

Condensed light, quantum black holes and L-CDM cosmology: Experimentally suggested and tested unified approach to dark matter, dark energy, cosmogenesis and two-stage inflation

Victor Borsevici

*International Informatization Academy, branch Rep. of Moldova,
Chisinau, Rep. of Moldova
E-mail: borsevici@mail.ru*

Based on the fundamental last discoveries (such as gravitational waves, Higgs field, gravitating Bose-Einstein photon condensates etc.), guided by the last A. Einstein's conjecture (about algebraic approach to the combined quantum and continuum description of reality) and directed by the J. Bekenstein-V. Mukhanov approach to the quantum black holes physics and astrophysics, the L. Susskind's "one photon – one bit" principle and V. Gribov-S. Hawking tunneling effect, one can easily find the simple but consistent algebraic relations and physical laws that govern birth and death of black holes in the Early Universe, its accretion and two-particle emitting (both outside black hole) accompanied with gravitational radiation.

From these basic relations in a natural way derive the Planck units system that confirm the theoretical consistence of presented approach and directly leads to the Planck Scale physics where standard models of the particle physics and cosmology are "blinded". Presented approach elucidates the physical background of dark matter (black holes), dark energy (gravitational waves), hot Big Bang and two-stage inflation of the Universe. Moreover, it solves many old unsolved problems, such as "cosmic censorship" conjecture, information loss paradox, black hole "end-point" problem, "cosmological constant" problem and so on.

The great importance for the validation of presented approach and its results had the comparison with "Planck-2018" data and other relevant information from the sources of observational cosmology, astrophysics, gravitational waves astronomy and numerical relativity.

Keywords: Planck scale physics, dark matter, dark energy, Bose-Einstein photon condensates, Cosmology.

*To the blessed memory of Jacob Bekenstein
and Steven Weinberg*

Introduction

*"Raffiniert ist der HerrGott, aber boshaft ist er nicht."
(A. Einstein)*

What "banged"? What is dark matter? What is dark energy? What caused two-stage inflation? *etc.* Such problematic questions that provoked a lot of troubles for L-CDM cosmology are a consequence of unsolved problems in physics. L. Smolin called this stagnant situation "Einstein's unfinished revolution" and the more critical theoreticist S. Hossenfelder called it "Lost in math".

But over the last decade, experimental physics and observational cosmology have made many fundamental discoveries: gravitational waves (LIGO), Higgs boson (LHC), photon condensates with rest energy and rest mass trapped in “mirror cavities” (Bonn University).^{1,2}

Through these remarkable results, Nature suggests that at pressures and temperatures well above the Higgs field level (246 GeV), only the 2-d and 3-d photon condensates trapped in their own “gravitational cavities” should be unique sources of gravity during the Early Universe. Moreover, we conclude that the real “*prima materia*” at the “beginning of the times” was a 3-dimensional Planck photonic condensate as “explosive” accompanied by Planck fluctuations as “fuse”. This unambiguously postulates the fundamental role of the laws of conservation of energy and momentum in cosmology, as well as reflect the decisive role of quantum fluctuation in the formation of the modern Universe (see V. Mukhanov-G. Chibisov and S. Hawking – B. Carr approaches).

Note that after the discovery of gravitational waves, the famous Nobel Prize winner P. W. Anderson, in his prophetic “Four Last Conjectures” (2018), argues that “dark energy” is gravitational radiation which causes irreversible loss of mass in the Universe.

It is quite clear that in the swift initial process of Hot Bing Bang we have only three kinds of “actors” at the Planck scale: the 2-d spherical self-gravitating photonic condensates (primordial quantum black holes as “dark matter”), free hot photons and gravitational waves (both acting as massless stretched “dark energy” forces). Note that only gravitational radiation represents one irreversible repulsive force. According to our calculations, only a part of relic gravitational waves with Planck energies E_p and $E_p/2$ make up 93.38% of the dark energy in the modern Universe. As you can see, S. Weinberg was absolutely right: only the energy of Hot Bing Bang causes the universal expansion process.

Moreover, the irreversibility of the gravitational radiation energy prohibits all kind of “big crunch” and “bouncing” (cyclic) cosmological theories, proving instead the Hot Big Bang approach to the cosmogenesis.

We must point out separately that at the end of the initial phase of the Hot Big Bang process (at about $50\pi \times 5.391 \times 10^{-44}$ sec), when the strongest inequality $\Omega_M \ll \Omega_\Lambda$ was reached and stage I of hyperinflation began, a part of the surviving primordial black holes became “seed” of the primordial Cosmic Web, in which baryogenesis occurs and all astrophysical objects are formed.

Where do these conclusions follow? Taking into account the quantum nature of the 2-d spherical photon condensates, trapped in their own gravitational fields, guided by the J. Bekenstein’s fundamental dependence between the Compton wavelength and the gravitational radius,³ L. Susskind’s principle “one photon — one bit”,⁴ the V. Mukhanov’s quantum black holes “degeneracy” principle⁵ and the modified V. Gribov–S. Hawking tunneling effect,^{6,7} one can easily find any exact and adequate laws that govern birth and death, rise and downfall, accretion of light

and quantum two-particle emission (both outside black holes). All these processes are accompanied by the emission of gravitational waves.⁸

In what follows, we will demonstrate that the most important thing that distinguishes such natural black holes modelling from all other “membrane models” (see K. S. Thorne, F. Wilczek and M. Maggiore approaches) is that tremendous gravitational squeezed force is perfectly equilibrated by repulsive (“antigravitational”) quantum-mechanical one.

Note that only the 2-d photon condensate “construct” with its tremendous “band” of possible wavelengths can explain the incredibly wide range of sizes, lifetimes and masses of black holes, ranging from the Planck scale to tens of Pluto orbits and tens of billions of masses of the Sun (see the quasar *TON 618*). We will demonstrate that this approach is free from the “cosmic censorship” principle, the “no hair” theorem and the information loss paradox.

Using this unified approach, we can calculate that at the end of the epoch of baryogenesis, when stage I of inflation ends, the rest energies of black holes (dark matter) and energy of gravitational waves (dark energy) are 28.62% and 66.42%, respectively. After this epoch, there came a non-accelerated expansion. But 6–8 billion years ago, the II stage of inflation began. Now, accordingly to “Planck-2018” data, we find the 26.57% and 68.47%, respectively. Comparison with the end of the stage I clearly shows that the increase in dark energy is caused by a decrease in the energy associated with dark matter. This leads to the unambiguous conclusion that stage II of inflation is provided by binary coalescences of black holes.

The greatest discoveries of Λ -CDM cosmologies are namely such epiphenomenons as “dark matter”, “dark energy”, “Hot Big Bang” and “two-stage inflation”. It remains to find the deep physical background of these unsolved phenomenologies. The “Einstein’s Unfinished Revolution” must be continued. However, it is quite clear that this is impossible without taking into account the new discoveries of modern physics and cosmology, including the self-gravitating Bose–Einstein photonic condensates. We must demonstrate the new approach to the Planck Scale Physics, where standard models of physics and cosmology was “blinded”.

The new theoretical approach is consistent, when from its basic relations and equations directly derive any fundamental constants. In what follows, we will demonstrate that proposed basic equations generate the fundamental system of Planck units. This directly leads to the Terra Incognita of Planck Scale Physics and solves many puzzles of black hole physics, astrophysics and cosmology, such as “cosmic censorship” problem, information loss paradox, black hole “end-point” problem, “cosmological constant” problem, dark matter and dark energy problems and so on.

1. Principia

In 1916 A. Einstein wrote: “. . . das die Quantentheorie nicht nur die Maxwellische Elektrodynamik, . . . auch die neue Gravitationstheorie wird modifizieren müssen”.⁹

Already in those distant times, under the influence of Bohr's theory of hydrogen atom, he came to the conclusion that a spherical system should emit not only quanta of light, but also any portions of gravitational waves. This led him to believe that the new theory of gravity should be modified taking into account quantum electrodynamics.

Many years later, in 1954, in his last work, he wrote the closing lines that sound like a testament: "... a finite system with its finite energy can be described in full by a finite set of numbers (quantum numbers). Seemingly, it cannot be compliant with the continuum theory and requires a purely algebraic theory for a reality description. However now nobody knows, how to find a basis for such a theory".¹⁰ Actually, A. Einstein explicitly claimed in his "last will" that just the algebraic approach must be the magistral way to unify quantum physics and continuum general relativity.

Only two decades later, it was the young Israeli physicist J. Bekenstein who turned out to be the "nobody" who found a way to do this. In his revolutionary work "Black Holes and Entropy"³ he gave an amazing formula for the entropy of a black hole

$$S_s \text{ (Bekenstein)} = \frac{\ln 2}{8\pi} kc^3 \hbar^{-1} G^{-1} A_S ,$$

which unites all world constants: Boltzmann's constant (k), speed of light (c), reduced Planck constant (\hbar), gravitational constant (G), as well as the area of the Schwarzschild spherical singularity

$$A_S = 4\pi R_S^2 ,$$

where R_S is the Schwarzschild gravitational radius.

Bekenstein's formula was striking in that it linked together thermodynamics, quantum physics, the theory of gravity, and geometry. But the most surprising thing was that he connected all this within the framework of a unified information approach to the physics and astrophysics of black holes. In his famous formula, he associated the multiplier $\ln 2$ with Shannon's "bit". And this is not accidental, because he followed the famous principle of his great teacher J. A. Wheeler: "It from bit".

In 1986, the young Russian physicist V. Mukhanov, guided (as A. Einstein) by hydrogen atom analogy, came to the irrefutable conclusion that the emission mechanism of quantum black holes must obey the "degeneracy" principle.¹¹ He turns to J. Bekenstein and in 1995 their common outstanding work "Spectroscopy of the quantum black hole"¹² is published. In this work, it was shown that the true emission spectrum of quantum black holes *is discrete*.

With the advent of new fundamental experimental discoveries, this work of J. Bekenstein and V. Mukhanov began to acquire fundamental significance for physics and cosmology. Moreover, it turned out to be not just a "window" (as its authors was believed), but a whole "gateway" to *Terra Incognita* of Planck Scale Physics.

2. Basic Relations

With the discovery of 2-d photonic Bose-Einstein condensate¹ it became clear that the rejected Schwarzschild spherical singularity was becoming an obvious alternative to the black hole model in the form of a “point singularity” surrounded by an “event horizon” of Schwarzschild radius.

The biggest problem of all “membrane models” was their physical inconsistency. No one has been able to prove their resilience when facing the tremendous gravitational squeezed forces. From the “relativistic” point of view, this seemed to be an unsolvable problem. That is why, first of all, after justifying our approach, we will prove the strong equilibrium between squeezed gravitational forces and repulsive “antigravitational” quantum mechanical forces of quantum black hole. Note, that this strong equilibrium may be broken only by V. Mukhanov’s “degeneracy” effect.

Based on the Bekenstein–Mukhanov approach, we will show how all these unsolved problems are solved in terms of the original basic system of equations for spherical 2-d photon condensates, trapped in their own gravitational field:

$$\left\{ \begin{array}{l} \bar{\lambda}_{sm} = R_s, \quad (1) \\ \bar{\lambda}_{sm} = \frac{\hbar}{M_{sm}c} = \frac{\hbar c}{E_{sm}}, \quad (2) \\ R_s = \frac{2G}{c^2} M_s = \frac{2G}{c^4} E_s, \quad (3) \\ N_s = \frac{M_s}{M_{sm}} = \frac{E_s}{E_{sm}}, \quad (4) \\ N_s(\min) = 2. \quad (5) \end{array} \right.$$

Equations (1)–(3) bind together the reduced Compton wavelength, $\bar{\lambda}_{sm}$, which is associated to one photon with rest mass M_{sm} and rest energy E_{sm} , and Schwarzschild radius, R_s , which is expressed in terms of M_s (total rest mass of the condensate) and E_s (total rest energy).

Note that relation of the type of Eq. (1) was first time discovered by Bekenstein in his remarkable work Ref. 3 to determine the largest Compton wavelength of a particle absorbable by a black hole. In essence, general Eq. (1) can be called “Bekenstein relation”. This is the only one equation known to us that organically interconnects Quantum Physics and General Relativity. In fact, it reflect the deepest equivalence between the “quantum wavelength geometry” ($\bar{\lambda}_{sm}$) and the “relativistic geometry” (R_s) that occurs in black holes as spherical 2-d photon condensates trapped in their own gravitational fields.

Equation (4) organically make links between the number of photons in the condensate, N_s , and the quantities expressing its “individual” rest mass, M_{sm} , and rest energy, E_{sm} , as well as with “total” rest mass, M_s , and rest energy, E_s . We note in particular that N_s is a natural integer, that reflect a quantum nature of black holes physics.

Equation (5) sets a natural limit on the minimum number of photons in the condensate, equal to two: $N_s(\min) = 2$.

Note that for the evident lower boundary condition, system Eqs. (1)–(5) gives us (it is easy to verify) the following solutions:

$$M_s(\min) = \sqrt{\frac{\hbar c}{G}} = M_p, \quad E_s(\min) = \sqrt{\frac{\hbar c^5}{G}} = E_p, \quad R_s(\min) = 2\sqrt{\frac{\hbar G}{c^3}} = 2l_p, \quad (6)$$

where M_p and E_p are the Planck mass and energy, respectively, l_p — Planck length.

It is striking that we unexpectedly found ourselves in the field of Planck Scale Physics and absolutely accurately described the Planck black hole without any preliminary assumptions, hypotheses and conjectures. We have used only the most basic information from Quantum Physics and General Relativity enclosed in the base system Eqs. (1)–(5).

Note that we do not use the “quantum gravity” approach based on the “gravitons” hypothesis because of the gravitational waves (the “ripples” of spacetime) radically differ from the quantum-mechanical waves (the “waves of probability”).

3. Equilibrium

In order to make certain that a quantum black hole taken as spherical self-gravitating 2-d photon condensate is a quasi-stable astrophysical object, from all the solutions of the system Eqs. (1)–(5) we will pick the following one:

$$M_{sm} = \frac{\hbar}{cR_s} = \frac{\hbar c}{2GM_s} = \frac{M_p^2}{2M_s}, \quad (7)$$

$$E_{sm} = \frac{\hbar c}{\lambda_{sm}} = \frac{\hbar c}{R_s} = \frac{\hbar c^5}{2GE_s} = \frac{E_p^2}{2E_s}. \quad (8)$$

As we can see, according to Birkhoff’s Theorem the attractive gravitational force for one photon with rest mass M_{sm} is equal to

$$F_{sm, gr} = \frac{2GM_{sm}M_s}{R_s^2} = \frac{\hbar c}{R_s^2}. \quad (9)$$

But this attractive gravitational force must be perfectly equilibrated by quantum-mechanical “antigravitational” force:

$$F_{sm, qu} = \frac{dE_{sm}(\lambda_{sm})}{d\lambda_{sm}} = \frac{d(\hbar c/R_s)}{dR_s} = -\frac{\hbar c}{R_s^2}. \quad (10)$$

4. Quantum Information and Entropy: The Downfall of the “Point-Centered” Model

In his remarkable book “The Cosmic Landscape”,⁴ L. Susskind convincingly demonstrates that every photon is a carrier of one bit of quantum information. The point is that a photon can be in two spin states, $|0\rangle$ or $|1\rangle$.

In the case of a quantum black hole consisting of N_s photons, this means that the power of the set of its possible quantum states is 2^{N_s} . From the point of view

of information theory, a quantum black hole can be represented as a binary text consisting of N_s positions filled with symbols from the alphabet $\{|0\rangle, |1\rangle\}$. To each such text, θ_i , a probability, p_i , can be attributed. The maximum of C. E. Shannon's entropy is achieved for an uniform distribution $p_i = 2^{-N_s}$:

$$H_s \text{ (Shannon)} =_{max} - \sum_{i=1}^{2^{N_s}} 2^{-N_s} \log_2 2^{-N_s} = N_s \text{ (bit)} . \tag{11}$$

According to Shannon's approach, the maximum amount of quantum information, I_s , contained in a quantum black hole does not exceed this value. Moreover, it is equal to it:

$$I_s = H_s \text{ (Shannon)}_{max} = N_s \text{ (bit)} . \tag{12}$$

Now let's calculate the thermodynamic entropy of the black hole. To do this, we first need to calculate the temperature of the quantum black hole from the relation

$$E_{sm} = 2 \times kT_s , \tag{13}$$

where kT_s is the energy corresponding to vibrational freedom degree, 2 is the multiplier due to the two spin states, $|0\rangle$ and $|1\rangle$. From Eq. (13) directly follows the wanted relationship for the temperature

$$T_s = \frac{E_{sm}}{2k} = \frac{E_p^2}{4kE_s} = \frac{\hbar c^3}{4kGM_s} . \tag{14}$$

It is easy to see that we have obtained an expression very close to the one of S. Hawking:

$$T_s \text{ (Hawking)} = \frac{1}{2\pi} T_s . \tag{15}$$

However, the essential difference lies in the fact that the temperature T_s (Hawking) is the typical temperature of the black hole taken as a "black body", whereas we are dealing with a quantum black hole. On the other hand, this similitude of the results suggests that we are acting in the right direction.

From the Clausius relation $dS = dE/T$ and Eq. (14), by integration, we easily find the required expression for the thermodynamic entropy:

$$S_s = \int_0^{E_s} \frac{4k}{E_p^2} E dE = k2 \frac{E_s^2}{E_p^2} = kN_s . \tag{16}$$

In other words, we get a direct connection between the entropy of a black hole and the Shannon's entropy:

$$S_s = kH_s \text{ (Shannon)}_{max} = kN_s . \tag{17}$$

From Eqs. (3), (4) and (16) we can get another important relation:

$$S_s = kN_s = k \frac{R_s^2}{2l_p^2} = \frac{1}{8\pi} kc^3 \hbar^{-1} G^{-1} A_s , \tag{18}$$

where A_s is the area of the Schwarzschild spherical singularity. This immediately implies a connection with the cited above Bekenstein formula:

$$S_s \text{ (Bekenstein)} = \ln 2 \cdot S_s . \quad (19)$$

The remarkable physicist S. Hawking relied on a model of a black hole in the form of a “point-singularity” surrounded by a “event-horizon”. Actually, this means that in the center of symmetry of any black hole there should be matter with the limiting Planck density. Only 3-d Planck photon condensate can have such a density. It is not difficult to calculate the amount of quantum information, I_{s^*} , in this model:

$$I_{s^*} = N_{s^*} = \frac{M_s}{M_p} . \quad (20)$$

For a 2-d model, as we already know, this quantity will be equal to

$$I_s = N_s = 2M_s^2/M_p^2 .$$

Let's compare these amounts of quantum information for a black hole of solar mass M_\odot :

$$I_s/I_{s^*} = 2M_\odot/M_p = 1.827 \times 10^{38} \text{ (sic!) .}$$

As we can see, in the light of modern discoveries, this difference in the amounts of quantum information, is simply monstrous. That clearly shows the invalidity of “point-model”.

5. Natural Quantum Black Hole Description

We can continue J. Bekenstein–V. Mukhanov approach that in fact derive from the A. Einstein “testament” that was cited above.

From (4) and (5) we can conclude that the number of photons in quantum black hole taken as a spherical self-gravitating 2-d photon condensate must be presented in the following form:

$$N_s = 2n_s, \quad n_s = 1, 2, 3, \dots \quad (21)$$

where n_s is the *principal quantum number*.

Then it is easy to show that all our previous results can be represented as follows:

$$\bar{\lambda}_{sm} = R_s = 2l_p\sqrt{n_s} ; \quad (22)$$

$$M_s = M_p\sqrt{n_s}, \quad E_s = E_p\sqrt{n_s} ; \quad (23)$$

$$M_{sm} = M_p/2\sqrt{n_s}, \quad E_{sm} = E_p/2\sqrt{n_s} ; \quad (24)$$

$$\omega_{sm} \text{ (cyclic frequency)} = E_{sm}/\hbar = \omega_p/2\sqrt{n_s}, \quad (25)$$

where $\omega_p = 1/t_p$ is the Planck frequency, $t_p = \sqrt{\hbar G/c^5}$ is the Planck time;

$$A_s = 16\pi l_p^2 n_s ; \quad (26)$$

$$I_s = N_s = 2n_s \text{ (bit)} ; \quad (27)$$

$$S_s = kI_s = 2kn_s ; \quad (28)$$

$$T_s = E_p/4k\sqrt{n_s} = T_p/4\sqrt{n_s}, \quad (29)$$

where $T_p = E_p/k$ is the Planck temperature;

$$\rho_{sh}^I \text{ (surface information density)} = I_s/A_s = 1/8\pi l_p^2 = \frac{c}{\hbar\kappa} \left(\frac{\text{bit}}{\text{m}^2} \right), \quad (30)$$

where $\kappa = 8\pi G/c^2$ is the Einstein constant;

$$g_{sh} \text{ (surface gravity)} = G \frac{M_s}{R_s^2} = a_p/4\sqrt{n_s}, \quad (31)$$

$a_p = F_p/M_p = c/t_p$ is the Planck gravity *etc.*

Note, that this natural “emergence” of Planck Scale Physics, that was “automatically” arised from basics system Eqs. (1)–(5), clearly shows, that Planck Units System as such was organically and naturally emerge from experimental physics and observational cosmology. Moreover, as we see, the “great” unity between Quantum Physics and General Relativity was reached in natural way.

The famous physicist J. A. Wheeler once said: “Mass tells spacetime how to curve; spacetime tells mass how to move.” Inspired by this example, we can coin a new one: “Light quanta tells spacetime how to quantize; spacetime tells light quanta how to curve.”

6. Quantum Black Hole “Evaporation”: Two-Particle Emission and Gravitational Waves Radiation

As shown above, quantum black hole has the property of strong equilibrium between squeezed gravitational force and repulsive (“antigravitational”) quantum-mechanical force. This state of strong equilibrium can be broken only by the V. Mukhanov’s degeneracy effect, which consists in spontaneous quantum jump from n_s state to $n_s - 1$ state. In our case this means that in all relations Eqs. (21)–(31) the argument n_s must be replaced by $n_s - 1$. It’s obvious that quantum energy corresponding to the level $E_p\sqrt{n_s}$ must reduce to the one corresponding to the level $E_p\sqrt{n_s - 1}$. The difference

$$\Delta E_{s,s-1} = E_p (\sqrt{n_s} - \sqrt{n_s - 1}) = \frac{E_p}{2\sqrt{n_s}} \left(1 + \frac{1}{4n_s} + \frac{1}{8n_s^2} + \dots \right) \quad (32)$$

is equal to the sum of radiance energy of two-particle emission

$$\Delta E_{s,s-1 qu} = \frac{E_p}{2\sqrt{n_s}}$$

and gravitational waves radiation energy

$$\Delta E_{s,s-1 gr} = \Delta E_{s,s-1} - \Delta E_{s,s-1 qu} = \frac{E_p}{2\sqrt{n_s}} \left(\frac{1}{4n_s} + \frac{1}{8n_s^2} + \dots \right). \quad (33)$$

Note that energy of the quadrupolar mode is represented by term:

$$\left| \frac{\hbar}{c} \frac{dg_{sh}(n_s)}{dn_s} \right| = \frac{E_p}{2\sqrt{n_s}} \times \frac{1}{4n_s}. \quad (34)$$

From this result we easily understand that the next term $1/8n_s^2$ from the sequence in parentheses from Eq. (33) corresponds to octupolar mode and so on.

We come to the conclusion that in the parenthesis

$$\left(1 + \frac{1}{4n_s} + \frac{1}{8n_s^2} + \dots\right)$$

of the main expansion of polynomial in Eq. (32), the first term refers to the dipolar mode, the second term — to the quadrupolar mode, the third — to the octupolar and so on. In other words, all the terms, except the first one, are directly related to the gravitational radiation of a quantum black hole. It only remains to find out the physical nature of the dipolar mode. For this, we turn to the data of experimental physics. In 2011, physicists from Chalmers University of Technology experimentally proved the existence of Casimir–Unruh dynamical effect in the squeezed vacuum excited by the “moving mirror” with frequency ω , that produce a pair of non-entangled photons with associated frequency $\omega/2$ each.¹³

By this experiment Nature suggests that in general case the energy $\Delta E_{s,s-1 qu}$ of two-particle emission, (X_s, \bar{X}_s) , must be presented by the following form:

$$\Delta S_{s,s-1 qu} = \hbar\omega_{sm} = \hbar\omega_{sx} + \hbar\omega_{s\bar{x}} = E(X_s) + E(\bar{X}_s), \quad (35)$$

where ω_{sm} is the cyclic frequency Eq. (36),

$$\omega_{sx} = \omega_{s\bar{x}} = \omega_{sm}/2, \quad E(X_s) = E(\bar{X}_s) = E_{sm}/2, \quad (36)$$

where $\omega_{sx}, \omega_{s\bar{x}}$ are the frequencies associated with emitted particles.

The following quantum black hole “degeneracy” diagram can illustrate all these results:

$$BH_s \longrightarrow BH_{s-1} + X_s + \bar{X}_s + \Gamma_s, \quad (37)$$

where Γ_s is the generated gravitational wave with energy Eq. (33).

Note that the pair (X_s, \bar{X}_s) of non-entangled particles, depending on their energy, can be a pair of hot photons, this being especially important at energies well above the Higgs field level (246 GeV).

Remark 6.1. Using the expressions obtained above, it is easy to calculate the energy structure of quantum black hole at its “end-point” ($n_s = 1$):

$$BH_p \longrightarrow 2\gamma_{p,0} + \Gamma_{p,0} \quad (38)$$

with energies:

$$E(BH_p) = E_p\sqrt{1} = E_p, \quad 2E(\gamma_{p,0}) = E_p/2\sqrt{1} = E_p/2, \quad (39)$$

$$E(\Gamma_{p,0}) = E_p\left(\sqrt{1} - \sqrt{1-1} - 1/2\sqrt{1}\right) = E_p/2. \quad (40)$$

As it can be observed, at the “end-point” of the existence of any black hole, all ends with a spectacular explosion with emission of half of energy, $(E_p/2)$, as quantum electromagnetic radiation and another half, $(E_p/2)$ — as gravitational

radiation. Note that these last two photons carry the last two bits of quantum information. As we can see, in the presented unified approach so called “end-point” problem is solved.

In general case we can find that the total energy radiated by a quantum black hole “evaporating” from the state n_s to its “end-point” is equal to

$$E_{s\ qu} \text{ (rad)} = \sum_{n=1}^{n_s} E_p/2\sqrt{n} \text{ ,} \tag{41}$$

$$E_{s\ gr} \text{ (rad)} = \sum_{n=1}^{n_s} E_p \left(\sqrt{n} - \sqrt{n-1} - \frac{1}{2\sqrt{n}} \right) = E_p\sqrt{n_s} - \sum_{n=1}^{n_s} \frac{E_p}{2\sqrt{n_s}} \text{ ,} \tag{42}$$

$$E_s \text{ (total rad.)} = E_{s\ qu} \text{ (rad)} + E_{s\ gr} \text{ (rad)} = E\sqrt{n_s} = E_s \text{ .} \tag{43}$$

Note that with increase of n_s the total gravitational radiation quickly tends to its maximal value E_p . As we see, the proportionality

$$E_{s\ gr} \text{ (rad)} / E_{s\ qu} \text{ (rad)} = 1/4n_s + 1/8n_s^2 + \dots \tag{44}$$

clearly shows the great importance of gravitational radiance when the temperatures of black holes are near to Planckian scale.

7. Real Mechanism of Quantum-Gravitational Radiation and Removal of “Information Loss” Problem

Above we have showed that, in accordance with the V. Mukhanov’s degeneracy effect, the strong equilibrium of quantum black hole can be broken by quantum jump from n_s state to the $n_s - 1$ one. We have found, in Eq. (36), that each of the two emitted particles have the energy equal to $E_{sm}/2$. From this, we can easily calculate their non-reduced Compton wavelength:

$$\lambda_c (X_s) = \lambda_c (\bar{X}_s) = \frac{2\pi\hbar c}{E_{sm}/2} = 4\pi R_s \text{ .} \tag{45}$$

If we compare this value with quantum black hole gravitational radius R_s , we find that these two particles will immediately “escape” from tremendous gravitational “well” of quantum black hole via the V. Gribov–S. Hawking tunneling effect. However, the “breaking news” is: this quantum tunneling effect of two emitted particles is accompanied with radiation of gravitational waves that play an important role in the Planck Scale Physics. Note that quantum black hole radiates two bit of quantum information during “jumping” process:

$$\Delta I_{s,s-1\ qu} = I_s - I_{s-1} = 2n_s - 2(n_s - 1) = 2 \text{ (bit)} \text{ ,} \tag{46}$$

and these two bits were carried away by two “escaped” particles.

As we see above Eq. (52), the last, “remnant” Planck hole collapsed emitting the last two photons carrying the last two bits of quantum information. These

conclusions can be represented by the strict mathematical relation:

$$I_s \text{ (radiated)} = \sum_{n=1}^{n_s} \Delta I_{n,n-1 qu} = 2n_s = I_s, \quad (47)$$

that confirms the unitarity principle of quantum physics in the quantum black hole theory in the form of the law of quantum information conservation, that definitely solves the so-called “information loss” problem.

8. Entropy Production by Radiation

In conformity with Clausius relation, we can easily find the entropy production by quantum black hole radiation:

$$\begin{aligned} \Delta S_{s,s-1} &= \frac{\Delta E_{s,s-1}}{T_s} = \frac{\Delta E_{s,s-1 qu}}{T_s} + \frac{\Delta E_{s,s-1 gr}}{T_s} = \\ &= \Delta S_{s,s-1 qu} + \Delta S_{s,s-1 gr} = 2k + 2k \left(\frac{1}{4n_s} + \frac{1}{8n_s} + \dots \right). \end{aligned} \quad (48)$$

The quantum entropy term

$$\Delta S_{s,s-1 qu} = 2k = k\Delta I_{s,s-1} \quad (49)$$

is closely connected with two bits of radiated information Eq. (46). But what represents the gravitational radiance term $\Delta S_{s,s-1 gr}$? The form of the term

$$\Delta S_{s,s-1 gr} = 2k \left(\frac{1}{4n_s} + \frac{1}{8n_s^2} + \dots \right) \quad (50)$$

suggests that it is closely connected with quadrupolar, octupolar etc. modes of gravitational radiation that is the source of a dissipative and irreversible dark energy. We may call this term “gravitational waves entropy production”.

It is interesting that famous physicist T. Jacobson in his important work “Nonequilibrium Thermodynamics of Spacetime” named a similar term “bulk viscosity entropy production term” and connected it with Ricci flow. But what is more fascinating: the famous mathematician Grisha Perelman presented his extraordinary work “The entropy formula for Ricci flow and its geometric application”, in which he argues that the entropy is an effective tool for mathematical investigation.

9. Dynamics: Radiance Power and Lifetime

Before the V. Mukhanov’s “degeneracy” jump, quantum black hole must relax to gain the spherical form. Note that inside the 2-d photon condensate the “time flow” is the same as the “time flow” for external observer. This is not a deliberate assumption but a result of LIGO measurement in which the duration of ring-down phase is only a small part of a second. This is because in each point of the 2-d photon condensate all the tangential forces are equilibrated. The “stopped time-flow” effect takes place only in the nearest vicinity, but not inside the “stretched horizon”.

It is clear that the relaxation process needs a causal synchronism, which means that all the pairs of points located, one from another, at a distance of half a circle (πR_s) must be casually connected. This requires fulfillment of a lot of time at n_s state

$$\Delta t_s = \frac{\pi R_s}{c} = 2\pi t_p \sqrt{n_s}. \quad (51)$$

From this formula, we can easily calculate the lifetime of an “evaporated” quantum black hole:

$$\tau_s = \sum_{n=1}^{n_s} 2\pi t_p \sqrt{n}. \quad (52)$$

Using Bohr’s correspondence principle, a very simple relation is found:

$$\tau_s = \int_0^{n_s} 2\pi t_p \sqrt{n} \, dn = \frac{4\pi}{3} t_p n_s^{3/2} = \frac{4\pi}{3} t_p \left(\frac{E_s}{E_p} \right)^3. \quad (53)$$

If we compare it with the S. Hawking’s formula, we can see that

$$\tau_s = \frac{1}{3840} \tau_s \text{ (Hawking)}. \quad (54)$$

This big difference appears because S. Hawking associated black hole radiation with the blackbody radiation and with the Stefan-Boltzmann law, which means a continuous thermal spectrum. In contrast to this, following J. Bekenstein and V. Mukhanov, we have showed that this spectrum is discrete. Hence such a strong difference occurs.

Let’s find the radiance power:

$$P_s = \frac{\Delta E_{s,s-1}}{\Delta t_s} \cong \frac{E_p}{4\pi t_p n_s} = \frac{\hbar c^6}{4\pi G^2 M_s^2}. \quad (55)$$

Comparing this with the S. Hawking’s result, we find, respectively

$$P_s = 3840 P_s \text{ (Hawking)}. \quad (56)$$

The reason for such a large difference is the same.

We have a rare opportunity to qualitatively compare our results. It is known that the “baryonic epoch” lasted for about 400 thousand years. According to our calculations, in conformity with Eq. (53), the lifetime of the black hole, able of generating baryons, is about 240 thousand years, that quite “fits” into the mentioned interval of 400 thousand years. Whereas by S. Hawking model Eq. (54) it take as long as 920 millions (!) years, which is completely inconsistent with the data of observational cosmology. In what follows, we can find more details.

Moreover, such quantum black holes can produce pairs of ordinary neutrons via broken chiral symmetry that may solve the “old” problems of baryogenesis (followed by nucleosynthesis and leptogenesis) and matter–antimatter asymmetry in the Universe.

10. Dynamical Parameters of Particle Production

Whereas with him, the results obtained above can be easily extended to the processes of pair-production of any kind. To do this, it is enough to implement the condition for their generation

$$E_{s,s-1qu} \geq 2E_x, \quad (57)$$

where $E_x = m_x c^2$ is the rest energy of each particle, m_x being the rest mass. This condition is easily transformed into

$$E_s \leq \frac{E_p^2}{4E_x}. \quad (58)$$

The main result can be obtained by substituting Eq. (58) into Eq. (53):

$$\tau_s(x) \leq \frac{\pi}{48} t_p \left(\frac{E_p}{E_x} \right)^3. \quad (59)$$

Condition Eq. (58) is easily modified into

$$M_s(x) \leq \frac{M_p^2}{4m_x}, \quad (60)$$

which determines the extremely large mass of a black hole able to produce a pair of particles (X_s, \bar{X}_s) of a given type.

If it is known that, in the table of rest masses, a particle of type X_1 of mass m_{X1} is immediately followed by a particle of type X_2 of a larger mass m_{X2} , then we can determine the generation time of a particle of type X_1 by a black hole using the formula

$$\Delta\tau_1(X_1) = \tau_s(X_1) - \tau_s(X_2), \quad (61)$$

whereas the total number of generated particles and antiparticles of the type X_1 is easily found from relation

$$N_s(X_1) = \frac{M_p^2}{8} \left(\frac{1}{m_{X1}^2} - \frac{1}{m_{X2}^2} \right). \quad (62)$$

11. Birth and Rise of Quantum Black Holes at The Beginning of the Universe and at the Gravitational Collapse of Astrophysical Objects

“What I cannot create, I do not understand.”

(R. Feynman)

The natural “suppression” of the Higgs field at the energies well above 246 GeV exclude all Feynman’s diagrams that describe scattering processes in Standard Model. Even two photon scattering “box-diagram” do not work because it needs the presence of a non-zero Higgs field. Despite that quantum electromagnetic field do not

depends on Higgs field, two-photon scattering require intermediate virtual particle-antiparticle pair that is impossible when Higgs field is suppressed by energies of photons. Note that photons do not interact directly. Only gravitational field may save this situation but photons are massless particles. With one notable exception: photons do have rest masses in 2-d and 3-d photon condensates.

With these arguments, we conclude that Planckian black hole may be created only through the mediation of the gravitational field that acted in the 3-d Planck photon condensate. Note that a minimal photon condensate is represented by binary Planck system, (γ_p, γ_p) , with two Planck photons in a close contact. These distinct binary Planck systems were formed when local homogeneity and isotropy were broken through the Planckian fluctuations. The distance between the “centers” of this “fuzzballs” is equal to Planck length l_p . The total mass of Planck binary system is $2M_p$ and its rest energy is $2E_p$. The initial internal gravitational force is equal to the Planckian one:

$$F(\gamma_p, \gamma_p) = G \frac{M_p M_p}{l_p^2} = F_p . \quad (63)$$

This tremendous gravitational force provokes coalescence of this binary Planckian system. This tremendous Planckian coalescence inevitably generates a Planckian gravitational wave Γ_p that break the continuity of spacetime and forms a two-photon spacetime “bubble”. The energy of this Planckian gravitational wave is equal to Planckian work W_p :

$$E(\Gamma_p) = W_p = F_p l_p = E_p . \quad (64)$$

But what is the rest energy of this “bubble”? This rest energy is equal to difference between initial rest energy of binary Planck system and energy of radiated Planck gravitational wave:

$$E(\gamma_p, \gamma_p) - E(\Gamma_p) = 2E_p - E_p = E_p .$$

In Sec. “2. Basic Relations” we have found that a minimal spherical 2-d photon condensate with two photons have Planck rest mass M_p and Planck rest energy E_p , Eq. (6). Note that this result was obtained without any presumptions and hypotheses about the importance of the Planckian Units System. We may say that it emerged in natural way starting from basic system of relations Eqs. (1)–(5) for quantum black holes. This means that the produced spacetime “bubble” is nothing but the Planck black hole BH_p .

Thus, we can write the following fundamental scheme (diagram) of the birth of Planck black hole accompanied by Planck gravitational wave:

$$(\gamma_p, \gamma_p) \longrightarrow BH_p + \Gamma_p \quad (65)$$

with

$$E(BH_p) = E(\gamma_p, \gamma_p) - E(\Gamma_p) = E_p . \quad (66)$$

Expressions Eqs. (65) and (66) represent the fundamental conditions for the nucleation of Planck black holes, which, through accretion, can reach very significant sizes.

A very important question arises: where and when conditions of formation of Planck binary systems (γ_p, γ_p) could exist?

There are only two mutually reinforcing answers to this question:

- at the Beginning of the Universe, when *primordial* quantum black holes were created;
- in the nuclei of gravitationally collapsed astrophysical objects, such as neutron stars, where did the *secondary* quantum black holes emerged from.

Let us define the natural condition for accretion in the general form

$$\lambda_\gamma \leq \lambda_{sm} = 2\pi R_s, \quad (67)$$

which is equivalent to the condition

$$E_\gamma \geq E_{sm}. \quad (68)$$

Here E_γ is the energy of the absorbed photon, γ , and λ_γ is its wavelength.

Note that the velocity of accretion can be well above velocity of emission because it depends only on the intensity of captured photons flow.

Note one more very important circumstance: black holes consist of light and “feed” only on light. A clear understanding of the function of accretion disks and jets accompanying black holes immediately follows from this. Accretion discs perform the function of “thermal extraction” of light from the ordinary matter, while jets perform the functions of “evacuators” of rested plasma as a sort of “production wast”. The tremendous two jets from *Sagittarius A**, which is called “Fermi bubbles”, are the good illustrations and demonstration of these statements.

12. Cosmogenesis: Hot Big Bang and Pre-Inflation Stage

According to the law of conservation of energy, it accounts for an energy equal to the value of the energy of the modern Universe of the order of $10^{61} E_p$. Since in the future we will operate with energy densities, the exact value of this quantity is not important for us, only important being the fact that the total number of Planck photons in this condensate is tremendous large and has a value of the order of 10^{61} , that allows us to use in the following the theory of probabilities and the law of large numbers. At the same time, we will consider our diagrams as probabilistic events which should be connected in the casual and energetical sense.

Remark 12.1. Considering primordial Planck photon condensate trapped in its gravitational field as a “*prima materia*” at the “beginning of times” we automatically remove the synchronization problem, since the speed of physical processes

is the same in all domains of primordial condensate. Also, we are removing the homogeneity and isotropy problems due to the fact that the 3-d Planck condensate has the densest possible packing, close to the fuzzy spherical one, in which each Planck photon is densely “covered” by 12 other ones (interference is impossible due to the impossibility of exceeding the Planck energy density E_p/l_p^3). The “time flow” was homogenic and was determined by the Planckian time t_p . Note that it was this fact that caused the initial synchronization of all processes that occurred at the beginning of Hot Bing Bang.

Remark 12.2. The law of conservation of energy in the observable Universe guarantees us that it is the energy of primordial Planckian 3-d photon condensate trapped in its own gravitational field that cause all physical processes that occurred and will occur in the Universe.

Vacuum cannot serve as a “*perpetuum mobile*” of the Universe. All attempts to explain the so called “cosmology constant problem” by the cosmic vacuum energy was failed. However, it is the Planckian quantum fluctuations of vacuum in the primordial 3-d condensate that are the natural sources of “breaking” of its gravitational homogeneity and anisotropy.

We must consider one very important problem: not all Planckian photons in the primordial 3-d condensate can form the primordial binary Planckian systems (γ_p, γ_p) . A part becomes free.

The probability of such event to occur, due to its exceptional importance for cosmology, we will note as \aleph (*alef*: “divine”). This probability can be computed using the relation

$$\aleph = (1 - \aleph)^{12} , \tag{69}$$

where $(1 - \aleph)^{12}$ is the probability for all its twelve surrounding photons to become non-free, *i.e.* to take part at events of the type Eq. (65). Solving the Eq. (69) we obtain

$$\aleph = 0.14745 \dots \tag{70}$$

If we use the presented above diagrams Eq. (65) as events from the point of view of probability theory and laws of large numbers we can easily find the following numbers at the beginning of the Universe as a beginning of the Hot Big Bang:

$$\Omega_L = \aleph \cong 0.1474 , \quad \Omega_{BH} = \Omega_{GR} = \frac{(1 - \aleph)}{2} \cong 0.4263 , \tag{71}$$

where Ω_L — energy density of free light (free Planck photons, γ_p), Ω_{BH} — rest energy density of Planck black holes (BH_p), Ω_{GR} — energy density of Planck gravitational waves (Γ_p).

After about $50\pi t_p$ (*i.e.* $\sim 10^{-41}$ sec) the preinflation stage of Hot Big Bang is finished with following results:

$$\Omega_L = (7N - 1) \sum_{n=1}^7 \frac{1}{2\sqrt{n}} + (1 - 6N) \sum_{n=1}^9 \frac{1}{2\sqrt{n}} \cong 0.3358, \quad (72)$$

$$\Omega_{BH} \cong 0 \text{ (a very small value)}, \quad (73)$$

$$\Omega_{GR} = 1 - \Omega_L - \Omega_{BH} \cong 0.6642 \text{ (66.42\%)}. \quad (74)$$

Note that this irreversible density of energy (66.42%) remains practically unchanged till the beginning of II stage of inflation, when the binary collisions of black holes will become the source of the increase of this density (as we can see below).

From Eqs. (72)–(74) we find the strongest inequality

$$\Omega_{BH} \ll \Omega_L + \Omega_{GR}, \quad (75)$$

that denote the end of the pre-inflation stage of the Hot Big Bang and the beginning of the hyperinflation stage of the Universe.

The strong inequality Eq. (75) shows that at the end of the pre-inflation stage the weak energy of the squeezed gravitational forces were totally suppressed by the tremendous common energy of the quantum electromagnetic and gravitational radiation stretched forces (close to the whole energy of the Universe!).

Moreover, the very small value of Ω_{BH} is backed by the enormous absolute number of primordial black holes that were the “seeds” of the modern Cosmic Web.

As far as we know, all these results are the only physically substantiated confirmation of Hot Bing Bang and inflation theories.

In conformity with “Planck-2018” observational data¹⁴ the energy density of dark energy Ω_Λ represents 0.6847 (68.47%) of the total energy of the Universe. This means that the relic gravitational waves emitted in the pre-inflation phases of Hot Big Bang represent

$$\frac{\Omega_{GR}}{\Omega_\Lambda} \times 100\% = \frac{0.6642}{0.6847} \times 100\% \cong 97\% \text{ (sic!)} \quad (76)$$

of the nowadays dark energy.

Note that the only two kinds of primordial gravitational waves Γ_p and $\Gamma_{p,0}$ with E_p and $E_p/2$ energies that accompanied birth and death of Planck black holes taken together represents 93.39% of the nowadays dark energy.

These and others relic gravitational waves form the homogenic and anisotropic relic Cosmic Gravitational Waves Background (CGWsB) manifested only as the most of the irreversible and omnipresent dark energy shown in the “Planck-2018” data. Now these waves are of infinitesimal amplitudes and of frequency range of few kilohertz.

In fact, these results represent the veritable solutions of so called “cosmological constant problem”,¹⁵ dark matter and dark energy problems.

13. Two-stage Inflation

At the end of the baryogenesis, leptogenesis and nucleosynthesis epoch, when the stage I of inflation was ended, the remaining energy of ordinary matter reached the 4.96% of the total energy of the observable Universe (note that this value presented by “Planck-2018” data is unchanged in time). If we neglect the small energy density of light, we can determine the remaining energy density of black holes (dark matter) at the end of I-st stage of inflation:

$$\Omega_{BH}(\%) = 100\% - 4.96\% - 66.42\% = 28.62\% .$$

After this epoch, there came the non-accelerated stage of the expanse of the Universe. But 6–8 billions of years ago, the II-nd stage of small inflation began. Now, from “Planck-2018” data we know that the distribution of the energy in the nowadays Universe is roughly represented by the following numbers:

$$\begin{aligned} &4.96\% \quad (\text{for baryonic matter}), \\ &26.57\% \quad (\text{for dark matter}), \\ &68.47\% \quad (\text{for dark energy}). \end{aligned}$$

As we can see, a comparison with the end of the I stage of inflation clearly shows that the increase in the dark energy is caused by the decrease of the energy associated to dark matter. This leads to the unambiguous conclusion that the stage II of inflation is provided by the binary coalescences of black holes.

Conclusions

Presented approach to the Planck scale physics that governed the cosmological processes at the beginning of the Universe is based on the simple but consistent algebraic relations and schemes (diagrams) with such specific terms as Planck binary systems (γ_p, γ_p) quantum black holes BH_s , gravitational waves Γ_s *etc.* These physico-mathematical tools work in the area where the tools of standard models of physics and cosmology do not work because of tremendous temperatures and pressures that blocked the Higgs field activity. In a such terrible environment only 3-d photon condensates and 2-d spherical photon condensates (quantum black holes) are unique sources of gravity at the beginning of the Universe.

As it was showed above, presented results elucidate the decisive role of primordial black holes (as a dominated part of dark matter) in the generation of gravitational waves (a dominant part of dark energy) and clearly explain the physical background of Hot Big Bang and two-stage inflation. Moreover, these results solve the “old” unsolved problem of physics and cosmology such as “cosmic censure” problem, information loss paradox, black hole “end-point” problem, so called “cosmological constant” problem and so on.

From the theoretical point of view, presented approach derive from the last A. Einstein’s conjecture about algebraic combined quantum and continuum description of reality and from the deepest principles and conjectures of such

distinguished physicists and cosmologists as J. Bekenstein, S. Weinberg, S. Hawking, V. Mukhanov, L. Susskind, G. 't Hooft, W. Unruh, T. Jacobson, K. S. Thorne, F. Wilczek, M. Maggiore., J. Peebles, B. Carr, P. W. Anderson, L. Smolin, C. Rovelli, F. Vidotto (“Planck Star” concept), A. Kashlinsky, S. Hossenfelder and many other eminent theorists. But the decisive role was played the theoretical consequences from the last fundamental discoveries of experimental physics and observational cosmology.

Acknowledgments

The author expresses his sincere gratitude to prof. Gr. Vereshchagin, prof. S. Ansoldi and prof. A. Pittelli for the opportunity to present these results at the “Early Universe” session of the 16-th Marcel Grossmann Meeting (Rome, 2021).

References

1. J. Klaers, J. Schmitt, F. Vewinger and M. Weitz, Bose-Einstein condensation of photons in an optical microcavity, *Nature* **468**, 545-548 (2010).
2. Eberhard E. Muller, Note on Bose-Einstein condensation of photons, *arXiv:1801.05220 (quant-ph)*.
3. J. D. Bekenstein, Black Holes and Entropy, *Phys Rev* **D7 No. 8**, 2333 (1973).
4. L. Susskind, *The Cosmic Landscape* (Little, Brown and Co. NY, 2006).
5. V. Mukhanov, Quantum Black Holes, *arXiv:1810.03525 (physics)*.
6. L. V. Okun, “Vladimir Naumovich Gribov (1930-1991)”, *Physics Today* **51**, **3** 104 (1998). (“... Gribov insisted, long before Stephen Hawking proposal, that due to quantum tunneling, black holes must emit particles.”)
7. S. W. Hawking, Particle creation by black holes, *Commun Math Phys* **43**, 199 (1975).
8. V. Borsevici, Black holes and Genesis of the Universe: Physics and Astrophysics of Spherical Photon Condensates in Their Own Gravitational Fields (Chisinau (Rep. Moldova): Stratum Plus, 2019), 179 p. (ISBN 978-9975-3198-5-0).
9. A. Einstein, Naherungsweise Integration der Feldgleichungen der Gravitation, *Sitzungsber preuss. Akad. Wiss* **1**, 688-696 (1916).
10. A. Einstein, *Relativistic Theory of the non-symmetric Field. The Meaning of Relativity*. 5th edn. (Princeton, 1955).
11. V. Mukhanov, Evaporation and Entropy of Quantized Black Hole, *Lebedev Institute preprint N* **163** (1986).
12. J. D. Bekenstein, V. F. Mukhanov, Spectroscopy of the quantum black hole, *arXiv:gr-qc/9505012 (gr-qc)*.
13. C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori and P. Delsing, Observation of the dynamical Casimir effect in a superconducting circuit, *Nature* **479**, 376-379 (2011).
14. Planck 2018 results. VI. Cosmological parameters. *arXiv:1807.06209 (astro-ph)*.
15. S. Weinberg, The cosmological constant problem. *Reviews of Modern Physics* **Vol. 61**, **No. 1**, (January 1989).
16. L. Smolin, *The Trouble with Physics* (Houghton Mifflin, 2006).
17. S. Hossenfelder, *Lost in Math: How Beauty Leads Physics Astray*. (Basic Books, 2018).
18. A. Kashlinski *et al.*, Electromagnetic probes of primordial black holes as dark matter, *arXiv:1903.04424v2 (astro-ph.CO)*.