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Introduction to B physics

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Abstract This article represents a short introduction on why the physics of bottom-quark hadrons is currently one of most important topics in contemporary particle physics. We introduce the main theoretical tools to set the stage for the following, more detailed and more specialized articles in this collection.

1 Introduction

Our understanding of matter at shortest distances has improved tremendously over the last century [1]. Staring with quantum mechanics which lead us to understand atomic spectra and the periodic system of elements, the insight that atomic nuclei consist of protons and neutrons paved the way to our modern understanding of strong interaction, which binds protons and neutrons to form atomic nuclei.

The beginning of what we call nowadays flavour physics may be dated back to the discovery of radioactivity by Henry Bequerel about one century ago [2]. The observed β-decays of nuclei are interpreted in the modern language as transitions between up and down quarks, i.e. as flavour-changing transitions. Subsequently many new particles were discovered, starting with the discovery of the pions and their (strong) interactions with protons a neutrons.

It was noticed already in the 1930s that strong interactions exhibit a symmetry, which was called isospin [3]. In modern language, this is an $SU(2)$ symmetry where the fundamental doublet consists of the up and the down quark. This allowed us to classify nuclear interaction and spectra in terms of $SU(2)$ multiplets.

The discovery of “strange” particles (nowadays called Kaons and Hyperons) constituted another milestone on the way to our current understanding. Their strange behaviour was their unexpectedly long lifetimes, which found a natural explanation in postulating a new flavour-quantum number *strangeness*. This quantum number is conserved in strong interactions but can change through weak interactions, which implies that the ground-state particles with strangeness can only decay via weak interactions.

The extension of the $SU(2)$ isospin symmetry to strangeness, enlarging it to flavour-$SU(3)$, allowed us to classify the quickly growing “zoo” of particles in terms of multiplets of the $SU(3)$ group [4]. However, none of the existing particles fitted into the fundamental triplet, which eventually lead to the quark model of hadrons, such that the fundamental triplet are u, d and s quark. The $SU(3)$ flavour symmetry and the quark model were important milestones for the development of QCD, the theory of strong interactions.

Historically, Kaon physics has revealed more and important features of nature. Model building in fundamental physics was always guided by symmetry considerations, such as isospin and Flavour-$SU(3)$. In particular, discrete symmetries like charge conjugation C, inversion of the spatial coordinates (parity) P and time reversal T were originally considered to be conserved separately. However, puzzles in Kaon decays showed, that parity is not conserved in weak interactions [5], but the combination CP was still considered to be conserved. It took another decade to notice—again in Kaon physics—that also CP is violated in weak interactions [6], which was another milestone in the construction of what call nowadays the Standard Model (SM) of particle physics [7–9].

Subsequently, many more particles were discovered, forcing us to introduce more quark flavours. The development culminated in the discovery of the top quark which is the heaviest quark, and which is actually too heavy to form hadronic states, since its weak decay happens very fast. Considering the flavour structure of the quark sector of the SM, we can introduce a flavour symmetry $G_{\rm F} = SU(3)_{\rm Q} \times SU(3)_{\rm U} \times SU(3)_{\rm D}$, where the subscript Q denotes the three doublets of left-handed up-and down-type quarks, U denotes the three right-handed up-type quarks and

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D denotes the three right-handed down-type quarks. However, this flavour-symmetry is broken by the Yukawa-couplings of the quarks to the Higgs sector, which is related to the different quark masses. In the absence of Yukawa-couplings, G_{F} is the largest quark-flavour symmetry which is compatible with the $SU(2)_L \times U(1)_Y$ gauge symmetry of the SM.

Within the SM, the core quantity of quark flavour physics is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [10, 11], which encodes the mixing between the different families as well as CP violation. In fact, it is exactly 50 year ago that Kobayashi and Maskawa pointed out, that the SM with three families has a source of CP violation hidden in the CKM matrix, which can have an “irreducible” phase that cannot be removed by a phase re-definition of the fields. Since “strong CP violation” [12] seems to be absent, the CKM-phase is the only source of CP violation in the SM.

Due to their large mass, the quarks of the third generation (the top and the bottom quark) play a special role. However, out of those only the bottom quark can form bound states, such that a plethora of decays can be observed. In addition, according to the structure of the CKM matrix, the processes with bottom hadrons exhibit large CP asymmetries in the SM. This was the main motivation to construct dedicated accelerators and detectors to perform precision measurements of bottom-hadron decays. These experiments at the so-called flavour factories started about 25 years ago, and a large sample of data has been accumulated.

On the theoretical side, the tools have developed since more than 30 years. Until the late eighties of the last century, all theoretical predictions were relying on quark models for hadrons and thus were heavily model dependent. The qualitative game-changer was the observation that one can make use of the fact that the bottom-quark mass is large compared to the scale Λ_{QCD} which determines the running of the strong coupling. The formulation of this expansion in terms of an operator product expansion (OPE) within an effective (quantum) field theory an OPE opened the road to precision predictions for bottom-hadron processes, which are largely model-independent.

In the following sections, we describe first very briefly the developments from the experimental side and and in some more detail the theoretical tools which lead to the present-day precision predictions in bottom physics, followed by a concluding section on the current status and the perspectives.

2 Experimental facilities

The discovery of the bottom quark in 1977 [13] proceeded through the observation of bottomonia Υ, which are bound states of a bottom and an anti-bottom quark. While the lowest lying states $\Upsilon(nS)$, $n = 1$, 2, 3 lie below the threshold for the decays into a B and $\overline{B}$ (where B is one of the ground-state mesons $B^{\pm}$ od B^0) the $\Upsilon(4S)$ lies slightly above this threshold and decays instantaneously and practically exclusively into $B^0 + \overline{B}^0$ and $B^+ + B^-$. This opens the possibility to use the resonance enhancement in the process $e^+ + e^- \to \Upsilon(4S) \to B + \overline{B}$ to produce a large amount of Bottom mesons. In addition, the two Bottom mesons from the $\Upsilon(4S)$ decay are quantum entangled, which will become important when it comes to the measurement of time dependent CP violation.

In the “pre-B factory era” (i.e. before the turn of the millenium), two experiments were running at e^+e^- machines using the effect discussed above. One was the ARGUS experiment at DESY in Hamburg, Germany, at the DORIS accelerator, and a second one, called CLEO, was using the CESR accelerator at Cornell in Ithaca, USA. Both experiments together validated quite a few predictions from the SM, such as the presence of $b \to u$ transitions [14] and a first observation of the $b \to s\gamma$ transition [15]. One highlights clearly was the observations of B-$\overline{B}$ oscillations by the ARGUS experiment [16], which was actually the first indication that the top quark mass exceeds 100 GeV.

However, one decisive measurement, namely a time-dependent measurement of CP violation could not be done at ARGUS and CLEO, since DORIS and CESR were so called symmetric e^+e^- machines, i.e. the beam energies in the lab frame of the e^- beam was the same as to one of the e^+ beam. As a consequence, the resulting $\Upsilon(4S)$ is at rest in the lab frame, and thus the predicted time-dependent CP asymmetry, could not be observed, since a motion in the lab frame is required to serve as a “clock” [17]. Due to the quantum entanglement of the two bottom mesons from the decay of the $\Upsilon(4S)$, the CP quantum number of one of the two B mesons can only be fixed by observing the decay of the second one. Once this decay happens, the time dependent decay of the first one can be observed by starting the clock in the moment of the decays of the second one and a subsequent measurement the flight distance of the first one.

This eventually triggered a large, world-wide effort to build asymmetric machines, such that the $\Upsilon(4S)$ moves in the lab frame. Just at the turn of the millenium, two asymmetric accelerators with very high luminosity went into operation. One was located at SLAC in Stanford, USA, with the dedicated experiment BaBar, a second one started at KEK in Tsukuba, Japan, with the experiment Belle. These two experiments produced a large amount of data, mainly confirming the predictions of the SM, thereby establishing the CKM picture at the level of precision, including also CP violation.

Bottom physics can also be done at e^+e^- machines at higher energies, in particular on the Z^0 resonance. Due to the development of precise tracking using silicon detectors, a b-quark-physics program became possible at LEP, using $e^+ + e^- \to Z^0 \to b + \bar{b}$, where the final-state quarks hadronize. The main advantage is that all bottom hadrons can be produced, including B_s and bottom baryons. A second observation is, that the b quark from the Z^0 decays is highly polarized, leading to partially polarized bottom hadrons [18].

Bottom quarks are also produced in hadronic collisions as well as in ep-collisions, and so there were B Physics programs at HERA as well as at the TEVATRON. Like with e^+e^- machines at higher energies all kinds on bottom hadrons are accessible, and so e.g. the first measurement of B_s-$\overline{B}_s$ oscillations came from FERMILAB [19].

Currently, there are two dedicated bottom-physics experiments, which will produce the an overwhelming amount of data, which will allow us to test the flavour structure of the SM at the quantum level. At LHC at CERN in Geneva, Switzerland, the LHCb experiment performed measurements using bottom quarks produced in pp collisions, while the BelleII experiment uses the upgraded KEK accelerator using e^+e^- collisions at the $\Upsilon(4S)$. These experiments are foreseen to run for at least the next decade, opening the road to a new level of precision.

3 Theoretical tools

The goal in precision b physics is to find possible small deviations from the SM, hinting at some “new physics”(NP) which goes beyond the SM. This requires two inputs. First of all, one needs to have a SM prediction with a controllable estimate of the theoretical uncertainties, which have to be small enough to become sensitive to tiny deviations. Secondly, in order to give limits on the parameters of possible NP scenarios, one has to device simplified models or even full-fledged theories as a input to NP analyses of data.

The SM is formulated in terms of quarks and gluons, while the states observed in experiment are hadrons. Thus, the main problem in describing weak decays of hadrons is the determination of the hadronic matrix elements of operators expressed in terms of quarks and gluons, which are genuinely non-perturbative. The only way to compute these matrix elements directly is to use lattice-QCD calculations (LQCD), which has made tremendous progress over the last decades, mainly due to increasing computing power and improvements in the algorithms.

Nevertheless, not all matrix elements can be computed using LQCD, so other methods are needed. For weak decays of particles with heavy quarks, one can make use of the fact that the heavy-quark mass m_Q sets a large scale, and so $\Lambda_{\mathrm{QCD}}/m_Q$ becomes a reasonable expansion parameter. The leading term is the limit $m_Q \to \infty$ which in many cases turns out to be very simple, so some model independent statements can be inferred, pushing model dependence to higher orders in $\Lambda_{\mathrm{QCD}}/m_Q$. Since for the b quark, the relevant parameter is $\Lambda_{\mathrm{QCD}}/m_b \sim 0.1$, heavy quark methods have become a standard tool.

Finally, one may also apply QCD sum rules to constrain and even estimate hadronic matrix elements [20]. In the context of B physics, mostly light-cone QCD sum rules (LCSR) are employed, which allow us to estimate hadronic matrix elements in terms of light-cone distributions of hadrons, which constitute the necessary nonperturbative input. However, the precision of the LCSR approach is limited, but LCSR allow us to estimate matrix elements which cannot (yet) be computed on the lattice.

3.1 Heavy quark methods

One may distinguish four branches of heavy quark expansions:

Heavy quark effective theory (HQET) is derived from QCD assuming the presence of a single heavy quark, while all other degrees of freedom are “soft”, i.e. have four-momenta $k_\mu \sim \Lambda_{\mathrm{QCD}}$ [21–24]. The key idea is that the heavy quark moves like a “cannon ball”, which means that the four velocity of the heavy quark remains mainly unchanged by soft QCD interactions. In this limit, QCD exhibits additional symmetries [25, 26] which can be used to relate hadronic matrix elements. Its major application is the description of exclusive decays, in particular $B \to D\ell\bar{\nu}$ and $B \to D^*\ell\bar{\nu}$, where both the charm and the bottom quark are treated in the infinite mass limit. The additional symmetries relate the semileptonic form factors to a single function, for which also a normalization statement can be derived. Corrections of order $1/m$ can be systematically parametrized, and QCD corrections from hard gluons can be computed in terms of a perturbation theory in powers of $\alpha_s(m)$.

Non-relativistic QCD (NRQCD) is similar to HQET, except that a heavy quark and a heavy antiquark are assumed to be present [27]. This allows us to describe heavy Quarkonia, but also mesons like the B_c meson.

Heavy Quark Expansion (HQE) is the tool to compute inclusive processes, such as lifetimes and mixing parameters, but also rates and spectra of inclusive semileptonic decays. [28–31] The effective Hamiltonians H_{eff}

described below mediate transitions into final states $|X\rangle$, and the inclusive rate is defined to be

$$\begin{aligned}\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|H_{\text{eff}}(0)|H(v)\rangle|^2 \\ &= \int d^4x \, \langle H(v)|H_{\text{eff}}(x) H^\dagger_{eff}(0)|H(v)\rangle \\ &= 2 \text{ Im} \int d^4x \, \langle H(v)|T\{H_{\text{eff}}(x) H_{\text{eff}}(0)\}|H(v)\rangle \ , \end{aligned} \tag{1}$$

In order to exploit the fact that m_Q is a scale large compared with Λ_{QCD}, we perform a field redefinition according to

$$Q_v = e^{-im_Q(v\cdot x)} Q \ . \tag{2}$$

This leads to

$$\Gamma \propto 2 \text{ Im} \int d^4x \, e^{-im_Q vx} \langle H(v)|T\{\widetilde{H}_{\text{eff}}(x)\widetilde{H}_{\text{eff}}(0)\}|H(v)\rangle \ , \tag{3}$$

where $\widetilde{H}_{\text{eff}}$ is defined like H_{eff} but with the heavy-quark field Q replaced by Q_v. The large scale M_Q appears now explicitly in the exponent, and the operator appearing here can be expanded in inverse powers of $1/m_Q$ as

$$\int d^4x \, e^{im_Q vx} \, T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n \hat{C}_{n+3}(\mu)\mathcal{O}_{n+3}(\mu) \, , \tag{4}$$

where the $\mathcal{O}_n$ are operators of dimension n, with their matrix elements renormalized at scale μ, and $\hat{C}_n$ are the corresponding Wilson coefficients. Taking the forward matrix element yields the desired expansion in $1/m_Q$

$$\Gamma \propto 2\,\text{Im} \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n \hat{C}_{n+3}(\mu)\langle H(v)|\mathcal{O}_{n+3}(\mu)|H(v)\rangle, \tag{5}$$

in terms of perturbatively computable Wilson coefficients and hadronic matrix elements of local operators with increasing dimension.

This approach has opened the road to precise predictions for inclusive processes, such as lifetimes, mixing parameters, but also spectra in inclusive semileptonic decays. The expansion in powers of $1/m_Q$ as well as in $\alpha_s(m_Q)$ allows us an estimate the remaining theoretical uncertainties.

Soft Collinear Effective Theory (SCET) can be applied for specific kinematic situations, where light partons have large energies in the rest frame of the decaying B meson [32, 33]. The master example is the decay $B \to \pi\pi$, where the final state does not contain a heavy quark, but the pions have the energy $E_\pi = M_B/2$ (neglecting the masses of the pions), i.e. the energy of the light partons scale as m_b. Thus, the HQET assumption, that all components of the momenta of light quark and gluons scale as Λ_{QCD}, is violated.

To set up an expansion in $1/m_b$, we need to extract the large scale, which is done by decomposing the four velocity of the B meson into two light-cone vectors

$$v = \frac{1}{2}(n_+ + n_-) \ , \quad n_\pm^2 = 0 \ , \quad (n_- n_+) = 2 \tag{6}$$

where the light-cone vectors define the direction of the pions. The metric can be decomposed as

$$g^{\mu\nu} = \frac{1}{2}(n_+^\mu n_-^\nu + n_-^\mu n_+^\nu) + g_\perp^{\mu\nu} \ . \tag{7}$$

such that the momenta k of light quarks and gluons can be decomposed as

$$(n_- k) \sim \mathcal{O}(m_b) \qquad (n_+ k) \sim \mathcal{O}(\Lambda_{\text{QCD}}) \qquad k_\perp \sim \mathcal{O}(\Lambda_{\text{QCD}}) \ , \tag{8}$$

where we have indicated the power counting as it is used in SCET.

This ansatz leads to a quite complex effective field theory which is currently under heavy investigation. The leading term has lead to clean formulation of QCD factorization in exclusive nonleptonic B decays (see below), however, power corrections seem to be sizable and are not yet under theoretical control.

3.2 Classification of B decays and effective Hamiltonian

Due to the large mass of the bottom quarks, bottom hadrons have many decay channels. Starting from the SM, one usually derives an effective Hamiltonian by integrating out the heavy particles of the SM, i.e. the weak gauge bosons and the heavy top quark. The theoretically simplest class of decays are the leptonic decays mediated by the charged-current interaction ($\ell = e,\, \mu,\, \tau$):

$$\mathcal{H}_{\rm eff}(b \to c\ell\bar{\nu}_\ell) = \frac{4G_F}{\sqrt{2}} V_{cb}(\bar{c}_{\rm L}\gamma_\mu b_{\rm L})\big(\bar{\ell}_{\rm L}\gamma_\mu \nu_{\ell,\,\rm L}\big) + \text{ h.c. } , \tag{9}$$

$$\mathcal{H}_{\rm eff}(b \to u\ell\bar{\nu}_\ell) = \frac{4G_F}{\sqrt{2}} V_{ub}(\bar{u}_{\rm L}\gamma_\mu b_{\rm L})\big(\bar{\ell}_{\rm L}\gamma_\mu \nu_{\ell,\,\rm L}\big) + \text{ h.c.} \tag{10}$$

The decay amplitudes for $B \to \ell\bar{\nu}$ contain the hadronic matrix element

$$\langle B_q(p_B)|(\bar{q}_{\rm L}\gamma^\mu b_{\rm L})|0\rangle = f_{B_q} p_B^\mu \tag{11}$$

which is given in terms of the hadronic parameter f_{B_q}, the decay constant of the B_q-meson. The rate is

$$\Gamma(B_q \to \ell\bar{\nu}) = \frac{G_F^2 M_B^3 f_{B_q}^2}{8\pi} |V_{qb}|^2 x^2 (1-x)^2 \tag{12}$$

where $x = m_\ell / M_B$ is the ratio of masses. While this is the the simplest decay, it suffers from helicity suppression factor x^2, which at least for electrons an muons leads to a very small branching fraction.

Next in line of increasing theoretical complexity are the semileptonic decays. Exclusive semileptonic decays into a final state $|f\rangle$ require to deal with hadronic matrix elements of the form

$$\langle B_q(p_B)|(\bar{q}_{\rm L}\gamma^\mu b_{\rm L})|f\rangle \tag{13}$$

which are usually decomposed into their tensor structures multiplied by scalar functions, called form factors. The number of form factors depend on the spin of the final-state particle f. For example, the semileptonic B decays into the charmed ground states D and D^* are described by in total six form factors, which are the nonperturbative inputs needed to describe these decays. In the infinite mass limit, heavy-quark symmetries relate some of the form factors, and corrections to these relations can be expanded in $1/m_Q$. However, with the increasing precision of the data and the improvement of LQCD methods, considerations based on heavy-quark methods become less competitive.

Going further along the line of theoretical complexity we arrive at nonleptonic decays. In order to describe these we first look at the relevant effective Hamiltonian, which now involves four quark operators. Staring with the case of four different quark flavours (i.e. the quark decays $b \to c\bar{u}d$,$b \to c\bar{u}s$, $b \to u\bar{c}d$ and $b \to u\bar{c}s$), the effective Hamiltonian becomes (for the case $b \to c\bar{u}d$)

$$\mathcal{H}_{eff}(b \to c\bar{u}d) = \frac{4G_F}{\sqrt{2}} V_{cb}V_{ud}^*[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] + \text{ h.c. } , \tag{14}$$

with the operators

$$\hat{O}_1 = (\bar{c}_{L,\,i}\gamma_\mu b_{L,\,j})\big(\bar{d}_{L,\,j}\gamma_\mu u_{L,\,i}\big) , \tag{15}$$

$$\hat{O}_2 = (\bar{c}_{L,\,i}\gamma_\mu b_{L,\,i})\big(\bar{d}_{L,\,j}\gamma_\mu u_{L,\,j}\big) , \tag{16}$$

where $i,\, j = 1,\, 2,\, 3$ are the colour indices of the quarks, and $C_1(\mu)$ and $C_2(\mu)$ are Wilson coefficients that can be computed in QCD perturbation theory. They are known to next-to-leading order in QCD, details can be found in [34]. These coefficients depend on the scale μ, at $\mu = M_W$ we have $C_2(M_W) = -1 + \mathcal{O}(\alpha_s(M_W))$ and $C_1(M_W) = \mathcal{O}(\alpha_s(M_W))$. Renormalization group running is used to compute the coefficients at lower scales, usually

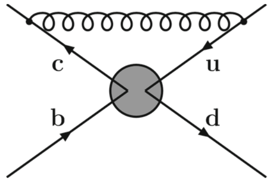

Fig. 1 Sample diagram illustrating the mixing between O_1 and O_2

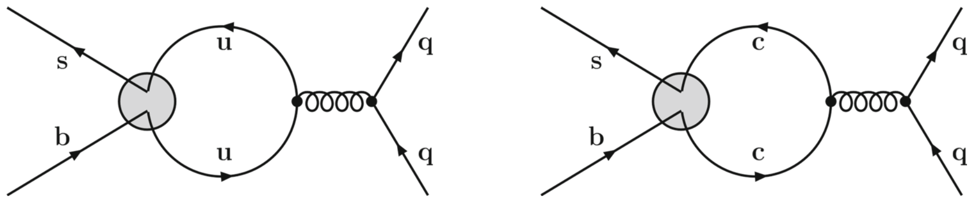

Fig. 2 Feynman Diagrams for the QCD penguin contributions, which emerge from contracting the up-type quark lines. We show the case for $b \to s$, the case $b \to d$ is obtained by replacing $s \to d$

the hadronic matrix elements in B decays are computed at $\mu = m_b$. In particular, QCD effects induce a mixing between $\hat{O}_1$ and $\hat{O}_2$ due to the Feynman diagrams shown in Fig. 1.

In case the two up-type flavours are the same, we find another possible contribution depicted in Fig. 2. these diagrams will force us to introduce more operators into the effective Hamiltonian, since they induce new operators $O_{3,..,6}$ called QCD penguins contributions $(q = s, d)$.

$$\mathcal{H}_{\text{eff}}(b \to q) = \frac{4G_F}{\sqrt{2}} \left\{ \sum_{q'=u,\,c} V_{q'b} V^*_{q'q} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] - V_{tb} V^*_{td} \sum_{i=3}^{6} C_i(\mu) O_i(\mu) \right\} + \text{ h.c.} \tag{17}$$

with

$$\begin{aligned}
O_1 &= (\bar{q}'_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,j})(\bar{q}'_{\mathrm{L},\,j} \gamma_\mu q'_{\mathrm{L},\,i}) \\
O_2 &= (\bar{q}'_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,i})(\bar{q}'_{\mathrm{L},\,j} \gamma_\mu q'_{\mathrm{L},\,j}) \\
O_3 &= (\bar{q}_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,i}) \sum_{q=u,\,d,\,s,\,c,\,b} (\bar{q}_{\mathrm{L},\,j} \gamma^\mu q_{\mathrm{L},\,j}) \\
O_4 &= (\bar{q}_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,j}) \sum_{q=u,\,d,\,s,\,c,\,b} (\bar{q}_{\mathrm{L},\,j} \gamma^\mu q_{\mathrm{L},\,i}) \\
O_5 &= (\bar{q}_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,i}) \sum_{q=u,\,d,\,s,\,c,\,b} (\bar{q}_{\mathrm{R},\,j} \gamma^\mu q_{\mathrm{R},\,j}) \\
O_6 &= (\bar{q}_{\mathrm{L},\,i} \gamma_\mu b_{\mathrm{L},\,j}) \sum_{q=u,\,d,\,s,\,c,\,b} (\bar{q}_{\mathrm{R},\,j} \gamma^\mu q_{\mathrm{R},\,i})
\end{aligned} \tag{18}$$

The operators appearing in the effective Hamiltonian mix under renormalization, i.e. the coefficients change their values when changing the renormalization scale μ. However, the matrix elements of the effective Hamiltonian are observable quantities (i.e. decay amplitudes), so H_{eff} cannot depend on μ. This implies that any change in the coefficients is compensated by the matrix elements of the O_i, which thus depend on μ. Since the Wilson coefficients can be computed in perturbation theory, a change in μ moves contributions from the operator matrix elements into the Wilson coefficients, and so the perturbative pieces of the matrix elements can be determined in this way. Details can be found in [34].

The calculation of amplitudes for exclusive non-leptonic decays requires to deal with the hadronic matrix elements of four quark operators,

$$\langle B | O_1 | f \rangle \quad, | f \rangle = | D\pi \rangle,\ | \pi\pi \rangle,\ | \pi\pi\pi \rangle,\ \cdots$$

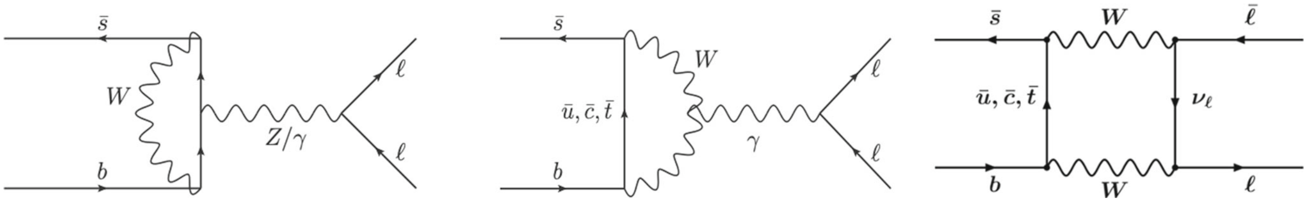

Fig. 3 SM contributions to the transitions $b \to s\ell^+\ell^-$

which is still a major theoretical challenge. It is tempting to interpret the four quark operators as a product of two quark currents, however, in a quantum field theory this does not hold, schematically for two quark currents J and J'

$$\lim_{x \to 0} J(x)J'(0) \neq O_i(0)$$

Nevertheless, it turns our that naive factorization turned our to be quite successful [35], describing at least the gross features of exclusive non-leptonic two body decays. Only at a later stage, this has been understood in terms of SCET and QCD factorization, establishing naive factorization as the leading term in a $1/m$ expansion, and QCD corrections to naive factorization became computable [36–38]. However, this holds only to leading order in $1/m$, and power corrections are seen to play a major role to describe the current data. Investigating this is a major topic in current research, and some of the articles in this collection deal with this problem.

Finally, we come to a specific feature in B decays which are radiative and (semi)leptonic flavour changing neutral current (FCNC) decays. While the effective Hamiltonian (17) is mediating effectively the FCNC transitions $b \to s$ and $b \to d$, there are additional contributions because of the CKM structure and the large top-quark mass. For the case of the semileptonic FCNCs $b \to s\ell^+\ell^-$ and $b \to d\ell^+\ell^-$, we have in the SM the Feynman diagrams shown in Fig. 3.

Although these diagrams are of second order in the electroweak interaction, the large mass of the top quark in fact compensates the suppression induced by the inverse of the mass of the weak bosons. In fact, the calculation of the diagrams in Fig. 3 yields functions that depend on the ratio of the masses of the up-type quark in the loop and the W mass M_W, which for the top quark mass is of order one. Together with the fact that $V_{tb} \sim 1$, i.e. there is no CKM suppression of these contributions, such that we have the unique situation for the b quark that these contributions become comparable to the ones already present in (17). The same argument holds for radiative FCNC processes, which are induced in the SM by the Feynman diagrams shown in Fig. 4.

To this end, the effective Hamiltonian for the transitions $b \to s$ and $b \to d$ is extended by more operators, which are given given by ($q = s,\, d$) [39, 40]

$$\begin{aligned}
O_7 &= \frac{e}{16\pi^2} m_b (\bar{q}_{\mathrm{L},\,\alpha} \sigma_{\mu\nu} b_{\mathrm{R},\,\alpha}) F^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2} m_b (\bar{q}_{\mathrm{L},\,\alpha} T^a_{\alpha\beta} \sigma_{\mu\nu} b_{\mathrm{R},\,\alpha}) G^{a\mu\nu} \\
O_9 &= \frac{1}{2} (\bar{q}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell}\gamma^\mu \ell), \\
O_{10} &= \frac{1}{2} (\bar{q}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell}\gamma^\mu \gamma_5 \ell)
\end{aligned} \tag{19}$$

and the total effective Hamiltonian is given by (17) where the last sum now runs over $i = 3, \cdots, 10$. We note that the coefficients $C_{7,\ldots,10}$ depend not only on the renormalization scale μ, but also on the ratio m_t^2/M_W^2.

As before, the calculation of decay amplitudes for processes such as $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$ or $B_s \to \ell^+\ell^-$ requires to deal with hadronic matrix elements, which are decay constants and form factors. However, there are

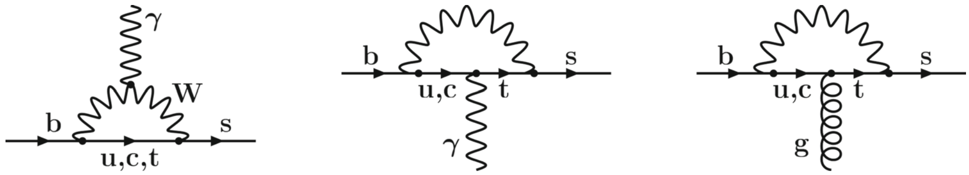

Fig. 4 SM contributions to the FCNC processes $b \to s\gamma$ and $b \to sg$

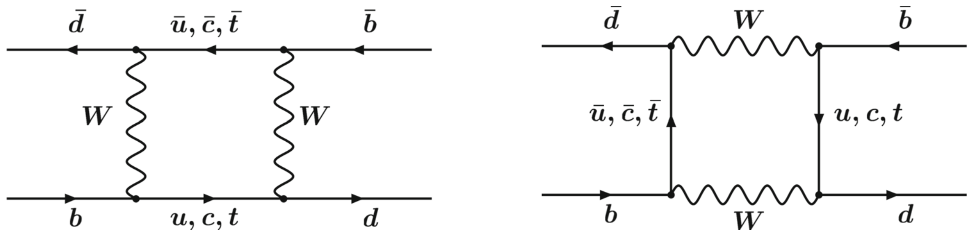

Fig. 5 Mix

also contributions of the form

$$\int d^4x\, e^{-iqx} \langle B|T[O_1(0)j^{\mu}_{\rm em}(x)]|K^{(*)}\rangle$$

where the electromagnetic current $j^{\mu}_{\rm em}$ generates a (virtual) photon of momentum q that decays into $\ell^+\ell^-$. These contributions are hard to compute and are subject of ongoing research [41, 42].

The FCNC processes are of special interest, since they may open a new window to observe effects beyond the SM. The absence of contributions at (partonic) tree level suppresses the SM contribution to these decays; hence, effects from heavy particles running in the loop can potentially be large enough to become visible in experiment.

Up to this point, we discussed only processes with $\Delta B = \pm 1$, i.e. the above effective Hamiltonians change the bottom quantum number by one unit. However, at second order in the weak interactions also $\Delta B = \pm 2$ processes can be mediated, the relevant Feynman diagrams are depicted in Fig. 5. These diagrams introduce particle-antiparticle oscillations for the B^0 and the B_s meson. As for the FCNC decays, the large top-quark mass compensates the suppression through the large weak boson mass, leaving us with a sizable, i.e. an observable oscillation. Due to the CKM structure, for B-$\overline{B}$ oscillations the short-distance contribution from the top quark dominates, allowing us a quite precise prediction of the oscillation frequency. Historically, the observation of B-$\overline{B}$ oscillations by the ARGUS collaboration in 1987 [16] was the first hint at a top-quark mass well above 100 GeV, while at that time the general believe was that the top-quark mass should lie below 30 GeV. In a similar way as this indirect discovery of a heavy top, one may use the loop-induced $\Delta B = \pm 2$ processes as a window to search for physics beyond the SM.

As pointed out in the introduction, the violation of CP symmetry is encoded in the SM solely in the CKM matrix, assuming absence of strong CP violation and ignoring possible CP in the leptonic sector. Since this requires the presence of a third generation, B physics is the best place to study CP violation [43]. Phenomenology tells us that CP violation for the quark sector is small in the SM. In fact, all CP violation in the quark sector is proportional to the imaginary part of

$$\Delta^{(4)}_{\alpha\rho} = V_{\beta\sigma}V_{\gamma\tau}V^*_{\beta\tau}V^*_{\gamma\sigma}\ , \qquad \text{where} \quad \begin{cases} \alpha,\, \beta,\, \gamma = u,\, c,\, t \text{ cyclic} \\ \rho,\, \sigma,\, \tau = d,\, s,\, b \text{ cyclic} \end{cases}, \tag{20}$$

which is the unique, dimension-four re-phasing invariant combination of CKM matrix elements [44]. We note that due to unitarity of the CKM matrix, $|{\rm Im}\Delta^{(4)}| \leq 1/(6\sqrt{3}) \approx 0.1$, but the observed value of $|{\rm Im}\Delta^{(4)}| \sim 10^{-4}$ is much smaller.

However, in B physics, we can have large CP asymmetries, however, in processes with small branching fractions. This becomes also manifest, since in the usual parametrization of the CKM matrix, the very small elements V_{ub} and V_{td} come with large phases, which on the one hand are related to ${\rm Im}\Delta^{(4)}$, on the other hand can be measured in various processes and compared to the SM prediction.

Many CP violating observables can be constructed for nonleptonic decays. However, here the calculation of the hadronic is still difficult, so most results rely on QCD factorization. In particular, charmless nonleptonic two-body decays have been investigated and measured, but for a conclusive test in this sector, the tool for the calculation of the hadronic matrix elements still need to be refined.

3.3 Effects beyond the standard model

B decays and flavour processes in general have a large sensitivity to test the presence of new physics at scales way higher than the mass of the bottom quark. This requires the well developed machinery for the accurate calculation of theoretical predictions, including estimates of theoretical uncertainties, which we have described in the last two subsections.

Searching for small deviations from the SM predictions requires a tool to implement such deviations in order to quantify them. A generic way to do this is to use again methods of effective field theory (EFT). As has been pointed out above, the SM is the unique renormalizable quantum field theory for the observed particle spectrum and interactions. Thus, it can be interpreted as the leading term of an EFT expansion of the form

$$\mathcal{L}_{\mathrm{tot}} = \mathcal{L}_{\mathrm{SM}} + \sum_{k=1}^{\infty} \frac{1}{\Lambda_{\mathrm{NP}}^k} \mathcal{L}^{(4+k)} \tag{21}$$

where $\mathcal{L}^n$ is a linear combination of operator of dimension n,

$$\mathcal{L}^n = \sum_i C_i^{(n)}(\mu) \mathcal{O}_i^{(n)} \tag{22}$$

where the summation runs over all operators of dimension n which are compatible with the SM symmetries, and Λ_{NP} is a scale of "new physics", e.g. a mass of a new, heavy particle.

This approach, called "Standard Model Effective Theory" (SMEFT) [45–47], is indeed very versatile, since any theory beyond the SM can be covered, i.e. for any theory one can compute the coefficients $C_i^{(n)}$. However, since we do not know the underlying theory, this virtue turns into a vice: The general expression up to $k = 2$ has more than 2000 independent coefficients, including a general flavour structure. To this end, it is hard to pin down any underlying theory solely by measurements at low energies and fits to the coefficients of SMEFT.

On top of this, one has to construct from the SMEFT the corresponding effective Hamiltonians at the scale of the b-quark mass, which sometimes is called "Weak Effective Theory" (WET). We have discussed the WET emerging from the SM in the last section, and any modification through the additional terms in SMEFT will end up as a deviation of the Wilson coefficients C_1, ..., 10 of the effective Hamiltonian (17). To this end, a generic way to quantify deviations from the SM is to use the Wilson Coefficients of (17) as fit parameters. This has been pointed out first in [48], and has become common practice for the analysis of the anomalies in B decays [49–51].

However, the deviations of the Wilson Coefficients of (17) can be expressed in terms of the SMEFT coefficients as some combinations of the $C_i^{(n)}$ of (22), which in general will not point uniquely to some more fundamental theory. This has motivated people to consider simplified models, a prominent one in the context of the recent discussion about the B decay anomalies is to introduce one or more leptoquarks, see e.g. [52]. In this case the deviations in the Wilson Coefficients of (17) can be interpreted in term of masses and couplings of leptoquarks, which may sharpen our intuition to eventually arrive at the full theory that lies behind the SM.

4 Status and outlook

Currently, flavour physics is one of the most exciting branches of contemporary particle physics. As it will be discussed in the subsequent articles, there are many observables which can be predicted with a sufficient accuracy to perform decisive test of the SM on the basis of present and future data. There are some intriguing tensions between the theoretical predictions and the present data, which are sometimes called anomalies. While the anomalies related to a possible violation of lepton-universality have disappeared through a recent re-analysis of data, there are still quite a few anomalies still persistent, and there are even a few new ones which need to be understood.

From the theory side, the tools and methods are continuously refined. In particular, there has been a lot of progress on the nonperturbative side through improvements in LQCD, which still continues and has visible potential for the future, e.g by better algorithms and possibly also through quantum computing. From the experimental side, we will have at least for the next ten years a continuous increase in the amount of data, which will certainly motivate the theory side to keep up in the precision of the predictions.

At the end, there is plenty of motivation to study b-quark physics or flavour physics in general. This is mainly due to the fact, that the obvious triplication of the particle spectrum remains to be a major mystery, and bottom physics will be at least one of the keys to understand this. It is worthwhile to point out that the flavour structure of the SM is quite generic and thus also agnostic: While the interactions of the quarks and leptons are fixed by the gauge principle, i.e. once the gauge group $SU(3)_{\mathrm{color}} \times SU(2)_{\mathrm{weak}} \times U(1)_{\mathrm{hypercharge}}$ and the multiplets of quarks and gluons are set, all interactions are fixed. But the flavour structure does not follow any principle but rather is just the most generic parametrization one can write down, assuming the triplication of the particle spectrum. In particular, the unitarity of the CKM matrix is a straightforward consequence of the gauge symmetry, which enforces that the gauge couplings are the same for the weak doublets in each family.

In the SM, nontrivial quark flavour physics is introduced by the fact, that the mass matrices of the up-type and down-type quarks do not commute. These mass matrices emerge in the SM from the Higgs Sector, which involves not only the Higgs particle itself, but also the longitudinal modes of the heavy gauge bosons of weak interaction. This suggests that the key to understand flavour physics may be hidden in the Higgs sector, which may mean that the simple Higgs sector we currently observe may be more complicated.

Another issue, possibly related to the above point, is the strong hierarchy of the parameters in the flavour sector. The most striking evidence is the very disparate mass scales of the quarks which in the SM is related to the very weak coupling of the quarks to the Higgs field. The only exception here is the top quark, which has a "natural" coupling to the Higgs field of order one. This has lead to speculations that actually only the top quark has a direct coupling to the Higgs field, while Higgs-couplings of the lighter quarks emerge as an effective vertices suppressed by some large scale. However, this has not yet been substantiated to the point that quantitative predictions become possible.

The next decade will certainly bring new insights into these problems, in particular, if some of the B anomalies turn into evidence for a real deviation from the SM. Together with other open problems, such as the presence of dark matter, the missing CP violation to generate the Matter–Antimatter asymmetry and the puzzle related to the strong CP problem, we will hopefully be able to get an insight of how to construct the theory underlying the SM. Bottom physics will certainly play a key role in this endeavour.

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