Nuclear Forces from Quarks and Gluons

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INTRODUCTION: NUCLEAR FORCES

When it was established that a nucleus is made of protons and neutrons, people realized the necessity of describing a new force that binds protons and neutron inside the nucleus. Electromagnetic force produces no attraction among protons and neutrons, while attractions induced by gravitational forces are too weak at such a short distance. As is well known, Hideki Yukawa introduced a new particle, nowadays known as a pion, to explain this new force, the nuclear force. According to his idea, protons and neutrons interact with each other by exchanging pions, whose mass was estimated to be 100~200 MeV (1 MeV=10^6eV) from the typical size of a nucleus. Since the mass of a pion lies between those of an electron and a nucleon (a generic name for protons and neutrons), the pion was called a meson at that time, and we now use the word meson as a generic name for pions and their relatives. The pion was indeed observed experimentally in 1947, and Dr. Yukawa received the Nobel Prize in Physics 1949.

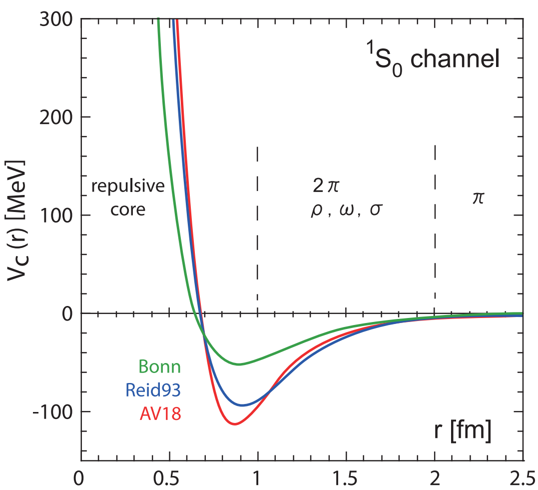

Fig. 1: Three examples of the phenomenological nuclear potential for the S-wave with the total spin 0. Taken from Ref. [3].

After Yukawa's success, nuclear forces have been investigated both theoretically and experimentally in detail, and are summarized as nuclear potentials. Three examples of nuclear potentials are given in Fig. 1, where a horizontal axis is a distance between nucleons in unit of fm (1fm=10^{-15} m) and a vertical axis represents a nuclear potential for an S wave with a total spin 0 in unit of MeV. The three potentials in the figure share the following common features. At a long distance (longer than 2 fm), there appears an attraction, which can be explained by the one pion exchange according to Yukawa's theory. At an intermediate distance (between 0.8 fm and 2 fm) the attraction becomes stronger, probably due to exchanges of multi pions and/or other heavier mesons. At a short distance (shorter than 0.8 fm), on the other hand, the attraction turns into a repulsion which gets larger and larger as the distance decreases, forming a "repulsive core" (strong repulsion at the short distance). The repulsive core is essential for the stability of atomic nuclei against collapse. Moreover, the repulsive core is an important ingredient for determining the maximum mass of neutron stars and for igniting Type II supernova explosions.

The complicated structure of nuclear potentials as a function of distance may suggest the existence of some internal structures in nucleons. Indeed, it is now established that a nucleon is made of three more fundamental particles, named quarks, which interact with each other by exchanging particles, called gluons. There are two species of quarks, up and down quarks, inside nucleons. A theory which governs a dynamics of quarks and gluons is called QCD (quantum chromodynamics). Whether QCD can explain the complicated structures of the nuclear potential in Fig. 1 is the main topic of this article and our answer to this question will be given below.

DIFFICULTIES

It is not so easy to deduce properties of nuclear potentials from the dynamics of quarks and gluons, as stated by Yoichiro Nambu in his book entitled "Quark: Frontiers in Elementary Particle Physics" (World Scientific, 1985): "Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."
First of all, the potential in classical mechanics is given by an energy for particles at rest as a function of distances. In quantum mechanics, however, the potential energy cannot be measured in this way, since a particle cannot have definite position and momentum at the same time due to the uncertainty principle. Instead, the potential is taken from the corresponding classical theory. For example, one usually uses the (classical) Coulomb potential to calculate a spectrum of a hydrogen atom. In the case of quantum field theories such as QCD, a notion of "potentials" becomes further ambiguous, since a number of particles are not conserved due to their creations and annihilations.

Secondly, even if the potential is defined, it is not so easy to calculate it in QCD. A perturbative expansion around the free theory, which is very successful in many other areas in physics, fails to work, since the interaction in QCD is so strong that it forms bound states of quarks, called hadrons (a generic name of nucleons and mesons). This failure of the perturbation theory is closely related to "quark confinement" phenomena that quarks never appear as free particles and are always confined inside hadrons. Therefore, a new method is required to perform non-perturbative calculations in QCD.

To overcome this difficulty, Ken Wilson proposed lattice QCD, which is a QCD defined on a 4-dimensional discrete lattice. Since lattice QCD in a finite volume is equivalent to quantum mechanics for a finite degree of freedoms, and thus can be defined non-perturbatively, analyses beyond the perturbation theory such as the strong coupling expansion and numerical simulations can be employed to investigate lattice QCD. In particular, thanks to continuous increases in the performance of super-computers and steady progress in simulation algorithms, hadron masses are now evaluated very accurately in lattice QCD, using numerical simulations based on Monte-Carlo methods. After taking a limit that a lattice spacing goes to zero, hadron masses in lattice QCD agree very well with experimental values, showing not only the correctness of QCD as a theory of the strong interaction among quarks and gluons but also the usefulness of lattice QCD combined with numerical simulations. See Ref. [1] for the latest results of hadron masses in lattice QCD.

DEFINITION OF NUCLEAR POTENTIAL IN QCD

Now let us go back to the first problem, the definition of the potential in QCD. One may wonder how potentials in Fig. 1 can be obtained if the definition of the potential is problematic. Nuclear potentials in Fig. 1 are extracted from experimental data of nucleon-nucleon scattering phase shifts as follows. One first chooses some form of the potential with several parameters, so that scattering phase shifts can be calculated by solving the Schroedinger equation with this potential. One then minimizes a difference of scattering phase shifts between theory and experiment by varying parameters of the potential. One finally determines the potential with the best choice of parameters. For this method to work, the collision energy of the nucleon scattering experiment must be smaller than the particle production threshold in order to prohibit inelastic scatterings such that two nucleons go to two nucleons plus one pion. It is then reasonable to assume that the scattering at such low energy can be described by quantum mechanics with some potential. Of course, an initial choice for a form of potential affects the final result, as seen in Fig. 1, where three similar but different potentials are plotted. These three potentials, however, reproduce the same experimental scattering phase shifts by construction.

As seen above, the potential is not unique. Therefore, as long as it reproduces the correct scattering phase shifts, we may adopt a convenient definition of the potential in QCD. Our proposal for the definition of the potential is similar but a little different from the one used to obtain potentials in Fig. 1. Let us explain our strategy to define and extract nuclear potentials in QCD. We first define a wave function for two nucleons, more specifically a Nambu-Bethe-Salpeter (NBS) wave function, which depends on a relative coordinate between two nucleons. This NBS wave function is shown to satisfy the Schroedinger equation for two free nucleons if a distance between two nucleons becomes sufficiently large. More importantly, the NBS wave function at such a large distance behaves as a free (partial) wave, sin(kr+d)/kr, where k, r are relative momentum and distance between two nucleons, while the phase shift d is exactly equal to the phase of the S-matrix for the nucleon-nucleon scattering in QCD. The NBS wave function therefore can be regarded as a scattering wave in quantum mechanics even though it is defined in QCD. If we multiply (H_{free}-E) to the NBS wave function, where H_{free} is a free Hamiltonian and E is a kinetic energy, the result becomes zero at a large separation, while it remains non-zero at a short distance (a small separation). We define our potential from this non-zero contribution as (H_{free}-E) (NBS wave function)=V (NBS wave function), where V is our nuclear potential. (A more detailed explanation can be found in Ref. [2].) By construction, our potential reproduces correct scattering phases shifts of QCD, since a solution to the Schroedinger equation with this potential must be the original NBS wave function, which gives the phase shift "d" in its large "r" behavior. In other words, we define the potential from the wave function through the Schroedinger equation, in contrast to the standard quantum mechanics, where the wave function is obtained after solving the Schroedinger equation with a given potential. Cause (potential) and effect (wave function) are reversed in our method to define potentials.

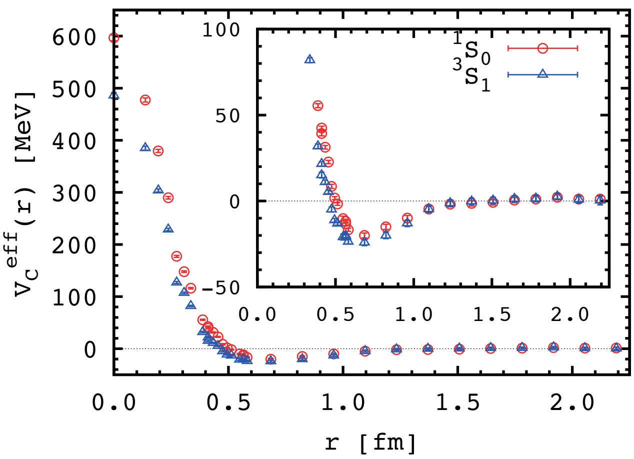

Fig. 2: The first result of nuclear potentials for S_0 and $^3S_1$ in quenched lattice QCD. Taken from Ref. [3].

RESULT

In Ref. [3], we have calculated the NBS wave function in lattice QCD with the quenched approximation, which neglects creations and annihilations of quark and anti-quark pairs in the vacuum, at the lattice spacing a=0.137 fm. Quarks in our calculation are much heavier than physical ones in nature, due to several technical difficulties in numerical simulations. A size of quark mass is related to the pion mass: the pion mass in our simulation is 530 MeV while it should be 135 MeV in nature.

According to our definition, we then have extracted nuclear potentials. Fig. 2 shows nuclear potentials obtained in this calculation for the S-wave with the total spin 0 and 1, which well reproduces qualitative features of the phenomenological nuclear potential given in Fig. 1, namely the repulsive core at a short distance surrounded by the attractive well at medium and long distances. In the figure, $^1S_0$ and $^3S_1$ denote spin-singlet and spin-triplet S wave states, respectively, where $^{2S+1}L_J$ represents a state with a total spin S, a orbital angular momentum L and a total angular momentum J. Although this calculation is still preliminary due to the heavier quark mass, a use of the quenched approximation and a lack of the continuum (a->0) extrapolation, this result is a significant step toward an understanding of nuclear forces from the 1st principle of the strong interaction, QCD. Fortunately, our first paper was selected as one of the 21 research highlights of 2007 in Nature (http://www.nature.com/nature/journal/v450/n7173/full/4501130a.html), with the following commentary: "This achievement is both a computational *tour de force* and a triumph for theory."

OUTLOOK

As shown in this article, it now becomes possible to extract nuclear potentials from the fundamental theory, QCD, using lattice formulation and numerical simulation techniques. The method described here can be easily extended to other hadronic interactions such as baryon-baryon, baryon-meson, and meson-meson interactions, where a baryon is a generalization of the nucleon, which contains not only up and down quarks but also a new species of quarks, called "strange". Moreover, three-nucleon forces, which appear only in systems with three or more nucleons can also be investigated by this method. See Ref. [4] for such calculations.

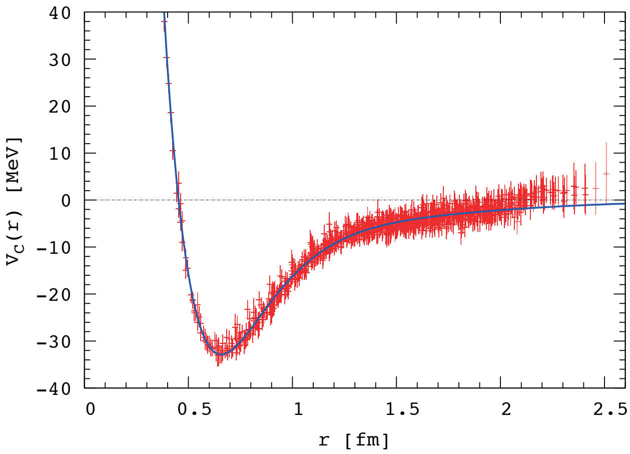

Fig. 3: Recent results of a nuclear potential for the $^1S_0$ channel obtained in full lattice QCD, together with the fit (solid line). Taken from Ref. [5].

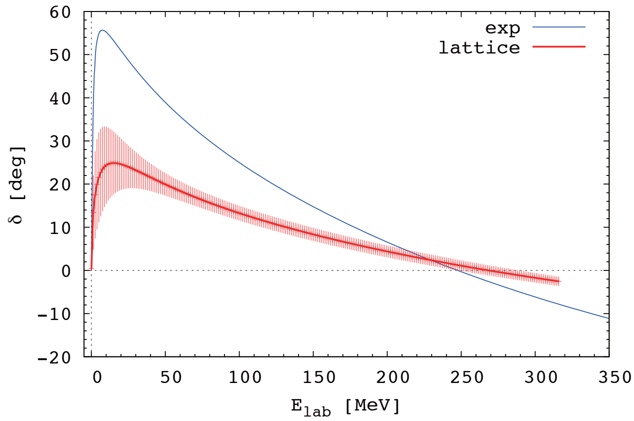

Fig. 4: The scattering phase shift for the $^1S_0$ channel in the laboratory system from the lattice nuclear potential in Fig. 3, together with experimental data. Taken from Ref. [5].

In Fig. 3, we give a recent result of the nuclear potential for the spin singlet channel, obtained in QCD without quenched approximation ("full QCD") at a=0.09 fm and 700 MeV pion mass[5], together with a fit (solid line) of numerical data. Solving the Schroedinger equation with the fitted potential, we then calculate the nucleaon-nucleaon scattering phase shift, which is shown in Fig. 4 as a function of collision energy in the laboratory system, together with corresponding experimental values. As seen from the comparison, the lattice QCD result well reproduces qualitative features of the experimental scattering phase shift. At the quantitative level, however, the strength of the increase near the origin is weaker for lattice data than the experimental one, probably due to the heavier pion (700 MeV) than the physical pion (135 MeV). Therefore, in order to investigate whether potentials obtained from QCD reproduce experimental scattering phase shifts or not, lattice QCD calculations will be needed to be performed at the physical pion mass. Such challenging calculations are being planned using the "Kei" computer, a 10 peta-flops super computer, located at Riken Advanced Institute for Computational Science (AICS), Kobe, Japan (http://www.aics.riken.jp/en/). We expect that nuclear potentials at the physical pion mass will be obtained within one year or two.

References

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Sinya Aoki received his Bachelor of Science, Master of Science and Doctor of Science degrees in physics from University of Tokyo. Until March, 2013, he was a professor of Graduate School of Pure and Applied Sciences, University of Tsukuba. In April, 2013, he becomes professor of Yukawa Institute for Theoretical Physics, Kyoto University. His research interests include lattice fermion problems, lattice QCD, and hadronic interactions from lattice QCD. He received the 1st JSPS Prize (2005), the 25th Inoue Prize for Science (2009), the 23rd Tsukuba Prize (2012), and 2012 Nishina Memorial Prize.