AN ESTIMATE OF THE LONGITUDINAL AND TRANSVERSE IMPEDANCES FOR THE SUPERCONDUCTING SUPER COLLIDER

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I. Introduction

We try to estimate the longitudinal impedance per harmonic Z_L/n as well as the transverse impedance Z_T for the 20 TeV Superconducting Super Collider (SSC). Effects due to space charge, wall resistivity, bellows, monitor plates, synchrotron radiation are considered. The resulting Z_L/n and Z_T are plotted in Figures 4 and 5 respectively. Such a knowledge of Z_L/n and Z_T is necessary in computing the limits of many types of instabilities for the bunched beam. To be more specific, in our estimation, we consider the special case of an injection energy of 1 TeV and assume a maximum field of 5 Tesla in the SSC dipoles, or

E (maximum) = 20 TeV, γ_p (maximum) = 21315.6,
E (injection) = 1 TeV, γ_p (injection) = 1065.78,
B = 5T,
ρ = radius of curvature = 13.345 km.

In some cases, we also assume a 60° FODO cell structure consisting of 4 dipoles and 2 quadrupoles each with 2 long straight sections.[1] The beampipe radius and beam radius are chosen as b = 1.0 in. and a = 0.05 cm respectively. Totally, the storage ring consists of 364 cells and has a mean radius of R = 17.38 km.

Our results show that when monitor plates matched at both ends (such as the ones used in the Tevatron) are used, their effects dominate both Z_L/n and Z_T. For example, for a bunch of R.M.S. length σ_ℓ = 50 cm, the effective Z_L/n due to monitor plates and resistive wall when averaged over bunch mode m = 0 are respectively $|Z_L/n|_{\text{plates}}$ = 0.18 ohm and $|Z_L/n|_{\text{wall}}$ = 0.0099 ohm. For the former, a set of two monitor plates of length 18 cm and characteristic impedance Z_c = 50 ohm have been assumed for each spool piece, while for the latter, a conductivity of $\sigma = 6.25\times10^7$ ohm^{-1} cm^{-1} has been used for the wall of the beampipe. The corresponding values for the transverse impedance are $|Z_T|_{\text{plates}} = 1.9\times10^5$ ohm/cm and $|Z_T| = 5.3\times10^3$ ohm/cm. It was pointed out in the Cornell Workshop[2] that at a luminosity of 10^{33} cm^{-2} sec^{-2}, for a 5 Tesla ring, the limit for single beam stability on longitudinal impedance Z_L/n was ∿1 ohm and that on transverse impedance was $\sim1\times10^6$ ohm/cm.

In Sections II to V, the longitudinal impedances due to various effects are calculated. The total Z_L/n is discussed in Section VI. The effective Z_L/n for longitudinal bunch modes m = 0 and 1 are computed in Section VII. Finally, in Section VIII, the transverse impedances due to space charge, wall resistivity, bellows and monitor plates are estimated.

II. Space Charge and Wall Resistivity

The space charge impedance is negligibly small at high energy due to the near cancellation between the electric part and the magnetic part. For harmonic $n \ll \gamma_W R/b = 7.29\times10^8$ at 1 TeV and 1.46×10^{10} at 20 TeV, it is given by (-i implies inductive)

$$\left(\frac{Z_L}{n}\right)_{\text{sp ch}} = \frac{iZ_0}{\beta_W\gamma_W^2}\left(\frac{1}{2}+\ln\frac{b}{a}\right) = \begin{cases} i\ 1.47\times10^{-3}\ \text{ohm} & 1\ \text{TeV}, \\ i\ 3.67\times10^{-6}\ \text{ohm} & 20\ \text{TeV}. \end{cases}$$

In above $Z_0 = 120\pi$ ohms is the free space impedance and we have used the fact that for a longitudinally perturbing wave the relativistic factors γ_W and β_W for the phase are the same as γ_p and β_p for the beam particle. When $n \gg \gamma_W R/b$, $(Z_L/n)_{\text{sp ch}}$ falls down rapidly.

In order to lower the resistivity of the beampipe and at the same time retain its rigidity, the beampipe will be made of stainless steel with a coating of commercial copper in the interior. The thickness of the coating is roughly one skin depth at frequency $\omega_1/2\pi \sim 500$ kHz. The impedance per harmonic at frequency $\omega/2\pi$ is given by

$$\left(\frac{Z_L}{n}\right)_{\text{wall}} = \frac{1-i}{n\sigma_1\delta_1}\frac{R}{b}\,\frac{1-\dfrac{1-\sqrt{\sigma_1/\sigma_2}}{1+\sqrt{\sigma_1/\sigma_2}}\,e^{-2(1-i)\sqrt{\omega/\omega_1}}}{1+\dfrac{1-\sqrt{\sigma_1/\sigma_2}}{1+\sqrt{\sigma_1/\sigma_2}}\,e^{-2(1-i)\sqrt{\omega/\omega_1}}},$$

where σ is the conductivity and δ the skin depth. The subscripts 1 and 2 denote copper and stainless steel respectively. For completeness, a derivation of the above formula is given in the Appendix.

We note that when $\omega/\omega_1 \gg 1$, the last factor is unity and we get

$$\left(\frac{Z_L}{n}\right)_{\text{wall}} = \frac{1-i}{n\sigma_1\delta_1}\frac{R}{b},$$

which implies that all the image current flows in the copper coating only. When $\omega/\omega_1 \ll 1$, the same factor gives $\sqrt{\sigma_1/\sigma_2}$ leading to

$$\left(\frac{Z_L}{n}\right)_{\text{wall}} = \frac{1-i}{n\sigma_2\delta_2}\frac{R}{b}$$

implying that the copper coating can be neglected.

Substituting the following conductivities at 4°K:

$\sigma_1 = 6.25\times10^7$ ohm^{-1} cm^{-1}, copper (102 OFHC)

$\sigma_2 = 2.00\times10^4$ ohm^{-1} cm^{-1}, stainless steel (304 LN)

the impedance is computed and is plotted in Figure 1. If the beampipe is made of one material only, we have instead

$$\left(\frac{Z_L}{n}\right)_{\text{wall}} = \begin{cases} 0.90\ n^{-1/2}\ \text{ohm} & \text{copper only}, \\ 50\ n^{-1/2}\ \text{ohm} & \text{stainless steel only}. \end{cases}$$

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We note that $(Z_L/n)_{wall}$ follows the curve for copper when $\omega/2\pi >> 500$ kHz and rises to meet the stainless steel values when $\omega/2\pi << 500$ kHz.

III. Synchrotron Radiation

The characteristic frequency of the synchrotron radiation is $\omega_c = \frac{3}{2}\gamma_p^2\omega_0$ where $\omega_0 = \beta_p c/R$ is the revolution frequency. Thus the characteristic harmonic is

$$n_c = \frac{\omega_c}{\omega_0} = \begin{cases} 1.82\times10^9 & 1\text{ TeV},\\ 1.45\times10^{13} & 20\text{ TeV}. \end{cases}$$

The cutoff harmonic for the beampipe is

$$n_{cutoff} = 2.405 \times \frac{R}{b} = 1.65\times10^6.$$

Below n_{cutoff}, the proton cannot radiate because there is no propagating mode in the beampipe. Above that, we have for $n << n_c$, the radiation impedance[3]

$$\left(\frac{Z_L}{n}\right)_{rad} = \frac{\Gamma(\frac{2}{3})}{3^{\frac{1}{3}}\beta_p}\frac{R}{\rho} Z_0 n^{-1/3}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$$

$$= 460\, n^{-1/3}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) \text{ ohm}.$$

For $n >> n_c$, $(Z_L/n)_{rad}$ falls off exponentially. Thus $(Z_L/n)_{rad}$ is bounded by $460\, n_{cutoff}^{-2/3} \approx 0.033$ ohms which is small. The result is plotted in Figure 1.

IV. Monitor Plates

In the Tevatron, a set of two monitor plates are installed in each spool piece.[4] Both ends of the plate are terminated with impedance Z_t equal to the characteristic impedance Z_c between the plate and the beampipe. If the same monitor plates are installed in each spool piece of the SSC, we will have approximately M = 1456 plates. When

$$n << n_c = \frac{\gamma_W R}{b} = \begin{cases} 7.3\times10^8 & 1\text{ TeV},\\ 1.5\times10^{10} & 20\text{ TeV}, \end{cases}$$

we have[5]

$$(Z_L)_{plates} = -iM\left(\frac{\phi_0}{\pi}\right)^2 Z_c \sin\frac{n\ell}{R}\, e^{in\ell/R},$$

where ϕ_0 is the half angle covered by each plate and ℓ the length of each plate. We take $Z_c = 50$ ohms, $\ell = 18$ cm and $\phi_0 = \pi/2$. From

$$\mathcal{I}m\,(Z_L)_{plates} = -\frac{1}{2}M\left(\frac{\phi_0}{\pi}\right)^2 Z_c \sin\frac{2n\ell}{R},$$

$$\mathcal{R}e\,(Z_L)_{plates} = M\left(\frac{\phi_0}{\pi}\right)^2 Z_c \sin^2\frac{n\ell}{R},$$

we find that $\mathrm{Im}\,(Z_L/n)_{plates}$ stays at -0.19 ohm and starts fluctuating between inductive and capacitive after $n \sim 7.6\times10^4$ and at the same time decreases as $1/n$. The real part increases linearly as n to ~ 0.19 ohm at $n \sim 10^5$ and then falls off as $1/n$. The results are plotted in Figure 2.

The reason that Z_L/n for the monitor plates is big is as follows. Although Z_L/n due to one plate is proportional to $1/R$, the number of plates M is proportional to the number of cells in the storage ring which is in turn proportional to $R^{1/2}$. Thus, comparing with the Tevatron, the contribution to Z_L/n by the monitor plates decreases only by a factor of $\sim 1/\sqrt{17.38} = 0.24$. This contribution, however, can be made smaller by not matching the terminated impedance Z_t to the characteristic impedance Z_c. In this case, we have[5]

$$\left|\frac{Z_L(r)}{Z_L(r=1)}\right| = \left[1 + \left(\frac{1-r^2}{2r}\right)^2 \sin^2\frac{n\ell}{R}\right]^{-\frac{1}{2}},$$

where $r = Z_t/Z_c$. For example, taking $Z_t = \infty$ can reduce the impedance to zero. However, one will lose all the nice features of the matched plate; namely, directional signals and the elimination of resonances.

V. Bellows

There are two quads and four dipoles in each cell and 364 cells in total. Thus, we expect to have M = 2184 bellows to join the elements together. If we consider the length and radius of each bellow to be $\ell = 3$ cm and $d = 3$ cm respectively, for $n << R/\beta_p d \sim 5.8\times10^5$, the contribution of the longitudinal impedance is[6]

$$\left|\frac{Z_L}{n}\right|_{bellows} = -i\, Z_0 M \frac{\beta_p \ell}{2\pi R}\, \ln\frac{d}{b} = -i \times 0.041 \text{ ohm}$$

where the radius of the beampipe is taken as b = 2.54 cm.

At higher harmonics, the bellow as a whole will resonate. The first resonance occurs[6] at $n \sim 1.5\times10^6$ with shunt impedance and quality factor

$$\left|\frac{Z_L}{n}\right|_{shunt} = \begin{cases} .012 \text{ ohms},\\ .65 \text{ ohms}, \end{cases}$$

$$Q = \begin{cases} 5.0\times10^3,\\ 2.8\times10^5 \end{cases}$$

for each bellow. The upper figures are for stainless steel bellow and the lower ones for bellow with an interior coating of copper. The length of each bellow will not be identical; thus, we expect each bellow to resonate at slightly different harmonics and therefore the shunt impedances will not add up. For higher resonant modes, the shunt impedance will fall as $n^{-3/2}$. In order to reduce the resonance peaks, it will be better not to coat the bellows with copper.

The wiggles in the bellows will contribute extra wall impedance and resonances. We approximate[7] the wiggles by square wiggles of depth τ and separation Δ. If we use the dimension of the wiggles in the bellows of the Tevatron: $\tau = 0.64$ cm, $\Delta = 0.098$ cm, there will be approximately 15 wiggles in each bellow and 32,760 wiggles altogether. We observe that the image current travels an extra distance of 2τ in each wiggle. So there is an extra impedance of

$$Z_W = \frac{2\tau}{2\pi d}\,\frac{1}{\sigma\delta}\,(1-i)$$

or a total of $Z_W/n = (1-i)\times 0.16\ n^{-1/2}$ ohm for all the wiggles; stainless steel has been assumed. To study the resonances, each wiggle is viewed as a transmission line. The k^{th} resonance occurs at

$$n_k \sim (2k-1)\pi R/2\tau = 4.3\times 10^6(2k-1),$$

$$\left(\frac{Z_L}{n}\right)_{shunt} = \frac{\Delta^2}{\pi\tau d} Z_0 \left(\frac{Z_0 R\sigma}{2}\right)^{\frac{1}{2}} n_k^{-3/2}$$

$$= 1.7\times 10^{-4}(2k-1)^{-3/2}\ ohm/wiggle,$$

$$Q_k \cong \Delta/\delta = 420(2k-1)^{\frac{1}{2}}.$$

If all the wiggles resonate at the same frequencies, the shunt impedance will add up to $(Z_L/n)_{shunt}$ = 5.6 $(2k-1)^{-3/2}$ ohm. It is reasonable to assume some spread in the depth τ of the wiggles. In this case, the resonance frequency will be spread out and $(Z_L/n)_{shunt}$ will be reduced by a factor $S_k = (\tan^{-1}2Q_k\delta\tau/\tau)/(2Q_k\delta\tau/\tau)$.

For the first mode, if we take $\delta\tau/\tau = 5\%$, $(Z_L/n)_{shunt}$ will be reduced to 0.20 ohm.

VI. Total Z_L/n

The total Z_L/n for all the contributions discussed above is plotted in Figure 4. We see that at low n, the real part is dominated by the resistive wall. At $n>10^3$, the effect of the monitor plates comes in and after n_{cutoff}, it is dominated by free space radiation. A few peaks due to the resonances of the bellows' wiggles are also seen.

The imaginary part starts off inductively from the contribution of the monitor plates and is dominated by free space radiation after n_{cutoff}.

The contribution due to the Lambertson magnets has not been included. It may be big because these magnets are warm and the conductivity of the laminations is small. The image current has to flow around each lamination and the total resistance can be big. Also the rf cavities have not been studied here. They will dominate at low frequencies.

VII. Effective Z_L/n For A Bunched Beam

For the stability criterion of a bunched beam, we need to compute Z_L/n for a coherent bunch mode:[8]

$$\left(\frac{Z_L}{n}\right)_m = \frac{\sum_{n=-\infty}^{\infty}\left[Z_L(\omega_n)/n\right] h_m(\omega_n)}{\sum_{n=-\infty}^{\infty} h_m(\omega_n)},$$

where h_m is the power density for mode m. For m = 0,

$$h_0(\omega) = e^{-\omega^2\sigma_\ell^2/c^2},$$

$$h_1(\omega) = \omega^2 e^{-\omega^2\sigma_\ell^2/c^2}.$$

We shall take the R.M.S. bunch length $\sigma_\ell \sim 50$ cm; the contributing frequencies will be up to 10^8 Hz or $n\sim 10^4$. Thus, for the resistive wall effect, we can assume the beampipe to be made of copper only. Then, only the imaginary part will contribute. Since $(Z_L)_{wall} \sim\sqrt{\omega}$, we get

$$\left|\frac{Z_L}{n}\right|_{\substack{wall\\ m=0}} = 2.05\sqrt{\frac{\omega_0\sigma_\ell}{c}}\ \frac{1}{b}\sqrt{\frac{RZ_0}{2\sigma}}$$

$$= 2.05\ \frac{1}{b}\sqrt{\frac{\sigma_\ell Z_0}{2\sigma}}$$

$$= 0.0099\ ohm$$

and

$$\left|\frac{Z_L}{n}\right|_{\substack{wall\\ m=1}} = 1.025\ \frac{1}{b}\sqrt{\frac{\sigma_\ell Z_0}{2\sigma}} = 0.0050\ ohm.$$

For the monitor plates, only the imaginary part contributes. Since the imaginary part is nearly constant up to $n\sim 7.6\times 10^4$, we have, for m = 0,

$$\left|\frac{Z_L}{n}\right|_{\substack{plates\\ m=0}} \sim M\left(\frac{\phi_0}{\pi}\right)^2 Z_0 \frac{\sigma_\ell\sqrt{\pi}}{2R}\ erf\left(\frac{\ell}{\sigma_\ell}\right) = 0.18\ ohm$$

and for m = 1,

$$\left|\frac{Z_L}{n}\right|_{\substack{plates\\ m=1}} \sim M\left(\frac{\phi_0}{\pi}\right)^2 Z_0 \frac{\ell}{R} \exp\left(-\frac{\ell^2}{\sigma_\ell^2}\right) = 0.17\ ohm.$$

Other contributions such as synchrotron radiation, bellows, etc. when weighted over a bunch mode are much smaller than the contribution of the monitor plates. For single beam stability, the limit[2] on longitudinal impedance Z_L/n is $\sim$1 ohm for a 5 Tesla ring at a luminosity of 10^{33} cm^{-2} sec^{-2}. We see that only the contribution of the monitor plates comes close to this limit. Therefore, care must be taken in their design.

VIII. Transverse Impedance

For a 5 Tesla ring at a luminosity of 10^{33} cm^{-2} sec^{-2}, the limit[2] on transverse impedance Z_T is $\sim 1\times 10^6$ ohm/cm.

The space charge contribution is

$$\left(Z_T\right)_{spch} = \frac{i Z_0 R}{\beta_p^2\gamma_p^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right).$$

The beam radius scales with $\gamma_p^{-1/2}$. Thus taking a = 0.05 cm at 20 TeV, we get

$$\left(Z_T\right)_{spch} = \begin{cases} 1.3\times 10^4\ ohm/cm & 1\ TeV, \\ 650\ ohm/cm & 20\ TeV. \end{cases}$$

The contribution of the resistive wall is

$$(Z_T)_{wall} = (1-i)\frac{2R^2}{n\beta_W b^3}\frac{1}{\delta\sigma}$$

$$= (1-i)\,4.9\times10^5 n^{-\frac{1}{2}}\ \mathrm{ohm/cm},$$

where $\sigma = 6.25\times10^7\ \mathrm{ohm}^{-1}\ \mathrm{cm}^{-1}$ for copper at 4°K has been used and $\beta_W c$ the phase velocity of the disturbance has been taken as $\sim c$. When averaged over bunch mode m = 0 as in Section VII,

$$|Z_T|_{\substack{wall\\ m=0}} = 4.1\frac{R}{b^3}\sqrt{\frac{\sigma_\ell Z_0}{2\sigma}} = 5.3\times10^3\ \mathrm{ohm/cm}$$

which is well below the stability limit.

The bellows of length ℓ = 3 cm and radius d = 3 cm, when considered as cavities, will give a contribution[9] of (M = 2184 bellows)

$$(Z_T)_{bellows} = -i\ \frac{MZ_0\ell}{\pi b^2}\ \frac{(d/b)^2-1}{(d/b)^2+1}$$

$$= -i\ 2.0\times10^4\ \mathrm{ohm/cm}$$

which is below the stability limit.

Each monitor plate of length ℓ will give a contribution of[5]

$$(Z_T)_{plate} = \left(\frac{\phi_0}{\pi}\right)^2\left(1+\frac{\sin 2\phi_0}{2\phi_0}\right)\frac{2Z_c\gamma_W^2 R}{\beta_p n b^2}\cdot$$

$$\cdot\left[C_1(\beta_W-\beta_p)+C_2(1-\beta_p\beta_W)\right],$$

where

$$C_1 = -\sin 2\phi\sin 2\beta_W\phi + i\sin 2\beta_W\phi\cos 2\phi,$$

$$C_2 = 1-\cos 2\phi\cos 2\beta_W\phi - i\cos 2\beta_W\phi\sin 2\phi,$$

$$2\phi = n\ell/R$$

For the slow wave that causes instability, the phase velocity of the disturbance is related to the particle velocity by $\beta_W \simeq (1-\nu/n)\beta_p$ where ν is the betatron tune. Thus, when $\nu/2 << n << 2\nu\gamma_p^2$, $\gamma_W^2 \simeq n/(2\nu\beta_p^2)$. As a result, the contribution of M plates becomes

$$(Z_T)_{plates} = M\left(\frac{\phi_0}{\pi}\right)^2\left(1+\frac{\sin 2\phi_0}{2\phi_0}\right)\frac{Z_c R}{n b^2}\left(2\sin^2\frac{2n\ell}{R} - i\sin\frac{4n\ell}{R}\right),$$

which is very similar to the formula for $(Z_L/n)_{plates}$ in Section IV. When $n << R/4\ell$, we get

$$(Z_T)_{plates} = (4.2\,n - i\,2.0\times10^5)\ \mathrm{ohm/cm},$$

where we have taken M = 1456, Z_c = 50 ohm, ℓ = 18 cm and b = 2.54 cm. For higher n, $(Z_T)_{plates}$ fluctuates and falls as 1/n. When averaging over bunch mode m = 0 with R.M.S. bunch length σ_ℓ = 50 cm, only the imaginary part contributes and $|Z_T|_{plates,\ m=0} = 1.9\times10^5$ ohm/cm, which is of the same order of magnitude as the stability limit. Thus, as in the situation of the longitudinal impedance, the largest contribution of Z_T comes from the monitor plates. We note that $|Z_T|_{plates,\ m=0}$ scales with $\ell R^{\frac{1}{2}} b^{-2}$ ($|Z_L/n|_{plates,\ m=0}$ scales with $\ell R^{-\frac{1}{2}}$). Further reduction of the beampipe radius will lead to an instability.

The total Z_T for all the effects discussed is plotted in Figure 5.

Appendix

In this appendix, the impedance due to the wall resistivity of the beampipe with a copper coating is derived. At a depth x inside the beampipe, the azimuthal magnetic field H_c satisfies the equation

$$\frac{\partial^2 H_c}{\partial x^2} + \frac{2i}{\delta^2}H_c = 0,$$

where δ is the skin depth of the beampipe material concerned. Assuming that the thickness of the coating of copper (subscript 1) is α and the thickness of the stainless steel part (subscript 2) is infinite, we get

$$H_c = \begin{cases} Ae^{-(1-i)x/\delta_1} + Be^{(1-i)x/\delta_1} & 0\le x\le\alpha, \\ Ce^{-(1-i)x/\delta_2} & \alpha\le x. \end{cases}$$

The electric field in the direction of the beampipe E_c at depth x is related to H_c by

$$E_c = -\frac{1}{\sigma}\frac{\partial H_c}{\partial x}$$

where σ is the electric conductivity. The continuity of H_c and E_c at the boundary of the two materials $x=\alpha$ enables us to solve for A and B in terms of C; i.e.,

$$\begin{Bmatrix} A \\ B \end{Bmatrix} = \frac{1}{2}C\left(1\pm\sqrt{\sigma_1/\sigma_2}\right)e^{\pm(1-i)\sqrt{\omega/\omega_1}\left(1\mp\sqrt{\sigma_2/\sigma_1}\right)},$$

where we have made use of the fact that the thickness of the copper coating is one skin depth ($\alpha=\delta_1$) at frequency $\omega=\omega_1$ or $\alpha=\delta_1\sqrt{\omega/\omega_1}$.

We are interested in H_c and E_c at the interior surface (z = 0) of the beampipe, which we denote by $H_{||}$ and $E_{||}$ respectively. We have $H_{||}$ = A+B and $E_{||} = (1-i)(A-B)/\sigma_1\delta_1$. Therefore

$$E_{||} = \frac{(1-i)H_{||}}{\sigma_1\delta_1}\ \frac{1-\dfrac{1-\sqrt{\sigma_1/\sigma_2}}{1+\sqrt{\sigma_1/\sigma_2}}e^{-2(1-i)\sqrt{\omega/\omega_1}}}{1+\dfrac{1-\sqrt{\sigma_1/\sigma_2}}{1+\sqrt{\sigma_1/\sigma_2}}e^{-2(1-i)\sqrt{\omega/\omega_1}}},$$

which is exactly the electric field due to wall resistivity that opposes the motion of the beam particles. When the longitudinal perturbing harmonic $n \ll \gamma_W R/b$, $H_{||}$ is related to the perturbing beam current I_n by $H_{||} = I_n/2\pi b$. Thus, dividing $E_{||}$ by I_n and integrating the result along the storage ring, the impedance due to wall resistivity is obtained.

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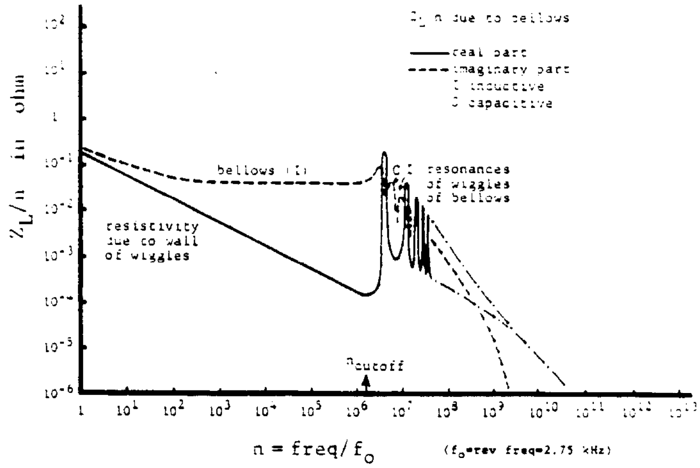

Figure 3

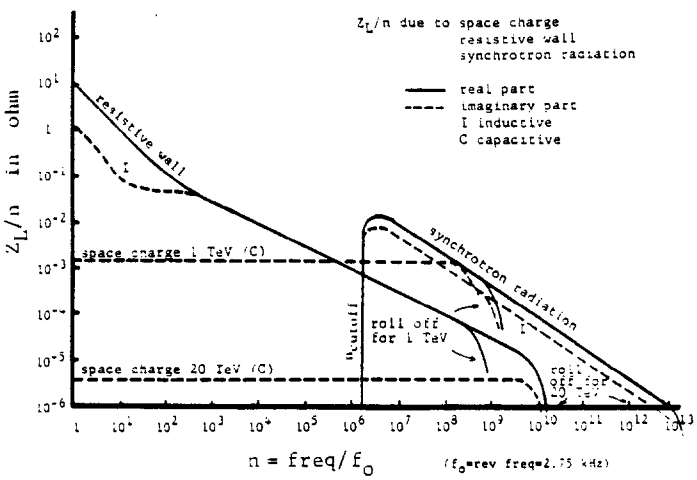

Figure 1

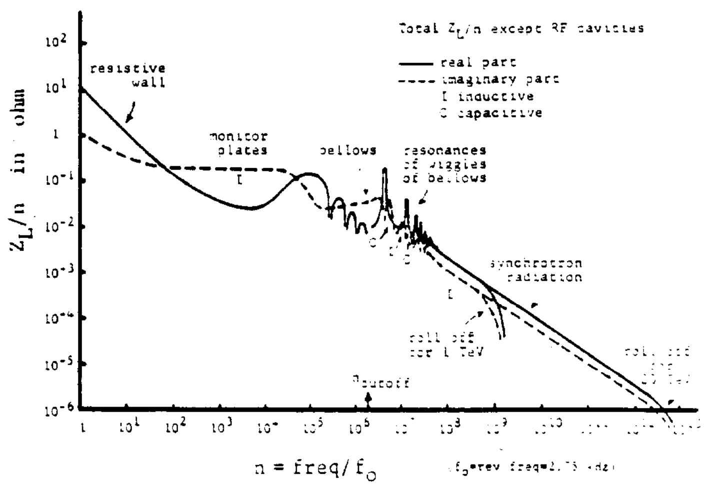

Figure 4

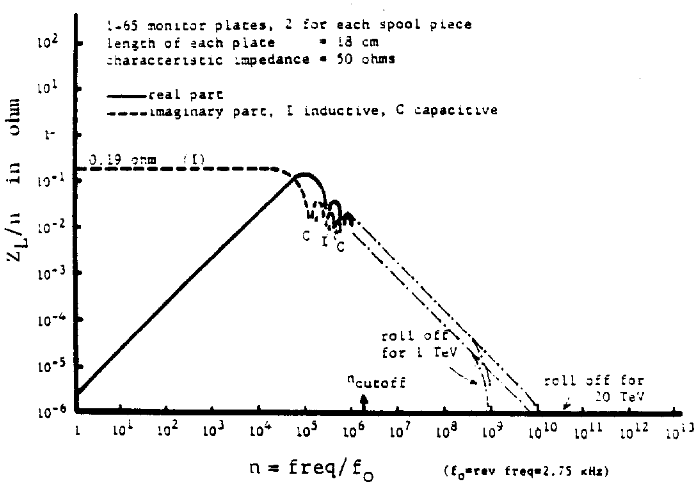

Figure 2

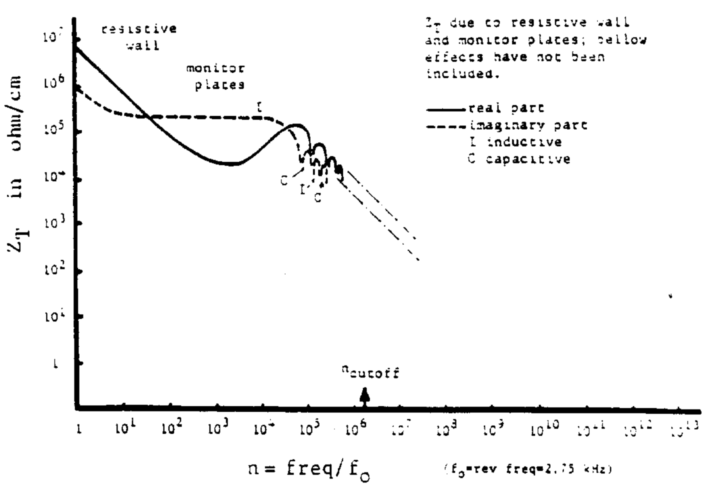

Figure 5