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# Quantum mechanical calculation of electron spin

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**Abstract:** The classical and quantum mechanical methods are used respectively to calculate the electron spin. It is shown that the classical method cannot derive the correct magnetic moment value. Assuming that the rest energy of the electron originates from the kinetic energy of the virtual particles, the electron spin motion equation and spin wave function can be derived. In the case of the quantum numbers of spin angular momentum and magnetic moment being 1/2 and 1 respectively, their correct values can be obtained. In the meanwhile, the anomalous magnetic moment is evaluated based on the wave function of the spinning electron. Suppose the probability of virtual photons converting into electron-positron pairs to be 0.00141, the result agrees with that of quantum electrodynamics. Given that the energy of the virtual photon obeys the classical Maxwell-Boltzmann distribution, the self-energy of the electron will be finite. In addition, the hierarchy problem can be solved with the same hypothesis.

**Keywords:** Electron spin; Wave function; Anomalous magnetic moment; Divergence of self-energy; Hierarchy problem

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## 1 Introduction

The mass origin and the radius of the electron are long standing problems in classical and modern physics. If the mass of the electron is totally due to electromagnetic origin, then the classical radius of the electron will be the order of $10^{-15}$m. However, if we wish to get the spin value based on that hypothesis, the rotational speed of the electron surface would be more than 100*c* [1], which is obviously unreasonable. To see the difficulties arising from regarding electron as a rotating rigid sphere, one may see Refs. [2, 3]. As the electron spin can be derived from Dirac equation, it is often owed to relativistic effect. In quantum field theory, electron is regarded as a point particle (bare electron) with a covering of virtual photons.

However, if electron was a point particle, and the spin arose from certain rotation effect, then its magnetic moment and angular momentum would always obey the classical relation, and the spin magnetic moment of the electron would be half of the correct value. Thus the electron cannot be a point particle and it must have different mass density and charge density. A number of electron spin models have been put forward to evaluate the values of angular momentum and magnetic moment. The works of Refs. [4–9] were based on the external motion equations of the electron, some of them could give the correct values of angular momentum and/or magnetic moment; while Refs. [10–12] discussed the internal structure of the electron and the distributions of the mass and charge, but failed to give the correct values of angular momentum and magnetic moment, and they could not explain what force binds the electric charge together inside the electron. At present, there is no unified model which can both elucidate the mass origin and give the correct values of angular momentum and magnetic moment.

In order to describe the electron spin motion at the level of quantum mechanics, we must get a spin wave function, just as the orbital wave function for the motion of the electron around the nucleus. The paper is organized as follows. Section 2 calculates the angular momentum and magnetic moment of the electron with classical method to show that the correct magnetic moment value cannot be obtained. In section 3, quantum mechanical method is given and the electron spin wave function is obtained. Section 4 proceeds to calculate the anomalous magnetic moment to explain its physical origin in classical manner. In section 5, a simple method is proposed to eliminate the infrared and ultraviolet divergences of electron self-energy as well as to solve the hierarchy problem. Discussion and conclusion are given in section 6.

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## 2 Classical calculations of angular momentum and magnetic moment

We first see the problems in classical models of the electron spin. In Ref. [10], a semi-classical model for electron spin was presented, which assumed that $\hbar\omega = m_0c^2$ and $R\omega = c$, where $\omega$ is the angular frequency of the spinning electron with a mass of $m_0$, thus the radius of the electron is

$$R = \frac{\hbar}{m_0 c} = \lambda\!\!\!^{-}_c, \tag{2.1}$$

where $\lambda\!\!\!^{-}_c$ is reduced Compton wavelength. Note that the above expression can also be derived by uncertainty principle. Ref. [10] assumed a mass distribution of the electron of $\rho(r) = kr^3$, where $k$ is a constant and $r$ the distance from the center of the electron. Then the spin value of $\hbar/2$ could be obtained. The ratio of magnetic moment to angular momentum is $1.055e/m_0$ based on the hypothesis that $\rho_e/\rho_m$ is a constant and $\rho_e$ is multiplied by a factor of $1/\sqrt{1-v^2/c^2}$, where $\rho_e$ is electric charge density and $\rho_m$ the mass density. This model cannot account for the origin of the electron mass. Besides, it cannot give the correct value of the magnetic moment.

Here we use a simple thin circular plate model to derive the values of spin angular momentum and magnetic moment. If electron is a sphere, its projection in any given direction is a thin circular plate. Suppose the mass density of the circular plate is uniform, while the electric charge occupies at the edge of the plate. In the case of the electron radius being $R = \lambda\!\!\!^{-}_c$, the spin angular momentum is

$$J = \frac{1}{2}m_0R^2\omega = \frac{1}{2}m_0Rc = \frac{1}{2}\hbar. \tag{2.2}$$

The magnetic moment is

$$\mu = IS = \frac{ec}{2\pi R} \times \pi R^2 = \frac{1}{2}ecR = \frac{e\hbar}{2m_0} = \mu_B, \tag{2.3}$$

where $\mu_B$ is Bohr magneton. Although this simple model can correctly derive the spin angular momentum and magnetic moment, it has two defects: It cannot elucidate the mass origin and give the three-dimensional distributions of the mass and electric charge. In the following, a simple thin spherical shell model is employed to evaluate the magnetic moment. Let the radius of the shell be $R$; it rotates with an angular velocity of $\omega$; the surface density of charge is $e/4\pi R^2$.

As shown in Fig. 1, there is a circular ring belt on the shell with the perimeter of $2\pi r = 2\pi R\sin\theta$ and the width of $Rd\theta$. The area of the ring belt is $dS = 2\pi R^2 \sin\theta d\theta$; the electric charge within the belt is $de = edS/4\pi R^2$; the electric current generated by the rotation of the belt is $dI = r\omega de/2\pi r$; the magnetic moment is $d\mu = \pi r^2 dI = \omega eR^2 \sin^3\theta d\theta/4$. The overall magnetic moment generated by the rotation of the shell is

$$\mu = \frac{\omega eR^2}{4}\int_0^{\pi} \sin^3\theta d\theta = \frac{\omega eR^2}{3}. \tag{2.4}$$

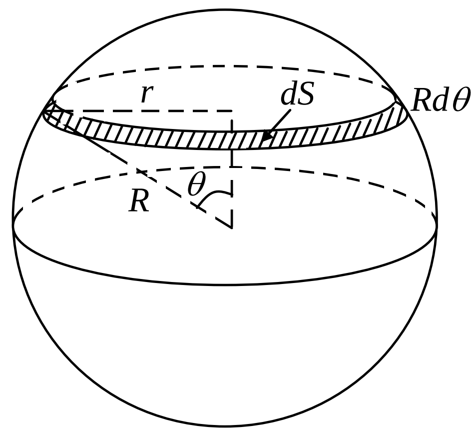


**Figure 1:** The rotating spherical shell model for the electron spin

Let $\omega R = c$ and $R = \lambda\!\!\!^{-}_c$, the result is

$$\mu = \frac{ec\lambda\!\!\!^{-}_c}{3} = \frac{2ec}{3} \times \frac{e\hbar}{2m_0c} = \frac{2}{3}\mu_B. \tag{2.5}$$

If electric charge distributes inside the electron, the magnetic moment will be less than the above value. Although the value is smaller than the correct value, the result greatly inspires us. The reason that we failed to derive the correct value is that we have used the classical model. If quantum mechanical method is adopted, we expect that the correct magnetic moment will be obtained.

## 3 Quantum mechanical calculations of angular momentum and magnetic moment

In order to calculate the spin angular momentum and magnetic moment, we first make some assumptions for the electron structure. Enlightenment can be achieved from the mass distribution of quarks. Particle physics tells us that the mass of up/down current quark (bare quark) is only several Mev, while the constituent mass of the quark is more than 300 Mev [13], so most of the quark mass comes from the effective masses of the sea quarks (virtual quark-antiquark pairs) and the virtual gluons. Similarly, we think that the electron is composed of point electric charge at the center, virtual electron-positron pairs inside the electron and virtual photons both inside and outside the electron, as shown in Fig. 2.

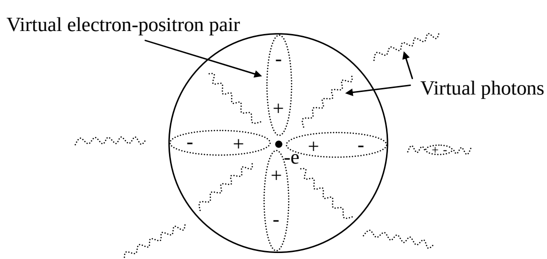


**Figure 2:** Illustration of the electron structure. -e is the point electric charge

We assume that the rest energy of the electron originates from the kinetic energies of the virtual electron-positron pairs and virtual photons. The virtual electron-positron pairs are induced by the point electric charge at the center of the electron; while the virtual photons are induced by the motion of the virtual electron-positron pairs. The virtual photons constitute the electromagnetic fields of the electron. There also exist virtual electron-positron pairs outside the electron due to vacuum polarization of the electromagnetic fields. But there is difference between the two instances. Inside the electron, there is a steady distribution of virtual electron-positron pairs due to the induction of the point electric charge. While outside the electron there is no inducing source, the presence of the virtual electron-positron pairs relies on the vacuum polarization, whose probability may be very small (as calculated in the later section). In the case of $R = \lambda_c$, the energy of the electromagnetic fields is the order of $\alpha$ (fine structure constant) of the electron rest energy, so the rest energy of electron mainly comes from the energy of the virtual electron-positron pairs.

Let the effective mass of the virtual electron-positron pairs be $m_p$ and their speeds be $c$. For the motion of the virtual electron-positron pairs, it results that

$$p^2c^2 = m_p^2c^4. \tag{3.1}$$

The above equation seems plausible at first glance. However, it is not the case. The electromagnetic fields are generated by the motion of the virtual electron-positron pairs, so the masses of the electromagnetic fields and the virtual electron-positron pairs are tied together and cannot be separated from each other. To change the motion state of the virtual electron-positron pairs implies to change the distribution of electromagnetic fields, so the effective mass of the electromagnetic fields should also be included in the inertial mass of the virtual electron-positron pairs. This is just like the instance where the earth moves with its aerosphere whose mass should also be included in the inertial mass of the earth. Thus the mass of the virtual electron-positron pairs in Eq. (3.1) should be replaced by the observable mass of the electron $m_0$.

In fact, if we think that the overall rest energy of the electron originates from the motion of the virtual particles, it directly leads to

$$p^2c^2 = m_0^2c^4, \tag{3.2}$$

without considering the internal structure of the electron. Quantizing the equation, i.e., substituting $p^2$ with operator $-\hbar^2\Delta$, we find

$$-\hbar^2\Delta\psi = m_0^2c^2\psi, \tag{3.3}$$

that is,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{m_0^2c^2}{\hbar^2}\psi = 0. \tag{3.4}$$

This is the wave equation for the electron spin. To solve the above equation, we refer to the wave equation of the orbital motion of the electron in hydrogen atom, which is

$$\Delta\psi + \frac{2m_0}{\hbar^2}(E + \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r})\psi = 0. \tag{3.5}$$

It can be seen that the wave equation of the spin motion is much simpler that that of the orbital motion. Following the solution to the wave equation of the orbital motion, we adopt separation of variables method. The spin motion is decomposed into angular motions and radial motion. The solutions to the spin angular wave equations are the same as those of orbital angular wave equations. The radial wave equation for the orbital motion is [1]

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + [\frac{2m_0}{\hbar^2}E + \frac{2m_0}{\hbar^2}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} - \frac{l(l+1)}{r^2}]R(r) = 0. \tag{3.6}$$

Similarly, the wave equation for the spin motion can be written as

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + \frac{m_0^2c^2}{\hbar^2}R(r) - \frac{l(l+1)}{r^2}R(r) = 0. \tag{3.7}$$

Eq. (3.7) is spherical Bessel equation. Before starting solving the equation, we first see how to derive the values of angular momentum and magnetic moment with wave function method. In the spherical coordinate system, the gradient operator is

$$\nabla = \mathbf{e_r}\frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{e}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}. \quad (3.8)$$

The probability current density in quantum mechanics is

$$j = \frac{\hbar}{2m_0 i}[\psi^\star \nabla\psi - \psi\nabla\psi^\star]. \quad (3.9)$$

The above expression applies to non-relativistic particles, but it can be verified that it also holds for the particles with the speed of light. In this case, $m_0$ denotes the effective mass. As a particle rotates around the specific z axis, as shown in Fig. 3, we obtain

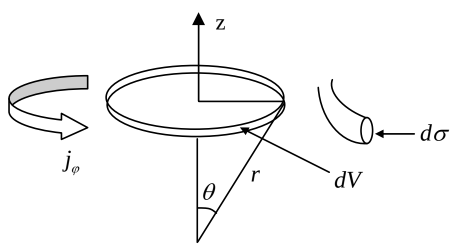


**Figure 3:** The rotation of a particle around z axis

$$\begin{aligned} j_\phi =& \frac{\hbar}{2m_0 i}[R(r)Y^\star_{lm}(\theta,\phi)\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(R(r)Y_{lm}(\theta,\phi)) \\ &- R(r)Y_{lm}(\theta,\phi)\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(R(r)Y^\star_{lm}(\theta,\phi))] \\ =& \frac{m\hbar}{m_0 r\sin\theta}R^2(r)|Y_{lm}(\theta,\phi)|^2. \end{aligned} \quad (3.10)$$

It can be seen from Fig. 3 that the differential angular momentum is $dJ_z = m_0 j_\phi r\sin\theta dV$, where $dV$ is the differential volume of the thin circular ring with the cross-sectional area of $d\sigma = rd\theta dr$ and the perimeter of $2\pi r\sin\theta$. The overall angular momentum is

$$J_z = \int_V dJ_z = m\hbar\int_V |\psi|^2 dV = m\hbar \quad (3.11)$$

We then turn to the calculation of magnetic moment. When $ej_\phi$ is multiplied by the differential area $d\sigma$, where $e$ is the charge of the electron, we get the differential current $dI = erj_\phi drd\theta$, which generates the magnetic moment of $d\mu_z = \pi r^2\sin^2\theta dI$. The overall magnetic moment for the spinning electron is

$$\begin{aligned} \mu_z &= \int e\pi r^2\sin^2\theta\frac{m\hbar}{m_0 r\sin\theta}|\psi|^2 d\sigma \\ &= \frac{em\hbar}{2m_0}\int 2\pi r\sin\theta|\psi|^2 d\sigma \\ &= \frac{em\hbar}{2m_0}\int_V |\psi|^2 dV = \frac{em\hbar}{2m_0} = m\mu_B. \end{aligned} \quad (3.12)$$

The above arguments may refer to Ref. [1]. For our electron spin model, if we assume a quantum number $l = 1/2$ for angular momentum while $l = 1$ for magnetic moment, we get the correct values. The reason that we use two different quantum numbers for the electron spin is that the mass distribution and the charge distribution inside the electron are different based on the model of Fig. 2. In contrast, the quantum numbers of the angular momentum and the magnetic moment can be regarded as equal for the orbital motion of the electron. This is because the electron radius is much smaller compared to the orbital radius; the orbital motion around the nucleus may be regarded as the motion of a point particle; while the spin magnetic moment is due to the rotation of electric dipole, which is the deep reason for a Landé $g$-factor of 2. For the electron structure model of Fig. 2, our goal is to solve for the distributions of mass and charge inside the electron. As we are more interested in the charge distribution, we solve the spherical Bessel equation for $l = 1$. In this case, the two specific solutions are

$$J_1(r) = \frac{\sin r}{r^2} - \frac{\cos r}{r}, \quad n_1(r) = -\frac{\cos r}{r^2} - \frac{\sin r}{r}, \quad (3.13)$$

respectively. Now consider the boundary condition at $r = 0$, where we should have $r\psi(r) \to 0$. It can be seen that $rJ_1(r) \to 0$ and $rn_1(r) \to \infty$, so we may assume $R(r) = k_1 J_1(r)$, where $k_1$ is a constant and can be derived by the normalization condition of the wave function. As

$$\begin{aligned} &\int r^2|R(r)|^2 dr \\ &= k_1[r(3+r^2)/6 + 3r\cos 2r/4 + (2r^2-5)\sin 2r/8 + C], \end{aligned} \quad (3.14)$$

if the upper limit of the integral is infinity, the above expression will diverge, so the electron must have a definite radius, which is obviously different from the orbital motion that may extend to infinity. We assume a radius of $\lambda_c$ for the electron. As $r$ is very small, we expand $R(r)$ into power series. As

$$\sin r \approx r - \frac{r^3}{6}, \quad \cos r \approx 1 - \frac{r^2}{2}, \quad (3.15)$$

we obtain

$$\int_0^{\lambda_c} k_1^2 \frac{r^4}{9} dr = 1. \tag{3.16}$$

It follows that $k_1^2 = 45/\lambda_c^5$, and

$$R(r) = \sqrt{\frac{45}{\lambda^5}}\left(\frac{\sin r}{r^2} - \frac{\cos r}{r}\right) \approx \sqrt{\frac{5}{\lambda^5}}\, r. \tag{3.17}$$

The wave function for the charge distribution inside the electron is

$$\psi(r,\theta,\phi) = \frac{\sqrt{3}}{2\sqrt{2\pi}} R(r)\sin\theta e^{i\phi} \approx \frac{1}{2}\sqrt{\frac{15}{2\pi\lambda^5}}\, r\sin\theta e^{i\phi}. \tag{3.18}$$

It can be seen that inside the electron the charge density increases with $r$ and reaches a maximum value at the surface. We know that the classical motion of a particle corresponds to the motion state with a maximum probability in quantum mechanics, so the thin circular plate model can obtain the correct magnetic moment. In the meanwhile, it is interesting to see that the charge density becomes zero at the center, which shows that the electric charge of the point charge is totally shielded by the virtual positrons.

Now consider the instance of $l = 0$, which represents the non-polarized state of the electron. In this case, Eq. (3.7) can be written as

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + \frac{m_0^2c^2}{\hbar^2}R(r) = 0, \tag{3.19}$$

whose two specific solutions are $\frac{1}{r}\cos\frac{r}{\lambda_c}$ and $\frac{1}{r}\sin\frac{r}{\lambda_c}$ respectively. Due to the boundary condition of $r\psi(r) \to 0$ at $r = 0$, we can only take the latter. It results that

$$R(r) = \frac{C_1}{r}\sin\frac{r}{\lambda_c}. \tag{3.20}$$

From the normalization condition of the wave function we find $C_1 = 1/\sqrt{\lambda_c(\frac{1}{2} - \frac{\sin 2}{4})}$. In the case of $l = 0$, $Y(\theta,\phi) = Y_{00}(\theta,\phi) = \frac{1}{2\sqrt{\pi}}$, thus the spin wave function is

$$\psi(r,\theta,\phi) = \frac{C_1}{2\sqrt{\pi}}\frac{1}{r}\sin\frac{r}{\lambda_c}. \tag{3.21}$$

## 4 Quantum mechanical calculation of anomalous magnetic moment

In quantum field theory, the anomalous magnetic moment is due to the electron vertex function. It is the higher-order correction as an electron experiences an external electromagnetic fields. We may explain the origin of anomalous magnetic moment in a classical manner. The spin of a free electron is unpolarized, thus a free electron has only electric field and has no spin magnetic moment. When an external magnetic field is exerted on the electron, its spin direction is parallel to the direction of the external magnetic field. In the first-order approximation, the magnetic moment of the electron may be regarded as generated by the rotation of the virtual electron-positron pairs inside the electron and is just a Bohr magneton. In higher-order correction, we need to take into account the magnetic moment generated by the rotation of the virtual electron-positron pairs due to vacuum polarization. This part of the magnetic moment is the anomalous magnetic moment in quantum electrodynamics. The virtual photons cannot always be in the vacuum polarization state; otherwise there will be no difference between the virtual electron-positron pairs inside and outside the electron. That is to say, there is a probability for the virtual photons to convert into virtual electron-positron pairs. When all the virtual photons have converted into virtual electron-positron pairs, we suppose the relation between the magnetic moment and the angular momentum of the electromagnetic fields is the same as that of virtual electron-positron pairs inside the electron. Thus for the magnetic moment arising from vacuum polarization, we find

$$\mu' = \eta e J_{em}/m_{em}, \tag{4.1}$$

where $\mu'$ is anomalous magnetic moment; $J_{em}$ is the angular momentum of the electromagnetic fields; $m_{em}$ is the effective mass of the electromagnetic fields; $\eta$ is the probability of the virtual photons converting into virtual electron-positron pairs. In the following, we first calculate $J_{em}$ and $m_{em}$ based on the wave function of charge distribution, then evaluate $\mu'$ based on Eq. (4.1).

We first derive the electromagnetic fields generated by an electrified thin circular ring at any space point. As shown in Fig. 4, the radius of the circular ring is $a$, and the electric charge is $q$. The electric fields are symmetrical with respect to $\phi_0$ and only relate to $r_0$ and $\theta_0$. For simplicity, we calculate the electrical fields at point p$(r_0, \theta_0, 0)$ located in the x-z plane. The detailed calculation process may refer to Ref. [14], the results are

$$\begin{cases} E_x = \frac{q}{4\pi^2\varepsilon_0}\frac{1}{r_0\sin\theta_0}\frac{1}{\sqrt{a^2+r_0^2+2ar_0\sin\theta_0}} \\ \quad \left[\frac{2r_0^2\sin^2\theta_0 - a^2 - r_0^2}{a^2+r_0^2-2ar_0\sin\theta_0}E(k_0) + K(k_0)\right] \\ E_y = 0 \\ E_z = \frac{q}{4\pi^2\varepsilon_0}\frac{2r_0\cos\theta_0}{\sqrt{a^2+r_0^2+2ar_0\sin\theta_0}}\frac{1}{a^2+r_0^2-2ar_0\sin\theta_0}E(k_0) \end{cases}, \tag{4.2}$$

where $E(k_0) = \int_0^{\pi/2}\sqrt{1-k_0^2\sin^2 x}\,dx$ is the elliptic integral of the second kind; $K(k_0) = \int_0^{\pi/2}\frac{dx}{\sqrt{1-k_0^2\sin^2 x}}$ is the elliptic

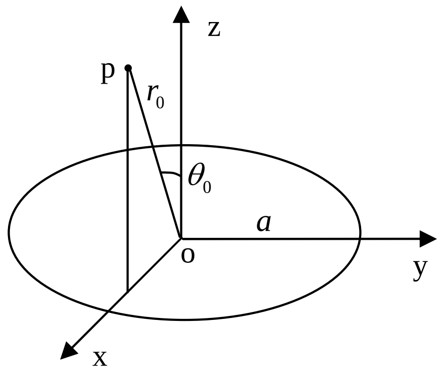


**Figure 4:** Electromagnetic fields generated by a thin circular ring

integral of the first kind, and

$$k_0 = \frac{4ar_0 \sin\theta_0}{a^2 + r_0^2 + 2ar_0 \sin\theta_0}. \tag{4.3}$$

Similarly, suppose the electric current in the circular ring is $I$, the magnetic fields are [15]

$$\begin{cases} B_x = \frac{\mu_0 I}{4\pi} \frac{2\cos\theta_0}{\sin\theta_0} \frac{1}{\sqrt{a^2+r_0^2+2ar_0\sin\theta_0}} \\ \left[\frac{a^2+r_0^2}{a^2+r_0^2-2ar_0\sin\theta_0} E(k_0) - K(k_0)\right] \\ B_y = 0 \\ B_z = \frac{\mu_0 I}{4\pi} \frac{2}{\sqrt{a^2+r_0^2+2ar_0\sin\theta_0}} \left[\frac{a^2-r^2}{a^2+r_0^2-2ar_0\sin\theta_0} E(k_0) + K(k_0)\right] \end{cases} \tag{4.4}$$

We now turn to the calculation of electromagnetic fields of a spinning electron. As shown in Fig. 5, the origin coincides with the center of the electron. The coordinates of point p in the x-z plane is p$(r, \theta)$. The space occupied by the distribution of wave function of the spinning electron is divided into lots of volume elements of thin circular ring with the cross-sectional area of $d\sigma_1 = r_1 d\theta_1 dr_1$ and the perimeter of $2\pi a$. The differential volume is $dV_1 = 2\pi r_1^2 \sin\theta_1 dr_1 d\theta_1$, which carries electric charge of $dq = e|\psi|^2 dV_1$; the electric current passing through the cross-section $d\sigma_1$ is $dI = ej_\phi d\sigma_1$. Then

$$|\psi|^2 = \frac{3k_1^2}{8\pi} \sin^2\theta_1 \left(\frac{\sin r_1}{r_1^2} - \frac{\cos r_1}{r_1}\right)^2 \approx \frac{k_1^2 r_1^2}{24\pi} \sin^2\theta_1, \tag{4.5}$$

$$j_\phi = \frac{\hbar}{m_0 r_1 \sin\theta_1} |\psi|^2 = \frac{k_1^2 \hbar}{24\pi m_0} r_1 \sin\theta_1, \tag{4.6}$$

$$dq = e|\psi|^2 dV_1 = \frac{k_1^2 e}{12} r_1^4 \sin^3\theta_1 dr_1 d\theta_1, \tag{4.7}$$

$$dI = ej_\phi d\sigma_1 = \frac{k_1^2 e\hbar}{24\pi m_0} r_1^2 \sin\theta_1 dr_1 d\theta_1. \tag{4.8}$$

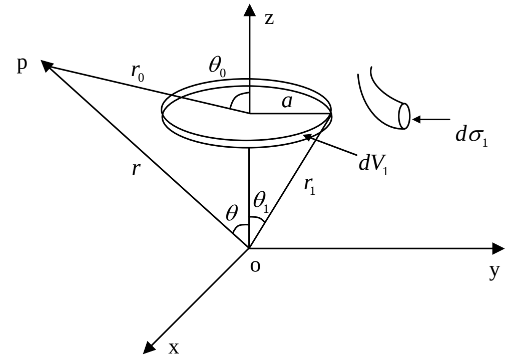


**Figure 5:** Electromagnetic fields generated by the spinning electron

The electromagnetic fields generated by a circular ring are shown in Eqs. (4.2) and (4.4). However, in the two expressions, the coordinates of point p are represented by $(r_0, \theta_0)$, which should be represented with $(r, r_1, \theta, \theta_1)$ for the convenience of subsequent integral calculation. According to the geometric relation in Fig. 5, we find

$$r_0 = \sqrt{r^2 + r_1^2 \cos^2\theta_1 - 2rr_1 \cos\theta \cos\theta_1}, \tag{4.9}$$

$$r_0 \sin\theta_0 = r \sin\theta, \tag{4.10}$$

$$\cos\theta_0 = \frac{r\cos\theta - r_1 \cos\theta_1}{r_0}, \tag{4.11}$$

$$a = r_1 \sin\theta_1. \tag{4.12}$$

We first simplify three expressions which will be used subsequently:

$$\Delta_1 = a^2 + r_0^2 + 2ar_0 \sin\theta_0 = r^2 + r_1^2 - 2rr_1 \cos(\theta + \theta_1), \tag{4.13}$$

$$\Delta_2 = a^2 + r_0^2 - 2ar_0 \sin\theta_0 = r^2 + r_1^2 - 2rr_1 \cos(\theta - \theta_1), \tag{4.14}$$

$$k_0 = \frac{4ar_0 \sin\theta_0}{a^2 + r_0^2 + 2ar_0 \sin\theta_0} = \frac{4rr_1 \sin\theta \sin\theta_1}{\Delta_1}. \tag{4.15}$$

The electromagnetic fields generated by the spinning electron at point p$(r, \theta)$ are

$$\begin{cases} E_x = & \frac{k_1^2 e}{48\pi^2\varepsilon_0} \int_0^\pi d\theta_1 \int_0^R dr_1 \frac{r_1^4 \sin^3\theta_1}{r\sin\theta} \frac{1}{\sqrt{\Delta_1}} \\ & \left[ \frac{2rr_1\cos\theta\cos\theta_1 - r_1^2 - r^2\cos 2\theta}{\Delta_2} E(k_0) + K(k_0) \right] \\ E_z = & \frac{k_1^2 e}{48\pi^2\varepsilon_0} \int_0^\pi d\theta_1 \int_0^R dr_1 \frac{r_1^4 \sin^3\theta_1}{\sqrt{\Delta_1}} \frac{2(r\cos\theta - r_1\cos\theta_1)}{\Delta_2} E(k_0) \\ B_x = & \frac{k_1^2 e\mu_0\hbar}{48\pi^2 m_0} \int_0^\pi d\theta_1 \int_0^R dr_1 \frac{r_1^2\sin\theta_1}{r\sin\theta} \frac{(r\cos\theta - r_1\cos\theta_1)}{\sqrt{\Delta_1}} \\ & \left[ \frac{r^2 + r_1^2 - 2rr_1\cos\theta\cos\theta_1}{\Delta_2} E(k_0) - K(k_0) \right] \\ B_z = & \frac{k_1^2 e\mu_0\hbar}{48\pi^2 m_0} \int_0^\pi d\theta_1 \int_0^R dr_1 \frac{r_1^2\sin\theta_1}{\sqrt{\Delta_1}} \\ & \left[ \frac{2rr_1\cos\theta\cos\theta_1 - r^2 - r_1^2\cos 2\theta_1}{\Delta_2} E(k_0) + K(k_0) \right] \end{cases} \tag{4.16}$$

The angular momentum of the electromagnetic fields is

$$\begin{aligned} J_{em} &= \int_{V_1} \mathbf{r} \times (\varepsilon_0 \mathbf{E} \times \mathbf{B}) dV_1 \\ &= \int_0^{2\pi} d\phi \int_0^\pi d\theta \int_0^\infty dr \varepsilon_0 r^3 \sin^2\theta (E_z B_x - B_z E_x). \end{aligned} \tag{4.17}$$

The energy of the electromagnetic fields is

$$\begin{aligned} E_{em} &= \int_{V_1} (\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}) dV_1 \\ &= \frac{1}{2\mu_0} \int_0^{2\pi} d\phi \int_0^\pi d\theta \int_0^\infty dr r^2 \sin\theta (\frac{E_x^2 + E_z^2}{c^2} + B_x^2 + B_z^2). \end{aligned} \tag{4.18}$$

The effective mass of the electromagnetic fields is

$$m_{em} = E_{em}/c^2. \tag{4.19}$$

The above expressions can be computed with Mathematica software. The results indicate that the angular momentum and electromagnetic energy of the electron concentrate mainly within the near region. For example, we obtain $m_{em} = 0.00641 m_0$ within the region of 0-10R and $m_{em} = 0.00674 m_0$ within the region of 0-100R. This is due to the fact that $E$ and $B$ decrease with $r^2$. Thus enough precision can be reached with an upper limit of 10000R for radial integrals. The computed results are

$$\begin{aligned} & m_{em} = 0.00678 m_0, \ J_{em} = 0.00278\hbar, \\ & \frac{e}{m_{em}} J_{em} = 0.8205 \mu_B. \end{aligned} \tag{4.20}$$

In order for the result of anomalous magnetic moment to agree with that of quantum electrodynamics, $\eta$ should be 0.0011596/0.8205=0.00141. It can be seen that the probability of the virtual photons converting into virtual electron-positron pairs is very small. Although the above method cannot directly derive the value of anomalous magnetic moment, it is helpful for us to understand its physical origin.

# 5 Proposal for the treatment of divergence of self-energy in quantum field theory

In quantum field theory, bare electron is a point particle, and its proper mass and electromagnetic mass are both infinite; but the sum of the two terms are finite, which is the observable mass of the electron [16]. Based on our electron structure model and calculations, the proper mass and the electromagnetic mass are both finite. The divergence of the electron self-energy in quantum field theory arises from the emission and absorption of virtual photons. As there are infinite number of virtual photons, the self-energy integral of electron diverges, which is the ultra-violet divergence. In addition, there exists infrared divergence, which arises from the virtual photons with zero frequency.

The electron continuously emits and absorbs virtual photons with various frequencies, which leads to the fluctuation of the electron energy. In general, the electron-virtual photon system is similar to a heat equilibrium system, so we may think that the probability of the electron emitting and absorbing a virtual photon with certain energy (or frequency) also obeys the statistical law of heat equilibrium system. In order to eliminate infinity of electron self-energy, we make two assumptions: (i) The overall energy of the virtual photons equals the electromagnetic energy. Virtual photons continuously appear and then disappear; during their mean surviving time, their energies constitute the energy of the electromagnetic fields. (ii) The probabilities of the electron emitting virtual photons with different frequencies are different. In quantum field theory, the probabilities are 1 for virtual photons with any frequency, which leads to the divergence of electron self-energy. In order to get rid of infrared and ultra-violet divergences, we must assume that the probabilities tend to be zero as frequency approaches zero and infinity. We now have two formulae for reference: One is Maxwell-Boltzmann distribution; the other is Planck's black body radiation law. The latter holds for the instance of discrete energy spectrum. For a free electron, the energy spectrum of the virtual photons should be continuous, so we refer to the former to write out the expression of frequency distribution of virtual photons. The Maxwell-Boltzmann distribution is

$$f(v) = 4\pi \left( \frac{m}{2\pi KT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2KT}}. \tag{5.1}$$

In the above expression, the kinetic energy of a molecule is $mv^2/2$; while the energy of a virtual photon is $\hbar\omega$, so we suppose the frequency distribution of the virtual photons to be

$$f(\omega) = C_0 \omega e^{-\frac{\omega}{x_0}}, \tag{5.2}$$

where $C_0$ and $x_0$ are determined by the following equations:

$$\int_0^\infty f(\omega) d\omega = 1, \tag{5.3}$$

$$\int_0^\infty \hbar\omega f(\omega) d\omega = E_{em}. \tag{5.4}$$

It's easy to obtain $x_0 = E_{em}/2\hbar$ and $C_0 = 1/x_0^2$. The infrared divergence is logarithmically divergent and the ultra-violet divergence is linearly divergent in the second-order correction of the electron self-energy [16]. If the probability distribution of Eq. (5.2) is imposed on the virtual photons, the self-energy of the electron will be finite. The similar method can be applied to the removal of divergence of self-energy of the photon.

It should be noted that our above prescription for the divergence of the electron self-energy introduces an extra parameter of $E_{em}$. Although it can be calculated with Eq. (4.18), it is only the classical approximate value. In order to obtain its precise value, we should consider higher-order effect. As a matter of fact, in the calculation of radiative corrections in quantum field theory, we don't need to know the electromagnetic mass of the electron. We only need to separate the finite terms from the divergent terms and take them as the radiative correction terms. Similarly, we don't need to know the exact value of $E_{em}$ after introducing Maxwell-Boltzmann distribution. What we need to do is separate the terms containing $E_{em}$ and incorporate them into the observable mass of the free electron. The remaining terms are the higher-order correction terms. In quantum field theory, the electromagnetic mass is included in the observable mass of free electron and the infinite terms are cancelled by counterterms. It can be seen that our method is basically the same as that of quantum field theory in essence. The infinite terms in quantum field theory correspond to the finite terms containing $E_{em}$ in our theory.

Another application of Maxwell-Boltzmann distribution in quantum field theory is the hierarchy problem. The Higgs boson mass is about 125 GeV [17, 18], but the quantum corrections will push its mass up to the gravitational scale, that is, the order of $10^{18}$ GeV. The most important contributions are the one-loop diagrams involving the top quark, the $SU(2.2)\times U(2.1)$ gauge bosons, and the Higgs boson itself. Supersymmetry provides a solution to this puzzle. However, the current LHC experiments have excluded large parameter regions of supersymmetric extensions of the standard model (see e.g. [19, 20]). Our above proposal offers an alternative solution to the hierarchy problem.

## 6 Discussion and conclusion

Modern high-energy experiments have revealed that the electron may be regarded as a point particle. But point particle model leads to infinite self-energy and cannot derive the correct spin magnetic moment; while Stern-Gerlach experiment demonstrated that the spin magnetic moment of the electron does exist, so the electron must have internal structure. In most of the literature the electron radius is taken as one reduced Compton wavelength. In our electron model, given that the angular quantum numbers of the mass distribution and the charge distribution are 1/2 and 1 respectively, the correct values of the angular momentum and magnetic moment can be obtained whatever the value of the electron radius is. However, if we take into other aspects into account, such as the electromagnetic energy of the electron, the rotational speed of the surface of the electron and the uncertainty principle, then it is reasonable that the electron has a radius of reduced Compton wavelength. In our model, the distribution of the electric charge is spherically symmetric for an unpolarized electron; its Coulomb interaction with other particles is the same as that of a point particle. Besides, the mass of the electron increases with its velocity. When the energy of the electron grows large, the radius of the electron decreases, it is more like a point particle, so the electron with a certain radius does not conflict with the point particle model as far as Coulomb interaction is concerned.

The aim of our electron spin model is to intuitively understand the internal structure of the electron and explore the origins of its mass and spin. Although the model can correctly derive the values of angular momentum and magnetic moment, it cannot directly evaluate the value of anomalous magnetic moment, which reveals the deficiency of quantum mechanics and the necessity of developing quantum field theory. However, the present formalism of quantum field theory suffers from the infinite particle self-energy. We expect that by introducing Maxwell-Boltzmann distribution, the divergence of the self-energy will disappear and the hierarchy problem will be solved.

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# Appendix A: Mathematica source program for evaluation of anomalous magnetic moment and the output results

```
c=299792458;
e=1.6021766208*10^-19;
m0=9.10938356*10^-31;
planckc=6.62606896*10^-34/(2*Pi);
u0=4*Pi*10^-7;
R=3.8615926764*10^-13;
BohrMagneton=9.274009994*10^-24;
AnoMagMoment=0.00115965218091*
    BohrMagneton;
k2=45/R^5;
f1[r_,r1_,xita_,xita1_]:=r*r+r1*r1-2*r
    *r1*Cos[xita+xita1];
f2[r_,r1_,xita_,xita1_]:=r*r+r1*r1-2*r
    *r1*Cos[xita-xita1]; k[r_,r1_,
    xita_,xita1_]:=4*r*r1*Sin[xita]*
    Sin[xita1]/f1[r,r1,xita,xita1];

Ex[r_?NumericQ,xita_?NumericQ]:=
    NIntegrate[r1^4*Sin[xita1]^3/Sqrt[
    f1[r,r1,xita,xita1]]*((2*r*r1*Cos[
    xita]*Cos[xita1]-r1*r1-r*r*Cos[2*
    xita])/f2[r,r1,xita,xita1]*
    EllipticE[k[r,r1,xita,xita1]]+
    EllipticK[k[r,r1,xita,xita1]]),{
    xita1,0,Pi},{r1,0,R}];
Ez[r_?NumericQ,xita_?NumericQ]:=
    NIntegrate[2*r1^4*Sin[xita1]^3*(r*
    Cos[xita]-r1*Cos[xita1])/Sqrt[f1[r
    ,r1,xita,xita1]]/f2[r,r1,xita,
    xita1]*EllipticE[k[r,r1,xita,xita1
    ]],{xita1,0,Pi},{r1,0,R}];
Bx[r_?NumericQ,xita_?NumericQ]:=
    NIntegrate[r1*r1*Sin[xita1]*(r*Cos
    [xita]-r1*Cos[xita1])/Sqrt[f1[r,r1
    ,xita,xita1]]*((r*r+r1*r1-2*r*r1*
    Cos[xita]*Cos[xita1])/f2[r,r1,xita
    ,xita1]*EllipticE[k[r,r1,xita,
    xita1]]-EllipticK[k[r,r1,xita,
    xita1]]),{xita1,0,Pi},{r1,0,R}];
Bz[r_?NumericQ,xita_?NumericQ]:=
    NIntegrate[r1*r1*Sin[xita1]/Sqrt[
    f1[r,r1,xita,xita1]]*((2*r*r1*Cos[
    xita]*Cos[xita1]-r*r-r1*r1*Cos[2*
    xita1])/f2[r,r1,xita,xita1]*
    EllipticE[k[r,r1,xita,xita1]]+
    EllipticK[k[r,r1,xita,xita1]]),{
    xita1,0,Pi},{r1,0,R}];

DateList[]
EMenergy=1/48/48*e*e*u0*k2^2/Pi^3*
    NIntegrate[(Ex[r,xita]^2/Sin[xita
    ]+Ez[r,xita]^2*r*r*Sin[xita])*c^2+
    planckc^2/m0^2*(Bx[r,xita]^2/Sin[
```

```
   xita]+Bz[r,xita]^2*r*r*Sin[xita])
   ,{xita,0,Pi},{r,0,10000*R}];
J=2/48/48*e*e*u0*planckc*k2^2/Pi^3/m0*
   NIntegrate[r*r*Sin[xita]*(Ez[r,
   xita]*Bx[r,xita]-Ex[r,xita]*Bz[r,
   xita]),{xita,0,Pi},{r,0,10000*R}];
EMmass=EMenergy/c^2;
u=e/EMmass*J;
eta= AnoMagMoment/u
UnitEMmass=EMmass/m0
UintAngularMomentum=J/planckc
UnitMagneticMoment=u/BohrMagneton
DateList[]

{2016,12,23,20,42,59.0931850}

NIntegrate::slwcon:
Numerical integration converging too
   slowly; suspect one of the
   following: singularity, value of
   the integration is 0, highly
   oscillatory integrand, or
   WorkingPrecision too small.
NIntegrate::ncvb: NIntegrate failed to
    converge to prescribed accuracy
   after 18 recursive bisections in r
    near {xita,r}= {2.23377,3.81608
   x10-13}. NIntegrate obtained
   4.473302206913984*^-95 and
   4.396409698419378*^-100 for the
   integral and error estimates.
NIntegrate::slwcon: Numerical
   integration converging too slowly;
    suspect one of the following:
   singularity, value of the
   integration is 0, highly
   oscillatory integrand, or
   WorkingPrecision too small.
NIntegrate::ncvb: NIntegrate failed to
    converge to prescribed accuracy
   after 18 recursive bisections in r
    near {xita,r} = {1.26097,3.56375
   x10-13}. NIntegrate obtained
   1.0209341758374409*^-112 and
   1.214693034211504*^-117 for the
   integral and error estimates.

0.00141337
0.00677574
0.0027797
0.820485
{2016,12,25,1,54,3.7845951}
```

(Note: The above results are obtained under the following running environment: 64bit Operating System of Windows 7; Mathematica 9.01; Intel(R) Core(TM) i5-4590 CPU @ 3.30GHz, 3.30GHz; 4GB Memory.)