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Method to test Lorentz invariance in electron-capture decay by measuring a neutrino recoil force

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Abstract. Due to hypothetical Loretz invariance violation, additional terms arise in the differential rate for neutrino radiation accompanying electron capture by polarized nuclei. These terms, as well as the parity-violating term, can be probed by measurement of a small recoil force acting on a radioactive sample. An expression for this force is obtained for the case of allowed Gamow–Teller transitions. We discuss prospects to measure the force by using the methods of the magnetic resonance force microscopy, present a list of the most suitable isotopes and give the numerical estimates for mass and activity of required radioactive samples.

Introduction

Recently a novel way to test Lorentz invariance in the weak interaction and, in particular, in orbital electron capture was proposed [1, 2]. The Lorentz invariance violation is taken into account by an addition of a general tensor $\chi^{\mu\nu}$ to the Minkowski metric $g^{\mu\nu}$. For electron capture by polarized nuclei, new terms appear in the differential probability of neutrino emission. The terms related both to Lorentz-violation and nuclear polarization are of special interest because they can be probed by switching on and off the polarization. Keeping only polarization-sensitive terms and considering pure Gamow–Teller transition $J_i \to J_f = J_i \pm 1$ (J_i and J_f are spins of parent and daughter nuclei) to the nth state of the daughter nucleus, one obtains [2]:

$$\frac{dw_{nEC}}{d\Omega} = \frac{w_{nEC}}{4\pi}\left(1 + B_n P\left(\mathbf{n}_\nu \mathbf{n}_J + \chi_r^{jk} n_{\nu j} n_{Jk} + \chi_i^{l0}\left[\mathbf{n}_\nu \times \mathbf{n}_J\right]_l\right)\right), \quad (1)$$

where $\mathbf{n}_\nu$ and $\mathbf{n}_J$ are unit vectors along neutrino momentum and nuclear polarization axis, P is the nuclear polarization, χ_r^{jk} and χ_i^{l0} $(j,k,l = 1,2,3)$ are real and imaginary parts of the components of a complex tensor $\chi^{\mu\nu}$ which parametrizes Lorentz violation, w_{nEC} is the electron-capture decay rate, while $B_n = J_i/(J_i+1)$ for $J_f = J_i + 1$ and $B_n = -1$ for $J_f = J_i - 1$ is the asymmetry coefficient. The term $B_n P\,\mathbf{n}_\nu \mathbf{n}_J$ describes the asymmetry of neutrino emission along and opposite nuclear polarization due to parity violation in the weak interaction; this term was first obtained in Ref. [3].

As a result of momentum conservation, there is a similar asymmetry for the recoil momenta of atoms. Clearly, the recoil momenta are transferred to the sample if the radioactive atoms are bound in it. Thus, for polarized nuclei, a recoil force emerges, acting on the sample as a whole. In Ref. [4] it was noted that such a force can be detected using an atomic force microscope. This means that the additional asymmetry terms in (1) may be probed as additional components of the recoil force.

In Ref. [5] we consider prospects to measure the neutrino recoil force by the use of modern micromechanical devices. We present numerical estimates for the force for a number of most suitable radioactive isotopes and specify potential applications for the weak interaction studies. In this work we discuss only the possibility to use this method to search for hypothetical Lorentz invariance violation.

Neutrino recoil force

Generally, an atom with a neutron-deficient nucleus is unstable with respect to electron capture and β^+-decay; let I_{nEC} and $I_{n\beta+}$ be the corresponding branching ratios for transition to the nth state of the daughter nucleus ($\sum_n I_{nEC} + \sum_n I_{n\beta+} = 1$). The decay rate $w_{nEC} = I_{nEC} \ln 2/T_{1/2}$ for the electron-capture transition is determined by the branching ratio I_{nEC} and by the half-life $T_{1/2}$ of the radioactive atom. Note that a sample containing N radioactive atoms has the activity $\alpha = N \ln 2/T_{1/2}$.

The z-component (the z axis is along $\mathbf{n}_J$) of recoil force $F_{nz} = \Delta P_{nz}/\Delta t$ is determined by the momentum

$$\Delta P_{nz} = -N\Delta t \oint \frac{E_{\nu n} \cos\theta}{c}\, dw_{nEC}(\theta), \tag{2}$$

transferred to the sample during the time Δt (in this calculation we take $\chi^{\mu\nu}$ equal to zero). The formula involves the averaged energy of the emitted neutrino $E_{\nu n}$, so that $p_{nz} = E_{\nu n}\cos\theta/c$ is the z-component of momentum for the neutrino emitted at the angle θ. Substituting the differential rate $dw_{nEC}(\theta)$ (1) into Eq. (2) and integrating over $d\Omega$, one obtains:

$$F_{nz} = -\frac{N I_{nEC} \ln 2\, E_{\nu n} B_n P}{3\, c T_{1/2}} = -\frac{\alpha I_{nEC} E_{\nu n} B_n P}{3\, c}\,. \tag{3}$$

Note that our value for the recoil force is three times lower than that obtained in Ref. [4].

In a constant magnetic field B at a temperature T, the polarization P arises from the Boltzmann distribution of nuclear states. Since a nuclear magnetic moment μ is of the order of the nuclear magneton μ_N, the value of $\beta = \mu B/(k_B T)$ is small even in a strong magnetic field B at a relatively low temperature T. Indeed, taking $B_0 = 1$ T and $T_0 = 1$ K one gets $\beta_0 \equiv \mu_N B_0/(k_B T_0) = 3.658 \cdot 10^{-4}$. In the case of $\beta \ll 1$, the polarization P and the z-component of the sample magnetic moment $M_z = N\mu P$ take the form

$$P \simeq \frac{\beta(J_i+1)}{3J_i}\,, \quad M_z \simeq \frac{N J_i(J_i+1)\hbar^2\gamma^2 B}{3k_B T}\,, \tag{4}$$

where $\gamma = \mu/(\hbar J_i)$ is the nuclear gyromagnetic ratio. Using Eq. (4) for the polarization P, we rewrite the recoil force (3) acting on a sample, which consists of one sort of radioactive atoms and has a mass m, as follows:

$$F_{nz} = -m\, \frac{B[T]}{T[K]}\, C_n f_n, \tag{5}$$

where $B[T]$ is the magnetic field measured in Tesla (T), $T[K]$ is the temperature measured in Kelvin (K), the coefficient $C_n = B_n(J_i+1)/J_i$ is determined by the initial and final nuclear spins and by the transition type, while the force parameter

$$f_n = \frac{\beta_0\, I_{nEC} \ln 2}{9\, T_{1/2}} \cdot \frac{E_{\nu n}}{m_a c} \cdot \frac{\mu}{\mu_N}, \tag{6}$$

depends on the transition characteristics and the initial atom properties, in particular, on its mass m_a.

The coefficient B_n (as well as the coefficient C_n) can be either positive or negative; therefore, the contributions from different transitions, generally speaking, will partially cancel each other. Because of this, the maximal force corresponds to the situation, when there is only one selected allowed transition with the branching ratio $I_{nEC} \geq 0.98$ (the contribution to the recoil force from neglected transitions will not exceed $\sim 2\%$, which is comparable with other uncertainties).

Prospects to measure a neutrino recoil force

The key element of an atomic force microscope is the cantilever, a micromechanical beam of length l, width w and thickness t, made of a material with Young's modulus E clamped at one end and with a tip at the other one (see, e.g., [6]). The force F acting on the tip and its displacement z are related by Hooke's law $z = F/k$, where the spring constant is given by $k \simeq Ewt^3/(4l^3)$.

When atomic force microscope is operated in noncontact mode, the tip on the free end oscillates with the fundamental frequency of the cantilever ω_c, while placed at a distance from the surface; this allows to measure very small forces. In one of the versions of the magnetic resonance force microscopy (MRFM) [6, 7, 8], a sample with a magnetic moment M_z is attached to a cantilever; the force acting on the sample results from a gradient $\nabla B(z)$ of inhomogeneous magnetic field. Note, that magnetic moment of the sample is due either to unpaired electrons or to nuclei with non-zero spins (and magnetic moments). Thus, the methods of electron paramagnetic resonance or nuclear magnetic resonance (NMR) can be applied. Namely, affecting the sample by a specifically modulated oscillating magnetic field (for NMR, with a frequency $\omega_{\rm rf}$ close to $\omega_{\rm NMR} = \gamma B$), one induces oscillations of magnetic moment M_z with the modulation frequency ω. In the case of NMR, one uses the method of cyclic adiabatic inversion (see details in Ref. [8]). Then the force

$$F_z = M_z \nabla B(z) \tag{7}$$

oscillates with the same frequency ω. When $\omega = \omega_c$, we get a resonance at which the amplitude of the cantilever displacement caused by the force of amplitude F_0 reaches its maximal value $x_0 = QF_0/k$, where Q is the quality factor. Hence, with a fixed accuracy of displacement measurement, the sensitivity to the force increases by a factor of Q. This is one of the methods of magnetic resonance registration, used in MRFM.

For our purposes, the following is important. Let us assume that a sample consisting of N electron-capturing atoms and attached to a cantilever is put in a constant and homogeneous magnetic field B. The nuclear polarization P in equilibrium is given by Eq. (4) in the case of $\mu B \ll k_B T$. Using cyclic adibatic inversion, one can initiate oscillations of the nuclear magnetic moment $M_z = N\mu P$ of the sample at the resonant frequency of the cantilever ω_c. But these oscillations are, in fact, the oscillations of polarization P. Therefore, the neutrino recoil force (3), proportional to P, will also oscillate. In this case, Eqs. (3) and (5) determine the amplitude $F_n = |F_{nz}|$ of this force.

Thus, the neutrino recoil force can be measured in the same manner as the force acting on a magnetized sample in MRFM. Of course, the homogeneity of the magnetic field has to be sufficiently high to ensure that the magnetic force (7) is much smaller than the recoil force.

The limitations of the method described above are related primarily to the thermal fluctuations [8]. At a given temperature T, the minimally measurable force is [7, Eq. (4.10a)] (see also [8, 9]): $F_{\rm min} = \sqrt{4kk_B T \Delta\nu / Q\omega_c}$, where $\Delta\nu$ is the measurement bandwidth. For estimates, let us assume it equal to the half width at half maximum of the resonance, $\Delta\nu = \omega_c/(2Q)$ (this is equivalent to a requirement $\Delta\nu = 1/\tau$, where $\tau = 2Q/\omega_c$ is the oscillator damping time); this leads to $F_{\rm min} = \sqrt{2kk_B T}/Q$. Evidently, the sensitivity to the force can be improved by increasing the quality factor Q, lowering the temperature T and reducing the spring constant k.

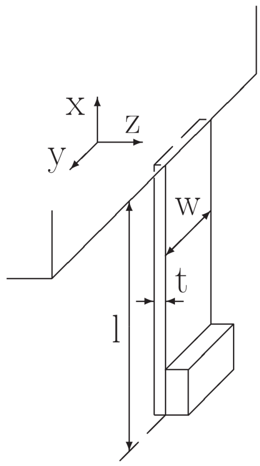

Figure 1. Scheme of a micromechanical resonator with a mass load on the tip; l, w and t are the length, the width and the thickness of the resonator.

In Ref. [9], a technology was presented to produce thin (up to $t = 50$ nm) and long (up to $l = 400$ μm) cantilevers, made of single-crystal silicon with a spring constant up to 10^{-5} N/m and a quality factor of 10^3–10^4; further development allowed to achieve $Q \sim 10^5$ [6, 8]. Since

$$F_{\min} \simeq 10^{-19}\ \text{N} \tag{8}$$

at $k = 10^{-5}$ N/m, $T = 1$ K and $Q = 10^5$, this technology opened a possibility to measure attonewton and sub-attonewton forces. In addition, a mass load to the free end of a cantilever to suppress oscillation modes of high orders was proposed in Ref. [10]. Note that the cantilever is positioned vertically; its upper end is clamped (see Figure 1). Ultra-thin cantilevers of this type with the mass load slightly exceeding the mass of the cantilever were successfully used, e.g., in studies [11, 12, 13]. The mass load determines, in fact, the effective mass of the oscillator m_{eff}, which, along with the spring constant k, gives the fundamental frequency of the cantilever oscillations:

$$\omega_c = \sqrt{\frac{k}{m_{\text{eff}}}}\,. \tag{9}$$

Let a radioactive sample of mass m be a mass load. For a sample with a mass, say, $m \simeq m_0 = 10^{-10}$ g, we consider a cantilever with $l = 100$ μm, $w = 2$ μm, $t = 50$ nm made of single-crystal silicon (with Young's modulus $E = 1.31$ GPa and the density $\rho = 2.33$ g/cm^3) and, therefore, with the mass $m_c \simeq 2.3 \cdot 10^{-11}$ g and the spring constant $k \simeq 10^{-5}$ N/m (similar cantilevers with a mass load $\sim 10^{-10}$ g were used in Refs. [11, 12, 13]). The oscillation frequency of such cantilever is $\nu_{c0} \simeq 1.5$ kHz. For definiteness, we assume that when a radioactive sample of mass $m < m_0$ is placed on the cantilever, it is additionally loaded up to the mass m_0 with any non-radioactive material (so its frequency is still ν_{c0}).

Taking into account the capability of modern superconducting magnets, we suppose $B = 10$ T. Let us find the minimal sample mass providing a detectable neutrino recoil force. To do this we substitute $B = 10$ T and $T = 1$ K into Eq. (5) and rewrite it in the form:

$$F_n = 10m|C_n|f_n \geq F_{\min} \;\Rightarrow\; m \geq \frac{F_{\min}}{10|C_n|f_n} \equiv m_1. \tag{10}$$

There is, however, an additional lower limit on the sample mass: during one period of cantilever oscillations, the average number of emitted neutrinos should be sufficiently large. Taking this

Table 1. List of Gamow–Teller transitions from the initial nucleus ${}^{A}X_i$ to the ground ($n = 0$) or excited nth state of the final nucleus ${}^{A}X_f$ due to electron capture. Here $T_{1/2}$ and μ are the half-life and the magnetic moment of the initial nucleus, E_n^* is the excitation energy of the final nucleus, $E_{\nu n}$ is the neutrino energy, f_n is the force parameter for the transition, $m_{\min}$ is the minimal value for the sample mass. Magnetic moment for ^{179}W is unknown; it was taken to be μ_N as an estimate.

${}^{A}X_i \to {}^{A}X_f$	$J_i^\pi \to J_f^\pi$	$T_{1/2}$	μ/μ_N	E_n^* (keV)	$E_{\nu n}$ (keV)	f_n (N/g)	$m_{\min}$ (g)
${}^{179}\mathrm{W} \to {}^{179}\mathrm{Ta}^*$	$7/2^- \to 9/2^-$	37.05 m	(1)	30.7	975	$2.2 \cdot 10^{-8}$	$4.5 \cdot 10^{-13}$
${}^{163}\mathrm{Er} \to {}^{163}\mathrm{Ho}$	$5/2^- \to 7/2^-$	75.0 m	+0.557	0	1164	$8.0 \cdot 10^{-9}$	$1.3 \cdot 10^{-12}$
${}^{135}\mathrm{La} \to {}^{135}\mathrm{Ba}$	$5/2^+ \to 3/2^+$	19.5 h	+3.70	0	1175	$4.1 \cdot 10^{-9}$	$3.6 \cdot 10^{-12}$
${}^{107}\mathrm{Cd} \to {}^{107}\mathrm{Ag}^*$	$5/2^+ \to 7/2^+$	6.50 h	−0.615	93.1	1301	$2.9 \cdot 10^{-9}$	$3.5 \cdot 10^{-12}$
${}^{119}\mathrm{Sb} \to {}^{119}\mathrm{Sn}^*$	$5/2^+ \to 3/2^+$	38.2 h	+3.450	23.9	542	$1.0 \cdot 10^{-9}$	$6.9 \cdot 10^{-12}$
${}^{111}\mathrm{In} \to {}^{111}\mathrm{Cd}^*$	$9/2^+ \to 7/2^+$	2.805 d	+5.503	416.6	420	$7.8 \cdot 10^{-10}$	$1.1 \cdot 10^{-11}$
${}^{165}\mathrm{Er} \to {}^{165}\mathrm{Ho}$	$5/2^- \to 7/2^-$	10.36 h	+0.643	0	332	$3.1 \cdot 10^{-10}$	$3.2 \cdot 10^{-11}$
${}^{131}\mathrm{Cs} \to {}^{131}\mathrm{Xe}$	$5/2^+ \to 3/2^+$	9.69 d	+3.543	0	325	$9.5 \cdot 10^{-11}$	$7.5 \cdot 10^{-11}$

number equal 100 (as an estimate), we get

$$\frac{\alpha}{\nu_c} = \frac{m \ln 2}{m_a T_{1/2} \nu_c} \geq 100 \, . \qquad (11)$$

The frequency $\nu_c = \omega_c/(2\pi)$ is determined by Eq. (9), where $m_{\text{eff}} = m$, if $m > m_0 = 10^{-10}$ g, and m_0, if $m < m_0$.

Neutrino recoil force for suitable isotopes

A list of the most suitable isotopes with highest values of the force parameter (6) decaying only ($I_{nEC} = 1$) or mainly ($I_{nEC} \geq 0.98$) by electron capture via Gamow–Teller transition to a single final nuclear state is presented in the Table 1. The basic properties of these isitopes are also shown, which were used to calculate the force parameter f_n and the minimal mass $m_{\min}$. All numerical values are taken from the website [14]. In practice, it turned out that $m_{\min} = m_1$ for all isotopes, except for the sample of ^{135}La (its minimal mass was found from Eq. (11)). The isotopes in the Table 1 are arranged in the descending order of the force parameter f_n that corresponds, as one can see, to the ascending order for the minimal mass $m_{\min}$. Only isotopes with $m_{\min} < m_0$ are included.

According to the calculations [5], the sample activity α varies from one isotope to the other but slightly: it is of the scale of 1 MBq. This is because of Eq. (3). Indeed, the sample activities α for different isotopes and transitions are to be comparable, if comparable are the recoil force and the values of I_{nEC}, $E_{\nu n}$, B_n, and P.

The situation is similar for the heat power due to secondary products, i.e. Auger electrons, conversion electrons, x-rays, and γ-rays [5]. Typically, their energies are of the scale of some tens of keV, thus the corresponding heat load is of the scale of nW for $\alpha = 1$ MBq (indeed, 1 MBq $\cdot$ 10 keV $\simeq$ 1.6 nW). It means that the radioactive decays contribution to the total heat load should not be a problem, because even for the temperature 25 mK the cooling power of modern dilution refrigerators is of the scale of tens μW [4].

Test of Lorentz invariance

There are strong restrictions on the real components of the tensor $\chi^{\mu\nu}$, in particular, $|\chi_r^{jk}| \leq 10^{-6}$ [1]. Thus, the term $\sim \chi_r^{jk} n_{\nu j} n_{Jk}$ in (1) is very small. However, the values of χ_i^{l0} are unconstrained [1, 2], therefore, the last term $\sim \chi_i^{l0} [\mathbf{n}_\nu \times \mathbf{n}_J]_l$ is of great interest.

One can see from Figure 1 that measuring the recoil force for the polarizing magnetic field B directed along the axis y allows to detect or to set upper limit on the value of χ_i^{l0}. The isotopes from Table 1 are suitable for such experiment. Some of them, namely, ^{165}Er and ^{131}Cs were discussed in Ref. [2] as the most appropriate. Notice that the method proposed in [2] requires a source with an activity of at least one Curie, i.e. 37 GBq. We see, first, that the six top isotopes from the Table 1 may be more suitable than ^{165}Er and ^{131}Cs because of the smaller sample masses, and, second, to measure the neutrino recoil force a sample with an activity of the scale of 1 MBq is enough.

Conclusion

A sample of radioactive atoms experiences a recoil force from neutrino radiation accompanying electron capture by polarized nuclei provided there is a directional asymmetry of neutrino emission. This recoil is of interest because the force is proportional to the asymmetry coefficient, i.e., the force measuring is equivalent to measuring of the neutrino angular distribution asymmetry. In particular, the recoil force can give information on a hypothetical Lorentz invariance violation resulting unique asymmetry terms in the neutrino angular distribution. It is shown to realize this one needs much less active samples than that discussed in [2].

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