Lepton Flavor Violation in the Standard Model with dimension 6 operators

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We investigate Lepton Flavor Violation (LFV) in the general extension of the Standard Model parametrized by the gauge invariant operators of dimension-6. We discuss 3-body charged lepton decays and decay of Z boson to lepton pair and compare the obtained analytical expression with the current experimental results. We derive the numerical bounds on the size of the Wilson coefficients parameterizing the allowed size of New Physics effects.

1 Parametrization of the SM extensions in terms of effective operators

The renormalizable Standard Model (SM) is probably an effective theory valid only up to some energy scale where a more fundamental theory could manifest itself. At the electroweak scale the effects of New Physics (NP) can be effectively parametrized by new interactions described by non-renormalizable operators of higher mass dimensions. Coefficients of such operators (called Wilson coefficients) are suppressed by heavy mass scale (Λ) at which New Physics should become effective. In general the dimension-4 renormalizable SM Lagrangian can be extended as follows:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \dots . \tag{1}$$

where $Q_k^{(n)}$ denote the higher-dimension operators and $C_k^{(n)}$ stand for the corresponding dimensionless Wilson coefficients.

Using such Lagrangian, the theoretical calculations of relevant physical observables can be performed in a model-independent way, with final formulae given in terms of Wilson coefficients. Having such expressions simplifies significantly a comparison of various SM extension with the experimental results, as now only the values Wilson coefficients of new operators need be calculated within a given model of NP - this part of analysis is always model-dependent.

In the extended SM with the neutrino mass term, the GIM mechanism makes the branching ratio of the charged lepton flavor violating (CLFV) very small due to smallness of the mass of the neutrino comparing to the mass of the heaviest particle in the loop. Experimentally, the CLFV decays have never been observed yet but there are many models beyond the SM predict sizable rates up to the current experimental bounds. Here, we investigate the $\ell_i \to \ell_j \ell_j \ell_j$ decay and $Z \to \ell_f^+ \ell_i^-$ decays in the extension of the SM with dimension 6 operators.The full list of operators of dimension 5 and 6 which can be constructed out of SM fields is given in [1].

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After simplification only 8 operators of dimension 6 give contribution to these decays at tree level:

- 2 dipole-type operators, contributing to tree-level flavor non-diagonal Z and photon couplings to leptons:

$$Q_{eW} = (\bar{\ell}_i \sigma^{\mu\nu} e_j)\tau^I \varphi W^I_{\mu\nu}, \quad Q_{eB} = (\bar{\ell}_i \sigma^{\mu\nu} e_j)\varphi B_{\mu\nu}.$$

- 3 operators modifying tree-level Z and W couplings:

$$Q^{(1)}_{\varphi\ell} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_i \gamma^\mu \ell_j), \quad Q^{(3)}_{\varphi\ell} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_i \tau^I \gamma^\mu \ell_j), \ Q_{\varphi e} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_i \gamma^\mu e_j)$$

- 3 four-lepton contact operators:

$$Q_{\ell e} = (\bar{\ell}_i \gamma_\mu \ell_j)(\bar{e}_k \gamma^\mu e_l), \ Q_{\ell\ell} = (\bar{\ell}_i \gamma_\mu \ell_j)(\bar{\ell}_k \gamma^\mu \ell_l), \quad Q_{ee} = (\bar{e}_i \gamma_\mu e_j)(\bar{e}_k \gamma^\mu e_l)$$

In [2] we list the Feynman rules arising from dimension 6 operators.

2 Lepton Flavor violation in 3-body charged lepton and Z to lepton pair decays

In this section we compare the analytical results with experimental bounds on the $\ell_i \to \ell_j \ell_j \ell_j$ decay and $Z \to \ell_f^+ \ell_i^-$ decays to constrain the Wilson coefficients.Such decays can be generated already at tree level. Our results for radiative lepton decays are given in [2,3]. The experimental bound on these decays are given in Tables 1 and 2.

Table 1: Experimental upper limits on the branching ratios of the three body charged lepton decays.

Process	Experimental bound
$\mathcal{B}[\tau^- \to \mu^-\mu^+\mu^-]$	2.1×10^{-8} [4]
$\mathcal{B}[\tau^- \to e^-e^+e^-]$	2.7×10^{-8} [4]
$\mathcal{B}[\mu^- \to e^-e^+e^-]$	1.0×10^{-12} [5]

Table 2: Experimental upper limits (95 % CL) on the lepton flavor violating Z decay rates.

Process	Experimental bound
Br $[Z^0 \to \mu^\pm e^\mp]$	1.7×10^{-6} [6]
Br $[Z^0 \to \tau^\pm e^\mp]$	9.8×10^{-6} [6]
Br $[Z^0 \to \tau^\pm \mu^\mp]$	1.2×10^{-5} [6]

In general, at the tree-level diagrams mediated by photon, Z^0 boson and 4-lepton contact interactions can contribute to the three body charged lepton decays $\mu \to 3e$, $\tau \to 3e$ and $\tau \to 3\mu$. The general expression for the $\text{Br}(\ell_i \to 3\ell_j)$ reads as:

$$\begin{aligned}\text{Br}(\ell_i \to \ell_j \ell_j \bar{\ell}_j) &= \frac{m^5_{\ell_i}}{12288\pi^3\Lambda^4\Gamma_{\ell_i}} \left(4\left(|C_{VLL}|^2 + |C_{VRR}|^2 + |C_{VLR}|^2 + |C_{VRL}|^2\right)\right. \\ &+ \left. |C_{SLR}|^2 + |C_{SRL}|^2 + 48X_\gamma\right) \qquad (2)\end{aligned}$$

where Γ^{ℓ_i} is the total decay width of the initial lepton. The X_γ (photon contribution) and C_X (Z and contact interactions) can be written in terms of Wilson coefficients of operators listed in previous Section as:

$$\begin{aligned}
X_\gamma &= -\frac{16ev}{m_{\ell_i}}\mathrm{Re}\left[\left(2C_{VLL}+C_{VLR}-\frac{1}{2}C_{SLR}\right)C^\star_{\gamma R}+\left(2C_{VRR}+C_{VRL}-\frac{1}{2}C_{SRL}\right)C^\star_{\gamma L}\right] \\
&+ \frac{64e^2v^2}{m^2_{\ell_i}}\left(\log\frac{m^2_{\ell_i}}{m^2_{\ell_j}}-\frac{11}{4}\right)\left(|C_{\gamma L}|^2+|C_{\gamma R}|^2\right) \qquad (3)
\end{aligned}$$

$$\begin{aligned}
C_{VLL} &= 2\left((2s^2_W-1)\left(C^{(1)ji}_{\varphi\ell}+C^{(3)ji}_{\varphi\ell}\right)+C^{jijj}_{\ell\ell}\right) \\
C_{VRR} &= 2\left(2s^2_W C^{ji}_{\varphi e}+C^{jijj}_{ee}\right) \\
C_{VLR} &= -\frac{1}{2}C_{SRL} = \left(2s^2_W\left(C^{(1)ji}_{\varphi\ell}+C^{(3)ji}_{\varphi\ell}\right)+C^{jijj}_{le}\right) \\
C_{VRL} &= -\frac{1}{2}C_{SLR} = \left((2s^2_W-1)C^{ji}_{\varphi e}+C^{jjji}_{\ell e}\right) \\
C^{ij}_{\gamma L} &= C^{ji\star}_{\gamma R} = 2\sqrt{2}s_W\left(c_W C^{ij\star}_{eB}-s_W C^{ij\star}_{eW}\right) \qquad (4)
\end{aligned}$$

Knowing that C_γ must be negligible, we neglect contribution from C_γ in order to constrain the Wilson coefficients by using the bounds on $\ell_i \to \ell_j\ell_j\ell_j$ decays. Normalizing their branching ratio to the limits in Table 1 we find following numerical equations constraining the Wilson coefficients:

$$\begin{aligned}
C_{\mu eee} &\leq 3.29\times10^{-5}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\mu\to eee\right]}{1\times10^{-12}}}, \\
C_{\tau eee} &\leq 1.28\times10^{-2}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\tau\to eee\right]}{2.7\times10^{-8}}}, \qquad (5) \\
C_{\tau\mu\mu\mu} &\leq 1.13\times10^{-2}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\sqrt{\frac{\mathrm{Br}\left[\tau\to \mu\mu\mu\right]}{2.1\times10^{-8}}},
\end{aligned}$$

with $C_{\ell_i\ell_j\ell_j\ell_j}$ given by

$$\begin{aligned}
C_{\ell_i\ell_j\ell_j\ell_j} = \Bigg(&2\left|\mathrm{C}^{jijj}_{\ell e}-0.54\left(\mathrm{C}^{(1)ji}_{\varphi\ell}+\mathrm{C}^{(3)ji}_{\varphi\ell}\right)\right|^2+2\left|\mathrm{C}^{jijj}_{ee}+0.46\mathrm{C}^{ji}_{\varphi e}\right|^2 \\
&+\left|\mathrm{C}^{jijj}_{\ell e}+0.46\left(\mathrm{C}^{(1)ji}_{\varphi\ell}+\mathrm{C}^{(3)fi}_{\varphi\ell}\right)\right|^2+\left|\mathrm{C}^{jjji}_{\ell e}-0.54\mathrm{C}^{ji}_{\varphi e}\right|^2\Bigg)^{\frac{1}{2}}. \qquad (6)
\end{aligned}$$

Similar analysis can be done for the LFV $Z \to \ell^+_f\ell^-_i$ decays. At the tree level the branching ratio is:

$$\mathrm{Br}\left(Z^0\to\ell^\pm_f\ell^\mp_i\right)=\frac{m_Z}{24\pi\Gamma_Z}\left[\frac{m^2_Z}{2}\left(|C^{ZR}_{fi}|^2+|C^{ZL}_{fi}|^2\right)+|\Gamma^{ZL}_{fi}|^2+|\Gamma^{ZR}_{fi}|^2\right], \qquad (7)$$

with $\Gamma_Z \approx 2.495$ GeV being the total decay width of Z boson.

where we included all tree-level contributions and

$$\begin{aligned}
\Gamma^{ZL}_{fi} &= \frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}\left(C^{(1)fi}_{\varphi l}+C^{(3)fi}_{\varphi l}\right)+\left(1-2s^2_W\right)\delta_{fi}\right), \qquad (8) \\
\Gamma^{ZR}_{fi} &= \frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}C^{fi}_{\varphi e}-2s^2_W\delta_{fi}\right), \qquad (9) \\
C^{ZR}_{fi} &= C^{ZL\star}_{if} = -v\sqrt{2}C^{fi}_Z \qquad (10)
\end{aligned}$$

where C_Z^{fi} is defined as

$$C_Z^{fi} = \left(s_W C_{eB}^{fi} + c_W C_{eW}^{fi} \right) . \tag{11}$$

In the experimental values for branching ratio are for the sum $Z \to \ell_f^{\pm}\ell_i^{\mp} + \ell_f^{\mp}\ell_i^{\pm}$ [7] while in this equation branching ratio is for the decay $Z \to \ell_f^{\pm}\ell_i^{\mp}$ or $\ell_f^{\mp}\ell_i^{\pm}$. Therefore this equation must be multiply by a factor 2 in order to compare into experimental value.

Again normalizing the formulae for the branching ratios to the current experimental bound listed in Table 2 we derived the numerical equations constraining the Wilson coefficients contributing to this decay:

$$\begin{aligned}
\sqrt{\left|C_{\varphi\ell}^{(1)12} + C_{\varphi\ell}^{(3)12}\right|^2 + \left|C_{\varphi e}^{12}\right|^2 + \left|C_Z^{12}\right|^2 + \left|C_Z^{21}\right|^2} &\leq 0.06 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \mu^{\pm}e^{\mp}\right]}{1.7 \times 10^{-6}}} , \\
\sqrt{\left|C_{\varphi\ell}^{(1)13} + C_{\varphi e}^{13}\right|^2 + \left|C_{\varphi e}^{13}\right|^2 + \left|C_Z^{13}\right|^2 + \left|C_Z^{31}\right|^2} &\leq 0.14 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^{\pm}e^{\mp}\right]}{9.8 \times 10^{-6}}} , \\
\sqrt{\left|C_{\varphi\ell}^{(1)23} + C_{\varphi\ell}^{(3)23}\right|^2 + \left|C_{\varphi e}^{23}\right|^2 + \left|C_Z^{23}\right|^2 + \left|C_Z^{32}\right|^2} &\leq 0.16 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^{\pm}\mu^{\mp}\right]}{1.2 \times 10^{-5}}} .
\end{aligned} \tag{12}$$

These constraints are less stringent than the ones from $\ell_i \to \ell_j\ell_j\ell_j$ decays.

3 Conclusions

We have calculated the rates of several lepton flavor violating decays in the Standard Model extended with dimension 6 operators. We present numerical expressions constraining the relevant Wilson coefficients, based on experimental bounds for the $\ell_i \to \ell_j\ell_j\ell_j$ decay and $Z \to \ell_f^+\ell_i^-$ decays. There are also some experiments searching for charged lepton flavor violation and are going to upgrade the sensitivity. Observation of charged lepton flavor violation at experiment would be a clear hint for physics beyond the standard model. We show that the bounds on lepton flavor violation couplings are already very strong if the scale of New Physics is low.

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