., that the wave-like behavior (the superposition, quantum nonlocality, and interference) is the property of each single particle (the  $\alpha$  particle, the photon and the atom discussed here, or the electron in Tonomura’s double-slit experiment [25]), and not due to a collective motion of, or correlation among, the particles in the beam.

The catchphrase “wave-particle duality” was used historically to describe the apparently schizophrenic behavior of the electrons, photons, and atoms. In hindsight, though, this familiar expression left space for ambiguity and misunderstanding. For instance, it is an entirely different story that a large number of identical bosons form collective wavelike motions, such as the classical electromagnetic waves (light), or Bose–Einstein-condensed cold atoms, which are described in terms of a macroscopic wave function, as are all macroscopic quantum phenomena such as superconductivity and superfluidity, quantum Hall effects, and so on.

The wave-particle duality, a core idea of QM, is the property of each single quantum particle. Indeed, a less poetic but more precise expression would be “quantum fluctuations of a *particle* described by the wave function.” The words “particle” and “wave” do not have the symmetric logical roles.

A last consideration: In this work, we took it for granted that the  $\alpha$  particle, the photon, and the silver atom, are all quantum particles. But what if a large molecule such as  $C_{70}$  is used instead? Is it still a quantum particle? See [16,28] for some developments in our understanding of these questions.

The quest to grasp the very essential factors which can tell quantum-mechanical particles (the elementary particles, atomic nuclei, atoms, small molecules, etc.) from classical ones (the center of mass of isolated macroscopic bodies at finite body temperatures) has led us recently to the concept of the Quantum Ratio [9,29].

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## References

1. Joos, E.; Zeh, H.D.; Kiefer, C.; Giulini, D.J.; Kupsch, J.; Stamatescu, I.O. *Decoherence and The Appearance of a Classical World in Quantum Theory*; Springer: Berlin/Heidelberg, Germany, 2002.
2. Wheeler, J.A.; Zurek, W.H. *Quantum Theory and Measurement*; Princeton University Press: Princeton, NJ, USA, 1983.
3. Bell, J.S. *Speakable and Unspeakable in Quantum Mechanics*; Cambridge University Press: Cambridge, UK, 1987.
4. Peres, A. *Quantum Theory: Concepts and Methods*; Kluwer: Dordrecht, The Netherlands; Boston, MA, USA; London, UK, 1995.
5. Konishi, K. Quantum fluctuations, particles and entanglement: A discussion towards the solution of the quantum measurement problems. *Int. J. Mod. Phys. A* **2022**, *37*, 2250113. [\[CrossRef\]](#)
6. Konishi, K. Quantum fluctuations, particles and entanglement: Solving the quantum measurement problems. *J. Phys. Conf. Ser.* **2023**, *2533*, 012009. [\[CrossRef\]](#)
7. Renninger, M. Messungen ohne Störung des Meßobjekts. *Z. FüR Phys.* **1960**, *158*, 417. [\[CrossRef\]](#)
8. Elitzur, A.C.; Vaidman, L. Quantum mechanical interaction-free measurements. *Found. Phys.* **1993**, *23*, 987. [\[CrossRef\]](#)
9. Konishi, K. Newton's equations from quantum mechanics for a macroscopic body in the vacuum. *Int. J. Mod. Phys. A* **2023**, *38*, 2350080. [\[CrossRef\]](#)
10. Aguilar, L.A.; García, R.V. The single-photon steering and the quantum mechanical free-interaction measurement are identical phenomena. *arXiv* **2024**, arXiv:2407.01587.
11. Vaidman, L. Lying particles. *Front. Quantum Sci. Technol.* **2024**, *3*, 1362235. [\[CrossRef\]](#)
12. Joos, E.; Zeh, H.D. The emergence of classical properties through interaction with the environment. *Z. Phys. B* **1985**, *59*, 223. [\[CrossRef\]](#)
13. Zurek, W.H. Decoherence and the Transition from Quantum to Classical. *Phys. Today* **1991**, *44*, 36. [\[CrossRef\]](#)
14. Tegmark, M. Apparent wave function collapse caused by scattering. *Found. Phys. Lett.* **1993**, *6*, 571. [\[CrossRef\]](#)
15. Zurek, W.H. Decoherence, einselection, and the quantum origins of the classical. *Rev. Mod. Phys.* **2003**, *75*, 715. [\[CrossRef\]](#)
16. Hackermüller, L.; Hornberger, K.; Brezger, B.; Zeilinger, A.; Arndt, M. Decoherence of matter waves by thermal emission of radiation. *Nature* **2004**, *427*, 711. [\[CrossRef\]](#) [\[PubMed\]](#)
17. Clauser, J.F.; Li, S. Talbot-vonLau atom interferometry with cold slow potassium. *Phys. Rev. A* **1994**, *49*, R2213. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Hansen, K.; Campbell, E.E.B. Thermal radiation from small particles. *Phys. Rev. E* **1998**, *58*, 5477. [\[CrossRef\]](#)
19. Frauchiger, D.; Renner, R. Quantum theory cannot consistently describe the use of itself. *Nat. Commun.* **2018**, *9*, 3711. [\[CrossRef\]](#) [\[PubMed\]](#)
20. Daneri, A.; Loinger, A.; Prosperi, G.M. Quantum theory of measurement and ergodic conditions. *Nucl. Phys.* **1962**, *33*, 297. [\[CrossRef\]](#)
21. Rosenfeld, L. The Measuring Processes in Quantum Mechanics. *Prog. Theor. Phys. Suppl.* **1965**, *65*, 222.
22. Jauch, J.M.; Wigner, E.P.; Yanase, M.M. Some Comments Concerning Measurements in Quantum Mechanics. *Il Nuovo C. B* **1967**, *48*, 144. [\[CrossRef\]](#)
23. Loinger, A. Comments on a recent paper concerning the quantum theory of measurement. *Nucl. Phys. A* **1968**, *108*, 245. [\[CrossRef\]](#)
24. Mott, N.F. The wave mechanics of  $\alpha$ -decay tracks. *Proc. Roy. Soc. Lond.* **1929**, *A126*, 79.

25. Tonomura, A.; Endo, J.; Matsuda, T.; Kawasaki, T.; Ezawa, H. Demonstration of single-electron buildup of interference pattern. *Am. J. Phys.* **1989**, *57*, 117. [[CrossRef](#)]
26. Alstrøm, P.; Hjorth, P.; Mattuck, R. Paradox in the classical treatment of the Stern-Gerlach experiment. *Am. J. Phys.* **1982**, *50*, 697. [[CrossRef](#)]
27. Platt, D.E. A modern analysis of the Stern-Gerlach experiment. *Am. J. Phys.* **1992**, *60*, 306. [[CrossRef](#)]
28. Arndt, M.; Hornberger, K. Testing the limits of quantum mechanical superpositions. *Nat. Phys.* **2014**, *10*, 271–277. [[CrossRef](#)]
29. Konishi, K.; Elze, H.T. The Quantum Ratio. *Symmetry* **2024**, *16*, 427. [[CrossRef](#)]

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