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Probing secular changes in the gravitational constant with white dwarf spectra

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Abstract Exploring variations in fundamental physics constants across cosmological space-time is of substantial importance in both theoretical and experimental physics. A crucial means of investigating such variations involves comparing laboratory measurements with astrophysical observations. In the present study, we employ a combination of laboratory data and observed Lyman transitions of H_2 identified in the white dwarf star GD133. Through this examination, we find the temporal variation of the gravitational constant, $\dot{G}/G = (0.016 \pm 0.098) \times 10^{-15}$ year^{-1} with a gravitational potential $\phi \approx 10^4$, and an average total redshift of H_2, $z_{abs} = 0.0001820(10)$. This newly determined constraint on the time variation of G serves as an important tool for advancing discussions within unified theories.

1 Introduction

One of the principles of General Relativity, known as the equivalence principle, claims the constancy of fundamental constants irrespective of their spatial location. However, contemporary grand-unification theories propose an important context for this notion, suggesting that the constants of nature might function as dynamic scalar fields associated with low mass [1]. Should these theories hold true, it implies the potential for slow variations in these constants across cosmological space and timescales, introducing a location-dependent dimension [2]. Recent years have produced significant efforts to constrain the variability of the fine structure constant, as extensively detailed in comprehensive reviews [3,4], establishing stringent upper limits on their rates of change. In contrast, to the extensive exploration of potential evidence for a variable fine structure constant, relatively limited attention has been devoted to probing the hypothetical variation of the gravitational constant, G. This limited focus likely stems from the inherent challenges associated with accurately measuring this constant [5]. Notably, G stands out as the fundamental constant with the least precise determination, marked by significant disparities in multiple measurements. The exploration of fundamental constants' variations in space-time constitutes a crucial avenue for advancing modern physics and understanding phenomena beyond the Standard Model (SM). Astrophysical and cosmological observations serve as powerful tools for considering these interesting phenomena. Recent studies have outlined unified scenarios involving space-time variations in fundamental physical constants, notably α, μ, and G. Consequently, investigations into the spatial/temporal effects on the gravitational constant (G) are deemed crucial for the development of unification scenarios. The most rigorous constraints on the rate of G's variation, denoted as $\dot{G}/G = (2 \pm 7) \times 10^{-13}$ year^{-1}, arise from Lunar Laser Ranging; however, these constraints are inherently local in nature [6]. Contributing to the constraints, the Hubble diagram of Type Ia supernovae at intermediate cosmological ages places $\dot{G}/G \sim \times 10^{-11}$ year^{-1} at $z \sim 0.5$ [7]. Lastly, Big Bang Nucleosynthesis limits on potential variations in the gravitational constant, span from -3×10^{-11} year^{-1} to 4×10^{-13} year^{-1} [8].

White dwarfs present an inherent method for constraining potential variations in the gravitational constant, G. This capacity stems from various factors. Firstly, white dwarfs possess exceptional longevity, making them sensitive indicators of even small rates of change in G. Secondly, as the final evolutionary stage for the majority of stars, white dwarfs are abundant in the cosmos. Thirdly, their compact nature enables them with a structure highly responsive to the precise value of G. Lastly, the well-understood evolution of white dwarfs can be precisely characterized as a straightforward gravothermal process, where their luminosity is predominantly dictated by the delicate equilibrium between thermal

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and gravitational energies. Consequently, any secular variation in G significantly influences the gravothermal balance of white dwarfs, thereby impacting their luminosities. One approach to determining white dwarfs for constraining variations in G involves estimating the dependence of the secular rate of change of the period of pulsation in variable white dwarfs on their cooling rate [9]. Studies suggest that this rate of change not only relies on the cooling rate but also on the rate of variation in the gravitational constant G. Applying this method to the extensively studied variable white dwarf G117-B15A yielded a somewhat broad constraint: -2.5×10^{-10} year$^{-1} \le \dot{G}/G \le 0$ [10]. An alternative study for constraining G-variation involves investigating the white dwarf luminosity function. The abundance of white dwarfs is intricately tied to the characteristic cooling time within the corresponding luminosity range, thereby influencing the cut-off position of the white dwarf luminosity function at low luminosities. Hence, the study employed a simplified model to explore the potential effects of a slowly varying G on the white dwarf luminosity function [11]. Assuming that G is sufficiently low for white dwarfs to rapidly adjust their mechanical structure compared to the cooling timescale, researchers determined the implications using energy conservation principles [11,12].

In this study, our investigation explores the potential impact of cosmological variations in space-time on the gravitational constant, as evidenced by spectral observations of Lyman transitions of H_2 within the white-dwarf star GD133 [13–15]. The spectrum of the white dwarf GD133 was captured using the *Cosmic Origins Spectrograph aboard the Hubble Space Telescope* and exhibited a gravitational potential approximately 10^4 times more robust than that observed in Earth-based experiments. Given this strong gravitational potential, it stands out as an optimal candidate for establishing an upper limit on the gravitational constant's variation over cosmological timescales. In this scenario, we can estimate potential cosmological deviations in the past with a level of $\dot{G}/G = (0.016 \pm 0.098) \times 10^{-15}$ year^{-1} precision comparable to that achieved in previous studies [16–19].

2 $\dot{G}/G$ with strong gravitational fields

With the spectra of white-dwarf stars, a category of models emerges to explicate phenomena beyond the Standard Model of cosmology and particle physics. This framework facilitates the identification of contemporary variations in several fundamental constants, notably the fine-structure constant (α), the proton-to-electron mass ratio (μ), and the gravitational constant (G). In prior investigations, the gravitational surface potential, denoted as ϕ at a distance r from the mass of an object M, was detailed $\phi = (GM/(Rc^2))$. These models involved the interaction of scalar fields and related fields, influenced by the gravitational redshift effect within the framework of general relativity. Consequently, the energy loss (E) of a photon escaping a gravitational surface (R) could be expressed as $z = -\Delta E/E = -\phi/c^2$, where the fractional change in energy is contingent upon the fractional change in observational wavelengths $-\Delta E/E = \Delta\lambda/\lambda \sim \Delta\alpha/\alpha$. This approach was employed to examine cosmological variations in fundamental constants, such as the fine-structure constant (α) under higher surface gravity [16–19]. An effective means to probe these variations lies in a specific dimensionless constant, typically denoted as α, grounded in Grand Unified Theories (GUTs), temporal or spatial variations in fundamental physical constants could potentially unify gravitational and electromagnetic forces. Assuming uniform variations in all related Yukawa couplings and leveraging dimensional transmutations to establish a weak scale. These couplings are expressed through a driven dilaton-type. Consequently, the relation between the variation of the fine-structure constant and the Quantum Chromodynamics (QCD) scale (Λ_{QCD}) is denoted as [20,21]:

$$\Delta\Lambda_{\rm QCD}/\Lambda_{\rm QCD} = R\Delta\alpha/\alpha \tag{1}$$

where R is determined by the GUT. The relation of $\alpha(M_{GUT}) = \alpha_s(M_{GUT})$ to identify the R-value relies on model-independence based on low energy. Simultaneously, changes in Yukawa coupling (h) induce alterations in the Higgs Vacuum Expectation Value (ν) at the Planck mass scale of GUTs. Consequently,

$$\nu = M_{Planck} \exp\left(-(8\pi^2 c/h^2)\right) \tag{2}$$

and

$$\begin{aligned}\Delta\nu/\nu = 16\pi^2 c\,(\Delta h/h) = S\,(\Delta h/h) \to \Delta\nu/\nu \\ = S\,(\Delta h/h)\end{aligned} \tag{3}$$

can be indicated through dimensional transmutation, denoted as $S = d\ln\nu/d\ln h$, and $\Delta h/h = (1/2)\Delta\alpha/\alpha$ ($c \simeq h \simeq 1$). Here,

$$\Delta m_e/m_e = (1/2)(1+S)\Delta\alpha/\alpha \tag{4}$$

and

$$\Delta m_p/m_p = [1.6R + 0.4(1+S)]\Delta\alpha/\alpha \tag{5}$$

correspond to the variations in electron mass and the proton-to-electron mass ratio, respectively. Utilizing a perturbative approach, the variations of neutron mass (m_n) and average nucleon mass (m_N) are determined by

$$\Delta m_n/m_n = \Delta m_N/m_N = \Delta m_p/m_p. \tag{6}$$

These assumptions lead to the derived relationship between α and G [16–21]:

$$\Delta G/G = [1.6R + 0.4(1+S)]\Delta\alpha/\alpha. \tag{7}$$

The parameters R and S are considered as free parametric quantities, and their absolute values are relative to the model being used. These values are constrained by observational data, emphasizing that variations are rooted in the parameters (α, μ, G). Notably, the outcomes exhibit a notable sensitivity to prior choices. In particular, a broader parameter space conduces to yield preferred values for R and S that are smaller, as a more general initial choice increases the fraction of the spatial parameter volume with large R or S values. Consequently, our study focuses on constraining an assumption of a uniform prior in R and S parameters. In this and other studies, observational data are employed to determine R or S, treating them as free phenomenological parameters within unification scenarios. Alternatively, both phenomenological parameters (R, S) can be obtained directly from astrophysical data or laboratory measurements. We have analyzed models corresponding to specific values of both R and S, but in the general case, they can vary unless marginalized. In our analysis, we explored models with varying values of parameters R and S to understand their impact on the gravitational constant (G). However, in the absence of a specific constraint or preference, these parameters are allowed to vary within certain limits unless marginalized. To narrow down our focus and provide a more accurate foundation for our investigation, we carefully considered the range of possibilities. To establish the most likely values of R and S, we considered a set of constraints derived from relations specific to our study. These constraints, expressed as mathematical equations, represented the intricate interplay between the parameters and the underlying physical phenomena. The relations, specifically:

$$0.80R - 0.30(1 + S) = -0.81 \pm 0.85 \tag{8}$$

$$0.10R - 0.04(1 + S) = -1.96 \pm 1.79 \tag{9}$$

introduced degeneracy directions in our study. Our statistical evaluation involved a rigorous exploration of the error determination, considering the potential presence of multiple minima. The nonlinear-least-squares algorithm was crucial in minimizing the discrepancies between our theoretical predictions and the observed data. We carefully selected a specific function within the family of nonlinear-least-squares algorithms, ensuring its suitability for our study objectives. Furthermore, we conducted a comprehensive examination of the distributional characteristics of the data, acknowledging the positivity inherent in both spectra and wavelengths. The assumption of a Gaussian distribution for error estimation was examined, and any observed patterns or non-Gaussian features were explicitly considered. This detailed analysis aimed to provide a robust foundation for the statistical significance of our results. Therefore, our rigorous analysis encompassed the application of advanced optimization techniques, constraints derived from relations, exploration of error estimation, and careful consideration of the distributional characteristics of the data above. These steps collectively contributed to the robustness and statistical significance of our parameter estimation process. After rigorous analysis and statistical evaluation, we identified that the parameters $R = 273 \pm 86$ and $S = 603 \pm 230$ best align with our research objectives. These values are not arbitrarily chosen but are determined through a systematic process of optimization, taking into account both the theoretical framework and compatibility with observational data. Therefore, the parameters $R = 273 \pm 86$ and $S = 603 \pm 230$ within the range explored in Refs. [16–21] are chosen as the best-fit for our purpose. While our estimations aim to yield best-fit values, the exact selection of R and S with prior choices depends on the detailed nature of the unified scenarios. By selecting these specific values, we aim to establish a well-defined set of conditions that allows us to delve into the gravitational constant's (G) secular variation within a meaningful and manageable parameter space. This approach enhances the robustness of our study and ensures that our results are grounded in a careful consideration of both theoretical expectations and empirical constraints. Comparing different lines of Lyman transitions of H_2 in the laboratory with their observational values from the white-dwarf GD133 spectrum allows us to determine the effect of time variation of G over cosmological timescales. This spectrum, recorded by the *Cosmic Origins Spectrograph aboard the Hubble Space Telescope*, includes a gravitational potential $\phi \approx 10^4$, and an average total redshift of H_2, $z_{abs} = 0.0001820(10)$. Fitting procedures deduce that the parameters (R, S) are consistent with other works [16–23]. Utilizing our fitting program, one can obtain an approximate fitted value for $\dot{G}/G$. In the procedure of our analysis, we applied the nonlinear-least-squares algorithm-a sophisticated optimization technique known for its effectiveness in finding the best-fit parameters for mathematical models. The nonlinear-least-squares algorithm, a well-established optimization technique, was employed for parameter estimation in our analysis. This method has been widely utilized in various fields, including astrophysics, as detailed in previous studies [24,25]. This algorithm is particularly well-suited for refining the parameters of our model through an iterative process, minimizing the discrepancies between our theoretical predictions and the actual white dwarf spectrum data. Its adaptability allows for a comprehensive exploration of the parameter space, ensuring a precise fit to the observed data. The nonlinear-least-squares algorithm is widely recognized for its capability to handle complex relationships and optimize model parameters, making it a valuable tool in our analysis.

To comprehensively assess the reliability of our results, we employed the Gaussian distribution. This statistical framework not only facilitated the estimation of statistical errors but also served as a versatile tool to gauge systematic errors.

By leveraging the Gaussian distribution, we were able to capture the accurate interplay between various sources of uncertainty, providing a detailed and precise characterization of the error estimation associated with our analysis. This useful approach enhances the robustness and validity of our conclusions, offering a more thorough understanding of the uncertainties inherent in our study of the secular variation of the gravitational constant within the strong gravitational fields of white dwarf stars. The white dwarf GD133 exhibits highly valuable spectral lines, particularly those of the Lyman transitions of H_2, which were utilized to find Doppler shifts based on their velocity scales. The analysis revealed that each selection of lines is narrow with symmetric shapes, suggesting that the same velocities could produce the H_2 lines. Chosen for their suitability in characterizing spectral lines, Gaussian line-fitting profiles were employed to determine linewidths and central velocities during the analysis. Single velocity components were fitted with a single Gaussian, while multiple Gaussians were fitted with various velocity components. Each component is characterized by three parameters $(N, z_{abs}, b = \sqrt{2}\sigma)$, where N is the column density, z_{abs} is the absorption redshift, and b is the Doppler linewidth, with σ being the root-mean-square of the velocity distribution. Initially, the H_2 lines were simulated to identify specific characteristics, referred to as $\Delta\alpha/\alpha$-values. Subsequently, the fitting parameters ($\Delta\alpha/\alpha$, R, S) were used to determine the $\dot{G}/G$-values.

The fitting procedure utilized only χ^2 and minimum χ^2_{min} to determine $\dot{G}/G$ for the data, incorporating a reduced approximation-fitting $\Delta\chi^2 = 1$. The standard statistical process assigned one-sigma error to the best-applied value of $\dot{G}/G$, with $\Delta\chi^2 = \chi^2 - \chi^2_{min} = 1$ used to estimate variations in $\dot{G}/G$. The maximal change rates of $\dot{G}/G$ were determined based on $\Delta\chi^2 = 1$ for the evaluation of errors. Minimal $\dot{G}/G$ values were employed to fit each line, providing an insight into the potential effect of cosmological variations in the gravitational constant (G). Table 1 presents the statistical and systematic errors calculated ($\sigma^2_{\text{total}} = \sigma^2_{\dot{G}/G} + \sigma^2_{\text{sys}}$) for the $\dot{G}/G$ values. Figure 1 illustrates the distribution of z versus gravitational redshift, allowing for the testing of bounds on the dimensionless fundamental physical constants (α, G). Various inconsistencies are observed in different methods used for determining systematic errors. It is acknowledged that the evidence is not fully controlled or sufficiently understood in some cases. Observational studies have produced different results due to variations in analysis methods and limitations imposed by uncertainties in laboratory wavelength measurements [13,16,17,19–21,24–27]. Utilizing the nonlinear-least-squares algorithm, the observed spectra were combined with an uncertainty of $\sim 1 : 10^6$ and laboratory wavelengths with an uncertainty of $\sim 1 : 10^7$. The Gaussian distribution provides a useful means to estimate the α-errors with high precision, improving the accuracy in measuring possible α-changes compared to previous works. In our analysis, we employed the nonlinear-least-squares algorithm for parameter optimization, with a crucial consideration being the assumption of a Gaussian distribution for error estimation. It is imperative to address the nature of our data, as both spectra and wavelengths inherently involve positive values, while the Gaussian distribution extends across both positive and negative ranges. To evaluate the appropriateness of the Gaussian assumption, we conducted a thorough examination of the distributional characteristics of our data. Acknowledging the positivity of our data and its potential deviation from a strictly Gaussian form, we are committed to enhancing the clarity and transparency of our methodology by conducting a detailed analysis of the distributional characteristics. This involves a thorough examination of the observed spectra and wavelengths, considering their inherently positive values. To address potential deviations from a Gaussian distribution, we will provide a clear description of any observed patterns or non-Gaussian features in our data. Additionally, we will explicitly state any adjustments made to account for the positivity of the data in our modeling and analysis, ensuring a comprehensive understanding of our approach. This analysis complements and extends previous studies, proposing that highly-resolved white-dwarf spectra offer one of the tightest constraints on the possible spatial/temporal variation of G, assuming potential Grand Unified Theories (GUTs) scenarios linking $\Delta\alpha/\alpha$ to $\dot{G}/G$.

The primary objective of our study was to demonstrate that white dwarfs can constrain variations in fundamental constants, including the fine-structure constant, the proton-to-electron mass ratio, and the gravitational constant. Our analysis supports the idea of potential cosmological variations in the gravitational constant over time. Specifically, based on our examination of the Lyman transitions of H_2, our results establish a fractional change in the gravitational constant of $\dot{G}/G = (0.016 \pm 0.098) \times 10^{-15}$ year^{-1}over the gravitational redshift $z_{abs} = 0.0001820(10)$. Our findings, considering current constraints from observational data and laboratory measurements, suggest a promising approach towards understanding variations in fundamental physical constants [13,16,17,19–21,24–27].

3 Discussions

Theoretical frameworks in fundamental physics propose changes in fundamental constants across cosmic time and space. However, the interactions among light scalar fields responsible for these fluctuations could lead to variations in fundamental constants influenced by local gravitational fields. Recently, researchers have investigated these phenomena to explore potential links between fundamental constants,

Table 1 The analysis yielded the following measurements of $\dot{G}/G$ at various redshifts with an average total redshift of H_2, $z_{abs} = 0.0001820(10)$

Line	λ_{Obser} (Å)	$\dot{G}/G$ [10^{-15} year^{-1}]	$\sigma_{\dot{G}/G}$ [10^{-15} year^{-1}]
R(9)	1313.376434 (6)	0.10837	0.11054
P(9)	1324.595010 (6)	0.10630	0.10843
R(11)	1331.963484 (6)	0.15556	0.15017
P(11)	1345.177888 (6)	– 0.23190	– 0.10054
P(13)	1368.662949 (7)	0.06882	0.07020
R(7)	1356.487606 (7)	0.18749	0.10624
P(7)	1366.395252 (7)	0.21569	0.13500
R(9)	1371.422414 (7)	0.17535	0.09386
P(9)	1383.659161 (7)	– 0.18579	– 0.16401
R(11)	1389.593797 (8)	0.12120	0.10662
P(11)	1403.982606 (8)	0.19592	0.16448
R(13)	1410.648000 (1)	– 0.14761	– 0.15056
P(13)	1427.013400 (8)	– 0.03138	– 0.11871
R(15)	1434.177000 (1)	0.01634	0.09889
P(15)	1452.354000 (1)	– 0.03765	– 0.12340
R(11)	1447.613000 (1)	0.20675	0.12589
R(11)	1310.900080 (6)	0.20758	0.12673
R(13)	1332.315000 (1)	0.24899	0.13497
R(19)	1410.759000 (1)	– 0.10449	– 0.08958
R(19)	1338.350000 (1)	0.01941	0.07930

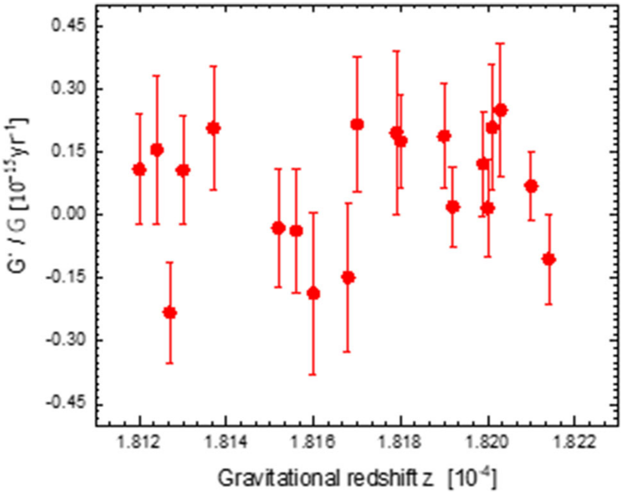

Fig. 1 Distribution of $\dot{G}/G$ versus gravitational redshift. Each point corresponds to one of the 20 strongest lines from the spectral lines of the white dwarf spectra GD133, which were utilized in the analysis of the Lyman transitions of hydrogen observed in the white dwarf spectrum of GD133. Our final result, $\dot{G}/G = (0.016 \pm 0.098) \times 10^{-15}$ year^{-1}, was determined through analysis of the data presented in Fig. 1. This figure summarizes essential information integral to our study. By closely examining the plotted data, we identified the observed trends that led to our conclusive finding

such as the fine structure constant α, the proton-electron mass ratio μ, and the gravitational constant G, within the intense gravitational fields present in the photosphere of white dwarf stars [7,10,28]. Pulsating white dwarfs, such as G117-B15A, and white dwarf asteroseismology have contributed to the exploration of G-variations with $|\dot{G}/G| \leq 4.10 \times 10^{-10}$ year^{-1} [29–35]. Gravitational waves and pulsar binaries, such as PSR1913+16, have also been employed to set constraints on $\dot{G}/G = 1.3 \times 10^{-10}$ year^{-1} and $\dot{G}/G \sim -1.8 \times 10^{-10}$ year^{-1} [36–39]. Comparison data from six telescopes, particularly through mode spectra, has provided further constraints on the time-dependent variation of $\dot{G}/G \leq 10^{-10}$–10^{-11} year^{-1} and $\dot{G}/G \sim -1.8 \times 10^{-12}$ year^{-1}. Various studies have explored the impact of time-varying G using different approaches and astronomical objects. The white dwarf cooling theory approach has provided results indicating time variations in G within the range of $\dot{G}/G \leq 10^{-10}$–10^{-11} year^{-1} and $\dot{G}/G \sim -1.8 \times 10^{-12}$ year^{-1} [40–47], while other investigations, particularly those utilizing pulsating white dwarfs like G117-B15A and R548, have yielded less stringent limits on the time variation of $\dot{G}/G \sim -1.3 \times 10^{-10}$ year^{-1} [48–50]. Notably, gravitational waves have been employed to set a new limit on the time variation of $\dot{G}/G \sim 10^{-11}$ year^{-1}, presenting interesting findings [51]. A similar analysis was conducted for the pulsar binary PSR1913+16 within the framework of Brans–Dicke theory, resulting in a limit denoted as $\dot{G}/G = (1.0 \pm 2.3) \times 10^{-11}$ year^{-1}. These limits were further refined by subsequent studies, denoted as $\dot{G}/G = (4 \pm 5) \times 10^{-12}$ year^{-1} and $\dot{G}/G = (-0.9 \pm 1.8) \times 10^{-11}$ year^{-1}, as they reexamined and combined data from various sources, including the quasar

PSR B1855+09 [1,52]. Furthermore, the potential effect of space or time-varying G has been investigated through a comparison of data from six telescopes. Notably, the application of mode spectra has led to more substantial constraints denoted as $\left|\dot{G}/G\right| < 1.6 \times 10^{-12}$ year^{-1} at a two-sigma level. It is consistently reported across these studies that the effect of time-dependent $\dot{G}/G = (-6 \pm 42) \times 10^{-13}$ year^{-1} does not exceed the 10^{-12} year^{-1} level, emphasizing the robustness of this conclusion [53]. Measuring the wavelength shift of absorption lines in the photosphere, based on the equivalent ratio of stellar mass to radius, provides a means to estimate the gravitational redshift. However, isolating the gravitational redshift is challenging due to Doppler shifts caused by random stellar motions along the line of sight. The use of co-moving companions helps identify the degeneracy between Doppler shift and gravitational redshift. This enables the constraint of velocity components like the radial velocity for white dwarfs. By measuring the mean gravitational redshift across a sample of field white dwarfs, and accounting for the Doppler effect based on random motions relevant to the Sun, it allows for the determination of white dwarf mass-radius relationships and surface temperatures. White dwarf spectra, characterized by absorbed photospheric lines, pressure, and effective temperature conditions, provide crucial information. Leveraging the previous best-fitting for the white dwarf star GD133 spectrum, physical conditions can be estimated. The spectrum of H_2 is then applied and related to each object before the fitting procedure for gravitational redshift estimation, considering uncertainties around $1 : 10^6$ and laboratory wavelength uncertainty of $1 : 10^7$. Parameters $(N, z_{abs}, b = \sqrt{2}\sigma)$ are applied in the fitting, assuming that components are the same transitions for all the Lyman transitions of H_2 lines. The redshift scale or velocity is used to determine the positions of H_2 lines. Typically, $\dot{G}/G$ is used as a fit parameter, and $\dot{G}/G$ values are determined by the shifted transitions of H_2 for each line. lines prove to be a suitable choice for analysis as they are frequently found in white dwarfs and offer high sensitivity for testing physical constants like the gravitational constant. The small line separations of H_2 lines in the present analysis allow for the practical reduction of systematic effects, enabling the identification of spatial or temporal variations in G with high accuracy.

4 Conclusions

In this study, we explore the secular variation of the gravitational constant (G) using the formula $\Delta G/G = [1.6R + 0.4(1 + S)]\Delta\alpha/\alpha$ where $\Delta G/G$ represents the relative change in gravitational constant, R and S are specific parameters obtained from our fitting program, and $\Delta\alpha/\alpha$ signifies the relative change in the fine-structure constant.

Our analysis, grounded in white dwarf spectra, has yielded a more precise upper limit for the secular variation of G compared to previous methods such as Pulsar timing, Lunar Laser Ranging, Big Bang Nucleosynthesis, and Ages of globular clusters. Importantly, the results are not only based on empirical data but also incorporate assumptions related to spatial/temporal extra-dimensions at a specified level [1,50–56]. In comparison to the study by Garcia-Berro et al. [57], our analysis offers several advancements and improvements. First, our study utilizes more recent and higher-quality spectral data from the white dwarf GD133. This data provides enhanced precision in measuring the Lyman transitions, which is crucial for determining the secular variation of the gravitational constant, G. Second, we have implemented advanced modeling techniques and used updated atomic data, resulting in a more accurate spectral analysis. While Garcia-Berro et al. [57] conducted a broad analysis on the secular variations of G using various astrophysical methods, our research focuses specifically on the white dwarf spectra, allowing for a more targeted and refined approach. Additionally, our findings present a more precise upper limit for $\dot{G}/G$, specifically $\dot{G}/G = (0.016 \pm 0.098) \times 10^{-15}$ year^{-1}, which offers a narrower uncertainty range compared to previous studies. These improvements underscore the novelty and significance of our work in providing more accurate constraints on the temporal variations of the gravitational constant.

In a broader theoretical context, exploring spatial or temporal variations of fundamental dimensionless couplings (α, μ, and G) presents opportunities to advance beyond conventional astrophysical/cosmological models. The inclusion of the formula above enhances the clarity of our approach and directly links our findings to the theoretical framework. Our investigation also suggests that deviations in G may offer a potential explanation for the incompleteness of the Equivalence Principle (EEP).

Moving forward, we provide for further studies to refine our understanding of these constants, utilizing astrophysical sources, such as Cosmic Microwave Background data, Big Bang Nucleosynthesis data, and exploring brane-world-type unification scenarios. As astrophysical observations and data quality improve, we anticipate that our study will contribute additional insights into the realms of Grand Unified Theories (GUTs) [53,58,59].

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Declarations

Conflict of interest The authors declare that we have no conflict of interest.

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