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REGULAR PAPER

Inner structure of leptons, nature of dark matter, and non-Higgs origin of elementary particle masses

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Abstract There is a solid evidence that polarization analysis of gravitational waves detected by LIGO-Virgo interferometers (Svidzinsky and Hilborn in Eur Phys J Spec Top 230:1149, 2021. https://doi.org/10.1140/epjs/s11734-021-00080-6) rules out general relativity in favor of vector theory of gravity (VG). Motivated by this result, we study charged leptons in the framework of VG, modeling leptons as bound states of the spinning gravitational and electromagnetic fields. We find nonsingular bound state solutions corresponding to the electron and muon, and, with no free parameters, obtain for their mass values 3% smaller than experimental result. We show that the 3% difference is consistent with the QED self-energy correction not included in our analysis. This striking agreement with experiment indicates that VG gives correct microscopic description of leptons. It also indicates that lepton mass has the gravitoelectromagnetic origin, rather than generated by the Higgs mechanism. We show that bound states describing the tau lepton and W boson appear if we include weak interaction. VG yields small value of particle's mass on the Planck scale, because in VG, the spinning gravitational field can have negative energy density, which screens the large positive contribution to the mass from the electromagnetic field. Spin is what makes charged particles light. We also find a nonsingular bound state formed solely from the gravitational field, which is VG prediction for the dark matter particle. Moreover, we show that weak and Higgs boson fields naturally appear in VG as the fields restoring the gauge symmetry of gravity at low energy, and the emerging scalar particle has properties of the Higgs boson discovered in LHC. Finally, our theory predicts at least three new elementary particles heavier than 1 TeV in the electroweak sector.

Keywords Vector gravity · Structure of leptons · Origin of mass · Origin of charge · Dark matter · Higgs boson

Contents

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1 Introduction

The Standard Model of particle physics treats elementary particles as excitations of quantum fields which interact with each other locally. In particular, electrons are thought of as the point-like quanta of an electron field. This description combined with postulates of symmetry (in the context of the Standard Model, Lorentz, and gauge invariance) turns out to be amazingly powerful, and accurately accounts for many physical observations.

Since gravity is excluded, the Standard Model does not explain the inner structure of charged particles (leptons, quarks and W bosons) and treat them as elementary. Namely, it approximates these particles as point-like, structureless objects characterized by charge, spin, flavor, mass, etc., and introduces free parameters (masses, coupling constants, etc.) that are not predicted by the theory. The assumption of point-like particles leads to divergences in calculated quantities, in particular, to a divergent self-energy (mass). To treat these infinities, renormalization methods were introduced into the quantum field theory to compensate for the effects of self-interactions.

This effective phenomenological description of particles is expected to work well at relatively "large" scales when particle inner structure is not important. For example, in the case of the electron, gravity substantially modifies the spacetime metric within distances of order 10^{-25}m [see Eq. (36)]. At larger distances, gravity can be neglected, and the electron behaves as a point-like constituent of matter possessing an electric charge e, weak charge e_w, magnetic moment μ, spin $S = \hbar/2$, and mass m_e.

A search for a contact interaction at the large electron–positron collider at CERN probes for electron structure at the 10 TeV energy scale and yields an upper bound on the electron size of 2×10^{-20} m [1]. Measurement of the electron $g-$factor and comparing it with the precise QED calculations (assuming absence of the electron substructure g_{QED}) can also set a limit on the electron size r_e [2,3]

$$r_e \approx \frac{\hbar}{c m_e} \left| g - g_{\mathrm{QED}} \right| .$$

If, however, the electron is composed of constituent particles bound together by some unknown attraction, then we would expect that the point-like electron model would not accurately predict the measured magnetic moment. Such an estimate yields $r_e < 6 \times 10^{-24}$ m [1].

Until now, the electron mass m_e was a free parameter in the Standard Model, whose value is determined from experiment. In electroweak theory, the Higgs boson generates the mass of the electron (as well as masses of muon and tau lepton) through Yukawa interactions between the fermions and the Higgs field. Without Higgs field, the Standard Model predicts that all elementary particles are massless due to gauge symmetry of the electroweak and strong interaction originating from the charge conservation.

However, a realistic model of electron must include the fact that electric and magnetic fields produced by the point-like electron contribute to its mass. Such contribution must be calculated in the framework of the theory of gravity. In general relativity (GR), for a static spacetime with metric $g_{\mu\nu}$ the particle's mass is given by the Komar mass [4]

$$Mc^2 = \int dV\,(2T_{00} - g_{00}T)\,, \tag{1}$$

where the integration is over the spatial volume, $T_{\mu\nu}$ is the energy-momentum tensor of the electron's electromagnetic field, and T is its trace. Since current experiments give only the upper bound on the electron size r_e, we estimate the contribution to the electron mass from the region $r > r_e$. For $r \geq 6 \times 10^{-24}$ m, the effect of gravity is negligible and the right-hand side of Eq. (1) can be taken in Minkowski metric. As a result, we obtain for the contribution to the electron mass due to the energy of the electric and magnetic fields

$$M_E = \frac{e^2}{2\pi\varepsilon_0 c^2 r_e}, \quad M_B = \frac{\mu_0\mu^2}{3\pi c^2 r_e^3}, \tag{2}$$

respectively. For $r_e = 6 \times 10^{-24}$ m, Eqs. (2) yield $M_E = 8.6 \times 10^{-22}$ kg and $M_B = 0.6$ kg.

In GR, the contribution to the mass due to spin is also positive. Thus, in the framework of GR, there is no mechanism to compensate the enormous contribution to m_e from the electromagnetic field. Therefore, GR predicts that $m_e \gtrsim 1$ kg which is 10^{30} times greater than the experimental value.

GR prediction for the neutrino mass is also way off. For example, in GR, the minimum value of the gravitational mass of a point particle with an angular momentum S is

$$M_{spin} = \sqrt{\frac{Sc}{G}}, \tag{3}$$

which for $S = \hbar/2$ yields $M_{spin} = m_{Pl}/\sqrt{2}$, where $m_{Pl} = 2.18 \times 10^{-8}$ kg is the Planck mass. This is 10^{28} times greater than the experimental bound on the electron neutrino mass, $m_{\nu_e} < 1.4 \times 10^{-36}$ kg [5].

GR has emerged as a somewhat successful model of large-scale gravitation and cosmology. However, GR is unable to describe phenomena on microscopic scales, and is not part of the Standard Model. A correct theory of gravity must properly describe observations at all scales, and explain the structure of charged "elementary" particles and their masses.

The arguments mentioned above give us a hint on what properties the correct theory of gravity must possess to explain the observed small value of the particle masses. Namely, in the framework of such a theory, the gravitational field generated by the particle's spin should compensate the large positive contribution to the mass from the electromagnetic field of a charged particle. Recall that all elementary particles in the Standard Model have spin, apart from the Higgs boson, where the latter carries no charge.

To figure out what a possible alternative to GR could be we ought to re-examine the construction of general relativity. GR is an extension of the special theory of relativity, which postulates that in the absence of gravity, the geometry of our universe is the four-dimensional Minkowski spacetime. A point particle with a rest mass m, freely moving in such spacetime with velocity $\mathbf{V}$, is described by the action

$$S_m = -mc\int\sqrt{\eta_{ik}dx^i dx^k} = -mc^2\int dt\sqrt{1 - \frac{V^2}{c^2}}, \tag{4}$$

where $\eta_{ik} = \text{diag}(1, -1, -1, -1)$ is Minkowski metric.

In 1907 A. Einstein noticed a physical equivalence of a gravitational field and the corresponding acceleration of the reference system [6]. Namely, the gravitational force experienced locally is the same as the force experienced by an observer in a non-inertial (accelerated) frame of reference. Nowadays this is known as Einstein's equivalence principle. Mathematically, the principle implies that in a gravitational field, particles move along geodesics of some metric tensor g_{ik}. That is, the action for a point particle moving in the gravitational field is given by Eq. (4), in which Minkowski metric η_{ik} is replaced with g_{ik}

$$S_m = -mc \int \sqrt{g_{ik} dx^i dx^k}. \tag{5}$$

Since special relativity and Einstein's equivalence principle are very well tested experimentally, we do not question their validity.

To complete formulation of GR, we need to make another assumption; namely, assume that spacetime metric g_{ik} (spacetime geometry) is a dynamical gravitational field. Mathematically, this means that all 10 independent components of the symmetric tensor g_{ik} are independent fields. The gravitational field action S_{gravity}, and the Einstein field equations are constructed in a unique way from these postulates, following the self-consistency requirement that S_{gravity} and S_m must possess the same symmetries.

One should note that the assumption about g_{ik} being the dynamical gravitational field does not follow from Einstein's equivalence principle. There is another possibility. Namely, universe has a fixed background geometry, and g_{ik} in Eq. (5) is an equivalent metric through which the gravitational field is coupled to matter. Symmetry arguments suggest that the background geometry of the universe is completely isotropic 4-dimensional Euclidean space with the fixed metric

$$\delta_{ik} = \text{diag}(1, 1, 1, 1), \tag{6}$$

and the gravitational field is a 4-dimensional vector field A_k ($k = 0, 1, 2, 3$), which lives in the Euclidean space and breaks the Euclidean symmetry of the background space. The direction of A_k is preferred, and this direction becomes the time coordinate. Directions perpendicular to A_k are the three spatial coordinates. This assumption is appealing, because it removes the obstacle of theory nonrenormalizability. Recall that GR is nonrenormalizable, because the background spacetime is not fixed, but rather a dynamical field.

The equivalent metric can be uniquely obtained in terms of δ_{ik} and A_k using the equivalence principle which yields (see Ref. [7])

$$g_{ik} = -e^{-2\phi}\delta_{ik} + 2\cosh(2\phi)u_i u_k, \tag{7}$$

where for convenience we introduced a scalar ϕ and a 4-dimensional unit vector u_k ($\delta^{ik}u_i u_k = 1$) according to

$$A = e^{2\phi}, \quad u_k = \frac{A_k}{A},$$

where A is the norm of A_k ($A^2 = \delta^{ik}A_i A_k$). Thus, g_{ik} is a functional of the vector gravitational field, which effectively alters the geometry of the Universe. Plugging Eq. (7) into Eq. (5) gives the action for a particle moving in the four-vector gravitational field.

As in the case of GR, the field action S_{gravity} can be constructed in a unique way using the self-consistency requirement that S_{gravity} and S_m must possess the same symmetries. This procedure yields a unique alternative theory of gravity that has been developed in Refs. [7,8], which is known as vector gravity (VG). We want to emphasize that the theory developed in Refs. [7,8] is the only theory of VG that can be constructed following the "symmetry protocol", and failure to follow this protocol does not yield a viable theory.

According to Will's classification, VG is a Lagrangian-based metric theory of gravity with fixed background geometry [9]. In VG, the action for the vector gravitational field in the background Euclidean space reads [7]

$$S_{\text{gravity}} = \frac{c^3}{8\pi G}\int d^4x\left[\frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^k}\left(-\delta^{ik}+\left(1-3e^{-4\phi}\right)u^i u^k\right)\right.$$
$$\left.+\cosh^2(2\phi)\frac{\partial u_i}{\partial x^k}\frac{\partial u_m}{\partial x^l}\left(\delta^{im}\delta^{kl}-\delta^{il}\delta^{km}-\left(1+e^{-4\phi}\right)\delta^{im}u^k u^l\right)+2\left(1+e^{-4\phi}\right)\frac{\partial\phi}{\partial x^i}\frac{\partial u_m}{\partial x^k}\delta^{im}u^k\right], \tag{8}$$

where G is gravitational constant. The action (8) is written in the background metric δ_{ik}, which means that raising and lowering of indexes are carried out using δ_{ik}.

Variation of the total action $S_{\text{gravity}}+ S_m$ with respect to ϕ and the unit vector u_k gives equations for the gravitational field (see Appendix A) [7]. Plugging the solution of these equations (ϕ and u_k) in Eq. (7) yields the equivalent metric. The motion of particles in the vector gravitational field is described by the same equations as in GR, in which metric tensor g_{ik} is replaced with the equivalent metric (7) (see Ref. [7] for details). Similarly, in terms of the equivalent metric, Maxwell's equations have the same form as in GR.

1.1 Past successes of vector gravity

Despite fundamental differences, VG also passes all available gravitational tests, as shown in Ref. [7] with necessary details. For example, in the post-Newtonian limit, GR and VG are equivalent [7]. The post-Newtonian limit is sufficiently accurate to encompass all solar-system tests of gravity performed so far; and, hence, VG also passes all those tests. In VG and GR, the power of gravitational wave emission by binary systems is given by the same quadrupole formula, and gravitational waveforms produced by the merger of two compact objects fit the LIGO and Virgo data with a similar accuracy [7].

Moreover, VG provides an explanation of dark energy as the energy of the longitudinal gravitational field induced by the universe expansion. In VG, such induced gravitational field has negative energy density and produces apparent acceleration of the universe expansion. With no free parameters, VG yields the value of $\Omega_\Lambda = 2/3$ [7,8]. The VG prediction agrees with the results of Planck collaboration [10], and with the results of the Dark Energy Survey within a small experimental uncertainty $\Omega_\Lambda = 0.686 \pm 0.02$. Thus, VG solves the dark energy problem.

In strong fields, VG substantially deviates from GR and yields no singularities such as black holes. In particular, the end point of a gravitational collapse is a nonsingular "point-like" compact object, see Sect. 3.1. Properties of supermassive objects at galactic centers can be explained well in the framework of VG, including their images by the Event Horizon Telescope (EHT) see Sect. 9.

Moreover, it has been found that the data on gravitational wave (GW) detection of binary neutron star event GW170817—the only event so far whose source location is determined precisely by concurrent electromagnetic observations, are inconsistent with the tensor GW polarization predictions of GR and Einstein's theory is ruled out at 99% confidence level [12]. At the same time, vector GW polarization, predicted by the vector theory of gravity, is supported by the GW detection data [12]. Future GW detections for which the source is identified are expected to reinforce this conclusion with greater accuracy.

1.2 Findings of the present paper

Inspired by the success of VG in describing the large-scale gravitation and cosmology, here we study elementary particles in the framework of VG. In particular, we explore the inner structure of leptons. We find that electron and muon are different bound states of the spinning gravitational and electromagnetic fields. In contrast to GR, in VG, such bound states have no singularities, and yield a finite value for the particle mass.

Moreover, we show that the total energy of the gravitational and electromagnetic fields of such objects agrees with the experimental values of the electron and muon masses up to 3% QED correction; the latter is not included

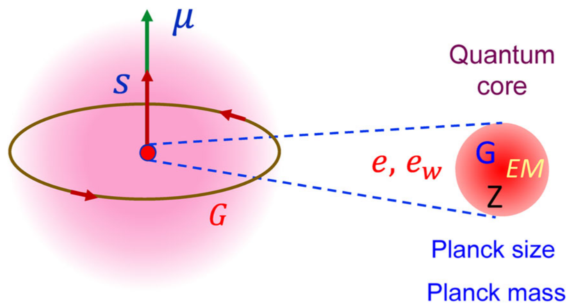

Fig. 1 Structure of charged leptons from the Vector Gravity (VG) perspective. The particle center contains a quantum core of Planck size and Planck mass, which is the lowest energy bound state of electromagnetic, weak (Z), and gravitational (G) fields. At distance greater than Planck length the core behaves as a point electroweak charge. In VG, the spinning gravitational field can have negative energy. To decrease the particle energy (mass), the field around the core spins, which reduces the mass from the Planck scale to the orders of magnitude smaller value of elementary particle masses we observe in experiment. In the presence of the electric field, the spinning gravitational field induces a magnetic moment due to the field dragging effect. For the given electroweak charge and spin, a spinning gravitational field can be attached to the core in different bound state configurations, yielding different generations of charged leptons, W boson, and much heavier particles not yet discovered

in our classical analysis. In VG, the gravitational field induced by the electron spin has negative energy, which compensates the large positive contribution to the mass from the EM field. Thus, lightness of elementary particles and dark energy of the universe have the same physical origin. For the latter, the gravitational field induced by the universe expansion has negative energy, which yields an apparent acceleration of the universe expansion [7,8]. Similarly, the negative energy of the spinning gravitational field reduces the mass of elementary particles from the Planck scale to the orders of magnitude smaller values we observe in experiments.

We also show that if the weak interaction is added to the EM field, there are additional bound states corresponding to the tau lepton and W boson. VG suggests an appealing picture of the structure of charged elementary particles. Namely, we find that particles consist of a point-like quantum core of Planck size and mass—the elementary charge—which produces the corresponding gauge boson field surrounding the charge (see Fig. 1). The latter has positive energy. Since, in VG, the spinning gravitational field can have negative energy, it is energetically favorable for the particle to spin and be in a state with a "large" angular momentum.

The lowest energy bound states correspond to the first generation of elementary fermions, which are stable particles. We find that for a given charge and spin, there are several bound states with increasing energy (mass). The excited bound states describe the next generations of elementary fermions and the W boson, that are unstable. Particles with no charge (photon, graviton, Higgs, and Z bosons) are fields that do not behave as point-like objects.

We also show that the present "microscopic" model of elementary particles yields the same relation between the particle's magnetic moment μ, electric charge q and spin $\mathbf{S}$ as the field theory approach of the Standard Model (if we disregard small radiative corrections)

$$\mu = -\frac{q}{m}\mathbf{S}. \tag{9}$$

This is remarkable, because in our approach, Eq. (9) is obtained using the theory of gravity, while the field theory derivation of the Standard Model does not include gravity at all—but rather based on symmetries. This example demonstrates that the "microscopic" description of charged particles as point-like constituent of matter and the description in terms of fields are complimentary to each other; and that the particle description can be a guidance for the field description, and vice versa.

Also, the present "microscopic" model explains the origin of lepton's magnetic moment. Namely, magnetic moment is not produced by electric current, but rather appears because of "dragging" the electric field by the spinning gravitational field [see Eq. (62)].

Modification of Coulomb's law by gravity is another prediction of the present model. Namely, electric field produced by a charge q with *no spin* is given by

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{r},$$

where a is the charge gravitational radius (see Eq. (21)). That is, gravity eliminates the divergence of the electric field at the origin and yields a finite mass of the *spinless* charge

$$M = \frac{q^2}{4\pi c^2 \varepsilon_0 a} = \frac{|q|}{\sqrt{4\pi\varepsilon_0 G}}.$$

The gravitational attraction force between two such point charges is precisely equal to the Coulomb force. That is, contrary to the conventional wisdom, the force of gravity is as strong as the Coulomb force.

VG also explains the origin of elementary charges (see Sect. 7). For example, the electric charge is a nonsingular bound state of the free electric and gravitational fields having Planck mass and confined to a Planck-size region. The balance between the quantum pressure and the gravitational self-attraction of the fields determines the magnitude of elementary charges. Because quantum uncertainty in the charge position is about Planck length, and lepton charges are small (small value of the fine structure constant), for modeling the lepton structure, such a bound state can be approximated as a classical point charge when viewed from a distance greater than the Planck length.

As a result, outside the Planck volume, one can accurately model the field configuration within the context of the classical field theory. We adopt this approach in our model of the lepton structure, and investigate possible nonsingular configurations of classical fields that can be attached to a point-like classical charge in the framework of VG. We find that for the field configurations with large spin, the negative energy of the spinning gravitational field compensates the big mass of the charge, and yields an exponentially small value for the total lepton mass in the Planck scale.

Finally, our results indicate that elementary particle masses are not generated by the Higgs mechanism. The latter mechanism was introduced into the Standard model to spontaneously break the gauge symmetry of the electroweak and strong interactions to obtain massive particles. However, if we include gravity, the gauge symmetry is inevitably broken, and particle masses emerge in a natural way.

In Sect. 8, we show that the weak interaction massive Z and Higgs fields naturally appear in VG as the fields restoring the gauge symmetry of gravity at low energy; and thus, emerging scalar particle indeed has properties of the Higgs boson discovered in LHC. Namely, VG predicts that the Higgs boson is coupled to the rest mass. The Z boson field can form a bound state together with EM and gravitational fields, yielding charged leptons and W boson.

According to our findings, the following picture of elementary particles emerges in the electroweak sector. The fundamental building blocks are four fields that carry no charge: Gravitational, EM, Weak (Z), and Higgs fields. The first three are real vector fields, while the last one is a real scalar field. The mass of the Z and Higgs fields is generated by the Stueckelberg mechanism (see Sect. 8), while the gravitational and EM fields are massless. Charged particles, which are considered as elementary in the Standard Model (leptons and W boson), are actually not elementary—they are bound states formed from the four fundamental fields.

Finally, we show in Sect. 10 that in the electroweak sector, VG predicts a fourth generation of leptons, and an additional charged vector boson—each with mass greater than TeV.

One should mention that the interpretation of bound states of classical fields as particles within the context of quantum field theory has been suggested by Skyrme in 1958 [13], and since then has been discussed in many publications (for a review, see Ref. [14] and citations to it). It has been shown that such classical solutions can be fitted into the quantum field theory provided that we allow the bound states to be part of the Hilbert space. These solutions signal the presence of particle states which cannot be seen in ordinary perturbation theory. It was shown that the full self-consistent quantum theory can be obtained if the field operator is expanded about the classical solution and quantum corrections are found in a series expansion [15, 16]. These nonperturbative calculations have put into evidence novel effects, such as emergence of fermion particles from Bose fields; and they have provided new mechanisms for spontaneous symmetry breaking without Goldstone bosons [14].

Finally, in Sect. 9, we show that there exist bound states formed solely from the gravitational field. The corresponding particle does not carry electroweak or color charges and couples only to the mass through gravity. Thus, such a particle very weakly interacts with the ordinary matter, and is the VG prediction for the dark matter in the universe.

2 Why general relativity is successful in describing many experiments

GR has passed many gravitational tests with flying colors. However, the message we are trying to convey in this paper is that GR is not the theory of gravity that describes nature. Instead, we argue that VG is the way to go until evidence against VG is found. We want to emphasize that the construction of VG based on symmetries gives only one theory of Vector gravity, which was developed in Ref. [7]; we adopt this theory in the present paper. Previous attempts to construct VG without using the "symmetry protocol" [17–20] were based on guessing the field action, and yielded no viable theory.

The reader might ask a reasonable question: Why is GR so successful in describing many experiments, including those in which gravitational field is not weak? A comparison of GR and VG shows that this happens, because GR mimics VG on those "successful" occasions, and both theories pass the corresponding gravity tests. For these "successful" tests, the experimental accuracy is not sufficient to distinguish between the two theories.

For example, as shown in Ref. [7], VG and GR are equivalent in the post-Newtonian limit and, thus, both theories pass the solar-system tests of gravity, such as gravitational redshift of light; deflection of light by the Sun; precession of the perihelion of Mercury; time delay of a radar signal traveling near the Sun; and the Lense–Thirring precession measured by Gravity Prob B. As shown in Appendix I of Ref. [7], VG (as well as GR) predicts no preferred frame nor preferred location effects—at least these effects are too small to be detected in available observations. Such effects are suppressed in VG due to the exponentially large universe expansion during the stage of cosmic inflation [7].

Moreover, in GR and VG, the power of gravitational waves emitted by binary systems is given by the same quadrupole formula, and predictions for radiation waveforms, produced by the merger of compact objects, are indistinguishable within the sensitivity limit of LIGO and Virgo interferometers (see Figs. 7 and 8 in Ref. [7]). This is the case, because the Schwarzschild spacetime of GR mimics the exponential metric (12) of VG even at distances of the order of the gravitational radius (see Fig. 6 in Ref. [7]). Thus, both theories can explain the measured radiation waveforms equally well.

According to VG, the gravitational wave events interpreted in GR as merger of black holes are produced by the merger of collapsed "point-like" objects with exponential geometry (12) which mimic black holes, see Sect. 3.1. Similarly, according to VG, supermassive compact objects at galactic centers are such collapsed objects with large mass. Current angular resolution of images of objects at the centers of the Messier 87 galaxy and the Milky Way, obtained by EHT, cannot capture the 4% difference in prediction of the image properties (see Sect. 9 and Fig. 7 for details). To notice the 4% difference between GR and VG predictions for the size of the accreting disk and the radius of the photon sphere, the EHT angular resolution must be increased by an order of magnitude.

In situations where GR and VG predictions are substantially different and can be noticed experimentally, GR fails to describe observations. For example, GR is

- incompatible with quantum mechanics
- predicts spacetime singularities
- does not explain dark energy
- does not explain elementary particles and origin of charges
- does not predict dark matter particle
- does not explain matter generation at Big Bang

- does not explain why universe is spatially flat.

These "unsuccessful" occasions are usually explained by arguing that GR is incomplete. For example, to eliminate singularities, various extensions of GR has been proposed—string theory, loop quantum gravity, etc. However, so far, there is no experimental evidence for the latter.

In contrast, VG is compatible with quantum mechanics, predicts no spacetime singularities, and answers the other questions listed above (see Refs. [7,8], and the present paper). In particular, a spatially flat universe is the only solution of VG equations in the cosmological model; while dark energy is simply the energy of the gravitational field induced by the expanding universe. The latter yields the value of the cosmological constant Λ in perfect agreement with observations without free parameters [7,8].

Moreover, there is strong evidence that gravitational wave detection of the binary neutron star event GW170817 (the only event so far for which GW polarization can be measured) supports the vector gravitational wave polarization predicted by VG, and rules out the tensor polarization predicted by GR [12].

In the present paper, we focus on the structure of charged "elementary" particles in VG using the gravitational field equations obtained in Refs. [7,8], and summarized here in Appendix A. It is remarkable that these equations yield—with no free parameters—the values for the particle masses (electron and muon) in agreement with experiment. This is a "smoking gun" in favor of VG.

3 Elementary particles with no spin in vector gravity

In the absence of spin, the gravitational field produced by a particle at rest is static. In this case, $u_k = (1, 0, 0, 0)$, and the equivalent metric (7) is diagonal and has an exponential form [7]

$$ds^2 = e^{2\phi} c^2 dt^2 - e^{-2\phi} \left(dx^2 + dy^2 + dz^2 \right), \tag{10}$$

where $c^2\phi$ has a meaning of the gravitational potential. For static field, the gravitational field equation [Eq. (A1)] reduces to

$$\nabla^2 \phi = \frac{8\pi G}{c^4} \left(T^{00} - \frac{T}{2} g^{00} \right). \tag{11}$$

In Eq. (11), $T_{\mu\nu}$ is the energy-momentum tensor of matter, T is the trace of the energy-momentum tensor $T = T_{\mu\nu} g^{\mu\nu}$, and $g^{\mu\nu}$ is the equivalent metric inverse to $g_{\mu\nu}$

$$g^{\mu\nu} = \text{diag} \left(e^{-2\phi}, -e^{2\phi}, -e^{2\phi}, -e^{2\phi} \right).$$

The metric (10) is invariant under the global transformation $\phi \to \phi + \phi_0$, $t \to e^{-\phi_0} t$, and $\mathbf{r} \to e^{\phi_0} \mathbf{r}$, where ϕ_0 is a constant. Thus, by rescaling coordinates, one can make ϕ to satisfy the condition $\phi \to 0$ at infinity (away from the field sources). This is similar to imposing the condition $\phi(\infty) = 0$ for the gravitational potential in Newtonian gravity.

Next we discuss gravitational field produced by particles with no spin.

3.1 Metric produced by point mass

In VG, the metric produced by a point mass m located at $\mathbf{r} = 0$ in spherical coordinates reads [7]

$$ds^2 = e^{-r_g/r} c^2 dt^2 - e^{r_g/r} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right), \tag{12}$$

which is known as the exponential metric. In Eq. (12) $r_g = 2Gm/c^2$ is the mass gravitational radius and r is the radial coordinate. Metric (12) has no event horizon and no singularities. Indeed, scalar invariants (Ricci, Weyl, and Kretschmann scalars) are finite everywhere in the exponential spacetime [21]. In particular, they decay to zero both as $r \to \infty$ and as $r \to 0$. E.g., the Ricci scalar is given by

$$R = \frac{r_g^2}{2r^4} e^{-r_g/r}.$$

It was pointed out in [21] that the metric (12) represents a traversable wormhole in the sense of Morris and Thorne [22,23]. The metric (12) appears in GR as a solution to the Einstein equations with an exotic matter, a massless scalar field with negative kinetic energy [24] and, therefore, is not acceptable. In contrast, in VG, the exponential metric describes a physical system—a static mass with positive energy.

In VG, metric (12) approximates the endpoint of gravitational collapse of matter and yields an exponentially large gravitational redshift at the central region. The size of such collapsed objects is much smaller than the gravitational radius and can be approximated as a δ-function source in the gravitational field equation (11). The exponentially large redshift z reduces the radiation power coming out from the interior by a factor of $(1+z)^2$, which mimics black holes. Such "point-like" massive objects can be formed by gravitational collapse of massive stars, dark matter clumps, direct collapse of gas clouds in early universe, etc.

In VG, from the perspective of a distant observer, surface of a collapsing dust sphere reaches the radial coordinate $r \ll r_g$ during an exponentially large time

$$t = \frac{r^2}{cr_g} e^{r_g/r}$$

Thus, the dust sphere continues collapsing "forever", and there is no rebounce off the center during the age of the universe.

An observer moving together with the sphere surface sees that sphere radius $R = re^{rg/2r}$ undergoes contraction until $r = r_g/2$. At smaller r, contraction swaps into exponential expansion of the sphere into "infinite" volume and negligible matter density. The latter occurs during finite proper time of the observer because surface velocity of the expanding sphere approaches the speed of light. Space in the vicinity of $r = 0$ is what is exponentially expanding during the gravitational collapse in VG, and the stellar material embedded within it moves apart in response to this expansion. Since space expands exponentially, an observer located at $r = 0$ sees formation of a horizon - a boundary beyond which light can not reach the observer. The latter is analogous to the cosmological horizon in the de Sitter universe for which the scale factor undergoes exponential expansion with time, and the physical distance between any points will eventually be growing faster than the speed of light.

Exponential expansion cools the stellar material to a very low temperature, similarly to the cooling of the cosmic microwave background radiation as the universe expands. Thus, from the perspective of a local observer, gravitational collapse leads to formation of a very large, cold and dilute stellar remnant emitting no radiation. This is analogous to the Big Freeze cosmological scenario as the end of the universe evolution. In contrast, in GR, the collapsing star shrinks to zero volume leading to formation of unphysical singularity (infinite matter density and infinite spacetime curvature).

Due to exponential expansion of space, gravitational collapse in VG generates "infinite" volume of essentially flat space in the vicinity of $r = 0$, separated from the exterior world by a region with large spacetime curvature at $r \sim r_g$. From the exterior region, the generated volume looks like a point. Thus, for a distant observer, the cold stellar remnant appears as a point-like dark object with an exponentially large gravitational redshift. Collapsing star expands into this self-generated "infinite" volume in the vicinity of $r = 0$, leaving behind a very cold cloud of gas and dust that produce no radiation, resembling black hole interior.

3.2 Electric charge as a bound state of static electric and gravitational fields

Electromagnetic field disturbances held together by gravitational forces were first discussed within the context of GR by Wheeler [25]. Wheeler named such bound states geons (*g* for "gravity", *e* for "electromagnetism", and *on* as the word root for "particle") [26]. Wheeler envisioned that the mass of such particles is pure energy holding itself together.

However, in GR, such bound states are singular and, hence, cannot exist in nature. Here, we show that in VG, the electromagnetic and gravitational fields do form nonsingular bound states—electric charges—and the mass of these objects is the total energy of the bound fields.

According to Eq. (11), energy of the electric field generates gravitational potential. In curved spacetime, the energy-momentum tensor for the electromagnetic field is given by by [27]

$$T_{\mu\nu} = \frac{1}{\mu_0}\left(-g^{\delta\gamma}F_{\mu\delta}F_{\nu\gamma} + \frac{1}{4}g_{\mu\nu}F_{\delta\gamma}F_{\varepsilon\lambda}g^{\delta\varepsilon}g^{\gamma\lambda}\right), \tag{13}$$

where

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}$$

is the electromagnetic field tensor. Introducing the electric field vector $E_\alpha = cF_{0\alpha}$ ($\alpha = x, y, z$), we obtain for the static electric field

$$T_{00} = \frac{1}{2}\varepsilon_0 e^{2\phi}E^2, \quad T^{00} = \frac{1}{2}\varepsilon_0 e^{-2\phi}E^2, \tag{14}$$

where $E^2 = E_x^2 + E_y^2 + E_z^2$, μ_0 is the vacuum permeability, and ε_0 is the vacuum permittivity. Taking into account that for the electromagnetic field the trace $T = 0$, Eq. (11) for the gravitational potential reduces to

$$\nabla^2\phi = \frac{4\pi G\varepsilon_0}{c^4}e^{-2\phi}E^2. \tag{15}$$

This equation should be solved together with the Maxwell's equations for the static electric field in the metric (10) [27]

$$\mathrm{div}\left(e^{-2\phi}\mathbf{E}\right) = 0, \quad \mathrm{curl}\mathbf{E} = 0, \tag{16}$$

where $\mathbf{E}$ is covariant vector E_α, while *div* and *curl* are defined as conventional operators in the three-dimensional space ($\mathrm{div}\mathbf{E} = \partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z$, etc.).

Please note that we did not include electric charge in Eqs. (16). The charge emerges as a nonsingular solution of the charge-free equations for $\mathbf{E}$ and the equivalent metric (see Sect. 7 for details).

For spherical symmetry, $\mathbf{E}$ has only the radial component $E_r(r)$ and, in spherical coordinates, Eqs. (15) and (16) reduce to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = \frac{4\pi G\varepsilon_0}{c^4}e^{-2\phi}E_r^2, \tag{17}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 e^{-2\phi}E_r\right) = 0. \tag{18}$$

Equation (18) yields

$$E_r = \frac{e^{2\phi} q}{4\pi \varepsilon_0 r^2}, \tag{19}$$

where q is an integration constant which has a meaning of the electric charge located at $\mathbf{r} = 0$. Plugging E_r into Eq. (17), we obtain equation for ϕ

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = \frac{e^{2\phi} a^2}{r^2}, \tag{20}$$

where

$$a = \sqrt{\frac{G}{4\pi \varepsilon_0} \frac{|q|}{c^2}} \tag{21}$$

is the charge gravitational radius. If q is equal to the electron charge $e = -1.602 \times 10^{-19}$C then $a = 1.381 \times 10^{-36}$m, which is a factor $\sqrt{\alpha} = 0.085$ smaller than the Planck length (here α is the fine structure constant).

Solution of Eq. (20) satisfying the proper boundary conditions reads

$$\phi(r) = -\ln\left(1 + \frac{a}{r}\right). \tag{22}$$

The corresponding electric field

$$E_r = \frac{q}{4\pi \varepsilon_0 (a + r)^2} \tag{23}$$

is finite everywhere, while the equivalent metric in spherical coordinates has the form

$$ds^2 = \frac{c^2 dt^2}{\left(1 + \frac{a}{r}\right)^2} - \left(1 + \frac{a}{r}\right)^2 \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right). \tag{24}$$

One should mention that for a point electric charge with no intrinsic mass, GR yields the Reissner–Nordström metric

$$ds^2 = \left(1 + \frac{a^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 + \frac{a^2}{r^2}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2, \tag{25}$$

which represents an unphysical naked singularity at $r = 0$ with Weyl and Kretschmann scalars diverging as $1/r^8$ [28,29]. In contrast, the VG equivalent metric (24) has no singularities. Namely, scalar invariants (Ricci, Weyl, and Kretschmann scalars) are finite everywhere in the spacetime (24). In particular, for the metric (24), the Ricci scalar is

$$R = 0,$$

while the nonzero Weyl scalar reads

$$\psi_2 = -\frac{ra}{(r + a)^4}.$$

For the metric (24), the area of the spherical surface of the constant r coordinate is

$$A(r) = 4\pi r^2 \left(1 + \frac{a}{r}\right)^2 = 4\pi\, (r + a)^2\,,$$

which monotonically decreases to the value $4\pi a^2$ as r goes to zero. Thus, the metric (24) does not represent a wormhole.

The gravitational mass M of the electric charge bound state is determined by the asymptote of the metric at $r \to \infty$. Equation (24) yields

$$M = \frac{ac^2}{G} = \frac{q^2}{4\pi c^2 \varepsilon_0 a} = \frac{|q|}{\sqrt{4\pi \varepsilon_0 G}}. \tag{26}$$

Thus, if two charges q are located at a distance $r \gg a$ from each other, the gravitational attraction force between charges

$$F_g = \frac{GM^2}{r^2} = \frac{q^2}{4\pi \varepsilon_0 r^2} \tag{27}$$

is equal to the force of Coulomb repulsion. That is, VG yields that force of gravity between electric charges with no spin is as strong as the Coulomb force.

If $q = e$, then $M = \sqrt{\alpha} m_{Pl}$, where $m_{Pl} = 2.18 \times 10^{-8}$ kg is the Planck mass. Thus, VG predicts that charged elementary particles with no spin should be very heavy and, hence, their generation requires lots of energy.

The present solution demonstrates the equivalence between the mass M and the energy W in the presence of static gravitational field. Namely, the mass of the particle is equal to the total energy of the electric and gravitational fields $W = Mc^2$. Indeed, the net energy density of the static fields reads [8]

$$w = \frac{c^4}{8\pi G}\, (\nabla\phi)^2 + \frac{1}{2}\varepsilon_0 e^{-2\phi} E^2.$$

Plug here Eqs. (22), (23), and (21) yields that the energy densities of the electric and gravitational fields are equal to each other, and the net energy density is

$$w = \frac{1}{16\pi^2 \varepsilon_0} \frac{q^2}{r^2\, (r + a)^2},$$

which in the limit $r \ll a$ reduces to an expression independent of q

$$w = \frac{c^4}{4\pi G r^2}.$$

The total energy of the particle is given by the integral over the spatial volume

$$W = 4\pi \int_0^\infty dr r^2 w = \frac{q^2}{4\pi \varepsilon_0 a}, \tag{28}$$

where we took into account that

$$\int_0^\infty \frac{dr}{(r + a)^2} = \frac{1}{a}.$$

Comparing Eqs. (26) and (28), we obtain $W = Mc^2$.

If the charge has a point intrinsic mass m at its center, the equation for the gravitational potential becomes

$$\nabla^2\phi = \frac{a^2 e^{2\phi}}{r^4} + \frac{4\pi G m}{c^2}\delta(\mathbf{r}), \tag{29}$$

which has a solution

$$\phi(r) = -\ln\left(\cosh\left(\frac{r_0}{2r}\right) + \sinh\left(\frac{r_0}{2r}\right)\sqrt{1+\frac{4a^2}{r_0^2}}\right), \tag{30}$$

where $r_0 = 2Gm/c^2$ is the gravitational radius of the intrinsic mass m. The corresponding equivalent metric (10) yields Ricci scalar

$$R = \frac{r_0^2}{2r^4\left(\cosh\left(\frac{r_0}{2r}\right) + \sinh\left(\frac{r_0}{2r}\right)\sqrt{1+\frac{4a^2}{r_0^2}}\right)^2}, \tag{31}$$

which is finite everywhere. The total gravitational mass of such a particle is

$$M = \frac{c^2}{G}\sqrt{a^2 + \frac{r_0^2}{4}}.$$

Equation (30) shows that for $r \ll r_0$, the spatial dependence of ϕ is governed by the point intrinsic mass m, namely, $\phi \approx -r_0/2r$.

3.3 Why are there no elementary dipoles and scalar field charges in nature?

As we show above, if we include gravity, EM field can exist not only as EM waves but also form monopole bound states—electric charges. These bound states are nonsingular and, thus, can occur in nature. Next, we explore the other possible bound state configurations of the EM field—a magnetic dipole and an electric dipole. We show that such bound states are singular and, therefore, can not exist in nature.

3.3.1 Absence of magnetic dipole bound state

Here, we calculate the gravitational field of a bound state configuration describing a static magnetic dipole with the magnetic moment μ located at $\mathbf{r} = 0$. Introducing $\mathbf{B} = \mathrm{curl}\mathbf{A}$ (please note that $A_k = (A_0, -\mathbf{A})$ [27]), Eq. (13) yields for the energy-momentum tensor of the static magnetic field in curved spacetime

$$T_{00} = \frac{1}{2\mu_0}e^{6\phi}B^2, \quad T^{00} = \frac{1}{2\mu_0}e^{2\phi}B^2,$$

where $B^2 = B_x^2 + B_y^2 + B_z^2$.

Taking into account that for the electromagnetic field the trace $T = 0$, the equation for the gravitational potential (11) reduces to

$$\nabla^2\phi = \frac{4\pi G}{\mu_0 c^4}e^{2\phi}B^2, \tag{32}$$

which should be solved together with the Maxwell's equations for the static magnetic field

$$\mathrm{curl}\left(e^{2\phi}\mathbf{B}\right)=0,\quad \mathrm{div}\mathbf{B}=0, \tag{33}$$

where $\mathbf{B}$ is a covariant vector B_α, while *div* and *curl* operators are defined in the conventional way (curl$\mathbf{B}$ = $\hat{e}_k\varepsilon^{klm}\partial B_m/\partial x^l$, etc.). Taking the magnetic moment μ to point along the polar axis of spherical coordinates r, θ, φ, Eqs. (32) and (33) have a solution in the form

$$\mathbf{B}=\frac{\mu\mu_0}{4\pi r^3}\left(2\hat{r}\cos\theta+\hat{\theta}\sin\theta\right),\qquad \phi=\phi\left(\frac{\cos\theta}{r^2}\right), \tag{34}$$

where the gravitational potential obeys equation

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)=\frac{b^4e^{2\phi}}{r^4}\left(1+3\cos^2\theta\right). \tag{35}$$

In this equation

$$b=\frac{1}{c}\left(\frac{G\mu_0\mu^2}{4\pi}\right)^{1/4} \tag{36}$$

is the magnetic dipole gravitational length. For electrons, $\mu = 9.285\times10^{-24}J/T$, and thus, we obtain $b = 5.17\times10^{-25}$m, which is 10 orders of magnitude grater than the Planck length $l_{pl}=1.616\cdot10^{-35}$m. Please note that Eq. (33) is satisfied provided ϕ depends on the combination $\cos\theta/r^2$ and $\mathbf{B}$ is given by Eq. (34).

Equation (35) has the following solution:

$$\phi(r,\theta)=-\ln\left(1+\frac{b^2\cos\theta}{r^2}\right), \tag{37}$$

and the corresponding equivalent metric reads

$$ds^2=\frac{c^2dt^2}{\left(1+\frac{b^2\cos\theta}{r^2}\right)^2}-\left(1+\frac{b^2\cos\theta}{r^2}\right)^2\left(dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\right). \tag{38}$$

For the metric (38), the Ricci scalar vanishes everywhere, however, the other scalar invariants are singular on the surface $r^2+b^2\cos\theta=0$. This implies that solution (38) is unphysical and, therefore, according to VG, elementary particles possessing only intrinsic magnetic moment can not exist in nature. This is indeed the case—all known elementary particles with magnetic moment also have a spin. In fact, as we show below, magnetic moment is generated by spin. It turns out that even a small value of the particle's spin eliminates singularity in the metric (38). We discuss this in Sect. 4.

3.3.2 Absence of electric dipole bound state

Next we calculate the gravitational field of a bound state configuration describing a static electric dipole with the electric dipole moment $\mathbf{d}$ located at $\mathbf{r}=0$. Taking $\mathbf{d}$ to point along the polar axis of spherical coordinates r, θ, φ, Eqs. (15) and (16) have a solution in the form

$$\mathbf{E}=\frac{d}{4\pi\varepsilon_0 r^3}e^{2\phi}\left(2\hat{r}\cos\theta+\hat{\theta}\sin\theta\right),\qquad \phi=\phi\left(\frac{\cos\theta}{r^2}\right), \tag{39}$$

where the gravitational potential obeys equation

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)=\frac{b_e^4 e^{2\phi}}{r^4}\left(1+3\cos^2\theta\right), \tag{40}$$

and

$$b_e=\frac{1}{c}\left(\frac{Gd^2}{4\pi\varepsilon_0}\right)^{1/4} \tag{41}$$

is the electric dipole gravitational length. Equation (40) has the following solution

$$\phi(r,\theta)=-\ln\left(1+\frac{b_e^2\cos\theta}{r^2}\right), \tag{42}$$

which, as in the case of the magnetic dipole, describes a singular spacetime.

3.3.3 Absence of scalar field charges

Here, we show that, in contrast to the vector EM field (or other vector fields, e.g., weak field), gravity cannot hold together a scalar field in a static configuration, which leads to lack of the scalar field charges. Indeed, in the presence of gravity, equation for a massive real scalar field ψ reads

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\psi}{\partial x^\nu}\right)+m^2\psi=0, \tag{43}$$

which in the static case and the exponential spacetime (10), reduces to

$$-\nabla^2\psi+e^{-2\phi}m^2\psi=0. \tag{44}$$

Equation (44) does not have bound state solutions. To show this, we multiply both sides of Eq. (44) by ψ, and integrate over the spatial volume. Taking into account that, for a bound state, ψ vanishes at infinity, and integrating the first term by parts, yields equation

$$\int d^3r\left[(\nabla\psi)^2+e^{-2\phi}m^2\psi^2\right]=0,$$

which has only trivial solution $\psi=0$. Thus, the scalar and gravitational fields do not form bound states.

This result explains absence of Higgs field charges in nature.

4 Particles with spin in vector gravity

In VG, the gravitational field produced by matter current can have negative energy density [7,8]. In particular, such negative energy gravitational field induced by the universe expansion accelerates the expansion of the universe, yielding the value of the cosmological constant in agreement with observations [7,8]. In this section, we study particles with nonzero spin in the framework of VG and show that the negative energy of the spinning gravitational field explains the large angular momentum of elementary particles and their small masses compared to the Planck scale. That is, the physics behind dark energy and the lightness of elementary particles is similar.

We model elementary particles as a point-like charge, which generates the corresponding gauge boson field. The latter has nonzero energy density and produces a gravitational field. As we show, to reduce the total mass, the gravitational field needs to spin, which in turn induces a magnetic-like moment of the particle due to field dragging.

To be specific, we consider charged leptons which consist of a point-like electric charge e that generates the static electric field. In this section, we show that spinning gravitational field can be "attached" to the electric charge, forming a spinning bound state. In the presence of the electric field, the spinning gravitational field induces the magnetic moment of the lepton. Negative energy of the spinning gravitational field compensates the positive electromagnetic contribution to the mass, yielding a light particle.

It turns out that for description of elementary particles with spin, one can make an approximation $u_k \approx (1, \mathbf{u})$, where $\mathbf{u}$ is a three-dimensional vector satisfying condition $u \ll 1$. Then, for the case of a stationary *transverse* ($\mathrm{div}\mathbf{u} = 0$) gravitational field, the equations of VG (A1) for ϕ and $\mathbf{u}$ read

$$\nabla^2\phi = -\sinh(4\phi)\,\mathrm{curl}^2\mathbf{u} + \left(1 + e^{-4\phi}\right)\mathrm{div}\,[(\mathbf{u}\cdot\nabla)\mathbf{u}] + \frac{8\pi G}{c^4}\left(T^{00} - \frac{e^{-2\phi}}{2}T + 2T^{0\alpha}u_\alpha\right), \tag{45}$$

$$\mathrm{curl}\left[\cosh^2(2\phi)\mathrm{curl}\mathbf{u}\right] = 2\left(1 + e^{-4\phi}\right)\left[(\mathbf{u}\cdot\nabla)\nabla\phi + \frac{2\pi G}{c^4}\mathbf{j}\right], \tag{46}$$

where the three-dimensional vector $\mathbf{u}$ is analogous to the vector potential in classical electrodynamics, and the three-dimensional current is given by the components of the energy-momentum tensor of matter

$$j^\alpha = T^{0\alpha} - T^{00}u^\alpha + T^{\alpha\beta}u_\beta.$$

In terms of the scalar ϕ and vector $\mathbf{u}$, the equivalent metric (7) for $u \ll 1$ reduces to

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}d\mathbf{r}^2 + 4c\cosh(2\phi)\mathbf{u}\cdot d\mathbf{r}dt. \tag{47}$$

For problems with cylindrical symmetry, the vector $\mathbf{u}$ in cylindrical coordinates ρ, φ, z can be written as $\mathbf{u} = u(\rho, z)\hat{\varphi}$, where $\hat{\varphi}$ is the azimuthal unit vector. In this case, the equations for the free gravitational field read

$$\nabla^2\phi = -\sinh(4\phi)\,\mathrm{curl}^2\mathbf{u} - \frac{1}{\rho}\left(1 + e^{-4\phi}\right)\frac{\partial u^2}{\partial\rho}, \tag{48}$$

$$\mathrm{curl}\left[\cosh^2(2\phi)\mathrm{curl}\mathbf{u}\right] = 2\left(1 + e^{-4\phi}\right)\frac{\partial\phi}{\partial\rho}\frac{\mathbf{u}}{\rho}. \tag{49}$$

Contrary to the exponential metric for the static field (10), the gravitational potential ϕ for the metric (47) has preferred values. As we showed in [8], according to VG, the transition between the Euclidean geometry of the equivalent metric and the geometry with Minkowski signature occurred at the moment of Big Bang. Before the Big Bang, the 4-vector gravitational field had no preferred direction, and was undergoing quantum fluctuations. The Big Bang is the point of quantum phase transition at which the gravitational field 4-vector acquires a nonzero expectation value on the "macroscopic" scales. This expectation value serves as a transition order parameter.

The Big Bang occurred at a point where $\phi = \ln(3)/4 \approx 0.27$ [8]. At such point, a small ordering of the vector gravitational field caused by fluctuations produces an average equivalent metric with the Minkowski character.

The subsequent universe expansion resulted in an exponentially large deviation of ϕ from the initial value, such that shortly after the Big Bang and in the present epoch $e^{-\phi} \ggg 1$. If we disregard the exponentially small terms e^{ϕ} compared to the exponentially large terms of the order of $e^{-\phi}$, the gravitational field is no longer "absolute". Namely, shift of ϕ by a constant is equivalent to the rescaling of coordinates [8].

However, when we calculate the metric produced by a point spin, the approximation $e^{-\phi} \ggg 1$ is not applicable, because gravitational potential undergoes a large change at small r. As a consequence, we must keep the hyperbolic functions in Eqs. (48) and (49).

The right-hand side of Eq. (48) is the source for ϕ produced by the spinning gravitational field. Integration of both sides of Eq. (48) over the spatial volume yields the total gravitational mass of the object

$$M = m_0 - \frac{c^2}{8\pi G}\int d^3 r \left(\sinh(4\phi)\,\mathrm{curl}^2\mathbf{u} + 4e^{-4\phi}\frac{\partial\phi}{\partial\rho}\frac{u^2}{\rho}\right), \tag{50}$$

where m_0 is the mass of the point matter source at the origin. Since $\sinh(4\phi)$ can be both positive and negative, in VG, the spinning gravitational field can screen the matter contribution to the mass and reduce the total mass of the particle.

4.1 Spinning gravitational field bound to a point mass

As an insightful exercise, next, we show that spinning gravitational field can be "attached" to a point mass forming a bound state. In the case of GR, such bound states are known as spinning black holes, and are described by a singular spacetime geometry. Here, we show that in the case of VG, such bound states have no singularities.

In the following, we assume that the spinning gravitational field is weak and does not affect ϕ produced by the point mass. We perform calculations in spherical coordinates r, θ, φ. If $\phi = \phi(r)$, Eq. (49) has a solution in the form

$$\mathbf{u}(r,\theta) = r\sin(\theta)\chi(r)\hat{\varphi}, \tag{51}$$

where $\hat{\varphi}$ is the azimuthal unit vector and the radial function $\chi(r)$ obeys equation

$$r\chi'' + 4\chi' + 4r\tanh(2\phi)\phi'\chi' + \frac{2\left(1+e^{4\phi}\right)}{\cosh^2(2\phi)}\phi'\chi = 0, \tag{52}$$

in which prime denotes derivative with respect to r.

As we mentioned above, in the present epoch $e^{-\phi} \ggg 1$. Under this assumption, one can disregard the last term in Eq. (52), and obtain equation

$$r\chi'' + 4\chi' - 4r\phi'\chi' = 0,$$

which can be solved analytically. Solution, satisfying the boundary conditions $u \to 0$ as $r \to \infty$, and u is finite at $r \to 0$, reads

$$\chi(r) = C - \frac{C}{N}\int_0^r dr\frac{e^{4\phi}}{r^4}, \tag{53}$$

where C is an arbitrary integration constant, and

$$N = \int_0^\infty dr\frac{e^{4\phi}}{r^4}.$$

Since we assume that $e^{-\phi} \ggg 1$, by shifting ϕ and rescaling coordinates one can make ϕ satisfy the condition $\phi \to 0$ at $r \to \infty$ [8]. Then, for a field attached to a point mass $\phi = -m/r$, and we obtain $N = 1/32m^3$,

$$\chi = C - C\left(\frac{8m^2}{r^2} + \frac{4m}{r} + 1\right)e^{-\frac{4m}{r}}. \tag{54}$$

The solution (54) has no singularities (provided $m > 0$). For this solution, the large-distance asymptotics reads

$$\mathbf{u} \approx \frac{32Cm^3}{3r^2}\sin(\theta)\hat{\varphi},$$

while for $r \ll m$

$$\mathbf{u} \approx Cr\sin(\theta)\hat{\varphi}.$$

Taking into account that the large-r asymptotic of the metric determines the particle's angular momentum $\mathbf{S}$, namely

$$ds^2 \approx \left(1 - \frac{2m}{r}\right)c^2dt^2 - \left(1 + \frac{2m}{r}\right)d\mathbf{r}^2 + \frac{4G}{c^2}\frac{(\mathbf{S}\times\hat{r})}{r^2}\cdot d\mathbf{r}dt, \tag{55}$$

one can write for $r \gg m$

$$\mathbf{u} \approx \frac{2G}{c^3}\frac{\mathbf{S}\times\hat{r}}{r^2},$$

where, in terms of S, the integration constant C is given by $C = 3GS/16c^3m^3$.

The right-hand side of Eq. (48) yields the correction to the mass produced by the spinning gravitational field. Plugging $\mathbf{u}$ from Eq. (51) into Eq. (48), yields for $e^{-\phi} \gg 1$

$$\nabla^2\phi = -2\chi^2 - r\sin^2\theta\frac{\partial\chi^2}{\partial r} + \frac{1}{2}r^2\sin^2\theta e^{-4\phi}\left(\frac{\partial\chi}{\partial r}\right)^2. \tag{56}$$

For solution (54), the right-hand side of Eq. (56) is finite everywhere, and decays as $1/r^6$ at large r. Therefore, the spinning gravitational field gives a finite correction to the mass.

Please note that in the present example, the particle's angular momentum is produced solely by the gravitational field. The point mass at the origin does not contribute to the spin due to its zero size.

4.2 Spinning gravitational and electromagnetic fields bound to electric charge

In Sect. 3.2, we showed that in VG, the electric charge bound state is described by a nonsingular equivalent metric, and gravity eliminates divergence in the Coulomb's law. Next, we show that spinning gravitational field $\mathbf{u}$ can be attached to the electric charge, and the emerging bound state is also nonsingular. In the presence of an electric field, $\mathbf{u}$ is coupled to the magnetic field via Maxwell's equations. Therefore, in the present problem, spinning gravitational field induces magnetic field.

Taking into account expression for the energy-momentum tensor for the electromagnetic field (13), current in Eq. (46) for the metric (47) reads

$$\mathbf{j}/\varepsilon_0 = -2\cosh(2\phi)(\mathbf{u}\cdot\mathbf{E})\mathbf{E} + ce^{2\phi}\mathbf{E}\times\mathbf{B} + \frac{1}{2}e^{2\phi}E^2\mathbf{u}, \tag{57}$$

where we used [27]

$$A_k = (A_0, -\mathbf{A}), \quad E_\alpha = cF_{0\alpha}, \quad \mathbf{B} = \mathrm{curl}\mathbf{A}.$$

As before, we will assume that $e^{-\phi} \ggg 1$. Then

$$\mathbf{j}/\varepsilon_0 \approx -e^{-2\phi}(\mathbf{u}\cdot\mathbf{E})\mathbf{E} + ce^{2\phi}\mathbf{E}\times\mathbf{B},$$

and equation for the gravitational field (46) reduces to

$$\mathrm{curl}\left[e^{-4\phi}\mathrm{curl}\mathbf{u}\right] = 8e^{-4\phi}\left[(\mathbf{u}\cdot\nabla)\nabla\phi + \frac{2\pi G\varepsilon_0}{c^3}e^{2\phi}\mathbf{E}\times\mathbf{B}\right]. \tag{58}$$

This equation should be solved together with Maxwell's equations for the electromagnetic field in curved spacetime [27]

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}F^{\mu\nu}\right) = 0, \tag{59}$$

which in the metric

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}d\mathbf{r}^2 + 2ce^{-2\phi}\mathbf{u}\cdot d\mathbf{r}dt \tag{60}$$

read

$$\mathrm{div}\left(e^{-2\phi}\mathbf{E}\right) = 0, \tag{61}$$

$$\mathrm{curl}\left(e^{2\phi}\mathbf{B}\right) = -\mathrm{curl}\left(e^{-2\phi}\mathbf{u}\times\mathbf{E}\right). \tag{62}$$

Please note that in Maxwell's Eqs. (59), we did not include electric charge and electric current. They appear as a solution of free field equations coupled to gravity.

Equation (62) shows that in the presence of an electric field, the spinning gravitational field **u** induces magnetic field **B**. We call this "electric field dragging" by analogy with the frame-dragging (Lense–Thirring) effect in the vicinity of rotating massive objects.

In the following, we will assume that the spinning gravitational field and the induced magnetic field are weak, and do not affect ϕ and **E** produced by the static charge; that is

$$\phi = -\ln\left(1+\frac{a}{r}\right), \quad E_r = \frac{q}{4\pi\varepsilon_0(a+r)^2}.$$

In the spherical coordinate system, vectors **u** and **A** have only azimuthal component

$$u_\varphi = u(r)\sin\theta, \quad A_\varphi = A(r)\sin\theta.$$

Plug this into Eqs. (58) and (62) yields the following equations for the radial functions $u(r)$ and $A(r)$:

$$-(ru)'' + 4\phi'(ru)' + \frac{2u}{r} = 8u\phi' - \frac{4ar^2}{(a+r)^4}\left(r\tilde{A}\right)', \tag{63}$$

$$\left(r\tilde{A}\right)'' + 2\phi'\left(r\tilde{A}\right)' - \frac{2\tilde{A}}{r} = \frac{a(a+r)^2}{r^2}\frac{\partial}{\partial r}\left(\frac{u}{r}\right), \tag{64}$$

where prime denotes derivative with respect to r, and we introduced a dimensionless vector potential $\tilde{A}$ according to

$$A = \frac{q}{4\pi\varepsilon_0 a}\tilde{A}.$$

Equations (63) and (64) can be solved analytically. The solution satisfying the proper boundary conditions reads

$$u_\varphi = \frac{Cr(2r+a)}{(a+r)^4}\sin\theta, \quad A_\varphi = -\frac{qC}{4\pi\varepsilon_0 a}\frac{\sin\theta}{r(a+r)}, \tag{65}$$

where C is an integration constant. Solution (65) can be written in a vector form

$$\mathbf{u} = \frac{G}{c^3}\frac{r(2r+a)}{(a+r)^4}\mathbf{S}\times\hat{r}, \quad \mathbf{A} = \frac{\mu_0}{4\pi}\frac{\mu\times\hat{r}}{r(a+r)}, \tag{66}$$

where **S** is the angular momentum (spin) of the particle determined by the large-distance asymptotics of the metric (55), and μ is the spin magnetic moment

$$\mu = -\frac{Gq}{c^2 a}\mathbf{S} = -\frac{q}{m}\mathbf{S}, \tag{67}$$

where we used Eq. (26) for the particle mass $m = ac^2/G$. Equation (67) is exactly the same as the gyromagnetic ratio predicted for a spinning particle by the Dirac equation, and for a spinning charged black hole in general relativity [30].

Solution (66) is regular and does not yield singularities in the metric and self-energy of the particle. In particular, for this solution, the energy density of the magnetic and that of the electric fields are given by

$$\frac{e^{2\phi}B^2}{2\mu_0} = \frac{\mu_0\mu^2}{32\pi^2 r^2(a+r)^4}\left(4\cos^2\theta + \frac{r^2}{(a+r)^2}\sin^2\theta\right), \tag{68}$$

$$\frac{1}{2}\varepsilon_0 e^{-2\phi}E^2 = \frac{q^2}{32\pi^2\varepsilon_0 r^2(a+r)^2}. \tag{69}$$

At $r \gg a$, Eqs. (68) and (69) reduce to the well-known formulas in the absence of gravity. For $r \ll a$, both energy densities scale as $1/r^2$ yielding a finite contribution to the mass. In this limit

$$w_B = w_E \left(\frac{2S}{\hbar\alpha}\right)^2 \cos^2\theta, \tag{70}$$

where

$$w_E \approx \frac{c^4}{8\pi G r^2}$$

is the energy density of the electric field, and α is a dimensionless parameter

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 \hbar c},$$

which reduces to the fine structure constant for $q = e$.

Equation (70) shows that electric field gives the dominant contribution to the energy if

$$S \lesssim \alpha\frac{\hbar}{2}. \tag{71}$$

We call it the electric field dominating regime. Solution (66) is valid in this regime. Particles for which the condition (71) is satisfied are very heavy and have a mass about $\sqrt{\alpha} m_{Pl}$, where m_{Pl} is the Planck mass.

4.3 Screening the electromagnetic energy by spinning gravitational field and generations of leptons

Next, we consider fast-spinning particles for which spin satisfies the condition

$$S \gg \alpha\frac{\hbar}{2}. \tag{72}$$

All elementary fermions and gauge bosons of the Standard Model are fast spinning particles. In this case, the magnetic field of the particle "blows up" due to the mass screening effect. Recall that, according to Eq. (67), the particle's magnetic moment goes as $1/m$. Screening effect reduces the mass from the Planck scale to the orders of magnitude smaller value of elementary particle masses we observe in experiments. This mass reduction increases the magnetic moment by many orders of magnitude. As a result, the magnetic field energy density becomes many orders of magnitude greater than that for the electric field.

Disregarding electric field in Eq. (45), we have

$$\nabla^2\phi = -\sinh(4\phi)\,\mathrm{curl}^2\mathbf{u} + \left(1 + e^{-4\phi}\right)\mathrm{div}\left[(\mathbf{u}\cdot\nabla)\mathbf{u}\right] + \frac{4\pi G}{\mu_0 c^4} e^{2\phi} B^2. \tag{73}$$

The last term in Eq. (73) is much larger than $\mathrm{curl}^2\mathbf{u}$. However, the $\sinh(4\phi)$ factor can amplify the energy density of the spinning gravitational field, and for $\phi \gg 1$ compensate the electromagnetic field contribution. Keeping in mind that the first term in the right-hand side of Eq. (73) is non negligible only for large ϕ, we can replace $\sinh(4\phi)$ with $e^{4\phi}/2$ and obtain equation

$$\nabla^2\phi = -\frac{1}{2}e^{4\phi}\mathrm{curl}^2\mathbf{u} + \frac{4\pi G}{\mu_0 c^4} e^{2\phi} B^2. \tag{74}$$

Please note that the first term in the right-hand side of Eq. (74), describing contribution to the mass due to the spinning gravitational field, is negative, while magnetic field (the second term) gives positive contribution.

Next we calculate the field at a distance greater than the Planck scale. In this case, Eqs. (46) and (62) reduce to

$$\mathrm{curl}\left(e^{2\phi}\mathbf{B}\right) = 0, \tag{75}$$

$$\mathrm{curl}\left[\cosh^2(2\phi)\mathrm{curl}\mathbf{u}\right] = 0. \tag{76}$$

By shifting ϕ, rescaling coordinates together with $\mathbf{u}$ and $\mathbf{B}$, one can make ϕ satisfy the condition $\phi \to 0$ at $r \to \infty$. Equations (75) and (76) have the following solution ($\mathbf{B}$ =curl$\mathbf{A}$)

$$\mathbf{u} = \frac{G}{c^3}\frac{\mathbf{S}\times\hat{r}}{r^2}, \quad \mathbf{A} = \frac{\mu_0}{4\pi}\frac{\mu\times\hat{r}}{r^2}, \quad \phi = \phi\left(\xi\right), \tag{77}$$

where $\mathbf{S}$ is the particle spin, μ is the spin magnetic moment parallel to $\mathbf{S}$

$$\xi = \frac{\cos\theta}{r^2},$$

and θ is the polar angle between $\mathbf{S}$ and $\mathbf{r}$ in the spherical coordinate system.

Plug Eq. (77) into Eq. (74) yields the following equation for $\phi\left(\xi\right)$:

$$\frac{\partial^2\phi}{\partial\xi^2} = -\frac{s^4}{2}e^{4\phi} + b^4e^{2\phi}, \tag{78}$$

where b is the magnetic dipole gravitational length given by Eq. (36), and s is the spin gravitational length

$$s = e^{2\phi_0}\sqrt{\frac{GS}{c^3}}.$$

Here, ϕ_0 is the value of ϕ in the present epoch before we made the shift of ϕ by ϕ_0 (recall that $e^{\phi_0} \lll 1$). For the electron, $b = 5.17\times10^{-25}$m and $s = 1.14\times10^{-35}e^{2\phi_0}$ m. Thus, the spin term in Eq. (78) is negligible unless ϕ has large positive values.

Integration of Eq. (78) leads to an equation analogous to the classical equation of motion of a particle in a potential well $V(\phi)$

$$\left(\frac{\partial\phi}{\partial\xi}\right)^2 + V(\phi) = -C^2, \tag{79}$$

where C is an integration constant and

$$V(\phi) = s^4e^{4\phi} - b^4e^{2\phi}.$$

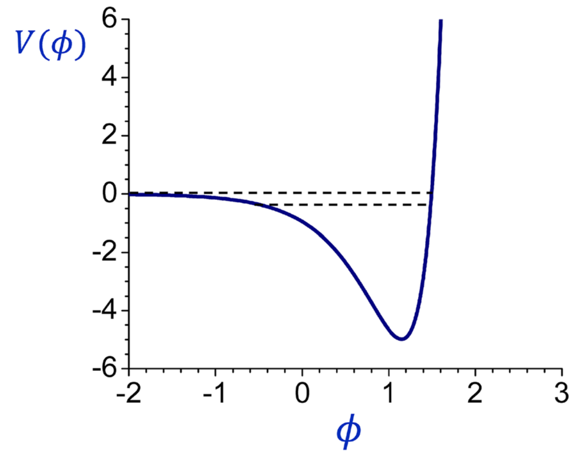

Fig. 2 Sketch of the effective potential in Eq. (79). The dashed line indicates the electron (top) and the muon (bottom) bound states

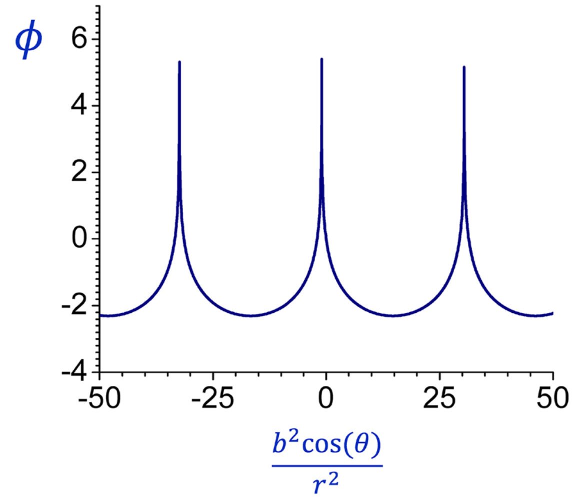

Fig. 3 Gravitational potential ϕ as a function of $\xi = \cos\theta/r^2$ given by Eq. (80) with $s = 0.05b$ and $C = 0.1b^2$

Potential $V(\phi)$ has a shape of a well for any nonzero value of the spin (see Fig. 2), which leads to the existence of bound solutions. A bound solution of Eq. (79), satisfying condition $\phi(r = \infty) = 0$, is

$$e^{-2\phi} = \frac{b^4}{2C^2}\left(1 - \sqrt{1 - \frac{4s^4C^2}{b^8}}\cos(2C\xi + C_1)\right), \tag{80}$$

where C_1 is given by

$$\cos(C_1) = \frac{b^4 - 2C^2}{\sqrt{b^8 - 4s^4C^2}}.$$

Solution (80) periodically oscillates as a function of ξ with a period π/C. Equation (80) shows that finite value of the spin eliminates singularities and makes ϕ finite everywhere. In Fig. 3, we plot $\phi(\xi)$ given by Eq. (80) for $s = 0.05b$ and $C = 0.1b^2$.

Asymptote of ϕ at $r \to \infty$ ($\xi \to 0$) goes as $1/r^2$. Thus, gravitational mass of the particle is equal to zero. That is, the negative energy of the spinning gravitational field totally compensates the positive energy of the magnetic field.

One can also see this by calculating the total energy of the particle by taking an integral over the volume from the right-hand side of Eq. (74), which is equal to the integral from the left-hand side. Mass accumulated from the region outside the sphere of radius r is given by

$$m(r) = -\frac{r^2}{2}\int_0^\pi d\theta \sin\theta \frac{\partial \phi}{\partial r} = r^3 \int_{-1/r^2}^{1/r^2} d\xi \xi \frac{\partial \phi}{\partial \xi}.$$

Integrating by parts yields

$$m(r) = r\left[\phi\left(\frac{1}{r^2}\right) + \phi\left(-\frac{1}{r^2}\right)\right] - r^3 \int_{-1/r^2}^{1/r^2} \phi d\xi.$$

Since ϕ is a periodic function of ξ, the last integral goes as ξ for large ξ, and in the limit $r \to 0$ we obtain $m(0) = 0$. Therefore, the electromagnetic field energy is fully compensated by the spinning gravitational field negative contribution yielding a particle with zero mass. This explains why elementary particles have negligible mass on the Planck scale making the gauge symmetry of the Standard Model a good approximation. However, an exponentially small residual is left over, which we discuss in the next section.

Equation (80) describes gravitational potential in the outer region (larger than the Planck length). To find the integration constant C and the corresponding bound states, we must match solution (80) with the inner solution. First, we note that if $C = 0$, Eq. (80) reduces to

$$e^{-2\phi} = \left(\sqrt{1 - \frac{s^4}{b^4}} + \frac{b^2\cos\theta}{r^2}\right)^2 + \frac{s^4}{b^4}. \tag{81}$$

Since $s \ll b$, one can approximately write

$$\phi \approx -\ln\left|1 + \frac{b^2\cos\theta}{r^2}\right|, \tag{82}$$

which yields the following asymptote for the angular averaged gravitational potential:

$$\bar{\phi}(r) = \frac{1}{2}\int_0^\pi \phi(r,\theta)\sin\theta d\theta \approx 2\ln r, \quad r \to 0.$$

Thus, for $C = 0$, at small r, the outer solution (80) matches with the solution of Sect. 3.2. Hence, Eq. (81) describes a fast-spinning bound state of the gravitational and electromagnetic fields. As we show below, this bound state corresponds to the electron.

For the electron solution (82), we obtain

$$m(r) = \frac{r^3}{b^2}\ln\left|\frac{b^2 + r^2}{b^2 - r^2}\right| - 2r.$$

While the accurate Eq. (81) gives

$$m(r) = \frac{r^3}{2b^2}\sqrt{1 - \frac{s^4}{b^4}}\ln\left(\frac{s^4r^4 + \left(b^4 + \sqrt{b^4 - s^4}r^2\right)^2}{s^4r^4 + \left(b^4 - \sqrt{b^4 - s^4}r^2\right)^2}\right) + \frac{s^2r^3}{b^4}\arctan\left(\frac{2s^2r^2}{r^4 - b^4}\right) - 2r. \tag{83}$$

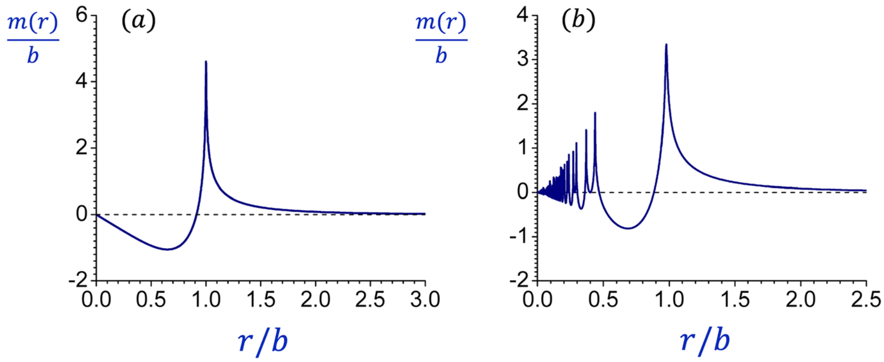

Fig. 4 Mass accumulated from the region outside the sphere of radius r for the electron-like bound state given by Eq. (83) for $s = 0.05b$ (**a**), and for the muon-like bound state with $s = 0.1b$ (**b**)

Equation (83) yields that for $r \ll b$, $m(r)$ is a linear universal function of r

$$m(r) \approx -2r. \tag{84}$$

In Fig. 4a, we plot $m(r)$, given by Eq. (83), for $s = 0.05b$. Figure 4a shows that negative contribution to the mass from the spinning gravitational field comes from the vicinity of $r = b$. This negative contribution totally compensates the positive contribution from the electromagnetic field, so that $m(0) = 0$.

Next, we find bound states with $C \neq 0$. For $C \neq 0$, solution (80) undergoes fast oscillations at $r \ll b$, and the total energy density of the gravitational and electromagnetic fields (the right-hand side of Eq. (74))—averaged over the fast oscillations—is equal to zero. Thus, for $C \neq 0$, the gravitational potential in the inner region obeys equation $\nabla^2\phi = 0$, and the inner solution is $\phi \approx \text{const}$.

Introducing the spatial average of the outer solution (80) at fixed angle θ

$$\bar{\phi}(\theta) = \frac{1}{r_0}\int_0^{r_0} dr\phi(r,\theta),$$

where $r_0 \ll b$, we find

$$\bar{\phi}(\theta) = \begin{cases} 0, \quad \theta = \pi/2 \\ \frac{C}{\pi}\int_0^{\pi/C}\phi(\xi)d\xi = \ln\left(\frac{\sqrt{b^8-4s^4C^2}}{4C^2}\right), \quad \theta \neq \pi/2. \end{cases}$$

That is, the averaged function $\bar{\phi}(\theta)$ undergoes a jump at $\theta = \pi/2$. The jump vanishes if

$$\ln\left(\frac{\sqrt{b^8-4s^4C^2}}{4C^2}\right) = 0,$$

which yields

$$C^2 = \frac{1}{8}\left(\sqrt{4b^8+s^8} - s^4\right) \approx \frac{b^4}{4}.$$

That is, solution (80) with

$$C \approx \frac{b^2}{2} \tag{85}$$

matches the isotropic inner solution and describes another fast-spinning bound state of the gravitational and electromagnetic fields. As we show below, this solution corresponds to the muon.

In Fig. 4b, we plot $m(r)$ for the muon-like bound state with $s = 0.1b$. Figure 4b shows that for $r \ll b$ the average total energy density of the spinning gravitational and electromagnetic fields oscillates around zero value and the negative contribution from the gravitational field fully compensates the positive contribution from the electromagnetic field.

5 Electron and muon masses

In this section, we calculate electron and muon masses in the framework of VG. The electron and muon are bound states of the spinning gravitational and electromagnetic fields attached to the electric charge $q = e$. Due to the electric field dragging effect, the bound states possess magnetic moment μ. Since calculations for both particles are similar, we explain the details keeping electron in mind. For the electron, the magnetic field energy density becomes greater than that of the electric field at a distance to the origin

$$\frac{b^2}{a} = \frac{\mu}{c|e|} = 1.9 \times 10^{-13}\ \mathrm{m}, \tag{86}$$

where a and b are the charge gravitational radius and the magnetic dipole gravitational length given by Eqs. (21) and (36), respectively.

As we showed in the previous section, at shorter distances, the spacetime metric is determined by the magnetic field energy and the energy of the spinning gravitational field. The latter has a negative contribution to the mass, which fully compensates the positive contribution from the electromagnetic field. The effects of gravity become large at spacing on the order of the magnetic dipole gravitational length

$$b = 5.17 \times 10^{-25}\ \mathrm{m}. \tag{87}$$

In the previous section, we omitted the electric field energy contribution, and found that for $r \gg b$, the gravitational potential, ϕ has the following asymptote for the electron-like bound state

$$\phi \approx -\frac{b^2 \cos\theta}{r^2} + \frac{b^4 \cos^2\theta}{2r^4}, \tag{88}$$

and

$$\phi \approx -\frac{\sqrt{3}}{2}\frac{b^2 \cos\theta}{r^2} + \frac{b^4 \cos^2\theta}{2r^4} \tag{89}$$

for the muon-like bound state. These asymptotes describe massless particles.

However, since the electric field has a monopole configuration, it decreases with r slower than the magnetic field. As a result, at a distance greater than $\mu/c|e|$, the electric field energy density becomes the dominant contribution to the mass, which is not compensated by the spinning gravitational field. Here, we calculate the electron and muon masses by including the electric field energy and solving the equation for the gravitational potential ϕ in the region $r \gtrsim b$. In this region, one can disregard the spinning gravitational field contribution to the energy. Then, the spacetime metric is diagonal and given by Eq. (10). The equation for ϕ reads

$$\nabla^2\phi = \left(\frac{a^2}{r^4} + \frac{b^4}{r^6}\left(1 + 3\cos^2\theta\right)\right)e^{2\phi}. \tag{90}$$

The first and the second terms in the right-hand side of Eq. (90) are due to the electric and magnetic field energy density, respectively. One should solve Eq. (90) subject to the asymptotic conditions (88) and (89) for $b \lesssim r \ll b^2/a$, and $\phi \approx -m/r$ for $r \gg b^2/a$. In Appendix B, we obtain solution by making expansion in the parameter $\ln(\gamma)$, where

$$\gamma = \frac{b^2}{a^2} = 1.39 \times 10^{23}\frac{\mu}{\mu_B},$$

and μ_B is the Bohr magneton. Keeping the leading terms in the expansion, we find with the logarithmic accuracy for the electron mass m

$$\frac{\mu m}{\hbar e} \approx \frac{1}{2^{5/4}}\alpha F(\gamma), \tag{91}$$

and

$$\frac{\mu m}{\hbar e} \approx \frac{1}{2^{1/4}\sqrt{3}}\alpha F(\gamma) \tag{92}$$

for the muon mass, where

$$F(\gamma) = \frac{1}{3}\ln^{3/2}(\gamma) + \left(\frac{3}{2}\ln(2) + 2 + \frac{1}{\sqrt{2}}\right)\ln^{1/2}(\gamma),$$

and

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

is the fine structure constant. We wrote Eqs. (91) and (92) in terms of $\hbar$ and the fine structure constant for convenience. In fact, Eqs. (91) and (92) are classical and do not depend on the Planck constant $\hbar$ implicitly.

Equations (91) and (92) show that for a given electric charge, the particle's mass depends on the magnetic moment μ. The latter is determined by the particle's spin S. To find m and μ, we need to specify the value of the spin, which is a quantum property of the particle. This gives an additional equation

$$\frac{\mu m}{e} = S. \tag{93}$$

Equations (91) or (92) must be solved together with Eq. (93), which for a given charge and spin uniquely determines m and μ. In Fig. 5, we plot $\mu m/\hbar e$ as a function of μ given by Eqs. (91) and (92) (solid lines), and by Eq. (93) for $S = \hbar/2$ and $\hbar$ (the horizontal dashed lines). The intersection of these lines yields m and μ for the electron and muon, as shown in the figure.

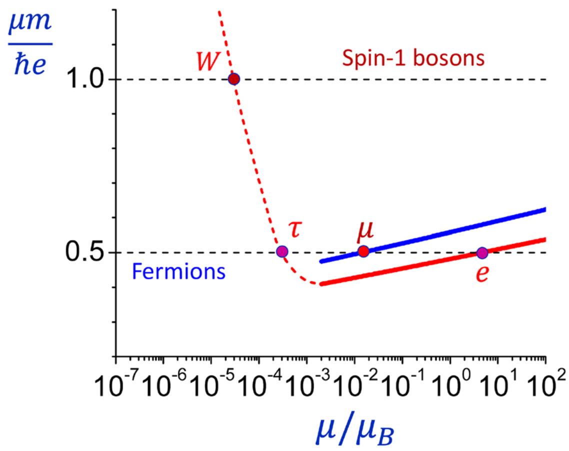

Fig. 5 $\mu m/\hbar e$ given by Eq. (91) (red solid line), Eq. (92) (blue solid line), and Eq. (93) for $S=\hbar/2$ and $\hbar$ (horizontal dashed lines) as a function of μ/μ_B, where μ_B is the Bohr magneton. Solid lines are plotted in the validity region of our analysis (95) which disregards the weak interaction. Continuation of Eq. (91) into the region where the weak interaction gives substantial contribution to the mass is sketched qualitatively as the red dashed line. Since the weak interaction eliminates the muon-like bound state, the blue solid line terminates at $\mu \sim \mu_w$. The lines' intersection with the horizontal dashed lines determines the mass and magnetic moment of elementary particles

To check agreement with experiment, we plug experimental values of μ for the electron ($\mu_e = 9.285\times 10^{-24}\, J/T$) and muon ($\mu_\mu = 4.49\times 10^{-26}$ J/T) in Eqs. (91) and (92), respectively, and obtain

$$m_e \approx 8.78\times 10^{-31}\ \text{kg},\quad m_\mu \approx 1.819\times 10^{-28}\ \text{kg},$$

which is 3% smaller than the experimental values of $m_e = 9.109\times 10^{-31}$ kg and $m_\mu = 1.884\times 10^{-28}$ kg. The striking agreement with experiment indicates that electron and muon masses have gravitoelectromagnetic origin.

The 3% deviation could be attributed to the QED correction to the mass Δm arising from the virtual emission and absorption of light quanta [31–33]. For a renormalizable theory, such as QED, it is expected that $\Delta m/m$ depend only on the fine structure constant α and, thus, $\Delta m/m$ should be the same for the electron and muon. We find that for both particles $\Delta m/m$ coincide within the logarithmic accuracy of our calculations (namely $\Delta m_e/m_e \approx 0.036$, $\Delta m_\mu/m_\mu \approx 0.035$) and, hence, the deviation of our results from the experiment is consistent with the QED self-energy correction.

Unfortunately, the dependence of $\Delta m/m$ on α is unknown exactly. However, $\Delta m/m$ can be estimated using various regularization schemes [31–33]. E.g., in the cut-off method of QED regularization, the correction depends logarithmically on a regularization mass [31,34]

$$\frac{\Delta m}{m} = \frac{3\alpha}{2\pi}\left(\ln\left(\frac{\Lambda}{m}\right)+\frac{1}{4}\right), \tag{94}$$

where Λ is an ultraviolet cut-off parameter. If we take Λ as the electroweak scale ($\sim$ 100 GeV), Eq. (94) yields $\Delta m/m \sim 0.03$, which is consistent with our findings.

6 Tau lepton and W boson

In the previous analysis, we included only gravitational and electromagnetic fields, and found two spinning bound states describing the electron and the muon. Here, we show that bound states corresponding to the tau lepton and W boson appear if we add weak interaction. The latter has a short range r_w determined by the mass m_w of the weak boson

$$r_w = \frac{\hbar}{m_w c} \approx 2.4 \times 10^{-18} \text{ m}.$$

Previous analysis yields that contribution to the particle mass comes from the region

$$r \gtrsim \frac{1}{\ln^{3/2}(\gamma)} \frac{b^2}{a}.$$

This estimate is obtained by comparing M_E of Eq. (2) with m in Eq. (91).

One can disregard weak interaction if

$$\frac{1}{\ln^{3/2}(\gamma)} \frac{b^2}{a} \gtrsim r_w,$$

which gives the following validity region of Eqs. (91) and (92):

$$\mu \gtrsim \mu_w = 0.004\mu_B. \qquad (95)$$

The corresponding solid lines in Fig. 5 are confined to this region.

For the smaller values of the particle's magnetic moment μ, Eqs. (91) and (92) must be modified by including the weak interaction. That is, in addition to the electric charge e, we need to add the lepton's weak charge. The weak charge e_w, determining the coupling to the weak bosons, is given by

$$e_w = \frac{e}{\sin(\theta_W)},$$

where θ_W is the weak mixing angle. Plugging the experimental value for θ_W, we obtain $e_w \approx 2.1e$. That is, intrinsically, weak interaction is stronger than the electromagnetic interaction. The weak charge e_w plays the same role in the weak interaction with $W^{\pm}$ and Z bosons as the electric charge does in electromagnetism.

According to Eq. (91), $\mu m/\hbar e$ goes as $\alpha \propto e^2$. To estimate the effect of weak interaction for $\mu \ll \mu_w$, in Eq. (91), we replace α with $\alpha + \alpha_w$, that is multiply the right-hand-side of Eq. (91) by the factor $1 + e_w^2/e^2 \approx 5.4$. This moves the red solid line in Fig. 5 up. The crossover region between the two limits ($\mu \ll \mu_w$ and $\mu \sim \mu_w$) is sketched by the red dashed line in Fig. 5. The line crosses the horizontal lines $S = \hbar/2$ and $S = \hbar$ at two points. Thus, taking into account weak interaction, it yields bound states corresponding to the tau lepton and W boson, respectively.

Next, we show that weak interaction eliminates the muon-like bound state, that is the blue solid line of Fig. 5 terminates at $\mu \sim \mu_w$, yielding no additional particles. The muon-like bound state is determined by the shape of the potential well $V(\phi)$ in Eq. (79). The latter equation describes classical motion of a particle between two turning points at which $V(\phi) = -C^2$. For $\mu \ll \mu_w$, the weak interaction yields an extra term in the potential, namely

$$V(\phi) = s^4 e^{4\phi} - b^4 e^{2\phi} \left[1 + \frac{e_w^2}{e^2} \Theta\left(|\phi| - \frac{b^2}{r_w^2}\right)\right], \qquad (96)$$

where the Heaviside step function Θ models the short range of weak interaction. Indeed, for $r \gg b$, $\phi \propto b^2/r^2$ and, hence, the requirement of vanishing contribution from the weak interaction at $r \gtrsim r_w$ implies no contribution to $V(\phi)$ for $|\phi| \lesssim b^2/r_w^2$.

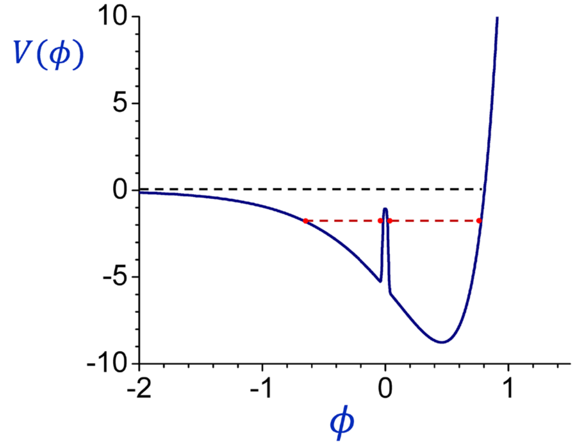

Fig. 6 Sketch of the potential (96) in the presence of weak interaction (solid line). Spike at $\phi = 0$ splits the muon-like bound state (lower dashed line) into two regions ($\phi > 0$ and $\phi < 0$) which eliminates the bound state. The electron-like bound state (upper dashed line) is not affected by the spike

In Fig. 6, we sketch potential (96) for $e_w/e = 2.1$. The potential has a spike at $\phi = 0$, which splits $V(\phi)$ into two regions: $\phi > 0$ and $\phi < 0$. As we showed in Sect. 4.3, for the muon-like bound state, C^2 in Eq. (79) is determined from the condition that the average value of ϕ over the particle motion in $V(\phi)$ vanishes. To meet this requirement, one turning point must be located at $\phi < 0$, while the other one at $\phi > 0$. Therefore, the muon-like bound state can exist only if $-C^2$ is above the spike level, that is

$$C^2 \leq -V(0) \approx b^4. \tag{97}$$

However, according to Sect. 4.3, in the absence of spike, for the muon-like bound state

$$C^2 \approx \frac{b^4}{4}\left(1 + \frac{e_w^2}{e^2}\right) \approx 1.4b^4. \tag{98}$$

Equation (98) is inconsistent with the condition (97) and, therefore, if we add the spike, the muon-like bound state no longer exists. On the other hand, the electron-like bound state for which $C = 0$ is not affected by the spike (see Fig. 6).

Thus, in the energy range studied experimentally, in the electroweak sector, VG yields only known charged particles—three charged leptons and W boson, plus the corresponding antiparticles.

7 Quantum origin of elementary charges

In classical electrodynamics, the electromagnetic field is generated by electric charges which enter Maxwell's equations. In particular, equation

$$\text{div}\mathbf{E} = \frac{\rho}{\varepsilon_0},$$

where ρ is the electric charge density, shows that electric charge is the source of the longitudinal ($\text{div}\mathbf{E} \neq 0$) electric field. So far, the physical origin of the electric charge was unknown. Here, we show that electric charge is a bound state of free electric and gravitational fields.

Indeed, if we include gravity, the longitudinal electric field can be produced without introducing charge into the Maxwell's equations. Namely, in the presence of gravity, Maxwell equation for the free field

$$\mathrm{div}\left(e^{-2\phi}\mathbf{E}\right) = 0$$

can be written as

$$\mathrm{div}\mathbf{E} = 2\nabla\phi \cdot \mathbf{E}. \tag{99}$$

The right-hand side of Eq. (99) plays the role of the electric charge density. In Sect. 3.2, we found that Eq. (99), together with Eq. (15) for the gravitational potential ϕ, has a solution describing a spherically symmetric nonsingular bound state of the static electric and gravitational fields

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 (a+r)^2}\hat{r}, \quad \phi(r) = -\ln\left(1 + \frac{a}{r}\right), \tag{100}$$

where q is an integration constant and a, which is given in terms of q by Eq. (21), determines the size of the bound state. For $r \gg a$, the electric field (100) reduces to that produced by a point electric charge q in classical electrodynamics.

Thus, electric charge is a nonsingular bound state of the free longitudinal electric and gravitational fields confined into the region of size a. In the classical treatment of the problem, q can have an arbitrary value. However, in the quantum description, q has a minimum value—the elementary electric charge.

The bound state can be represented as a superposition of a classical solution and quantum fluctuations of the field which give quantum correction to the mass [14]. Similar bound states of EM field (quantum solitons) have been studied in nonlinear optics [35–38].

The classical solution is accurate provided that the quantum corrections are small. The latter increase if the size of the bound state decreases. We estimate the minimum charge using the variational principle. If a massless field is confined into a finite volume of size a, this produces kinetic energy of quantum confinement $\sim \hbar c/a$. As a consequence, one can estimate the total energy of the object as

$$W \sim \frac{c^4}{G}a + \frac{\hbar c}{a}, \tag{101}$$

where the first term is the classical energy of the bound state given by Eq. (26), and the second term is the quantum correction. In the context of the quantum field theory, quantum correction to the energy has been accurately calculated for various classical solitons [14]. These accurate results show that the quantum correction is indeed of the order of $\hbar c/a$, where a is the size of the soliton.

The total energy (101) is minimum for $a \sim \sqrt{\hbar G/c^3}$, which is the Planck length. Using Eq. (21), we then obtain

$$q \sim \frac{e}{\sqrt{\alpha}} \approx 11.7e, \tag{102}$$

that is charge is independent of the gravitational constant.

Equation (102) gives an order of magnitude estimate of the electric charge of a particle composed of the electric and gravitational fields, which is somewhat greater than the electric charge of leptons. However, leptons are not composed of these two fields only, and the fine structure constant α in Eq. (102) should be replaced with

$$\alpha_{\mathrm{tot}} = \alpha + \alpha_w, \tag{103}$$

where $\alpha_w = e_w^2/4\pi\varepsilon_0\hbar c \approx 1/29$, and e_w is the weak charge. This gives a closer estimate $q \sim 4.9e$. The Higgs field also contributes to α_{tot}.

Results of the present section suggest that charged elementary particles have a quantum core with Planck mass and Planck size (see Fig. 1). The core is the bound state of the free gravitational and gauge boson fields which acts as a point charge when viewed from a large distance. Since quantum uncertainty in the charge position is about Planck length, for modeling the lepton's structure, the charge can be approximated as a classical point-like object when viewed outside the Planck volume.

For leptons, at distances greater than the Planck length from the core, quantum corrections to the lepton structure are small. Indeed, as we showed in Introduction [see Eq. (2)], at such distances, the magnetic field energy gives mass substantially larger than the Planck mass, which is compensated by the negative energy of the spinning gravitational field. As a result, outside the quantum core, the energy of quantum confinement is negligible compared to the energy of the classical stationary EM and gravitational fields, and one can accurately model the lepton's structure in the outside region as the classical field attached to the classical point-like charge. We do so in the present paper.

8 Origin of weak interaction and the Higgs boson

According to the Standard Model, sources of the electromagnetic, and weak and strong interactions are conserved charges. The symmetry corresponding to the charge conservation is the gauge invariance. Thus, if gravity is excluded, elementary particles must be described by a gauge invariant theory. However, a gauge invariant field theory yields that particles have a zero mass, because the mass term would break the gauge symmetry of the Lagrangian. This prediction of the theory disagrees with observations which show that most elementary particles are massive.

Since the source of gravity is not a conserved charge, gravity inevitably breaks the gauge symmetry and yields massive particles in a natural way. However, for Einstein's general relativity does not fit into the particle physics, physicists attempted to explain the particle masses without gravity. This leads to a somewhat artificial problem—how to break the gauge symmetry in a gauge invariant theory.

To resolve this problem, it has been proposed that the symmetry is broken spontaneously by the so-called Higgs mechanism with a help of an additional scalar field (the Higgs field). According to this model, an electroweak phase transition had occurred in the early universe, which made the Higgs field acquire a nonzero vacuum expectation value everywhere in space. Coupling to the Higgs condensate breaks the gauge symmetry and yields massive particles. This is somewhat analogous to the propagation of EM waves through plasma which makes photon (the gauge boson of EM interaction) massive. In the Standard Model, the Higgs field is pivotal in generating the masses of quarks and charged leptons (through Yukawa coupling), and the W and Z gauge bosons (through the Higgs mechanism).

In the present paper, we show that coupling of the three fundamental interactions to gravity breaks the gauge symmetry and yields massive particles. Thus, there is no need for the Higgs mechanism. The striking agreement of the calculated electron and muon masses with experiment indicates that gravity is the mechanism of mass generation.

Next we show that the weak and the Higgs bosons naturally appear in VG. More precisely, they naturally appear in any metric theory of gravity, in particular, GR or VG (see Appendix D). The point is that, equations of VG are not gauge invariant. However, at low energy the rest mass of a particle is approximately conserved and plays a role of a conserved charge. Consequently, there must exist fields which make equations of VG gauge invariant at low energy (here by the gauge transformation, we mean transformation of fields without transforming coordinates). They are the Z boson and the Higgs fields. These fields have the same physical origin as the fields restoring the gauge symmetry of gravity at low energy. The Z boson field (Z field for short) can form a bound state together with the EM and gravitational fields, yielding the charged leptons and the W boson.

We want to stress that in the present scenario, particles do not acquire mass due to interaction with the Higgs condensate, and the vacuum expectation value of the Higgs field is equal to zero.

Next, we discuss what the Higgs boson discovered in LHC is in detail according to VG. The ATLAS experiment essentially confirmed that the coupling of the discovered scalar particle with the second and third generations of the elementary fermions, as well as the W and Z bosons, is consistent with the Higgs mechanism of mass generation

[39]. The latter mechanism yields that coupling of the Higgs field h to the electroweak gauge bosons, e.g., the W boson, is described by the following term in the Lagrangian:

$$g^2(v+h)^2W^+W^- \approx g^2v^2W^+W^- + 2g^2vhW^+W^- = m_W^2W^+W^- + g_WhW^+W^-, \tag{104}$$

where v is the Higgs field vacuum expectation value, h is the field small perturbation, $m_W = gv$ is the mass of the W boson, and $g_W = 2g^2v$ is the coupling constant to the Higgs field. Equation (104) shows that the ratio

$$\frac{g_W}{m_W^2} = \frac{2}{v} \tag{105}$$

is the same for all gauge bosons.

One the other hand, the Higgs field couples fermion states of opposite helicity by the following term in the Lagrangian:

$$g_f(v+h)\left(\bar{f}_Lf_R + \bar{f}_Rf_L\right) = m_f\left(\bar{f}_Lf_R + \bar{f}_Rf_L\right) + g_fh\left(\bar{f}_Lf_R + \bar{f}_Rf_L\right),$$

where $m_f = g_fv$ and therefore

$$\frac{g_f}{m_f} = \frac{1}{v} \tag{106}$$

is the same for all elementary fermions. Combining Eqs. (105) and (106), we obtain that for all massive elementary particles

$$\frac{g_f}{m_f} = \frac{g_W}{2m_W^2}, \tag{107}$$

which is prediction of the Higgs mechanism. The ATLAS experiment confirmed relation (107) with a high precision [39] by measuring the decay rate of the Higgs boson into different particle channels, and the corresponding branching ratios. This result is considered by many as an experimental conformation of the Higgs mechanism.

One should note that a scalar boson, for which relation (107) is satisfied, must be introduced into the electroweak gauge theory in connection with the issue of divergence cancelations (see Sect. 6.1 in [40]). Namely, existence of such scalar boson is a necessary ingredient for achieving the tree-level unitarity (which, in turn, is necessary for renormalizability) in a gauge model incorporating mass terms for vector bosons and fermions. It also arises in theories with extra dimensions as radion field.

Next we show that a massive scalar field, for which relation (107) is satisfied without the Higgs mechanism, naturally emerges in VG. Since at low energy, the rest mass of particles is approximately conserved, it plays a role of a conserved charge. Consequently, there must exist fields which make gravity gauge invariant at the low energy.

Let us consider a point mass m located at $\mathbf{r} = 0$. The energy-momentum tensor of such a particle reads [7]

$$T^{00} = e^{\phi}mc^2\delta(\mathbf{r}), \qquad T = e^{3\phi}mc^2\delta(\mathbf{r}).$$

Linearizing Eq. (20) in Ref. [8], and keeping the terms first order in the time derivative, yields the following equation for the gravitational field:

$$\operatorname{div}\left(\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{u}}{\partial t}\right) = \frac{4\pi G}{c^2}me^{\phi}\delta(\mathbf{r}). \tag{108}$$

The left-hand side of Eq. (108) is invariant under the gauge transformation $\phi \to \phi + \partial f/\partial t$, $\mathbf{u} \to \mathbf{u} + c\nabla f$, but the right-hand-side is not. Since the gravitational potential ϕ is a real scalar field, to make Eq. (108) gauge invariant, we need to introduce two fields—a real scalar field ψ (we call it the Higgs field paying the tribute to the history) and a 4-dimensional real vector field Z_k (the Z field). Equation (108) for the stationary problem is then replaced with the gauge invariant equations

$$\mathrm{div}\left(\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t}\right) = \frac{4\pi G}{c^2} m e^{\phi+\psi}\delta(\mathbf{r}), \tag{109}$$

$$\mathrm{div}\left(\nabla\psi - \frac{m_Z c}{\hbar}\mathbf{Z}\right) = \frac{4\pi G_\psi}{c^2} m e^{\phi+\psi}\delta(\mathbf{r}), \tag{110}$$

where m_Z is a free parameter (mass of the Z field). One should also add equations for Z_k and $\mathbf{u}$; however, they are not relevant at this point of discussion.

Equations (109) and (110) can be obtained by taking variation of the action

$$S = -\frac{c^3}{8\pi G}\int d^4x \left(\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t}\right)^2 - \frac{c^3}{8\pi G_\psi}\int d^4x \left(\nabla\psi - \frac{m_Z c}{\hbar}\mathbf{Z}\right)^2 - mc\int d^4x e^{\phi+\psi}\delta(\mathbf{r}) \tag{111}$$

with respect to ϕ and ψ. Equations (109) and (110) [and the action (111)] are invariant under the gauge transformation $\phi \to \phi + \partial f/\partial t$, $\mathbf{u} \to \mathbf{u} + c\nabla f$, $\psi \to \psi - \partial f/\partial t$, and $\mathbf{Z} \to \mathbf{Z} - (\hbar/m_Z c)\,\partial\nabla f/\partial t$.

Equations (109) and (110) show that interaction with the scalar field ψ alters the rest mass of the particle according to

$$m \to e^{\psi} m \approx m + m\psi. \tag{112}$$

As a result, the mass term for the gauge boson is modified as

$$e^{2\psi} m_W^2 W^+ W^- \approx m_W^2 W^+ W^- + 2m_W^2 \psi W^+ W^-.$$

That is, the ratio of the coupling constant $g_W = 2m_W^2$ to m_W^2

$$\frac{g_W}{m_W^2} = 2 \tag{113}$$

is the same for all gauge bosons. One the other hand, the fermion mass term changes into

$$e^{\psi} m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right) \approx m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right) + m_f \psi \left(\bar{f}_L f_R + \bar{f}_R f_L\right),$$

and therefore

$$\frac{g_f}{m_f} = 1 \tag{114}$$

is the same for all fermions, where g_f is the coupling constant of the fermion and the scalar fields.

Combining Eqs. (113) and (114), yields relation (107). Thus, the results of the ATLAS experiment are actually not a conformation of the Higgs mechanism, but rather evidence that the discovered scalar particle couples to the rest mass.

Next we add a gauge invariant term describing the free field Z_k to the action (111). Disregarding gravity, we then obtain the following relativistic action for the free field:

$$S = -\frac{c^3}{16\pi G_\psi}\int d^4x \left(\frac{\partial Z^\nu}{\partial x_\mu} - \frac{\partial Z^\mu}{\partial x_\nu}\right)\left(\frac{\partial Z_\nu}{\partial x^\mu} - \frac{\partial Z_\mu}{\partial x^\nu}\right) + \frac{c^3}{8\pi G_\psi}\int d^4x \left(\frac{\partial \psi}{\partial x_k} - \frac{m_Z c}{\hbar} Z^k\right)\left(\frac{\partial \psi}{\partial x^k} - \frac{m_Z c}{\hbar} Z_k\right), \tag{115}$$

where

$$Z^i = \eta^{ik} Z_k,$$

and η_{ik} =diag$(1, -1, -1, -1)$ is Minkowski metric. The first term in the action (115) is the same as for the EM field, but with the coupling constant G_ψ. The action (115) is known as the Stueckelberg action [41]. The Stueckelberg extension of the Standard Model does not destroy the renormalizability of the theory and provides a mechanism for mass generation (the Stueckelberg mechanism), that is distinct from the Higgs mechanism in the context of Abelian gauge theories [42]. In our model, the masses of the ψ and Z fields are generated by the Stueckelberg mechanism.

Variation of the action (115) with respect to Z_k and ψ yields coupled equations

$$\Box Z_k - \frac{\partial}{\partial x^k}\frac{\partial Z^i}{\partial x^i} + \frac{m_Z^2 c^2}{\hbar^2} Z_k - \frac{m_Z c}{\hbar}\frac{\partial \psi}{\partial x^k} = 0, \tag{116}$$

$$\Box \psi - \frac{m_Z c}{\hbar}\frac{\partial Z^i}{\partial x^i} = 0, \tag{117}$$

where we introduced d'Alembert operator

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2.$$

Please note that Eqs. (116) and (117) are not invariant under the parity transformation $\mathbf{r} \to -\mathbf{r}$. Consequently, weak interaction violates parity.

Equations (116) and (117) have a solution for which $\psi = 0$, and the field Z_k obeys equations

$$\Box Z_k + \frac{m_Z^2 c^2}{\hbar^2} Z_k = 0, \qquad \frac{\partial Z^i}{\partial x^i} = 0, \tag{118}$$

describing a massive vector field Z_k with mass m_Z.

The mass of the scalar field ψ depends on the choice of gauge. Choosing the Lorentz invariant gauge as

$$\frac{\partial Z^i}{\partial x^i} + \frac{m_\psi^2 c}{m_Z \hbar}\psi = 0, \tag{119}$$

we obtain equations

$$\Box \psi + \frac{m_\psi^2 c^2}{\hbar^2}\psi = 0, \tag{120}$$

$$\Box Z_k + \frac{m_Z^2 c^2}{\hbar^2} Z_k + \frac{c}{m_Z \hbar}\left(m_\psi^2 - m_Z^2\right)\frac{\partial \psi}{\partial x^k} = 0. \tag{121}$$

Equation (120) describes a massive scalar field with mass m_ψ. Similarly to photon, the corresponding quantum excitation of the fields (the Higgs and the Z bosons) carry no charge and coincide with its own antiparticle.

Please note that the massive fields ψ and Z_k had arisen from the gauge invariant action (115) without spontaneous symmetry breaking mechanism. However, the gauge invariant coupling of ψ to the rest mass requires gravity and holds only at low energy. Absence of the gauge invariant coupling at high energy makes the field ψ physical. If the coupling would be gauge invariant at any energy, ψ can be absorbed into the other fields and produce no effect. To have ψ as a physical field, we need a mechanism breaking the gauge invariance. In the present model, gravity provides such a symmetry breaking mechanism.

In Appendix D, we obtain classical field equations describing weak interaction taking into account the field sources. For a point mass m with no weak charge, located at $\mathbf{r} = 0$, we find the following static solution for the fields ψ and Z_k in spherical coordinates:

$$\psi = \frac{G_\psi m}{c^2}\left(\frac{m_\psi c}{2\hbar} - \frac{1}{r}\right) e^{-c m_\psi r/\hbar}, \tag{122}$$

$$\mathbf{Z} = -\frac{G_\psi m m_\psi^2}{2c\hbar m_Z} e^{-c m_\psi r/\hbar} \hat{r}, \tag{123}$$

which describes a short-range interaction with the range $\hbar/m_\psi c$.

At $r \ll \hbar/m_\psi c$, the solution (122) is similar to the gravitational potential produced by a point mass m. However, despite coupling to the rest mass, ψ is not the gravitational field we are used to. Instead, ψ is a carrier of a new short-range fundamental interaction between masses, which is coupled to the mass much stronger than the conventional gravitational field. The corresponding coupling constant G_ψ can be estimated from the measurement of the Higgs boson decay, which yields

$$G_\psi = \frac{G}{4\pi}\frac{m_{Pl}^2}{v^2} = 1.95 \times 10^{32} G, \tag{124}$$

where $v = 246.22$ GeV is the vacuum expectation value of the Higgs field in the Higgs model, and $m_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass.

Since the interaction is short range (with the range about 10^{-18} m), we usually do not experience this force. The new fundamental interaction is analogous to the weak interaction which has a similar range. Hence, the Higgs boson can be treated as a carrier of a new type of the weak force. However, as we show in Appendix D, for leptons, interaction produced by the Higgs field is much weaker than that due to the EM and Z fields.

Sources of the Higgs and Z fields are different. The source of the Higgs field is the particle rest mass, which can be expressed as the trace of the energy-momentum tensor for the particular particle. The source of the Z field is the weak charge, which is analogous to the electric charge in electrodynamics. According to Eq. (119), the Higgs field is accompanied by the Z field and, hence, the Higgs field can also interact with the weak charges.

As we showed in the previous section, charge is a bound state of a gauge boson field and gravity. Similarly to the EM field, the Z field can form a bound state with the gravitational field. Such bound state is the weak charge. It has the quantum origin, and the size of the bound state is given by the Planck length. The spinning gravitational and Z fields can be attached to the weak charge in different states, yielding three types of neutrinos (and the corresponding antineutrinos).

A bound state can also be formed from the gravitational, Z and EM fields, which yields a Planck-size object possessing both the weak and the electric charges. The spinning gravitational, Z and EM fields can be attached to such electroweak charge in different states, yielding the charged leptons and $W^{\pm}$ bosons.

One should note that neutral elementary particles (photon, graviton, Higgs, and Z bosons) are fields, rather than bound states. They do not behave as point-like objects. Their size is governed by the quantum mechanical uncertainty principle. However, charged particles have a quantum core of Planck size and Planck mass, even though their total energy is negligible on the Planck scale (see Fig. 1). Thus, charged particles can be viewed as point-like constituents of matter.

9 Nonsingular alternative to black holes in vector gravity and nature of dark matter

Black hole is a bound state solution for the free gravitational field that exists in GR. Such a solution has an event horizon, and a curvature singularity at the origin. The mass of the black hole comes from the energy of the gravitational field. Here, we explore the corresponding solution in VG.

In Appendix C, we find a spherically symmetric bound state solution of the free field equations with zero spin, formed solely from the longitudinal gravitational field

$$\phi = -\ln\left(1+\frac{M}{r}\right), \quad \mathbf{u} = \pm\frac{Mr}{(r+M)^2}\hat{r}, \tag{125}$$

where $\hat{r}$ is the unit vector along the radial coordinate, and $M > 0$ is the mass of the bound state which can be arbitrary in the classical description of the problem. The corresponding equivalent metric reads

$$ds^2 = \frac{c^2}{\left(1+\frac{M}{r}\right)^2}dt^2 - \left(1+\frac{M}{r}\right)^2 d\mathbf{r}^2 \pm 2\frac{cM}{r}drdt. \tag{126}$$

The metric (126) has no event horizon and no singularities. Namely, scalar invariants (Ricci, Weyl and Kretschmann scalars) are finite everywhere in the spacetime (126). In particular, for the metric (126), the Ricci scalar is

$$R = -\frac{2\left(M^3+M^2r-r^3\right)M^2}{\left(M^2+r^2\right)^2(M+r)^3}.$$

Physically, the bound state (126) is a spacetime region with nonzero curvature confined into a volume of size M. It appears due to the self-interaction of the gravitational field caused by the nonlinear structure of the gravitational field equations.

By making a coordinate transformation, one can rewrite the metric (126) in a diagonal form

$$ds^2 = \frac{c^2}{\left(1+\frac{M}{r}\right)^2}dt^2 - \left(1+\frac{M}{r}\right)^2\left(1+\frac{M^2}{r^2}\right)dr^2 - (M+r)^2\left(d\theta^2+\sin^2\theta d\varphi^2\right). \tag{127}$$

The nonsingular solution (127) replaces static black holes in GR.

For a slow rotation with spin $S \ll M^2$

$$\mathbf{u} \approx \pm\frac{Mr}{(r+M)^2}\hat{r} + 2r\frac{\mathbf{S}\times\hat{r}}{(r+M)^3},$$

and the term

$$4cr\frac{\mathbf{S}\times\hat{r}}{(r+M)^3}d\mathbf{r}dt$$

is added to the right-hand side of Eq. (127).

In the classical treatment of the problem, M can have any positive value. In quantum treatment, due to additional energy of quantum confinement, the bound state has a minimum mass of the order of Planck mass. If this is the minimum energy state, the corresponding particle is stable. One can think of such bound state as an elementary "u" charge. The sign $\pm$ in Eqs. (125) and (126) imply that the u-charge can be positive or negative, while the mass of the particle is always positive. The positive and negative elementary u-charges describe the particle and antiparticle, respectively. When collide, they annihilate emitting two or more gravitons and, if heavy enough, Higgs bosons.

Similarly to the electric charge, to reduce the energy (mass), the gravitational field around the u-charge might spin, and the stable bound state might be a fermion. We call this hypothetical particle *t*eon [*t*e for tenebrae, which means darkness in Latin, and *on* as the word root for particle]. Teon does not carry electroweak or color charges, and couples only to the mass through gravity. Thus, such a particle very weakly interacts with ordinary matter. Teon is the VG prediction for the dark matter particle in the universe. Similarly to the baryon asymmetry, we expect that only one type of u-charges makes up the vast majority of the universe.

Conservation of the u-charge prevents teon from decaying into gravitons and Higgs bosons (the latter is somewhat strongly coupled to mass and, hence, interacts with teons). That is, teon is a stable particle. If mass of the teon is smaller than that of the Higgs boson, the latter can decay into teon-antiteon pairs. These decays can be detected at LHC or future electron-positron colliders as invisible decay modes.

Dark matter forms halo around galaxies. A part of the galactic halo coalesces, creating dark matter clumps or compact halos around galactic centers, which, when massive enough, undergo gravitational collapse forming compact objects with the core size much smaller than the gravitational radius. The compact dark matter halos are described by the exponential metric (10) (or by its spinning generalization), while the metric (12) approximates the collapsed objects, both baryonic and dark matter, see Sect. 3.1. The latter could also have a primordial origin.

Such formations (compact dark matter halos, "point-like" collapsed objects made of baryonic or dark matter) with large masses could be the supermassive compact objects residing in galactic centers. Merger of the low mass collapsed objects could produce gravitational wave events detected by LIGO-Virgo interferometers. GR interprets these events as merger of black holes, while in VG they are mergers of point-like collapsed objects described by the nonsingular exponential metric (12).

One should note that the exponential metric (12), produced by the collapsed objects in VG, resembles the black hole spacetime even at a distance somewhat close to the gravitational radius (see Fig. 6 in [7]), and the accuracy of current observations is insufficient to distinguish between the exponential and Schwarzschild geometries. For example, as it was shown in Figs. 7 and 8 of Ref. [7], gravitational waveforms detected by LIGO-Virgo interferometers can be quantitatively explained within the experimental uncertainty as being produced by a merger of two compact objects in the exponential geometry.

Accretion disks of hot baryonic matter encircle supermassive compact objects at galactic centers. The inner radius of the accretion disk represents the innermost orbit matter can be in before plunging into the center. In the curvature coordinates, the innermost stable circular orbit for massive particles in the Schwarzschild geometry is $r_s = 6M$. For the exponential metric (12) of VG, $r_s = 6.34M$ [21], which is close to that for the black hole. Accretion disks around supermassive objects at the center of the Milky Way and M87 galaxies have been imaged by the EHT [43,44]. However, the image angular resolution is insufficient to distinguish between the close predictions for the innermost stable orbit radius in the black hole and the exponential spacetimes (see Fig. 7).

A similar situation takes place with gravitational lensing and shadows of the compact objects produced by the gravitational bending of light. For example, the shadow size in the Schwarzschild geometry is $R_{sh} = 3\sqrt{3}M \approx 5.2M$, while for the exponential spacetime it is only 4% larger $R_{sh} = 5.4M$ [45]. The radii of the photon sphere also differ by 4% [45]. From the EHT observation of SgrA*, the 2σ constraint on the shadow radius is about 40%

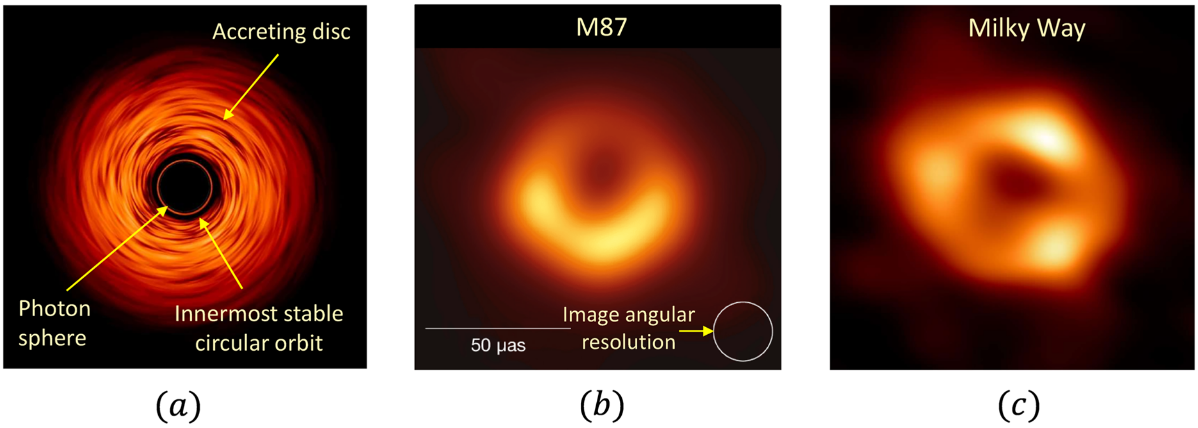

Fig. 7 **a** Sketch of the luminous accretion disk of hot material that encircles a dark compact object. The inner radius of the disk corresponds to the innermost stable orbit, while the bright circle inside the disk is the photon sphere. **b**, **c** EHT images of supermassive objects at the center of M87 (**b**) and the Milky Way (**c**) galaxies with colors indicating the brightness temperature [43,44]. The EHT image angular resolution of 20 micro-arcseconds [shown as a circle in (**b**)] is insufficient to capture the 4% difference between GR and VG predictions for the size of the accreting disk and the radius of the photon sphere

of the radius mean value [46]. Thus, to notice the difference between GR and VG predictions, one should increase the EHT angular resolution by an order of magnitude.

The exponentially large redshift reduces the radiation power coming out from the central region of the collapsed object by a factor of $(1+z)^2$, which mimics black holes. In addition, since scale factor $e^{r_g/2r}$ grows exponentially at small r , accreting matter undergoes exponential expansion and cools down to low temperature, which reduces its luminosity (see Sect. 3.1). Thus, EHT images of the galactic centers can also be explained as the images of compact objects in the exponential geometry, rather than black holes.

10 Summary and outlook

General relativity (GR) has emerged as a somewhat successful model of large-scale gravitation and cosmology. However, GR is unable to describe phenomena on microscopic scales, and is not part of the Standard Model of particle physics. As we show in Introduction, GR predicts that mass of the electron is greater than 1 kg, which is 30 orders of magnitude larger than experimental value.

There exists a viable alternative to GR, the vector theory of gravity (VG), which has been proposed in Refs. [7,8]. It was shown in Ref. [7] that VG passes all available gravitational tests, including the measured radiation power of gravitational waves emitted by binary systems, and gravitational waveforms produced by merger of two compact objects and detected by LIGO-Virgo interferometers. In addition, VG yields— with no free parameters—the value of the cosmological constant that agrees with observations within the 2% experimental uncertainty [7,8]. Thus, VG solves the dark energy problem. Moreover, it has been found that the data on gravitational wave (GW) detection are inconsistent with the tensor GW polarization predictions of GR, while the vector GW polarization, predicted by VG, is supported by the data [12].

Despite these experimental evidence in favor of VG, the theory did not capture attention of the physics community. The reason is that GR is successful in describing many experiments, including those in which gravitational field is not weak. This is considered by many as overwhelming evidence in favor of GR. In Sect. 2, we explained that GR mimics VG in those "successful" occasions, but fails to describe observations in numerous situations when GR and VG predictions are substantially different.

The present paper provides a "smoking gun" in favor of VG. Namely, here we show that VG yields, with no free parameters, for the electron and muon mass, the values that agree with experiment up to the 3% QED self-energy

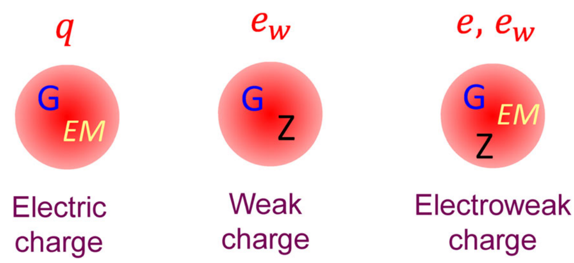

Fig. 8 Charges are the lowest energy bound states of the electromagnetic, weak (Z), and gravitational (G) fields, which are glued together by the gravitational attraction. The elementary charge has a Planck size and at a distance greater than the Planck length behaves as a point electric, weak, or electroweak charge

correction. The latter is not included in our classical analysis. Such striking agreement with experiment indicates that VG gives the correct microscopic description of leptons, and lepton's mass has the gravitoelectromagnetic origin. Thus, VG fits into the particle physics and can explain the inner structure of elementary particles. In contrast to GR, VG predicts no singularities and no black holes. All bound state solutions describing elementary particles we obtained in this paper are regular. This is what is expected from a physical theory.

The present paper is the first step in understanding the inner structure of leptons. Namely, in accurate calculations, we included only the EM and gravitational fields, which yield only two bound state solutions describing the electron and the muon. We showed that bound states describing the tau lepton and W boson appear if we include the weak interaction in our analysis, and estimated their mass by an order of magnitude, which agrees with experiment (see Fig. 9 below). An accurate calculation of the mass of the tau lepton, W boson, and neutrinos in the framework of VG will be the next step.

However, the phenomenological "macroscopic" description of the weak interaction provided by the Standard Model is not very helpful for such calculations, because, in the Standard Model, the W boson, mediating the weak interaction, exists from the beginning. That is, the Standard Model treats the W boson as an elementary particle. Recall that modern phenomenological electroweak theory takes four vector fields as an input, mix them together using symmetries by a non-Abelian gauge theory, and obtain the Z, $W^{\pm}$ bosons and photon as an output. That is, the existence of the W boson-like vector field is assumed from the beginning. In contrast, our analysis shows that the bound state solution corresponding to the W boson arises from the more fundamental fields, and the W boson is a particle composed of these fields.

According to VG, the following picture of elementary particles emerges in the electroweak sector. The fundamental building blocks are four fields that carry no charge—the gravitational, EM, Z, and Higgs fields. The first three are the real vector fields, while the last one is the real scalar field. As we shown in Appendix D, the Z and Higgs fields naturally arise as the fields restoring the gauge symmetry of gravity at low energy, which yields that Higgs field is coupled to the rest mass of particles. That is, quantum excitation of the Higgs field has properties of the Higgs boson discovered in LHC. The mass of the Z and Higgs fields is generated by the Stueckelberg mechanism [41,42], as we discussed in Sect. 8; while the gravitational and EM fields are massless. Elementary quantum excitations of these fields are known as the Z and Higgs bosons, graviton, and photon respectively. The masses of the Z and Higgs fields remain free parameters in our theory, and obtained from experiment.

In VG, the static gravitational, EM, and Z fields can form a nonsingular bound state of Planck size and Planck mass. This bound state is an elementary electroweak charge, which is a very heavy particle (see Fig. 8). At the distance larger than the Planck length, it acts as a point-like charge producing the longitudinal electric and Z fields, described by the Coulomb and the short-range Yukawa potentials, respectively.

The spinning gravitational field can be attached to the elementary electroweak charge, forming a new nonsingular bound state (see Fig. 1). The latter has a lower energy and, thus, the state with a spin is energetically favorable. Similarly to the gravitational field induced by the universe expansion (acting as the dark energy), the spinning gravitational field has a negative energy density and reduces the mass of the charge from the Planck scale to the exponentially small value of elementary particle masses we observe in experiments. Spin is what makes charged particles light. The spinning gravitational field induces magnetic moment of the particle due to the electric field dragging (see Sect. 4.2).

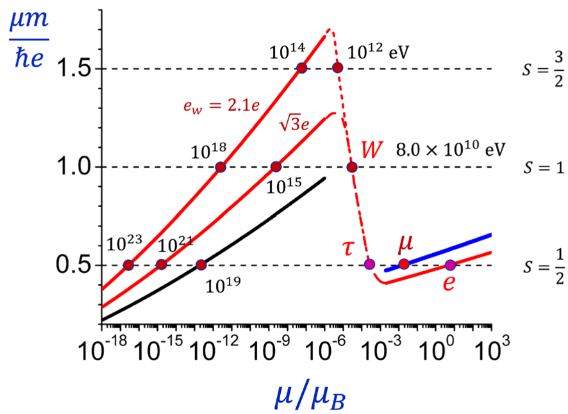

Fig. 9 Extension of Fig. 5 to the region of higher mass (smaller μ) for $e_w = 2.1e$ and $e_w = \sqrt{3}e$. The solid lines are plotted in the validity region of our analysis. Interpolation between the two asymptotes is shown as the red dashed line. The line intersection with the horizontal lines $S = 1/2$, 1 and $3/2$ yield bound states describing charged particles. The number next to the crossing point indicates the mass of the corresponding particles in eV. The intersection of the black solid line with the horizontal dashed line yields a heavy neutrino-like bound state with spin $1/2$

The spinning gravitational field can be attached to the electroweak charge in different bound state configurations corresponding to the three generations of the charged leptons and W boson. Electron is the least massive bound state, which is a stable particle. The muon, tau lepton, and W boson are excited bound states which are unstable. The total energy of the fields forming the bound state yields the mass of the elementary particles. The striking agreement between the calculated electron and muon masses and experiment provides a solid foundation for this physically intuitive result. Thus, particle masses are not generated by the Higgs mechanism.

The static gravitational and Z fields can form a bound state of the Planck size and mass without EM field. This is an elementary weak charge. The spinning gravitational field with negative energy can be attached to the weak charge in different bound state configurations yielding the three generations of neutrinos.

Neutral elementary particles (photon, graviton, Higgs, and Z bosons) are fundamental fields, rather than bound states. They do not behave as point-like objects. In contrast, the charged particles (the elementary fermions and W boson) have a quantum core of Planck size and Planck mass (see Fig. 1), even though their total energy is negligible on the Planck scale. Thus, charged particles can be viewed as point-like constituents of matter. In the present picture, the charged particles are not elementary—they are bound states of the four fundamental fields.

We also find a nonsingular bound state formed solely from the gravitational field, which is the VG alternative to the black holes (see Sect. 9). Since the bound state is nonsingular, and does not have an event horizon, the corresponding objects do not evaporate by emitting Hawking radiation and, thus, can have a very small mass. The static bound state with the smallest (Planck) energy is an elementary charge (we call it u-charge). To reduce the mass, the gravitational field around the u-charge might spin, and the stable bound state might be a fermion. The corresponding particle does not carry electroweak or color charges and interacts only gravitationally. It is the VG prediction for the dark matter particle.

The present approach describes particles at a more fundamental level compared to the Standard Model. Namely, it shows that charged particles, considered as elementary in the Standard Model, are actually bound states formed from the four fundamental fields. Moreover, our theory predicts the existence of bound states (charged particles) that were not yet discovered. The mentioned above dark matter particle is an example.

The theory also predicts new particles in the electroweak sector. To show this, we extend the red line of Fig. 5 to the higher mass (smaller μ). If the main contribution to the mass comes from the region $r \ll r_w$, that is

$$\mu \ll 0.004\mu_B, \tag{128}$$

the mass of the Z field can be disregarded, and the Z field is described by the same equations as the massless EM field. Therefore, Eq. (91) is valid in the limit (128), but now the electric charge e should be replaced with the total electroweak charge of the particle, $e^2 \to e^2 + e_w^2$. However, the weak charge of leptons is not known accurately. Measurement of the muon β decay allows us to estimate the strength of the weak interaction, which yields $e_w \approx 2.1e$. However, this value also includes the contribution due to the exchange of the W bosons. That is, the weak charge producing the Z field is somewhat smaller than $2.1e$.

According to Eq. (98), the weak charge should be greater than $\sqrt{3}e \approx 1.73e$. Otherwise, the weak interaction would not eliminate the muon-like bound state, and the theory would predict an additional lepton having mass between the muon and the tau lepton, in contradiction with experiment. Thus, e_w lies between $1.73e$ and $2.1e$.

Figure 9 shows an extension of the red line of Fig. 5 to the higher mass for $e_w = 2.1e$ and $e_w = 1.73e$. The red dashed line depicts a crossover between the two limits (two regions of validity of Eq. (91)). The intersection of the red curve with the horizontal lines $S/\hbar = 1/2$, 1, 3/2 yields a set of bound states corresponding to charged particles.

Figure shows that, apart from the known particles (the electron, muon, tau lepton, and W boson), there are additional bound states with spin 1/2 and 1. The number next to the crossing points indicates the mass of the new particles in eV. If $e_w = 2.1e$, the theory also predicts two new particles with the spin 3/2. The predicted particles are substantially heavier than the W boson. The lightest among them (the spin 3/2 particle) has a mass of the order of 1 TeV. These new particles are not part of the Standard Model.

Similarly, there are heavy neutrino-like bound states. These particles possess only the weak charge, and in the limit (128), they are described by Eq. (91), in which e is replaced with e_w. The resulting curve is shown as the black solid line in Fig. 9 for $e_w = 1.73e$, which yields a heavy neutrino with the spin 1/2. Extension of the black curve into the region of smaller mass (larger μ) yields bound states describing known light neutrinos, and will be discussed elsewhere.

What about bound states formed from the EM and gravitational fields only? These states should be studied in the models of strong interaction. As we show in Sect. 7, electric charge of such particles is of the order of $q = e/\sqrt{\alpha}$ and, thus, coupling constant $q^2/4\pi\varepsilon_0\hbar c$ is of the order of 1. In this regime, the quantum effects are no longer small, and the quantum correction to the mass is of the order of the mass itself. It has been shown that charges of this magnitude do not obey Coulomb's law due to induced strong vacuum polarization, and can behave as QCD gluons [47,48].

One should also keep in mind that quantum cores of charged particles, described in Sect. 7 can also be in excited states, yielding a larger value of the particle charges. It would be neat if quarks emerge as charged leptons with an excited quantum core, and gluons appear as bound states of the EM and gravitational fields. In this case, the four fundamental fields would also describe the strong interaction, and no new building blocks will be required to explain the whole picture. However, at the moment, this is only a speculation.

Certainly, results obtained in this paper are not the last word in understanding the elementary constituents of matter, and the picture will undergo changes revealing more details. However, our findings, for the first time, make gravity part of the particle physics which, as we see, provides a powerful tool for exploring the particle zoo and predicting new elementary particles not yet discovered.

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Data Availability Statement No new data were generated or analyzed in this study.

Declarations

Conflict of interest The author does not have any conflict of interest to declare.

Appendix A: Gravitational field equations in vector gravity

Equations for the vector gravitational field (the scalar ϕ and the unit vector u_k) in the background Euclidean space were obtained in Ref. [7], and read

$$\begin{aligned}
&\left[\delta^{mk}u^i - 2\delta^{im}u^k + \left(1+3e^{-4\phi}\right)u^m u^k u^i\right]\frac{\partial^2\phi}{\partial x^m \partial x^k}\\
&\quad +2\left[\delta^{im} - \left(3e^{-4\phi}+1\right)u^m u^i\right]\frac{\partial\phi}{\partial x^m}\frac{\partial\phi}{\partial x^k}u^k + 2\left[e^{4\phi}\left(\delta^k_l\delta^{im} - \delta^i_l\delta^{mk}\right) + \delta^i_l\delta^{mk} - \delta^m_l\delta^{ik}\right]\frac{\partial\phi}{\partial x^k}\frac{\partial u^l}{\partial x^m}\\
&\quad +\left[2\left(e^{4\phi} - 2e^{-4\phi} - 1\right)\delta^i_l u^m u^k - \left(1-3e^{-4\phi}\right)\delta^m_l u^i u^k - \left(2e^{4\phi} - 3e^{-4\phi}+1\right)\delta^k_l u^m u^i\right]\frac{\partial\phi}{\partial x^k}\frac{\partial u^l}{\partial x^m}\\
&\quad +\cosh(2\phi)\left[e^{2\phi}\frac{\partial}{\partial x^k}\left(\frac{\partial u^k}{\partial x_i} - \frac{\partial u^i}{\partial x_k}\right) + e^{-2\phi}u_m u^i\frac{\partial^2 u^m}{\partial x_k \partial x^k} + 2\cosh(2\phi)u^k u^l\frac{\partial^2 u^i}{\partial x^l \partial x^k} - \left(e^{2\phi}+2e^{-2\phi}\right)u^m u^i\frac{\partial^2 u^k}{\partial x^k \partial x^m}\right]\\
&\quad +2\cosh^2(2\phi)\left[\frac{\partial u^i}{\partial x^k}\frac{\partial}{\partial x^m}\left(u^k u^m\right) - \frac{\partial u_k}{\partial x_i}\frac{\partial u^k}{\partial x^l}u^l - \frac{\partial u^k}{\partial x^m}\frac{\partial u^m}{\partial x^k}u^i + \left(1+2e^{-4\phi}\right)\frac{\partial u^k}{\partial x^m}\frac{\partial u_k}{\partial x^l}u^m u^l u^i\right]\\
&= \frac{8\pi G}{c^4}\left(T^{ik} - \frac{T}{2}g^{ik}\right)u_k, \qquad \text{(A1)}
\end{aligned}$$

where T^{ik} is the energy-momentum tensor of matter, and $T = T^{mk}g_{mk}$ is the trace of the energy-momentum tensor. Equations (A1) for ϕ and u_k are written in the background Euclidean metric δ_{ik}, which means that raising and lowering of indexes are carried out using δ_{ik} =diag(1, 1, 1, 1). The field equations (A1) and the action (8) are not generally covariant. However, they are invariant under coordinate transformations that leave the background Euclidean metric δ_{ik} intact.

In Eq. (A1), g_{ik} is the equivalent metric which is a functional of the vector gravitational field and the background geometry δ_{ik}

$$g_{ik} = -e^{-2\phi}\delta_{ik} + 2\cosh(2\phi)u_i u_k, \qquad \text{(A2)}$$

while g^{ik} is the metric inverse to g_{ik}, defined as $g^{ik}g_{im} = \delta^k_m$

$$g^{ik} = -e^{2\phi}\delta^{ik} + 2\cosh(2\phi)u^i u^k,$$

$u^i = \delta^{ik}u_k$, $u_k u^k = 1$ and $i, k = 0, 1, 2, 3$.

Equations (A1) are applicable for the problems in which the gravitational field is stationary, that is, they are suitable for the description of elementary particles. However, for the problems involving emission of gravitational waves (GWs), Eqs. (A1) must be modified (see Ref. [7]). The point is that the classical equations (A1) predict that energy of GWs is negative [7]. This appealing property of VG provides a natural mechanism of matter generation at the Big Bang. Universe would not be created if GWs energy would be always positive (unless there is another yet unknown "natural" mechanism). In contrast, GR does not explain how matter was generated at the Big Bang (in GR, the energy of GWs is always positive).

According to VG, just after the Big Bang, matter with positive energy was generated together with GWs that carry negative energy. So that the total energy of the universe is conserved. This instability is analogous to the excitation of the Glauber's inverted oscillators [49,50], and caused the stage of cosmic inflation, leading to the creation of a new field vacuum (stable "true vacuum" state) for which GWs have positive energy (the present epoch) [7]. For a discussion of the Big Bang phase transition from the VG perspective, see Ref. [8].

Appendix B: Calculation of the electron and muon masses

Introducing a new function f according to

$$\phi = -\ln(1+f), \tag{B1}$$

Eq. (90) can be written as

$$(\nabla f)^2 - (1+f)\nabla^2 f = \frac{a^2}{r^4} + \frac{b^4}{r^6}\left(1 + 3\cos^2\theta\right). \tag{B2}$$

The function f has the asymptote $f \to 0$ for $r \to \infty$. Taking into account that f depends only on r and θ, in spherical coordinates r, θ, φ, we have

$$(\nabla f)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2,$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right).$$

Adopting b^2/a as a unit of length, one can rewrite Eq. (B2) as

$$r^2\left[(\nabla f)^2 - (1+f)\,\nabla^2 f\right] = \frac{1}{\gamma^2}\left(\frac{1}{r^2} + \frac{1}{r^4}\left(1 + 3x^2\right)\right), \tag{B3}$$

where

$$\gamma = \frac{b^2}{a^2} \ggg 1,$$

and $x = \cos\theta$.

As we show below, one can divide the radial distance r into three characteristic regions $r \lesssim 1/\sqrt{\gamma}$, $1/\sqrt{\gamma} \lesssim r \lesssim 1/\sqrt{\ln(\gamma)}$ and $r \gtrsim 1/\sqrt{\ln(\gamma)}$. Solution of Eq. (B3) in the outer region is

$$f(r,x) = \frac{1}{\gamma^2}\left(\frac{M}{r} - \frac{1}{2r^2} - \frac{x^2}{2r^4}\right), \qquad r \gtrsim \frac{1}{\sqrt{\ln(\gamma)}}, \tag{B4}$$

where M is the particle mass in units c^2a^3/Gb^2 that we need to find.

In the region $r \lesssim 1/\sqrt{\ln(\gamma)}$, we look for a solution of Eq. (B3) as an expansion in r, which yields

$$f(r,x) \approx \frac{px}{\gamma r^2} + \frac{1}{4p\gamma}\ln\left(\frac{1+x}{1-x}\right) + \frac{\left(p^2-1\right)x^2}{2\gamma^2 r^4} + f_1(r,x), \tag{B5}$$

where the function $f_1(r,x)$ vanishes at $r \to 0$ as $f_1 \propto r$, and we kept terms upto $1/\gamma^2$. In Eq. (B5), $p = 1$ for the electron and $p = \sqrt{3}/2$ for the muon (see Eqs. (88) and (89)). Plug Eq. (B5) in Eq. (B3) yields the following equation for f_1:

$$r^2\nabla^2 f_1 = \frac{1}{4p^2\gamma^2\left(1-x^2\right)}. \tag{B6}$$

The first two terms in the right-hand-side of Eq. (B5) are odd functions of x. Thus, averaging f over the angles yields

$$\bar{f}(r) = \frac{p^2-1}{6\gamma^2 r^4} + \bar{f}_1(r), \tag{B7}$$

and for the averaged function

$$\bar{f}_1(r) = \frac{1}{2}\int_{-1}^{1} dx f_1(r,x),$$

we find equation

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \bar{f}_1}{\partial r}\right) = \frac{1}{4p^2\gamma^2}\int_0^1 \frac{dx}{1-x^2} \approx \frac{1}{8p^2\gamma^2}\ln\left(\frac{1+x_{\max}}{1-x_{\max}}\right), \tag{B8}$$

where $x_{\max}$ is the cut-off value of x which is obtained from the condition of the validity of the perturbation expansion. Namely, the right-hand-side of Eq. (B6) should be smaller than the right-hand-side of Eq. (B3). The latter condition yields

$$\frac{1}{4\gamma^2\left(1-x^2\right)} \lesssim \frac{1}{\gamma^2 r^4}\left(1+3x^2\right),$$

or

$$1 - x_{\max} = \frac{r^4}{32}.$$

Thus, Eq. (B8) for $\bar{f}_1$ reduces to

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \bar{f}_1}{\partial r}\right) = \frac{1}{p^2\gamma^2}\left(\frac{3}{4}\ln(2) - \frac{1}{2}\ln(r)\right). \tag{B9}$$

Solution of Eq. (B9) is

$$\bar{f}_1 = \frac{1}{p^2\gamma^2}\left[\left(\frac{3}{4}\ln(2)+\frac{1}{2}\right)\ln(r) - \frac{1}{4}\ln^2(r) + C\right], \tag{B10}$$

where C is an integration constant, and we disregarded a diverging term $1/r$. The integration constant C is obtained by matching the solution (B10) with the inner region solution

$$\bar{f}_1(r) \propto r, \qquad r \lesssim \frac{1}{\sqrt{\gamma}}. \tag{B11}$$

Matching implies that functions $\bar{f}_1$ and $\partial \bar{f}_1/\partial r$ should be continuous at the matching point $r = 1/\sqrt{\gamma}$. This yields

$$C = \frac{1}{2}\left(\frac{1}{8}\ln^2(\gamma) + \left(\frac{3}{4}\ln(2)+1\right)\ln(\gamma) + \frac{3}{2}\ln(2)+1\right). \tag{B12}$$

To find M, one should match solution (B7), where $\bar{f}_1$ is given by Eq. (B10), with the outer asymptote (B4), which after the angular averaging yields

$$\bar{f} \approx \frac{1}{\gamma^2}\left(\frac{M}{r} - \frac{1}{2r^2} - \frac{1}{6r^4}\right).$$

We denote the matching point as r_0. The unknown parameters M and r_0 are obtained from the requirement that $\bar{f}$ and $\partial \bar{f}/\partial r$ are continuous at $r = r_0$, which gives two equations

$$\frac{M}{r_0} - \frac{1}{2r_0^2} - \frac{p^2}{6r_0^4} = \frac{1}{p^2}\left[\left(\frac{3}{4}\ln(2)+\frac{1}{2}\right)\ln(r_0) - \frac{1}{4}\ln^2(r_0) + C\right],$$

$$-\frac{M}{r_0} + \frac{1}{r_0^2} + \frac{2p^2}{3r_0^4} = \frac{1}{p^2}\left[\frac{3}{4}\ln(2) + \frac{1}{2} - \frac{1}{2}\ln(r_0)\right].$$

In the limit $C \gg 1$, one can write solution of these equations as an expansion in $1/C$. Keeping the first two terms in the expansion, we obtain

$$\frac{r_0}{p} \approx \frac{1}{(2C)^{1/4}} + \frac{2^{1/4}}{8C^{3/4}}, \qquad pM \approx \frac{2}{3}(2C)^{3/4} + \frac{C^{1/4}}{2^{3/4}}.$$

Plug here Eq. (B12), we find the following expression for the particle mass as an expansion in $\ln(\gamma)$:

$$M \approx \frac{2^{3/4}}{4p}\left[\frac{1}{3}\ln^{3/2}(\gamma) + \left(\frac{3}{2}\ln(2) + 2 + \frac{1}{\sqrt{2}}\right)\ln^{1/2}(\gamma)\right].$$

Appendix C: Bound state of free gravitational field with zero spin

Due to expansion of the universe shortly after the Big Bang, the spatial scale $a = e^{-\phi}$ has been magnified in an exponentially large factor, and in the present epoch $e^{-\phi} \ggg 1$. As we showed in the main text, for elementary particles with spin, ϕ changes substantially near the particle center, and the condition $e^{-\phi} \gg 1$ is not satisfied.

Here, we consider a spinless particle, and assume that $e^{-\phi} \ggg 1$ everywhere. Then, one can disregard the exponentially small terms e^{ϕ} compared to the exponentially large terms of the order of $e^{-\phi}$, and obtain equations for the gravitational field which are invariant under transformations [8]

$$\phi \to \phi + \phi_0, \quad \mathbf{r} \to e^{\phi_0}\mathbf{r}, \quad t \to e^{-\phi_0}t, \quad \mathbf{u} \to e^{2\phi_0}\mathbf{u}, \tag{C1}$$

where ϕ_0 is an arbitrary constant. We choose ϕ_0 be equal to the value of the gravitational potential ϕ in the present epoch; then, ϕ vanishes far away from the particle center.

We further assume that $u \ll 1$, and consider a longitudinal gravitational field ($\mathrm{curl}\mathbf{u} = 0$) independent of t. Then, keeping terms linear in u, Eq. (21) of Ref. [8] for the free field reduces to

$$(\nabla\phi \cdot \nabla)\,\mathbf{u} + [(\mathbf{u}\cdot\nabla)\,\phi - \mathrm{div}\mathbf{u} - (\mathbf{u}\cdot\nabla)]\,\nabla\phi = 0, \tag{C2}$$

while Eq. (20) of Ref. [8], up to the terms quadratic in u, reads

$$\nabla^2\phi + 3e^{-4\phi}\left[(\mathbf{u}\cdot\nabla) + \mathrm{div}\mathbf{u} - 2(\mathbf{u}\cdot\nabla)\phi\right](\mathbf{u}\cdot\nabla)\phi - e^{-4\phi}\mathrm{div}\left[(\mathbf{u}\cdot\nabla)\mathbf{u}\right] = 0. \tag{C3}$$

A nonsingular solution of Eqs. (C2) and (C3) gives a bound state of the free gravitational field with zero spin. We will look for a spherically symmetric solution in the form

$$\phi = \phi(r), \quad \mathbf{u} = u(r)\hat{r},$$

where $\hat{r}$ is the radial unit vector. Then, Eq. (C2) becomes

$$\phi' u' + \left[u\phi' - \frac{1}{r^2}\left(r^2 u\right)'\right]\phi' - u\phi'' = 0,$$

or

$$\left(\phi'\right)^2 = \frac{2}{r}\phi' + \phi'', \tag{C4}$$

where the prime denotes the derivative with respect to r. Equation (C4) has the following solution satisfying the boundary condition $\phi(\infty) = 0$:

$$\phi = -\ln\left(1 + \frac{M}{r}\right), \tag{C5}$$

where $M > 0$ is an integration constant equal to the mass of the bound state.

On the other hand, for the spherical symmetry, Eq. (C3) becomes

$$\frac{e^{4\phi}}{r^2}\left(r^2\phi'\right)' + 3u\left(u\phi'\right)' + \frac{3}{r^2}\left(r^2 u\right)' u\phi' - 6\left(u\phi'\right)^2 - \frac{1}{r^2}\left(r^2 u u'\right)' = 0. \tag{C6}$$

Plug here Eq. (C5), yields

$$r^2\left(uu'\right)' - \frac{r(4M - 2r)}{r + M}uu' + \frac{M^2\left[3(r+M)^4 u^2 - r^4\right]}{(r+M)^6} = 0. \tag{C7}$$

Solution of Eq. (C7), satisfying the proper boundary conditions, reads

$$u = \pm\frac{Mr}{(r+M)^2}. \tag{C8}$$

Equations (C5) and (C8) describe a bound state of the free gravitational field with zero spin. For this solution, the spacetime metric is nonsingular. The size of the bound state (C8) is $\approx M$.

In the classical treatment of the problem, M can have any positive value. In the quantum treatment, the bound state describing elementary excitation has a finite mass of the order of Planck mass.

Appendix D: Classical theory of weak interaction

In a self-consistent theory of fundamental forces, the necessity for having fields describing weak interaction arises from gravity. Here, we show that such fields naturally appear in any metric theory of gravity, including GR and VG.

In the post-Newtonian limit, a small deviation of the metric g_{ik} from the Minkowski metric is described by four functions, which we denote as ϕ and the three-dimensional vector $\mathbf{u}$. In terms of these functions, the square of the interval reads

$$ds^2 = (1+2\phi)\, c^2 dt^2 - (1-2\phi)\, d\mathbf{r}^2 + 2c\mathbf{u}\cdot d\mathbf{r}dt. \tag{D1}$$

Let us consider a point particle with the rest mass m moving with a small velocity $\mathbf{V}$. In Newtonian gravity, equation for the gravitational field reads

$$\nabla^2\phi = \frac{4\pi G}{c^2} m\delta(\mathbf{r}-\mathbf{r}_0(t)), \tag{D2}$$

where $\mathbf{r}_0(t)$ is the particle trajectory, and $c^2\phi$ is the gravitational potential. In metric theories, the source of gravity is energy, rather than the rest mass. Therefore, we need to take into account that in the presence of gravitational potential the energy of the particle changes. Namely, in the linear approximation, the change is

$$mc^2 \to mc^2(1+\phi),$$

and Eq. (D2) becomes

$$\nabla^2\phi = \frac{4\pi G}{c^2} m(1+\phi)\delta(\mathbf{r}-\mathbf{r}_0(t)). \tag{D3}$$

For a slow motion, the mass of the particle is approximately conserved. Thus, in this limit, equations should be gauge invariant. To make the left-hand-side gauge invariant, Eq. (D3) is replaced with

$$\mathrm{div}\left(\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{u}}{\partial t}\right) = \frac{4\pi G}{c^2} m(1+\phi)\delta(\mathbf{r}-\mathbf{r}_0(t)), \tag{D4}$$

where $\mathbf{u}$ is the three-dimensional vector entering the metric (D1), which in the post-Newtonian limit obeys equation

$$\nabla^2\mathbf{u} - \nabla\mathrm{div}\mathbf{u} + \frac{4}{c^2}\nabla\frac{\partial\phi}{\partial t} = \frac{16\pi G}{c^3} m\mathbf{V}\delta(\mathbf{r}-\mathbf{r}_0(t)). \tag{D5}$$

Here, we disregarded terms with the second-order time derivative. Equations (D4) and (D5) describe the post-Newtonian limit of GR and VG (see, e.g., Refs. [7,9]).

The left-hand side of Eqs. (D4) and (D5) is invariant under the gauge transformation $\phi \to \phi + \partial f/\partial t$, $\mathbf{u} \to \mathbf{u} + c\nabla f$ upto the terms first order in the time derivative. However, the right-hand side of Eq. (D4) is not. To restore the gauge symmetry at slow motion, we need to introduce two additional fields—a real scalar field ψ, and a real four-vector field Z_k (Z field). Paying the tribute to the history, we call ψ the Higgs field, even though the Higgs field of the Standard Model is a two-component complex scalar, rather than a one-component real field.

The scalar field ψ is introduced into the right-hand side of Eq. (D4) as

$$\mathrm{div}\left(\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{u}}{\partial t}\right) = \frac{4\pi Gm}{c^2}(1+\phi+\psi)\delta(\mathbf{r}-\mathbf{r}_0(t)), \tag{D6}$$

which makes the equation gauge invariant. Equation (D6) shows that the Higgs field ψ couples to the rest mass m, and hence, the rest mass is the source of the Higgs field. Equation (D6) is valid for small ϕ and ψ. For large values, according to VG, the combination $1+\phi+\psi$ should be replaced with $e^{\phi+\psi}$.

Introduction of the vector field Z_k is necessary to make the equation for ψ gauge invariant. Weak charges (bound states of the Z and gravitational fields) are sources of the Z field, but there are no Higgs charges (see Sect. 3.3.3). Physically, ψ and Z_k arise as fields restoring the gauge symmetry of gravity in the limit of slow motion.

Since for the problems involving weak interaction, gravity usually produces tiny effect, in the following, we disregard gravity and construct a classical theory of the weak interaction keeping in mind that the Higgs field couples to the rest mass. As a classical field theory, the weak interaction is described by a real four-vector field Z_k and a real scalar field ψ in Minkowski spacetime. Let us consider a particle with the rest mass m and the weak charge q_w moving in the Minkowski spacetime along the trajectory $\mathbf{r} = \mathbf{r}_0(t)$, and assume that the weak charge is conserved. By analogy with electrodynamics, we introduce a four-vector of the weak current as

$$j_w^k = \rho_w \frac{\partial x^k}{\partial t} = (c\rho_w, \mathbf{j}_w),$$

where ρ_w and $\mathbf{j}_w = \rho_w \mathbf{V}$ are the weak charge and the weak current densities, respectively. Disregarding gravity, the total Lorentz invariant action reads

$$S = -\frac{c^3}{16\pi G_\psi}\int d^4x \left(\frac{\partial Z^\nu}{\partial x_\mu} - \frac{\partial Z^\mu}{\partial x_\nu}\right)\left(\frac{\partial Z_\nu}{\partial x^\mu} - \frac{\partial Z_\mu}{\partial x^\nu}\right) + \frac{c^3}{8\pi G_\psi}\int d^4x \left(\frac{\partial \psi}{\partial x_k} - \frac{m_Z c}{\hbar} Z^k\right)\left(\frac{\partial \psi}{\partial x^k} - \frac{m_Z c}{\hbar} Z_k\right)$$
$$-\int d^4x Z_k j_w^k - c\int d^4x e^\psi \rho_m \sqrt{1 - \frac{V^2}{c^2}}, \tag{D7}$$

where $\rho_m = m\delta(\mathbf{r} - \mathbf{r}_0(t))$ is the rest mass density, m_Z is the mass of the Z field,

$$Z^i = \eta^{ik} Z_k,$$

and η_{ik} =diag$(1, -1, -1, -1)$ is Minkowski metric.

The last term in the action (D7) describes coupling between ρ_m and ψ. This term is not gauge invariant. A gauge invariant coupling of ψ to the rest mass requires gravity, and holds only at low energy. Thus, variation of the action (D7) must be carried out in a fixed gauge. As we showed in Sect. 8, the mass of the Higgs field m_ψ is determined by the choice of the gauge. We add the following gauge fixing term to the action:

$$\frac{c^4 m_Z}{4\pi\hbar G_\psi}\int d^4x \Lambda \left(\frac{\partial Z^k}{\partial x^k} + \frac{m_\psi^2 c}{m_Z \hbar}\psi\right),$$

where $\Lambda(t, \mathbf{r})$ is the Lagrange multiplier.

Variation of the action with respect to Z_k, ψ, and Λ yields the following equations:

$$\Box Z_k + \frac{m_Z^2 c^2}{\hbar^2} Z_k - \frac{\partial}{\partial x^k}\left[\frac{\partial Z^i}{\partial x^i} + \frac{m_Z c}{\hbar}(\psi + \Lambda)\right] = \frac{4\pi G_\psi}{c^3} j_{wk}, \tag{D8}$$

$$\Box \psi - \frac{m_Z c}{\hbar}\frac{\partial Z^i}{\partial x^i} - \frac{m_\psi^2 c^2}{\hbar^2}\Lambda = -\frac{4\pi G_\psi}{c^2} e^\psi \rho_m \sqrt{1 - \frac{V^2}{c^2}}, \tag{D9}$$

$$\frac{\partial Z^i}{\partial x^i} + \frac{m_\psi^2 c}{m_Z \hbar}\psi = 0, \tag{D10}$$

where we introduced d'Alembert operator

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2.$$

One can rewrite Eqs. (D9) and (D10) as

$$\Box\psi + \frac{m_\psi^2 c^2}{\hbar^2}(\psi - \Lambda) = -\frac{4\pi G_\psi}{c^2} e^\psi \rho_m \sqrt{1 - \frac{V^2}{c^2}}, \tag{D11}$$

$$\Box\Lambda + \frac{m_\psi^2 c^2}{\hbar^2}\Lambda = \frac{4\pi G_\psi}{c^2} e^\psi \rho_m \sqrt{1 - \frac{V^2}{c^2}}. \tag{D12}$$

For the free field, Eqs. (D8), (D11) and (D12) have solutions in the form

$$\Box Z_k + \frac{m_Z^2 c^2}{\hbar^2} Z_k = 0, \qquad \frac{\partial Z^i}{\partial x^i} = 0, \qquad \psi = \Lambda = 0,$$

describing the massive vector field Z_k with the mass m_Z, and

$$\Box\psi + \frac{m_\psi^2 c^2}{\hbar^2}\psi = 0, \qquad \Lambda = 0,$$

describing the massive scalar field with the mass m_ψ.

For a point mass m carrying a weak charge q_w, and located at $r = 0$, the static solution of Eqs. (D8), (D11), and (D12) in the spherical coordinates reads

$$\psi = \frac{G_\psi m}{c^2}\left(\frac{m_\psi c}{2\hbar} - \frac{1}{r}\right) e^{-cm_\psi r/\hbar}, \quad \Lambda = \frac{G_\psi m}{c^2 r} e^{-cm_\psi r/\hbar},$$

$$Z_r = -\frac{G_\psi m m_\psi^2}{2c\hbar m_Z} e^{-cm_\psi r/\hbar}, \quad Z_0 = \frac{G_\psi q_w}{c^2 r} e^{-cm_Z r/\hbar}, \tag{D13}$$

which represents a short-range interaction.

Similarly to the classical electrodynamics, in the present model, the weak interaction is described by the classical fields ψ and Z_k which carry no charge. In such classical picture, there is no exchange of virtual bosons between charges. Instead, the weak charge generates the longitudinal Z field (see Eq. (D13)), which applies a force to another weak charge. Since the Z field is massive, the interaction is short range.

In the quantum treatment, there is an additional contribution to the weak interaction due to the exchange of virtual pairs of charged $W^\pm$ bosons. Since the mass of the W boson is comparable to that of the Z field, this interaction channel gives a comparable contribution.

The action (D7), and Eqs. (D8), (D11), and (D12) are written from the perspective of gravity. That is, the weak charge q_w is measured in kilograms, and ρ_w has units kg/m^3. To make connection with electrodynamics, we introduce

$$\tilde{Z}_k = \frac{Z_k}{\beta}, \quad \tilde{\rho}_w = \beta\rho_w, \quad \text{where} \quad \beta = \sqrt{\frac{4\pi G_\psi \varepsilon_0}{c}}.$$

Then, the weak charge is measured in Coulombs, while $\tilde{\rho}_w$ has units C/m^3. $\tilde{Z}_k$ is analogous to the electromagnetic four-potential, and in the EM units, the solution (D13) looks like a short-range Coulomb's law

$$c\tilde{Z}_0 = \frac{q_w}{4\pi\varepsilon_0 r} e^{-cm_Z r/\hbar}.$$

In the following, to simplify notations, we omit tildes. In the EM units, the action (D7) reads

$$S = -\frac{1}{4\mu_0}\int d^4x \left(\frac{\partial Z^\nu}{\partial x_\mu} - \frac{\partial Z^\mu}{\partial x_\nu}\right)\left(\frac{\partial Z_\nu}{\partial x^\mu} - \frac{\partial Z_\mu}{\partial x^\nu}\right) - \int d^4x Z_k j_w^k - c\int d^4x e^\psi \rho_m \sqrt{1-\frac{V^2}{c^2}}$$
$$+\frac{1}{2\mu_0}\int d^4x \left(\frac{1}{\beta}\frac{\partial\psi}{\partial x_k} - \frac{m_Z c}{\hbar} Z^k\right)\left(\frac{1}{\beta}\frac{\partial\psi}{\partial x^k} - \frac{m_Z c}{\hbar} Z_k\right) + \frac{c^4 m_Z}{4\pi\hbar G_\psi}\int d^4x \Lambda \left(\beta\frac{\partial Z^k}{\partial x^k} + \frac{m_\psi^2 c}{m_Z \hbar}\psi\right), \tag{D14}$$

where $\mu_0 = 1/\varepsilon_0 c^2$. The first two terms in Eq. (D14) coincide with the action for the EM field.

Equation (D8) now becomes

$$\Box Z_k + \frac{m_Z^2 c^2}{\hbar^2} Z_k - \frac{\partial}{\partial x^k}\left[\frac{\partial Z^i}{\partial x^i} + \frac{m_Z c}{\hbar\beta}(\psi + \Lambda)\right] = \mu_0 j_{wk}.$$

Variation of the action (D14) with respect to the particle's trajectory yields the following equation of motion for the weak charge q_w with the rest mass m in the four-dimensional form:

$$mc\frac{d}{ds}\left(e^\psi \frac{dx_i}{ds}\right) = mce^\psi \frac{\partial\psi}{\partial x^i} + q_w\left(\frac{\partial Z_k}{\partial x^i} - \frac{\partial Z_i}{\partial x^k}\right)\frac{dx^k}{ds},$$

where $ds = cdt\sqrt{1-\frac{V^2}{c^2}}$. Introducing particle's generalized momentum ($\mathbf{V} = dx^\alpha/dt$)

$$\mathbf{p} = \frac{e^\psi m\mathbf{V}}{\sqrt{1-\frac{V^2}{c^2}}},$$

one can rewrite the equation of motion in the three-dimensional form as

$$\frac{d\mathbf{p}}{dt} = -\sqrt{1-\frac{V^2}{c^2}} mce^\psi \nabla\psi + q_w\left(\mathbf{E}_Z + \mathbf{V}\times \mathrm{curl}\mathbf{Z}\right), \tag{D15}$$

where

$$\mathbf{E}_Z = -c\nabla Z_0 - \frac{\partial \mathbf{Z}}{\partial t}.$$

The last term in Eq. (D15) is the Lorentz force acting on the weak charge.

To compare the interaction strength produced by the Higgs field, one can introduce an equivalent charge q_m corresponding to the rest mass m. For the electron, we have

$$q_{m_e} = \beta m_e = \sqrt{\frac{4\pi G_\psi \varepsilon_0}{c}} m_e = 4.0 \times 10^{-10} e,$$

where we used Eq. (124) for G_ψ. This value should be compared with the weak charge of the electron $e_w \approx 2.1e$.

Even for the heaviest lepton, the equivalent charge

$$q_{m_\tau} = 1.4 \times 10^{-6} e$$

is much smaller than the electric charge. As a result, the Higgs field gives a negligible contribution to the interaction between leptons. If so, one can disregard ψ, which reduces the equation for the Z field to the Proca equation [51]

$$\Box Z_k - \frac{\partial}{\partial x^k} \frac{\partial Z^i}{\partial x^i} + \frac{m_Z^2 c^2}{\hbar^2} Z_k = \mu_0 j_{wk}.$$

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