

# SPECTROMETER-BASED X-RAY FREE-ELECTRON LASER PULSE DURATION MEASUREMENTS OF CHIRPED BEAMS

R. Robles<sup>1\*</sup>, A. Halavanau, A. Lutman, D. Cesar, G. Stupakov,  
SLAC National Accelerator Laboratory, Menlo Park, USA  
<sup>1</sup>also at Stanford University, Stanford, USA

## Abstract

Accurate measurements of the X-ray pulse duration produced by X-ray free-electron lasers (XFELs) typically rely on longitudinal electron beam phase space diagnostics, e.g. in a transverse deflecting cavity or TCAV, or from measurements of spectral correlations. All of the known spectral methods share the weakness that they will underestimate the pulse length in the case that the FEL spectrum is broadened due to the electron beam having an energy chirp. We present a statistical analysis of FEL radiation in the presence of a linear electron beam energy chirp which extends previous results by including an accurate description of the FEL gain process. In doing so, we show that with measurements of the spectral intensity correlations and the average spectrum, one can reconstruct the X-ray pulse length, e-beam chirp, and spectrometer resolution. Our approach is validated by comparison with 1D FEL simulations.

## INTRODUCTION

Several methods exist to measure the pulse duration produced by XFELs. One can directly infer something about the XFEL pulse shape and duration if a longitudinal phase space diagnostic is available for the electron beam, such as a transverse deflecting cavity (TCAV) [1]. In practice, however, TCAVs can be difficult to operate and have a limited temporal resolution, about 2-4 fs, particularly at higher beam energies, and thus other methods which do not rely on knowledge of the e-beam phase space pose interest. It has been known for decades that the fluctuations of spectral intensity of the radiation emitted by electron beams stores information about the bunch length [2–5]. A technique applied to the XFEL by Lutman *et al* in [6, 7], showed that by evaluating the spectral intensity correlations, one can reconstruct not only the pulse duration but also the resolution of the spectrometer used to measure the XFEL spectra. A similar approach was recently taken in [8] to unveil slightly more information about the time-frequency correlations of the XFEL pulses. Both of these approaches, as well as other approaches based on studying intensity correlations, are inaccurate if the electron beam has a time-energy correlation or chirp. The presence of the e-beam chirp broadens the FEL spectrum and results in these measurements underestimating the pulse duration [9]. This inability stems from the assumption, in those models, that each electron emits

radiation centered around the same central frequency  $\omega_0$  defined by the FEL resonance condition.

The behavior of the FEL gain process in the presence of a linear e-beam energy chirp is well-understood, however, and an appropriate Green's function was derived by Krinsky and Huang in [9]. These spectral correlation-based measurements should be extendable to include the chirp. We present a revised analysis of FEL intensity fluctuations in the presence of a linear energy chirp and non-zero spectrometer resolution. By fitting to both the spectral intensity correlation function and the average spectrum, we are able to extract all three parameters - the X-ray pulse length, the electron beam chirp, and the spectrometer resolution. There is inherent uncertainty in this method which stems from ambiguity in the value of the SASE bandwidth, however, we demonstrate that the impact of that uncertainty is negligible on practical measurements. We validate our approach by using it to reconstruct the beam parameters of 1D FEL simulations.

## CALCULATION OF SPECTRAL INTENSITY CORRELATION

In an experiment, one typically has access to spectral intensity measurements of the FEL field via spectrometer. The measurement is a convolution of the true intensity spectrum with a spectrometer resolution function:

$$\tilde{S}(\omega) \equiv \int \frac{d\omega'}{2\pi} e^{-\frac{(\omega-\omega')^2}{2\sigma_m^2}} |\tilde{E}(\omega')|^2 \quad (1)$$

where  $\sigma_m$  is the spectrometer resolution. The measured spectral intensity correlation is then defined as

$$G_2(\delta\omega) \equiv \frac{\langle \tilde{S}(\omega - \frac{\delta\omega}{2}) \tilde{S}(\omega + \frac{\delta\omega}{2}) \rangle}{\langle \tilde{S}(\omega - \frac{\delta\omega}{2}) \rangle \langle \tilde{S}(\omega + \frac{\delta\omega}{2}) \rangle} - 1 \quad (2)$$

We utilize an integral form of this equation given by Eq. (A5), in [6], omitted here for brevity. We now compute two quantities: the spectral field correlation  $\langle \tilde{E}(\omega - \frac{\delta\omega}{2}) E^*(\omega + \frac{\delta\omega}{2}) \rangle$  and the spectral intensity correlation  $\langle |\tilde{E}(\omega - \frac{\delta\omega}{2})|^2 |E^*(\omega + \frac{\delta\omega}{2})|^2 \rangle$ . We will do this within the framework of [9], which treated a 1D FEL in the high-gain regime including the effects of a linear energy chirp. In this model, the SASE electric field takes the form

$$E(t) = \sum_j e^{i\omega_j(\frac{z}{c} - (t-t_j))} g(t-t_j) h_{1d}(t_j) \quad (3)$$

where  $\omega_j = \omega_0 + ut_j$  is the frequency of light emitted by the  $j$ -th electron which is the central frequency  $\omega_0$  offset by a

\* This work was supported by the Department of Energy, Laboratory Directed Research and Development program at SLAC National Accelerator Laboratory, under contract DE-AC02-76SF00515.

term proportional to the electron beam chirp  $u$ . Furthermore,

$$g(t) \propto \exp \left[ -b \left( t - \frac{z}{v_g} \right)^2 - \frac{i u}{2} \left( t - \frac{z}{v_0} \right) \left( t - \frac{z}{c} \right) \right] \quad (4)$$

is the time-independent gain function. The parameter  $b = \frac{3}{4} \left( 1 + \frac{i}{\sqrt{3}} \right) \sigma_\omega^2$  where  $\sigma_\omega^2 = \frac{3\sqrt{3}\rho}{k_w z} \omega_0^2$  is the SASE bandwidth. Furthermore,  $v_g = \omega_0 / (k_r + \frac{2}{3}k_u)$  and  $v_0 = \omega_0 / (k_r + k_u)$  with  $k_r = 2\pi/\lambda_r$  the radiation wavenumber,  $k_u = 2\pi/\lambda_u$  the undulator wavenumber, and  $\omega_0$  the central resonant FEL frequency. We note here that this model is valid only for small chirps where  $u \ll \sigma_\omega^2$ . The function  $h_{td}(t_j)$  is a stand-in for any time-dependent effect in the gain process, and can thus account for variations in the e-beam current profile, in principle, undulator taper, etc. With these definitions, the frequency domain field is defined by the Fourier transform

$$\tilde{E}(\omega) \equiv \int e^{i\omega t} E(t) dt = \sum_j e^{i\left(\frac{\omega_j}{c} z + \omega t_j\right)} \tilde{g}(\omega - \omega_j) h_{td}(t_j) \quad (5)$$

where  $\tilde{g}(\omega) \equiv \int e^{i\omega t} g(t) dt$ . The spectral field correlation is then evaluated as a double sum, however, under the assumption that the beam particle arrival times are independent we can reduce it to a single sum. Similarly, the intensity correlation is a four-way sum which can be reduced to two. The process is similar to that found in [6, 10], and we omit the details here. The resulting correlations are

$$\left\langle \tilde{E} \left( \omega - \frac{\delta\omega}{2} \right) \tilde{E}^* \left( \omega + \frac{\delta\omega}{2} \right) \right\rangle = \tilde{F}(\omega, \delta\omega) \quad (6)$$

$$\left\langle \left| \tilde{E} \left( \omega - \frac{\delta\omega}{2} \right) \right|^2 \left| \tilde{E}^* \left( \omega + \frac{\delta\omega}{2} \right) \right|^2 \right\rangle = \tilde{F} \left( \omega - \frac{\delta\omega}{2}, 0 \right) \tilde{F} \left( \omega + \frac{\delta\omega}{2}, 0 \right) + |\tilde{F}(\omega, \delta\omega)|^2, \quad (7)$$

where  $\tilde{F}(\omega, \delta\omega)$  is defined as:

$$\tilde{F}(\omega, \delta\omega) \equiv \left\langle \sum_j e^{-i\delta\omega t_j} \tilde{g} \left( \omega - \omega_j - \frac{\delta\omega}{2} \right) \times \tilde{g}^* \left( \omega - \omega_j + \frac{\delta\omega}{2} \right) |h_{td}(t_j)|^2 \right\rangle \quad (8)$$

With this definition we can write  $G_2$  as

$$G_2(\delta\omega) = \left[ \int d\Delta d\Omega e^{-\frac{\Delta^2}{4\sigma_m^2} - \frac{\Omega^2}{\sigma_m^2}} |\tilde{F}(\omega + \Omega, \delta\omega + \Delta)|^2 \right] / \left[ \int d\Delta d\Omega e^{-\frac{\Delta^2}{4\sigma_m^2} - \frac{\Omega^2}{\sigma_m^2}} \times \tilde{F} \left( \omega + \Omega + \frac{\Delta + \delta\omega}{2}, 0 \right) \tilde{F} \left( \omega + \Omega - \frac{\Delta + \delta\omega}{2}, 0 \right) \right] \quad (9)$$

Our expression for  $\tilde{F}$  can be simplified further by replacing the ensemble average of the sum with an integral over

the beam current distribution  $f(t)$ , as

$$\tilde{F}(\omega, \delta\omega) = \int dt_j f(t_j) |h_{td}(t_j)|^2 e^{-i\delta\omega t_j} \times \tilde{g} \left( \omega - \omega_j - \frac{\delta\omega}{2} \right) \tilde{g}^* \left( \omega - \omega_j + \frac{\delta\omega}{2} \right) \quad (10)$$

So far we have left  $h_{td}$  inexplicit, which is problematic for future applications. To resolve this, we may connect it to the average X-ray intensity profile  $\chi(t)$  by first writing

$$\chi(t) \equiv \langle |E(t)|^2 \rangle = \int dt_j f(t_j) |h_{td}(t_j)|^2 |g(t-t_j)|^2 \quad (11)$$

In the limit that the bunch is long compared to the inverse of the SASE bandwidth,  $\sigma_t \sigma_\omega \gg 1$ , the SASE Green's function  $g(t)$  is much narrower than the product  $f(t)|h_{td}(t)|^2$ , and thus behaves much like a delta function, allowing us to write the proportionality

$$\chi(t) \propto f(t) |h_{td}(t)|^2 \quad (12)$$

Thus, up to some multiplicative factor that will drop out of  $G_2$  in the end, we may write

$$\tilde{F}(\omega, \delta\omega) = \int dt_j \chi(t_j) e^{-i\delta\omega t_j} \times \tilde{g} \left( \omega - \omega_j - \frac{\delta\omega}{2} \right) \tilde{g}^* \left( \omega - \omega_j + \frac{\delta\omega}{2} \right) \quad (13)$$

One can easily carry out these calculations for the Green's function described by Eq. (4) for a gaussian X-ray intensity profile,  $\chi(t) = e^{-t^2/2\sigma_t^2}$ , which yields  $G_2$  in the form

$$G_2(\delta\omega) = \frac{1}{\sqrt{1 + 2\sigma^2\sigma_t^2}} \exp \left[ -\frac{\delta\omega^2 \sigma_t^2 \xi_0^2}{1 + 2\sigma^2\sigma_t^2} \right] \quad (14)$$

where to make an explicit connection to [6] we define

$$\sigma = \frac{\sqrt{2}\sigma_m\sigma_\omega}{\sqrt{\sigma_m^2 + (1 + \delta_u^2)\sigma_\omega^2}} \quad \xi_0 = \frac{\sigma_\omega^2 \sqrt{1 + \delta_u^2}}{\sigma_m^2 + (1 + \delta_u^2)\sigma_\omega^2} \quad (15)$$

where  $\delta_u^2 = \tilde{u} + \tilde{u}^2 (1 + 3\sigma_t^2\sigma_\omega^2)$  with  $\tilde{u} \equiv u/\sqrt{3}\sigma_\omega^2$ . If the bunch is long enough that  $\frac{u\sigma_t}{\sigma_\omega}$  is of order one, then  $\delta_u \approx u\sigma_t/\sigma_\omega$ . Finally, we also note that the average spectral intensity can be written as

$$F(\omega, 0) \propto \exp \left[ -\frac{(\omega - \omega_0)^2}{2(\sigma_m^2 + \sigma_\omega^2(1 + \delta_u^2))} \right] \quad (16)$$

Taking Eqs. (14) and (16) together, we have a total of four unknown parameters  $\sigma_m$ ,  $\sigma_t$ ,  $\sigma_\omega$ , and  $u$ . By fitting to the measured spectrum and the measured  $G_2$  we can extract three parameters in the form of the amplitude of  $G_2$ , the width of  $G_2$ , and the width of the spectrum. Assuming we can estimate the SASE bandwidth reasonably well, we can then solve for the three remaining parameters to extract the bunch length, beam chirp, and spectrometer resolution.

## VALIDATION WITH 1D FEL SIMULATIONS

To verify the equations derived in the previous section, we performed simulations using a one-dimensional FEL code. The parameters common to all simulations are shown in Table 1. We scanned the chirp of the electron beam over a wide range of values, performing 1000 statistically independent simulations at each working point in order to generate sufficient statistics for calculating  $G_2$  and the average spectral intensity. The undulator length of 30 m was chosen such that the simulation ends during the exponential gain regime where the equations explicitly apply. To mimic a true experimental measurement, we computed the spectral intensity  $|\tilde{E}(\omega)|^2$  and convolved it with a gaussian of width 0.2 eV, as in Eq. (1). As stated in the previous section, the method demands some prior knowledge of the SASE bandwidth, which we can estimate as  $\sigma_\omega = \sqrt{\frac{3\sqrt{3}\rho}{k_w z} \omega_0^2} = 4.55$  eV.

Table 1: 1D FEL Simulation Parameters

Parameter	Unit	Value
Beam energy	GeV	12.1
Beam energy spread	MeV	0.726
Norm. emittance	$\mu\text{m rad}$	0.6
Peak current	kA	3
$\beta$ function	m	25
Undulator period	cm	3
Undulator K		3.5
Radiation wavelength	nm	0.19061
Bunch shape		Gaussian
RMS bunch length	$\mu\text{m}$	2
Undulator length	m	30

The results of the reconstruction are shown in Fig. 1. In it we see that, in spite of the varying chirp and therefore varying bandwidth and  $G_2$ , our model can successfully extract the correct pulse length. A naive application of the model of [6] would on the other hand predict a successively smaller pulse length as the chirp was increased. In addition to that, the chirp itself is effectively reconstructed, as is the spectrometer resolution.

## SENSITIVITY TO THE KNOWLEDGE OF SASE BANDWIDTH

Since the method we have laid out leaves the SASE bandwidth unspecified, it is worth considering how sensitive the predictions are to the estimated value of  $\sigma_\omega$ . To understand this we should look explicitly at the fitted solutions. In particular, suppose from our measurements we extract  $G_2(\delta\omega) = A_G e^{-\delta\omega^2/2\sigma_G^2}$  and  $\tilde{F}(\omega, 0) \propto e^{-(\omega-\omega_0)^2/2\sigma_{BW}^2}$ . Equating these with Eqs. (14) and (16) allows us to solve explicitly for  $\sigma_m$ ,  $\sigma_t$ , and  $u$ . In doing so we find that

$$\sigma_m = \frac{\sigma_G \sigma_{BW} \sqrt{1 - A_G^2}}{\sqrt{2\sigma_{BW}^2 + (1 - A_G^2)\sigma_G^2}} \quad (17)$$

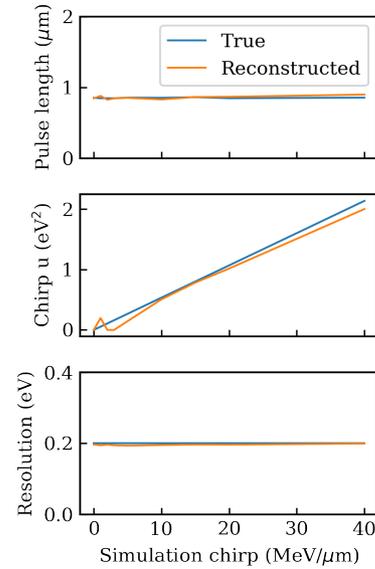


Figure 1: Reconstruction of X-ray pulse length, e-beam chirp, and spectrometer resolution from 1D simulations.

The spectrometer resolution prediction is thus independent of the SASE bandwidth estimate. The pulse length, on the other hand, is

$$\sigma_t = \frac{\sqrt{2\sigma_{BW}^2 + (1 - A_G^2)\sigma_G^2}}{2A_G\sigma_G\sigma_\omega} \quad (18)$$

and the chirp is

$$u = \frac{1}{\sigma_t} \sqrt{\frac{2\sigma_{BW}^4}{2\sigma_{BW}^2 + (1 - A_G^2)\sigma_G^2} - \sigma_\omega^2} \quad (19)$$

Now we can identify two regimes. When  $\delta u$  is large compared to one, Eqs. (14) and (16) show that  $\sigma_{BW}$  reduces to  $u\sigma_t$ , and  $\sigma_G$  becomes  $u^2/2\sigma_\omega^2$ . Then, neglecting for the moment the spectrometer resolution, our predictions for both  $\sigma_t$  and  $u$  (as in Eqs. (18) and (19)) become independent of the SASE bandwidth. In the opposite limit, the spectral bandwidth reduces to roughly the SASE bandwidth, while the width of  $G_2$  becomes roughly independent of the SASE bandwidth. As such,  $\sigma_t$  remains insensitive to  $\sigma_\omega$  while  $u$  becomes heavily sensitive to  $\sigma_\omega$ . Thus we conclude that regardless of the chirp value, our reconstructions of the bunch length and spectrometer resolution are relatively insensitive to our knowledge of the SASE bandwidth. The chirp, on the other hand, can be quite sensitive to  $\sigma_\omega$  if  $u\sigma_t/\sigma_\omega \approx 1$ , but becomes less and less sensitive as this parameter grows.

Let us consider this concretely in the case of the previous section. We plot the reconstructed pulse length and chirp as a function of the SASE bandwidth estimate in Fig. 2 for two different values of the chirp, indicated in the legend. Dashed lines represent the true value. In the top figure, it is clear that at both chirp working points the estimate of the pulse length is relatively insensitive to the SASE bandwidth. The chirp,

on the other hand, portrays a relatively weak dependence on bandwidth for the larger chirp case, but a stronger one for the weaker chirp case. We note that this reconstruction method will be most relevant when the chirp is larger anyways, so this does not substantially limit its applicability.

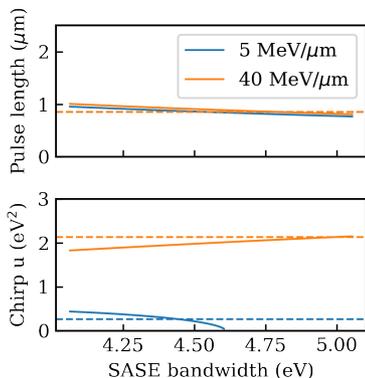


Figure 2: Dependence of pulse length and chirp reconstruction on the estimate of the SASE bandwidth.

## CONCLUSIONS

We have performed an analysis of FEL intensity fluctuations when the electron beam is chirped and demonstrated that the combination of the spectral intensity correlation and the average spectrum provides sufficient information to reconstruct the X-ray pulse length, the e-beam chirp, and the resolution of the spectrometer used for the measurement. The reconstructions are technically dependent on a guess of the SASE bandwidth but are found to be insensitive to it in regimes where the reconstruction would be useful. We have validated our method using 1D FEL simulations. Further validation with 3D FEL simulations and experiments will be the subject of future work.

## ACKNOWLEDGEMENTS

Authors would like to thank E. Hemsing and W. Fawley (SLAC) for useful discussions on the topic.

## REFERENCES

[1] C. Behrens *et al.*, “Few-femtosecond time-resolved measurements of x-ray free-electron lasers,” *Nat. Commun.*, vol. 5, no. 1, pp. 1–7, 2014. doi:10.1038/ncomms4762

- [2] J. Krzywinski, E. Saldin, E. Schneidmiller, and M. Yurkov, “A new method for ultrashort electron pulse-shape measurement using synchrotron radiation from a bending magnet,” *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 401, no. 2-3, pp. 429–441, 1997. doi:10.1016/S0168-9002(97)00987-X
- [3] M. S. Zolotarev and G. V. Stupakov, “Spectral Fluctuations of Incoherent Radiation and Measurement of Longitudinal Bunch Profile,” in *Proc. PAC’97*, Vancouver, Canada, May 1997.
- [4] F. Sannibale, G. Stupakov, M. Zolotarev, D. Filippetto, and L. Jägerhofer, “Absolute bunch length measurements by incoherent radiation fluctuation analysis,” *Phys. Rev. Spec. Top. Accel Beams*, vol. 12, no. 3, p. 032 801, 2009. doi:10.1103/PhysRevSTAB.12.032801
- [5] I. Lobach *et al.*, “Transverse beam emittance measurement by undulator radiation power noise,” *Phys. Rev. Lett.*, vol. 126, p. 134 802, 13 2021. doi:10.1103/PhysRevLett.126.134802
- [6] A. Lutman *et al.*, “Femtosecond x-ray free electron laser pulse duration measurement from spectral correlation function,” *Phys. Rev. Spec. Top. Accel Beams*, vol. 15, no. 3, p. 030 705, 2012. doi:10.1103/PhysRevSTAB.15.030705
- [7] A. Lutman, Z. Huang, J. Krzywinski, J. Wu, D. Zhu, and Y. Feng, “Statistical characterization of an x-ray FEL in the spectral domain,” in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, vol. 10237, 2017, paper 102370H, 102370H. doi:10.1117/12.2268918
- [8] S. Serkez *et al.*, “Wigner distribution of self-amplified spontaneous emission free-electron laser pulses and extracting its autocorrelation,” *J. Synchrotron Radiat.*, vol. 28, no. 1, pp. 3–17, 2021. doi:10.1107/S160057752001382X
- [9] S. Krinsky and Z. Huang, “Frequency chirped self-amplified spontaneous-emission free-electron lasers,” *Phys. Rev. Spec. Top. Accel Beams*, vol. 6, no. 5, p. 050 702, 2003. doi:10.1103/PhysRevSTAB.6.050702
- [10] E. L. Saldin, E. A. Schneidmiller, and M. Yurkov, “Statistical properties of radiation from vuv and x-ray free electron laser,” *Opt. Commun.*, vol. 148, no. 4-6, pp. 383–403, 1998. doi:10.1016/S0030-4018(97)00670-6